Transport Phenomena in Biological Systems
Prof. G.K. Suraishkumar
Department of Biotechnology Bhupat and Jyoti Mehta School of Biosciences building
Indian Institute of Technology, Madras

Lecture - 74
Kinetics of a Process with an Enzyme Immobilized on a Non-porous Slab

(Refer Slide Time: 00:16)

Transfer coefficient approach:
Simultaneous concentration gradient
and velocity gradient
– Kinetics of enzymes immobilized on a non-porous surface

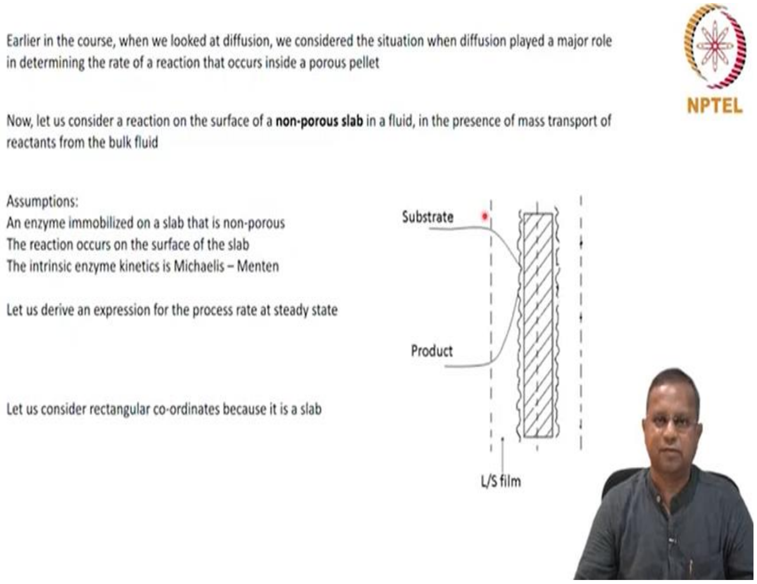
Welcome, we are looking at multiple driving forces causing fluxes, we looked at two approaches essentially an approach that involved simultaneous solution of the conservation equations and the constitutive equations as needed. That was one that needed extensive mathematical effort and the transfer coefficient approach which does not need so much of a mathematical effort but it would be applicable only to a small range of situations.

And therefore the confidence level associated with this approach is less compared to the equations approach. However, it is very useful, it gives us a lot, it even gives us insight into the processes and therefore we are picking up that approach also. Today we will continue with the transfer coefficient approach, simultaneous concentration gradient and velocity gradient. The particular example is that of kinetics of enzymes immobilized on a non-porous surface.

I think we did look at concentration gradient, velocity gradient even at the previous situation when we looked at k_La and so on and so forth. Then we took a slight detour and looked at some even research applications of the principles of transport phenomena, the basic principles of transport phenomena applied to novel research aspects and so on so forth, even existing research aspects before the use.

Throughout the course integrated well-meshed into the course, we had a lot of situations where you saw applications to industry application, industries situations, industry problems and so on so forth, therefore I wanted to show you even the application even in cutting edge research and that is what we saw through the previous few classes, let us begin the kinetics of enzymes immobilized on a non-porous surface. The transport aspects associated with it which impinge on the process dynamics itself.

(Refer Slide Time: 02:57)



Earlier in the course, when we looked at diffusion, we considered the situation when diffusion played a major role in determining the rate of a reaction that occurs inside a porous pellet

Now, let us consider a reaction on the surface of a **non-porous slab** in a fluid, in the presence of mass transport of reactants from the bulk fluid

Assumptions:
An enzyme immobilized on a slab that is non-porous
The reaction occurs on the surface of the slab
The intrinsic enzyme kinetics is Michaelis – Menten

Let us derive an expression for the process rate at steady state

Let us consider rectangular co-ordinates because it is a slab

Remember earlier in the course when we looked at diffusion back in the second chapter, we considered the situation when diffusion played a major role in determining the rate of reaction, so it was something like this it was heterogeneous situation, where you had a pellet and so on so forth, pellet in the in aqueous environment in a liquid environment and things like that, therefore the overall system is heterogeneous.

We looked only at the pellet which is some what heterogenous however what I mean by heterogeneous is from the process point of view. You had solid phase, liquid phase and so on so forth and things coming in to the porous pellet you remember the situation when we looked at enzymatic reaction occurring inside the pellet substrate had to get into the pellet through diffusion through its various pores get reacted there and then the product needs to move out.

So there is a process of substrate moving in product moving out or in other words there is transport of product in transport of product out and then there is reaction that is occurring. Both these aspects the transport and the reaction have different time constraints associated with them, the characteristic times could be different or they could be comparable, so depending on whether they are comparable or whether they are very different.

The kind of process kinetics could be very different could it could either be completely reaction based or it could completely be transport based and so on so forth. We looked at an effectiveness factor that gave us some ideas to what is happening there and so on. We are continuing on the same lines however here there is no diffusion inside the pellet it is something else. Earlier diffusion played a major role in determining the rate of a reaction that occurred inside of porous spherical pellet.

Now we are going to consider a reaction on the surface of a non-porous slab, there are no pores there, no pores of relevance there. So we have a non-porous slab present in a fluid and the substrate is present in the fluid. It goes to the non-porous slab surface reacts there and the product moves out, that is the process here. So in the presence of mass transport of reactants from the bulk fluid, so the reactants are moving from the bulk fluid to the surface.

They are getting reacted there and then they move out. So the assumptions to at least make the problem tractable and to make it meaningful in certain cases, an enzyme is immobilized on a slab that is non-porous that is a statement of the problem itself. The reaction occurs only on the surface of the slab. The intrinsic enzyme kinetics is Michaelis-Menten the liquid kinetics the solution kinetics is Michaelis-Menten.

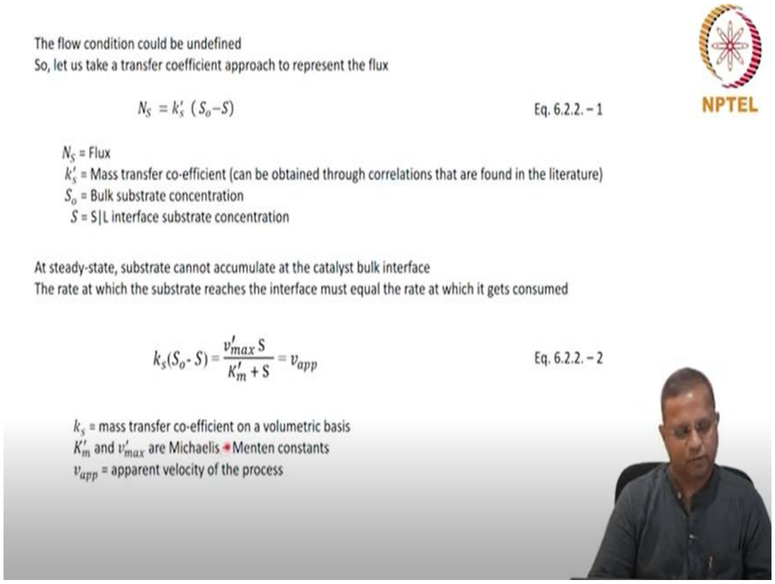
And therefore the immobilized constants the k'_m and v'_{max} would be different from the case of Michaelis-Menten, but that is okay that we already know the form of the equation would be the same and we are going to derive an expression for the process rate, process rate is the overall thing the individual aspects that contributed to the process rate such as transport and reaction is what we talked about.

Here we are going to derive an expression for the process rate at steady state. Lets begin this is the situation here you have these slab. The longitudinal section is going to be a rectangle you look at it from the end it is going to be a rectangle and you have y dimension that is going into the screen. We are looking only at the x and z dimensions maybe and so the hashed part is the slab.

You have the enzyme that is immobilized on either surface and you have a certain region which is the conceptual liquid-solid film; liquid here, solid here where the concentrations significantly change compared to the other regions, so this region where the concentrations change is the conceptual liquid solid film similar to the gas liquid film that we already see. So the substrate moves in here the concentration in the bulk could be a constant and here it drops it reacts here the product moves out.

Now let us consider the rectangular cartesian coordinates because it is a slab. x y z is good enough for a for an easy description of this geometry so we will consider that.

(Refer Slide Time: 08:04)



The flow condition could be undefined
So, let us take a transfer coefficient approach to represent the flux

$$N_S = k'_s\ (S_o - S) \qquad \text{Eq. 6.2.2. – 1}$$

N_S = Flux
k'_s = Mass transfer co-efficient (can be obtained through correlations that are found in the literature)
S_o = Bulk substrate concentration
S = S|L interface substrate concentration

At steady-state, substrate cannot accumulate at the catalyst bulk interface
The rate at which the substrate reaches the interface must equal the rate at which it gets consumed

$$k_s(S_o - S) = \frac{v'_{max}\,S}{K'_m + S} = v_{app} \qquad \text{Eq. 6.2.2. – 2}$$

k_s = mass transfer co-efficient on a volumetric basis
K'_m and v'_{max} are Michaelis Menten constants
v_{app} = apparent velocity of the process

We are unsure of the kind of flow in the liquid, nothing is mentioned about it, it could be slushing around it could be a nice laminar flow or it could be even still we do not really know, so to develop a reasonably general solution. Let us take it to be undefined and let us take a transfer coefficient approach, once we take a transfer coefficient approach then it becomes simple you use a different mass transfer coefficient.

And that would represent the conditions, how whether we have direct access to it and things like that is another aspect to be considered totally different aspect to be considered that may not be so however, let us take a transfer coefficient approach to represent the flux of the substrate from the bulk to the surface. So the flux which is modes per area, modes per time per area perpendicular to the direction of motion is some k'$_s$ times a difference in substrate concentrations $(S_0 – S)$ at the surface, so what is this case? or this it is on the form of a transfer coefficient times a concentration difference, so you can see the use of a transfer coefficient approach here. Let us call this equation 6.2.2-1. This of course is the flux as I mentioned. K'$_s$ is the mass transfer coefficient per unit area, because the area basis is relevant here, so this is per unit area, so that is why there is a ' there and it can be obtained through correlation that are found in the literature.

$$N_S = k'_s\ (S_o - S) \qquad (6.2.2\text{-}1)$$

where N_S is flux, k'_s is mass transfer coefficient, S_o is bulk substrate concentration and S is $S|L$ interface substrate concentration. k'_s can be obtained through correlations that are found in literature.

S_0 as the bulk substrate concentration of the liquid S is the solid liquid interface substrate concentration, and the solid liquid interface where the reaction happens of course nothing happens on the other side because there are no pores but the entire reaction happens in the surface and S represents that substrate concentration. At steady state the substrate cannot accumulate at the catalyst bulk interface.

At the catalyst bulk interface, at the enzyme bulk interface. The, which is here. I think this is where our coordinate begins. So, we are looking at the surface here, at this point where the enzyme ends we are looking at the concentrations. And what it says is at steady state, they cannot be any accumulation of either the substrate or the product. So, at steady states substrate cannot accumulate at the catalyst bulk interface.

Therefore, the rate at which substrate reaches the interface must equal the rate at which it gets consumed. That is essential for steady state otherwise it is not going to be at steady state. So if we are doing a steady state analysis, if we are forcing a steady state analysis, which is relevant for most of the time by the way then this condition must be satisfied. Therefore the rate at which it reaches the surface is k_s now $(S_0 – S)$ note that this k_s is different from k'_s, k'_s is a surface basis k_s is on a volumetric basis.

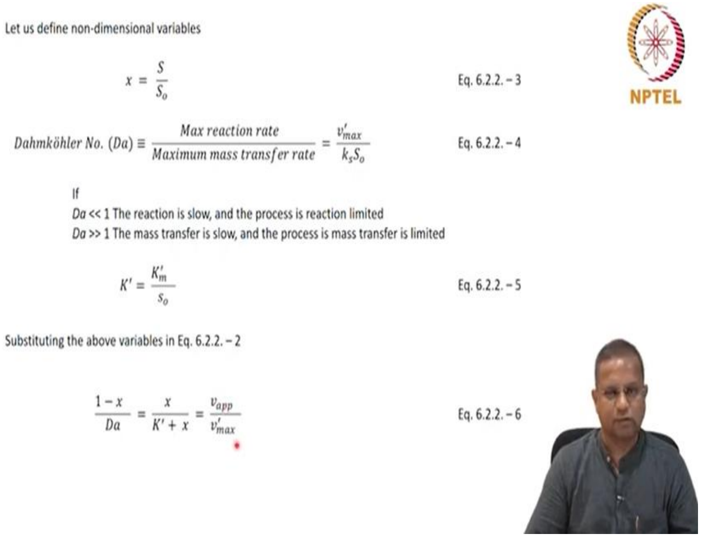
Because the right hand side here which is the Michaelis-Menten reaction is on a volumetric basis and therefore we need the same basis to compare, contrast, manipulate and so on so forth. So the rate at which the substrate reaches the surface on a volumetric basis k_s times S_0 - S equals the rate at which it reacts because it cannot go anywhere else $[v'_{max} S/(k'_s + S)]$ and we are going to call this as at apparent rate v_{app}.

$$k_s(S_o - S) = \frac{v'_{\max} S}{K'_m + S} = v_{\mathrm{app}} \tag{6.2.2-2}$$

where k_s is mass transfer coefficient on a volumetric basis, K'_m and $v'_{\max}$ are Michaelis-Menten constants and v_{app} is apparent velocity of the process.

Let us call this equation 6.2.2 - 2. As I mentioned k_s is the mass transfer coefficient on a volumetric basis. k'_m v'_{max} are Michaelis-Menten constants modified because of the or dashed here because these could be different from the intrinsic or the solution kinetic constants. This is the immobilized case here. And v_{app} is the apparent velocity of the process itself.

(Refer Slide Time: 13:01)



Let us define non-dimensional variables

$$x = \frac{S}{S_o} \qquad \text{Eq. 6.2.2.} - 3$$

$$\text{Dahmköhler No. } (Da) \equiv \frac{\text{Max reaction rate}}{\text{Maximum mass transfer rate}} = \frac{v'_{max}}{k_s S_o} \qquad \text{Eq. 6.2.2.} - 4$$

If

$Da << 1$ The reaction is slow, and the process is reaction limited

$Da >> 1$ The mass transfer is slow, and the process is mass transfer is limited

$$K' = \frac{K'_m}{s_o} \qquad \text{Eq. 6.2.2.} - 5$$

Substituting the above variables in Eq. 6.2.2. – 2

$$\frac{1-x}{Da} = \frac{x}{K' + x} = \frac{v_{app}}{v'_{max}} \qquad \text{Eq. 6.2.2.} - 6$$

We have been doing things with the non-dimensional variables, we know that it generalizes solutions to a larger extent, so let us do that here also. Let us define non-dimensional variables as follows. The substrate concentration non-dimensional variable $x = S/S_0$ normalize with respect to bulk substrate concentration equation 6.2.2 - 3. And let us bring in a Dahmkohler number Da which is mass reaction rate divided by the maximum mass transfer rate. Or in other words the reaction rate, this is the maximum reaction rate divided by the maximum mass transfer rate, that is not mass let me correct it right away. So its a ratio of maximum reaction rate divided by the maximum mass transfer rate So there, we said the two major processes one is transport and then the other one is the reaction. So the Dahmkohler number gives us an idea is to which one is more important.

In terms of the maximum possible rates that are there these may not be the rates that are actually present in the particular situation, the particular situation could have influences from transfer, could have influences from reaction together if they are comparable and so on so forth. So this gives us an idea as to if you consider the extremes in both cases, what would be the ratio? So that would give us some idea as to what to work with.

So its a ratio of the maximum reaction rate to the maximum mass transfer rate. The maximum reaction rate, you know is v'_{max} rate that is the maximum rate at which it can occur the Michaelis-Menten kinetics and k_s is not as a maximum mass transfer rate because k_s $(S_0 – S)$ is the actual mass transfer rate when s =0, you will have the maximum mass transfer rate. So you have k_sS_0 in the denominator here.

$$\text{Da} = \text{Damköhler number} = \frac{v'_{\max}}{k_s S_o} = \frac{\text{Mass reaction rate}}{\text{Maximum mass transfer rate}} \qquad (6.2.2\text{-}4)$$

If

- Da << 1, the reaction is slow, and the process is reaction limited.
- Da >> 1, the mass transfer is slow, and the process is mass transfer limited.

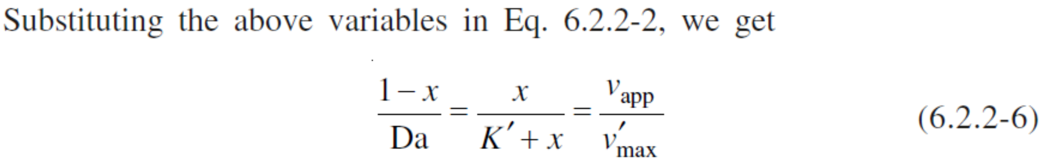
Let us call this equation. 6.2.2 - 4. If the Dahmkohler number is much-much less than 1 then the reaction rate is slow in comparison to the mass transfer rate and the processes reaction limited, remember the slowest step determines the rate. So if the rate is rate of one aspect of a serial processes slow compared to the other that one is going to determine the overall process of it.

So if the reaction is slow, the process is reaction limited. If Dahmkohler number is much-much greater than 1 it means that the mass transfer rate is much slower compared to the reaction rate and the process is mass transfer limited. So the mass transfer rate would determine the process rate. If the reaction is slow, then the reaction rate would determine the process rate. So that is the meaning of reaction limited and mass transfer limited.

Let us define another non-dimensional number that is needed k’ its nothing but k'_m / S_0, equation 6.2.2 - 5 and if we substitute x Da and k‘ into a expression here. Its nice we do not have differential equations here or as yet. This is our governing equation when steady state occurs and therefore we are substituting these variables with the non-dimensional variables.

$$K' = \frac{K'_m}{S_o} \qquad (6.2.2\text{-}5)$$

If we do that, we would get (1- x)/ Da equals x /(k‘ + x) equals (v_{app} / v'_{max}). I just thought I will show this to you and then ask you to pause the video here and go from the equation that has been given on with regular variables and replace the non-dimensional variables. See whether you get to whether you arrive at this answer it will be a nice practice, it will also give you some insights as to what is happening, please pause the video here and go forward.

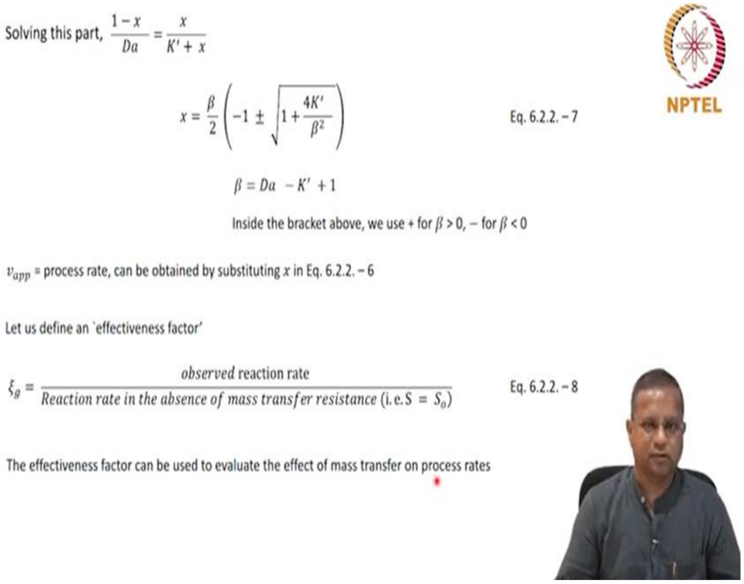


Substituting the above variables in Eq. 6.2.2-2, we get

$$\frac{1-x}{\mathrm{Da}} = \frac{x}{K'+x} = \frac{v_{\mathrm{app}}}{v'_{\mathrm{max}}} \qquad (6.2.2\text{-}6)$$

I am sure you would have arrived at this (1 – x) by Da equals x/(k‘ + S) equals v_{app} /v'_{max}, this is equation 6.2.2 - 6. Are we doing in terms of time, fine few more minutes maybe, so if you solve these two parts let us this is you know, there are 2 parts to this equation or 3 parts of this equation this, this, this, this, and then this, this will take this and this first (1 – x) / Da equals x /(k‘ + x).

(Refer Slide Time: 18:13)



Solving this part, $\frac{1-x}{Da} = \frac{x}{K'+x}$

$$x = \frac{\beta}{2}\left(-1 \pm \sqrt{1+\frac{4K'}{\beta^2}}\right) \qquad \text{Eq. 6.2.2.} - 7$$

$$\beta = Da \; - K' \; + 1$$

Inside the bracket above, we use + for $\beta > 0$, – for $\beta < 0$

v_{app} = process rate, can be obtained by substituting x in Eq. 6.2.2. – 6

Let us define an 'effectiveness factor'

$$\xi_g = \frac{\textit{observed}\text{ reaction rate}}{\textit{Reaction rate in the absence of mass transfer resistance}\text{ (i.e. } S = S_o)} \qquad \text{Eq. 6.2.2.} - 8$$

The effectiveness factor can be used to evaluate the effect of mass transfer on process rates

If we solve this we are looking for x an expression for x in terms of the other variables it is quite clear 1-x here k + x here cross multiply you are going to get a x^2 term, and you have x times Da, so it is a classic quadratic equation, therefore your solution, x is nothing but [-b ± √(b^2 - 4 ac)]/ 2a and so on so forth. That would turn out to be [- 1±(√1+4K'/β^2) the whole multiplied by (β/2) where β = Da -k‘ +1.

Solving $\frac{1-x}{\text{Da}} = \frac{x}{K'+x}$, we get

$$x = \frac{\beta}{2}\left(-1 \pm \sqrt{1+\frac{4K'}{\beta^2}}\right) \qquad (6.2.2\text{-}7)$$

where $\beta = \text{Da} - K' + 1$.

So inside the bracket above, we use + when β is greater than 0 and we use - when β is less than 0 that provides the realistic solution. Now the process rate, which is given by v_{app} can be found by substituting x, we have an expression for x and we have v_{app} / v_{max} equals (1-x)/Da or equals x / (k' + x) either one should be fine let us see what I would use here and before that we are going to look at our effectiveness factor.

An 'effectiveness factor' can be defined as

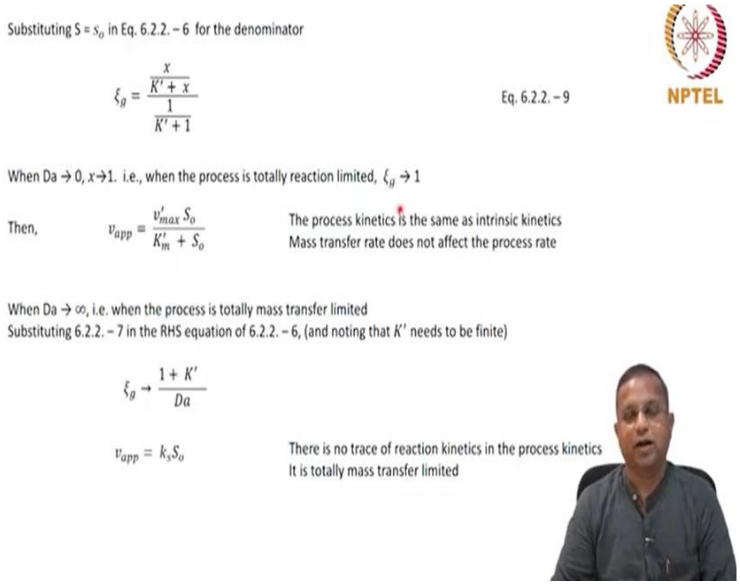
$$\xi_g = \frac{\text{Observed reaction rate}}{\text{Reaction rate in the absence of mass transfer resistance (i.e. } S = S_o)} \qquad (6.2.2\text{-}8)$$

We have already seen the effectiveness factor in the case of diffusion with reaction in a porous pellet, porous spherical pellet, in the second chapter this we are going to define an effectiveness factor on the same lines, its nothing but the observed reaction rate divided by the reaction rate in the absence of mass transfer resistance. Again observed reaction rate is what we observe. Reaction rate or observed process rate you can say.

Reaction rate in the absence of mass transfer resistance or process rate in the absence of mass transfer resistance, this is nothing but a conceptual rate the maximum possible under the given situations and this you can mathematically get by putting S equals S_0 because the entire substrate being available at the bulk concentration at the interface or the substrate concentration at the interface being the bulk substrate concentration.

Is the situation when there is no mass transfer resistance, and that would give the process rate of the absence of mass transfer resistance. 6.2.2 - 8. And the effectiveness factor can be used to evaluate the effect of mass transfer on process rates as we have already seen. We are reinforcing these concepts which are very powerful concepts, especially in heterogeneous processes heterogeneous, you know, its not the same homogeneous one phase system and so on.

(Refer Slide Time: 21:16)



Substituting $S = s_o$ in Eq. 6.2.2. – 6 for the denominator

$$\xi_g = \frac{\frac{x}{K' + x}}{\frac{1}{K' + 1}} \qquad \text{Eq. 6.2.2.} - 9$$

When Da → 0, $x \rightarrow 1$. i.e., when the process is totally reaction limited, $\xi_g \rightarrow 1$

Then, $v_{app} = \frac{v'_{max} S_o}{K'_m + S_o}$ The process kinetics is the same as intrinsic kinetics
Mass transfer rate does not affect the process rate

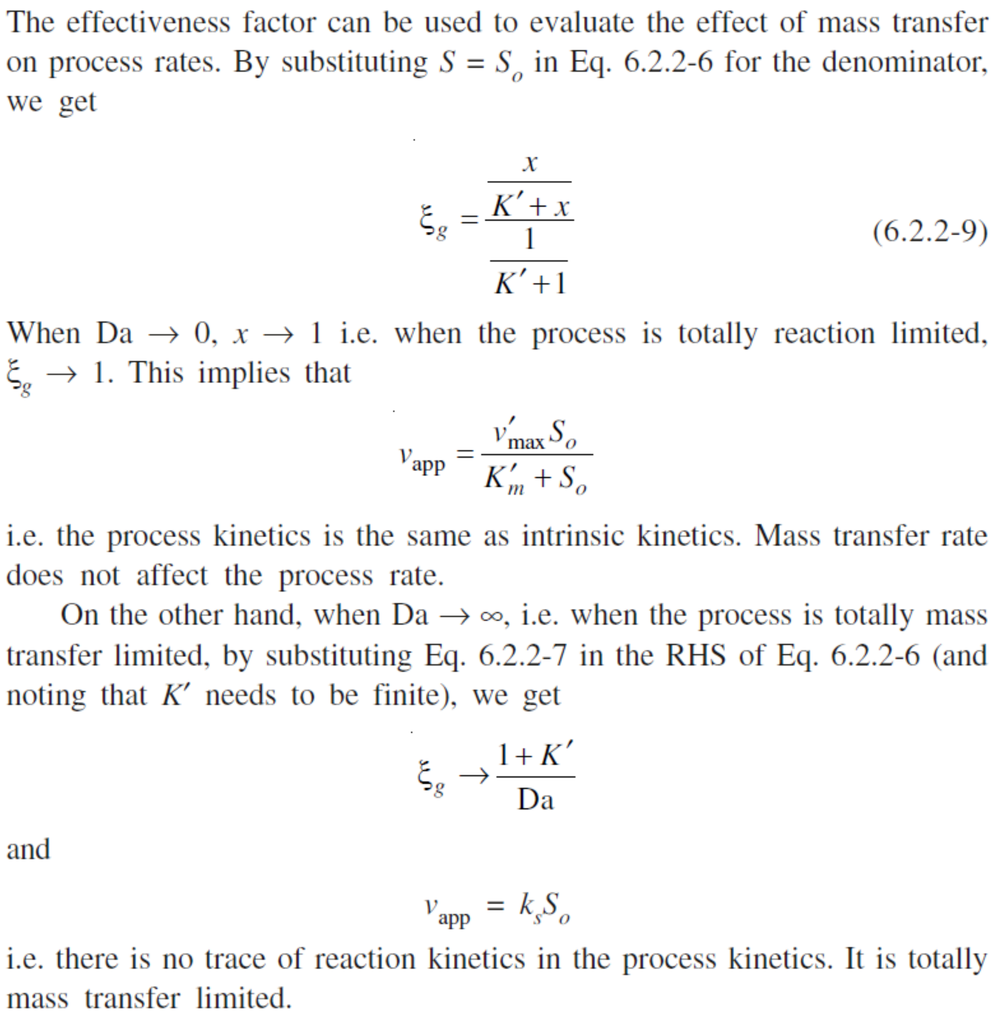
When Da → ∞, i.e. when the process is totally mass transfer limited
Substituting 6.2.2. – 7 in the RHS equation of 6.2.2. – 6, (and noting that K' needs to be finite)

$$\xi_g \rightarrow \frac{1 + K'}{Da}$$

$v_{app} = k_s S_o$ There is no trace of reaction kinetics in the process kinetics
It is totally mass transfer limited

Now we substitute S equals S_0 in 6.2.2 - 6 which is a regular expression here in terms of the non-dimensional variables, S equals 0 implies x equals 0, So S equals S_0 that gives you the maximum rate, then you get this effectiveness factor as x /(k‘ + x) divided by 1 /(k’+1), again, I would like you to stop the video here go back work this out convince yourself that this is indeed happening, you do not have to take my word for this.

Check whether there are any errors that are kept in here. If there are any errors let us know in the comments section or the discussion forum, ask the question forum. This is equation 6.2.2-9 please pause the video. See whether you had you spotted any errors here, if you did spot any error, then let us know. When Da tends to 0, what is Da maximum reaction rate divided by maximum mass transfer rate.



The effectiveness factor can be used to evaluate the effect of mass transfer on process rates. By substituting $S = S_o$ in Eq. 6.2.2-6 for the denominator, we get

$$\xi_g = \frac{\dfrac{x}{K' + x}}{\dfrac{1}{K' + 1}} \tag{6.2.2-9}$$

When Da $\rightarrow$ 0, $x \rightarrow 1$ i.e. when the process is totally reaction limited, $\xi_g \rightarrow 1$. This implies that

$$v_{\text{app}} = \frac{v'_{\max} S_o}{K'_m + S_o}$$

i.e. the process kinetics is the same as intrinsic kinetics. Mass transfer rate does not affect the process rate.

On the other hand, when Da $\rightarrow \infty$, i.e. when the process is totally mass transfer limited, by substituting Eq. 6.2.2-7 in the RHS of Eq. 6.2.2-6 (and noting that K' needs to be finite), we get

$$\xi_g \rightarrow \frac{1 + K'}{\text{Da}}$$

and

$$v_{\text{app}} = k_s S_o$$

i.e. there is no trace of reaction kinetics in the process kinetics. It is totally mass transfer limited.

What is effectiveness factor? It is the actual reaction rate divided by the reaction rate in the absence of mass transfer resistance. Therefore you are tending to the completely reaction controlling interchain when the effectiveness factor tends to it. Then you could write from the same equation there v_{app} as $[v'_{max} S_0 /(k'_m + S_0)]$; the process kinetics is the same as intrinsic kinetics here, microsmetic kinetics mass transfer does not affect the process rate.

There is no scent or there is no hint of any mass transfer related term here. The process rate has completely determined by the intrinsic kinetic rate. The mass transfer rate does not affect the process rate. Whereas the reverse happens when Da tends to infinity. When Dahmkohler number tends to infinity what does it mean, the mass transfer rate is much-much less the denominator is much-much less compared to the reaction rate.

And therefore the mass transfer rate is much slower compared to the reaction rate and therefore the process becomes completely mass transfer limited. You substitute that noting that k needs to be finite you would see that **ε_g the effectiveness factor** tends to (1+ k‘)/ Da. And the apparent rate becomes k_s times S_0, you see this completely mass transfer determined the apparent rate here the apparent rate is completely reaction determinant and there is no trace of reaction kinetics in the process kinetics here.

It is totally mass transfer limit. So this analysis comes in very handy in right from analysis to design to even operation at times and this is a well known analysis in chemical engineering terms here, we have applied it to the enzyme reaction situation, that to an enzyme that is immobilized onto a surface of a non-porous slab. I think that is all I have here for today. I think we have been at it for a while now, good to take a break.

In this class we looked at simultaneous velocity gradient and concentration gradient and we handled the velocity gradient part of it through transfer coefficient approach and got ideas as to what determines the process rate depending on the value of the effectiveness factor. Let us take things forward when we meet in the next class.