Transport Phenomena in Biological Systems

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Lecture-65
Gas-Liquid Interface Transport-Continued

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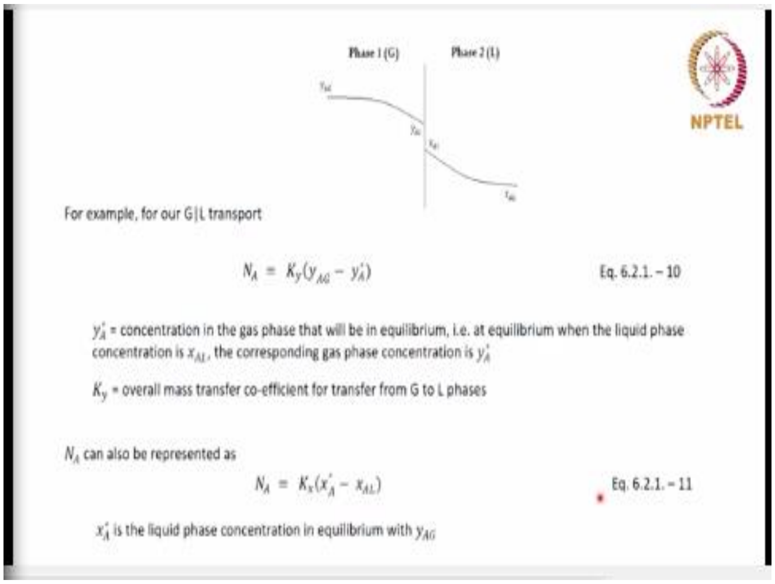

For example, for our G|L transport

$$N_A = K_y(y_{AG} - y_A^*) \qquad \text{Eq. 6.2.1.-10}$$

y_A^* = concentration in the gas phase that will be in equilibrium, i.e. at equilibrium when the liquid phase concentration is x_{AL}, the corresponding gas phase concentration is y_A^*

K_y = overall mass transfer co-efficient for transfer from G to L phases

N_A can also be represented as

$$N_A = K_x(x_A^* - x_{AL}) \qquad \text{Eq. 6.2.1.-11}$$

x_A^* is the liquid phase concentration in equilibrium with y_{AG}

Welcome back, we are looking at the non cell coefficient approach, we are looking at it in the context of oxygen supply to bioreactors. Now, we are in the middle of a generalized formulation, read maybe general formulation for this approach itself. We said that we are trying to represent the transport across phases through appropriate definitions for appropriate quantities. The essential difficulty being if you want to work on d with concentrations and not to k potentials.

Then concentrations you can do algebraic operation where concentrations that are in the same case. For example, we could add, subtract Y_{AG} Y_{Ai} and so on or I say I x A and so on. But it cannot add subtract Y AG say things like that for obviously because the base aspects are different and so on so forth. So, you cannot represent the difference as Y_{AG} - X_{AL} will directly and expected to go, ok.

To overcome that difficulty we are working or we are looking at a conceptual property. We said that we will define a quantity in the gas phase that will be in equilibrium with the bulk concentration of liquid base and thereby we could do the algebraic manipulations now. So, that is what is given here N_A the flux of A is a product of an overall parts of coefficient K capital K_y times $Y_{AG} - Y_A^*$, $Y_A^*$is the conceptual quantity from the real quantity.

This real quantity in a certain sense but it is not real in the way we are describing the process, that is all. The $Y_A^*$is the concentration in the gas phase that will be in equilibrium with the bulk liquid phase concentration. And thereby we have transformed the bulk liquid phase concentration into an equivalent gas phase concentration that we can do algebraic operations on that is called this.

So, N_A equals capital K_y times $Y_{AG} - Y_A^*$. Similarly we can do the same thing for a liquid side aspect, the flux of A equals a capital K x and overall transfer coefficient in terms of the liquid side X times $X_A^* - X_{AL}$, X_{AL} is the bulk concentration here, X_A^* represents to the concentration in the liquid phase. That will be in equilibrium with Y_{AG} and that provides the needed driving force

So, by this process we have express the flux in terms of a difference between concentration and concentration and a nuclear concentration in the other phase. And also in terms of an overall coefficient.

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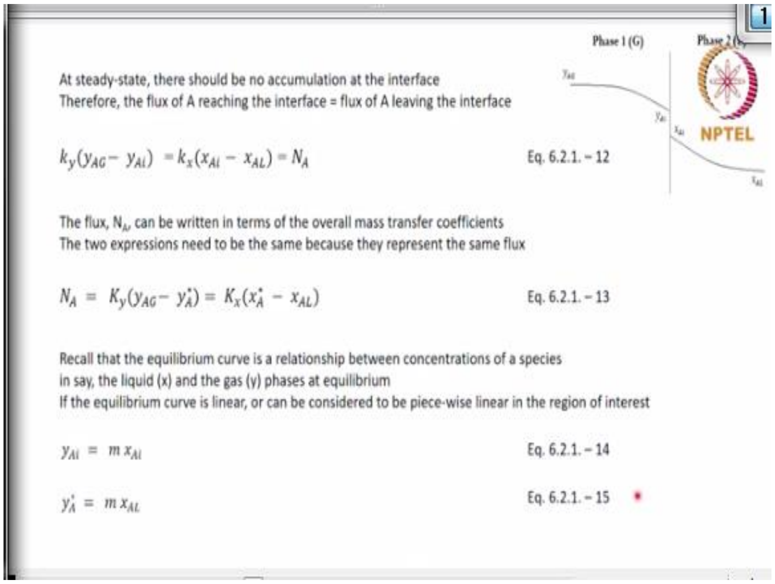


At steady-state, there should be no accumulation at the interface
Therefore, the flux of A reaching the interface = flux of A leaving the interface

$$k_y(y_{AG} - y_{Ai}) = k_x(x_{Ai} - x_{AL}) = N_A \qquad \text{Eq. 6.2.1.} - 12$$

The flux, N_A, can be written in terms of the overall mass transfer coefficients
The two expressions need to be the same because they represent the same flux

$$N_A = K_y(y_{AG} - y_A^*) = K_x(x_A^* - x_{AL}) \qquad \text{Eq. 6.2.1.} - 13$$

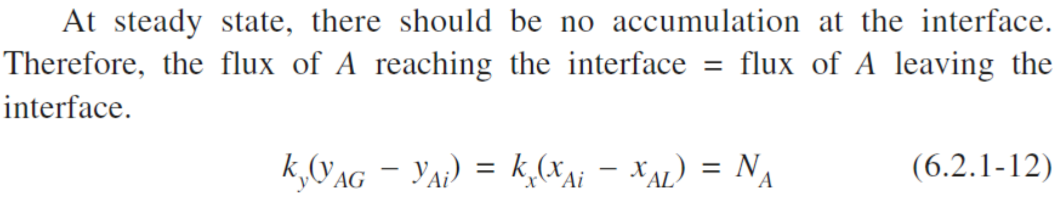
Recall that the equilibrium curve is a relationship between concentrations of a species
in say, the liquid (x) and the gas (y) phases at equilibrium
If the equilibrium curve is linear, or can be considered to be piece-wise linear in the region of interest

$$y_{Ai} = m\,x_{Ai} \qquad \text{Eq. 6.2.1.} - 14$$

$$y_A^* = m\,x_{AL} \qquad \text{Eq. 6.2.1.} - 15$$

At steady state, there should be no accumulation of the interface because the properties of interface concentration does not change the time. If there is accumulation, the concentration of the species will change the time. Therefore the flux of A reaching this interface must equal the flux of A leaving the interface, ok. The powerful view which we have already used, let us use it again, totally different context we use, recall variables.

Here at steady state there should be no accumulation of the interface and therefore the flux of A reaching that interface must equal the flux of A b the interface. So, what is the flux of A reaching the interface in terms of individual mass transfer coefficients, lower case k_y (Y_{AG} - Y_{Ai}) is the rate at which oxygen reaches the interface, K_X times X_{Ai} - X_{AL} is the rate at these interface.

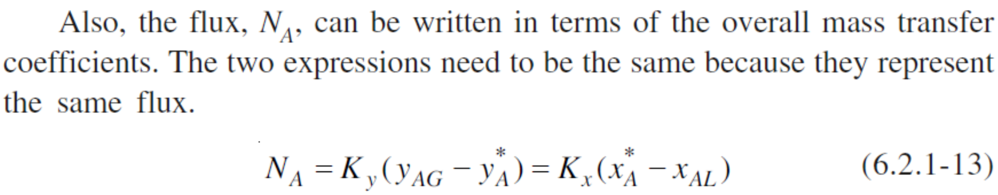


At steady state, there should be no accumulation at the interface. Therefore, the flux of A reaching the interface = flux of A leaving the interface.

$$k_y(y_{AG} - y_{Ai}) = k_x(x_{Ai} - x_{AL}) = N_A \qquad (6.2.1\text{-}12)$$

And both those must be equal and both those were equal to N_A from a expressing anyway I think ok. So, we are equating this at steady state, 6.2.1 -12, equation number. The flux N_A can be written in terms of the overall mass transfer coefficients. These are the individual side mass transfer coefficients in terms in the overall mass transfer coefficients. The 2 expressions need to be the same because they are represent the same flux.

And therefore N_A equals capital K_y the overall mass transfer coefficient times Y_{AG} - a concentration that is equivalent to the bulk liquid phase concentration by Y_A* that is an equilibrium with the liquid phase concentration equals K_x times X_A*, X_A*is the liquid state concentration which is in equilibrium with the bulk gas concentration. Therefore, it represents
Y_{AG} - X_{AL} ok.



Also, the flux, N_A, can be written in terms of the overall mass transfer coefficients. The two expressions need to be the same because they represent the same flux.

$$N_A = K_y(y_{AG} - y_A^*) = K_x(x_A^* - x_{AL}) \qquad (6.2.1\text{-}13)$$

So, this is the same flux N_A, this is written in terms of individual mass transfer coefficients, this is written in terms of the overall mass transfer coefficient, equation 6.2.1 - 13. Now, recall that the equilibrium curve is a relationship between concentrations of species in a liquid and gas phases and equilibrium y versus x. And the relationship is something like this, this is attitude, that is a typical equilibrium relationship or equilibrium curve.

If the equilibrium curve is linear or can be considered to be piecewise linear in the region of interest, Y_{Ai} the interpretation will concentration at the interface there is equilibrium between the gas and liquid phases instantaneous equilibrium. So, Y_{Ai} equals the constant m times X_{Ai} ok, a linear constant we have taken shear. In other words we have taken a region of relevance in the equilibrium curve which would be this curve kind of thing.

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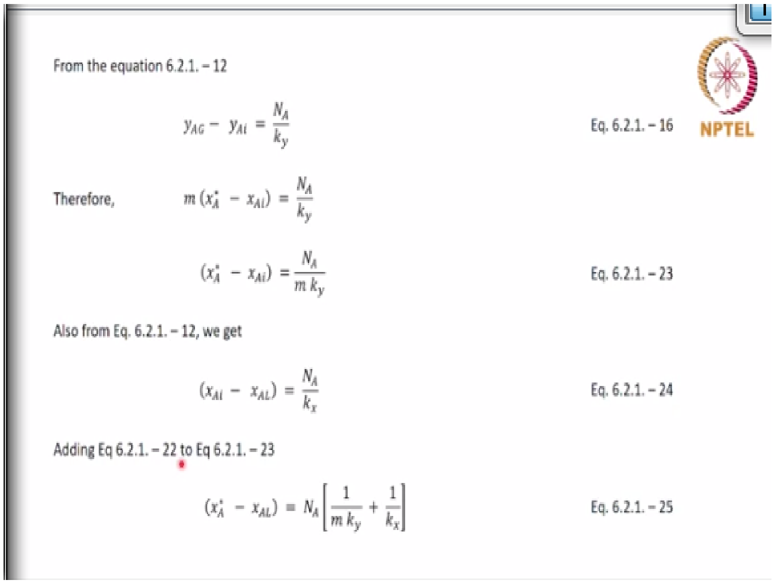

From the equation 6.2.1. – 12

$$y_{AG} - y_{Ai} = \frac{N_A}{k_y} \qquad \text{Eq. 6.2.1.} - 16$$

Therefore,

$$m\,(x_A^* - x_{Ai}) = \frac{N_A}{k_y}$$

$$(x_A^* - x_{Ai}) = \frac{N_A}{m\,k_y} \qquad \text{Eq. 6.2.1.} - 23$$

Also from Eq. 6.2.1. – 12, we get

$$(x_{Ai} - x_{AL}) = \frac{N_A}{k_x} \qquad \text{Eq. 6.2.1.} - 24$$

Adding Eq 6.2.1. – 22 to Eq 6.2.1. – 23

$$(x_A^* - x_{AL}) = N_A\left[\frac{1}{m\,k_y} + \frac{1}{k_x}\right] \qquad \text{Eq. 6.2.1.} - 25$$

If the equilibrium curve is linear, or can be considered to be piece-wise linear in the region of interest

$$y_{Ai} = m\ x_{Ai} \qquad (6.2.1\text{-}14)$$

$$y_A^* = m\,x_{AL} \qquad (6.2.1\text{-}15)$$

From Eq. 6.2.1-12

$$y_{AG} - y_{Ai} = \frac{N_A}{k_y} \qquad (6.2.1\text{-}16)$$

$$x_A^* - x_{Ai} = \frac{N_A}{m\,k_y} \qquad (6.2.1\text{-}23)$$

Also from Eq. 6.2.1-12, we get

$$x_{Ai} - x_{AL} = \frac{N_A}{k_x} \qquad (6.2.1\text{-}24)$$

Adding Eqs. 6.2.1-22 and 6.2.1-23, we get

$$x_A^* - x_{AL} = N_A\left[\frac{1}{m\,k_y} + \frac{1}{k_x}\right] \qquad (6.2.1\text{-}25)$$

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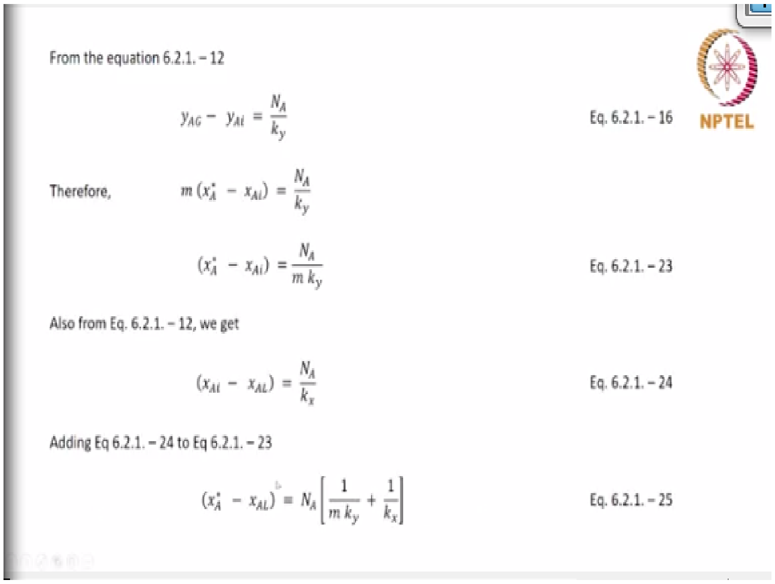

From the equation 6.2.1. – 12

$$y_{AG} - y_{Ai} = \frac{N_A}{k_y} \qquad \text{Eq. 6.2.1.} - 16$$

Therefore, $m\,(x_A^* - x_{Ai}) = \frac{N_A}{k_y}$

$$(x_A^* - x_{Ai}) = \frac{N_A}{m\,k_y} \qquad \text{Eq. 6.2.1.} - 23$$

Also from Eq. 6.2.1. – 12, we get

$$(x_{Ai} - x_{AL}) = \frac{N_A}{k_x} \qquad \text{Eq. 6.2.1.} - 24$$

Adding Eq 6.2.1. – 24 to Eq 6.2.1. – 23

$$(x_A^* - x_{AL}) = N_A\left[\frac{1}{m\,k_y} + \frac{1}{k_x}\right] \qquad \text{Eq. 6.2.1.} - 25$$

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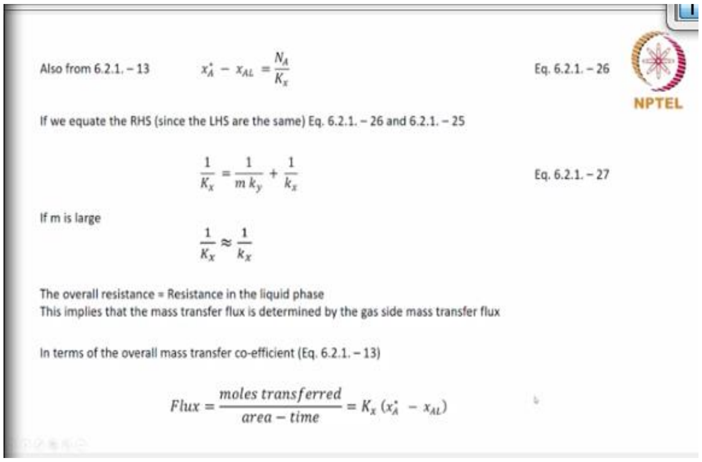

Also from 6.2.1. – 13 $x_A^* - x_{AL} = \frac{N_A}{K_x}$ Eq. 6.2.1. – 26

If we equate the RHS (since the LHS are the same) Eq. 6.2.1. – 26 and 6.2.1. – 25

$$\frac{1}{K_x} = \frac{1}{m\,k_y} + \frac{1}{k_x} \qquad \text{Eq. 6.2.1.} - 27$$

If m is large

$$\frac{1}{K_x} \approx \frac{1}{k_x}$$

The overall resistance = Resistance in the liquid phase
This implies that the mass transfer flux is determined by the gas side mass transfer flux

In terms of the overall mass transfer co-efficient (Eq. 6.2.1. – 13)

$$Flux = \frac{moles\ transferred}{area - time} = K_x\,(x_A^* - x_{AL})$$

Also, from Eq. 6.2.1-13

$$x_A^* - x_{AL} = \frac{N_A}{K_x} \qquad (6.2.1\text{-}26)$$

From Eqs. 6.2.1-26 and 6.2.1-25, we get

$$\frac{1}{K_x} = \frac{1}{m k_y} + \frac{1}{k_x} \qquad (6.2.1\text{-}27)$$

If m is large ok, what is m, m is the relationship between the gas phase concentration mole fraction and the liquid phase mole fraction, ok. In other words look at those sparingly soluble gas, sparingly soluble species, then at equilibrium the gas rate concentration will be much higher than the liquid phase concentration ok.

So, that is a situation here let us say spare a case of solid to gas, oxygen it is a sparingly soluble gas, you would recall that the thumb rule value is about 8 ppm and that now the conditions of temperature and pressure is the oxygen concentration in the liquid right at 8 ppm is small. So, and is indeed large, if m is large in comparison with the other things, this becomes small in comparison with this.

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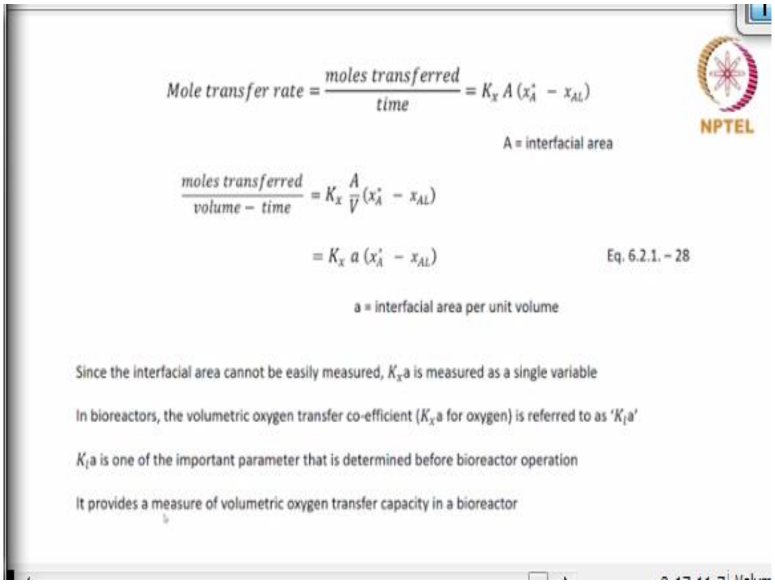

$$Mole\ transfer\ rate = \frac{moles\ transferred}{time} = K_x A\,(x_A^* - x_{AL})$$

A = interfacial area

$$\frac{moles\ transferred}{volume - time} = K_x \frac{A}{V}(x_A^* - x_{AL})$$

$$= K_x\, a\,(x_A^* - x_{AL}) \qquad \text{Eq. 6.2.1. – 28}$$

a = interfacial area per unit volume

Since the interfacial area cannot be easily measured, K_xa is measured as a single variable

In bioreactors, the volumetric oxygen transfer co-efficient (K_xa for oxygen) is referred to as 'K_la'

K_la is one of the important parameter that is determined before bioreactor operation

It provides a measure of volumetric oxygen transfer capacity in a bioreactor

If m is large

$$\frac{1}{K_x} \approx \frac{1}{k_x}$$

i.e. the overall resistance = resistance in the liquid phase. This implies that the mass transfer flux is determined by the gas side mass transfer flux.

Now, using the overall mass transfer coefficient (Eq. 6.2.1-13)

$$\text{Flux} = \frac{\text{Moles transferred}}{\text{Area} - \text{Time}} = K_x(x_A^* - x_{AL})$$

Thus

$$\begin{aligned}\text{The mass transfer rate} &= \frac{\text{Moles transferred}}{\text{Time}} \\ &= K_x A(x_A^* - x_{AL})\end{aligned}$$

where A is interfacial area.

Also

$$\begin{aligned}\frac{\text{Moles transferred}}{\text{Volume} - \text{Time}} &= K_x \frac{A}{V}(x_A^* - x_{AL}) \\ &= K_x a(x_A^* - x_{AL}) \qquad (6.2.1\text{-}28)\end{aligned}$$

where a is interfacial area per unit volume.

And again the interfacial area cannot be measured easy ok, interfacial area is termed as the area of the bubbles of the interface between the bubbles and the liquid, ok, that is not that easy to measure. Therefore, by an appropriate means K_xa is measured as a single variable, ok, and that is a reason why in bioreactors the volumetric oxygen transfer coefficient, right, moles transfer above the coefficient for that moles transferred per volume per time.

That is this times the concentration variance K_xa for oxygen is often referred to as K_La, ok. And of course, this is also written in terms of concentration difference rather than the mole fraction difference, that does not matter because total concentration times the mole fraction is the concentration of A, right, so it does not really matter. Of course the K_xa is going to be different from the K_La in terms of the concentration in the mole fraction but the form of this is same.

So, in bioreactors the volumetric coefficient times the coefficient K_xa is often referred to as K_La, K_La is one of the important parameters that is determined before each bioreactor operation. That gives us an idea as to how much oxygen can be transferred per volume to the cells in culture. So, that is a very important parameter to know, we need to know whether we are able to supply the oxygen that is required by the microbes for them to grow and produce the products, that is a measure of K_La. It provides a measure of the volumetric oxygen transfer capacity in the bioreactor.

I think we will stop here, you had things to soak in, you need to mull over them and then become comfortable with them. In the next class, next lecture, I am going to tell you about how to estimate this K_La, which is a very important thing for biological engineers something that is commonly done whether you run a bioreactor in the industry whether you run a bioreactor in the lab, that looks at appropriate as we see. You certainly need to measure K_La you can also ok, let us meet in the next class and take things forward, see you.