Transport Phenomena in Biological Systems

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Lecture-77
Design of Heat Exchangers-Continued

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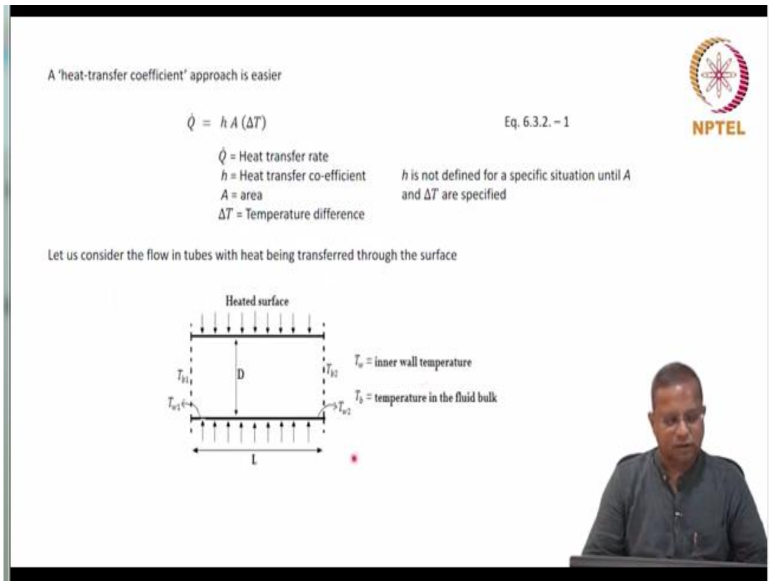

Welcome back, let us continue our heat exchanger design. As I mentioned towards the end of the previous lecture, a heat transfer coefficient approach is going to be used here because it is much easier mathematically speaking. And for that we are going to express the heat transfer rate as hA(ΔT), where this is the equation 6.3.2 - 1, where $\dot{Q}$ is the heat transfer rate amount of heat transfer per time.

H is the heat transfer coefficient, A is the area across which the heat is transferred and ΔT is the temperature difference, ok. Note that the heat transfer coefficient is not defined for a specific situation until the area and the temperature difference are specified ok. You need to say under these conditions of area because there are various ways by which you can look at areas, that are various temperature differences that you can look at in a particular situation.

The heat transfer is going to depend on those aspects. Let us consider the flow in tubes with heat being transferred through the surface that is the situation here ok. You have inner pipe of a double pipe heat exchanger there is heat in the radial direction ok there is coming through the wall of the inner pipe that is heating up the fluid inside, it is something like this ok. This is a tube that I am considering here this is the heated surface radially heated surface.

Let us say that we consider a length tube of length L. The bulk temperature at this cross section let us say T_{b1}, the bulk temperature at this cross section let us say is T_{b2} ok. T_b is a temperature in the fluid bulk average temperature here, average temperature here and the wall temperature would be different ok, heat has been transferred here.

The layer that is closest to the wall is expected to be at a different temperature if the heat transfer is happening. Otherwise the heat transfer will not happen there has to be a temperature difference. So, let us say that the wall temperature at the cross section 1, ok that is be close to this is T_{w1} and the wall temperature at cross section 2 is T_{w2} ok, so this is our terminology here.

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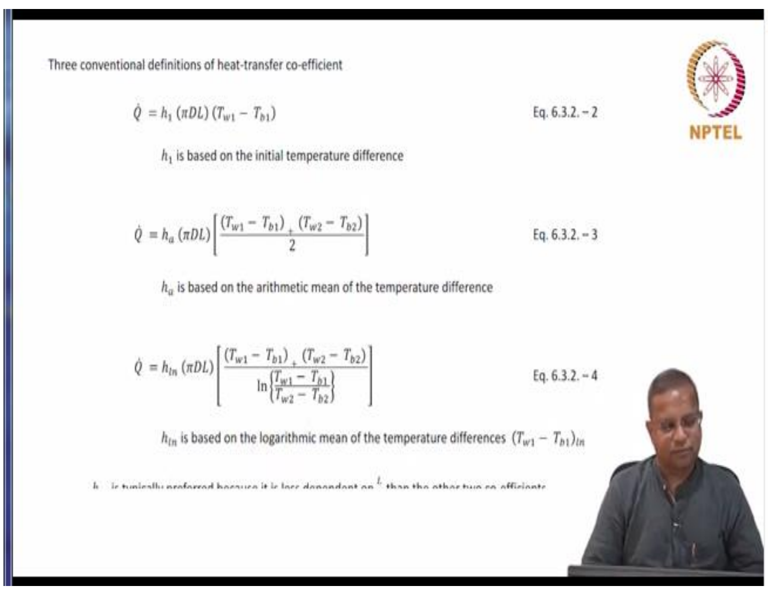

Three conventional definitions of the heat transfer coefficient this is what we going to see next. You could say that the heat transfer rate is some heat transfer coefficient times the cylindrical area πDL that is a cylindrical area times the difference between the wall temperature and the bulk temperature at cross section 1 ok, at this cross section alone T_{w1} - T_{b1} ok.

So, this h_1 is obviously based on initial temperature difference ok. So, the heat transfer coefficient that determines that can be used represent the heat transfer rate is of course based on the initial temperature difference here. Let us call this equation 6.3.2 - 2. It would also be based on something else a certain the temperature difference you take the temperature difference at cross section 1, you take the temperature difference at cross section 2, you take the average of these 2, it could also be based on that, right.

Three conventional definitions of the heat transfer coefficient are

$$\dot{Q} = h_1(\pi DL)(T_{w1} - T_{b1}) \quad (6.3.2\text{-}2)$$

where h_1 is based on the initial temperature difference.

$$\dot{Q} = h_a(\pi DL)\left[\frac{(T_{w1} - T_{b1}) + (T_{w2} - T_{b2})}{2}\right] \quad (6.3.2\text{-}3)$$

where h_a is based on the arithmetic mean of the temperature difference.

$$\dot{Q} = h_{\ln}(\pi DL)\left[\frac{(T_{w1} - T_{b1}) + (T_{w2} - T_{b2})}{\ln\left\{\dfrac{T_{w1} - T_{b1}}{T_{w2} - T_{b2}}\right\}}\right] \quad (6.3.2\text{-}4)$$

where $h_{\ln}$ is based on the logarithmic mean of the temperature differences $(T_{w1} - T_{b1})_{\ln}$. $h_{\ln}$ is typically preferred because it is less dependent on $\frac{L}{D}$ than the other two coefficients. Also, if the inside wall temperature is unknown, or if the fluid properties change appreciably along the pipe, then it becomes difficult to predict the heat transfer coefficient defined earlier.

So, these are the 3 conventional definitions of the heat transfer coefficient. Typically, the heat transfer coefficient based on the logarithmic mean of the temperature difference is preferred. Because of the operational aspects it is less dependent on L by D compared to the other things, you have a large length by diameter ratio ok. In other words, your length is very large compared to the diameter.

In other words, a very thin maybe a thin radius pipe long pipe or long pipe the reasonable diameter and so on so forth, that is the L by D factor. This $h_{\ln}$ is not usually dependent on L by D as much as the other 2, that is the reason why this is preferred, I think we are going to use this.

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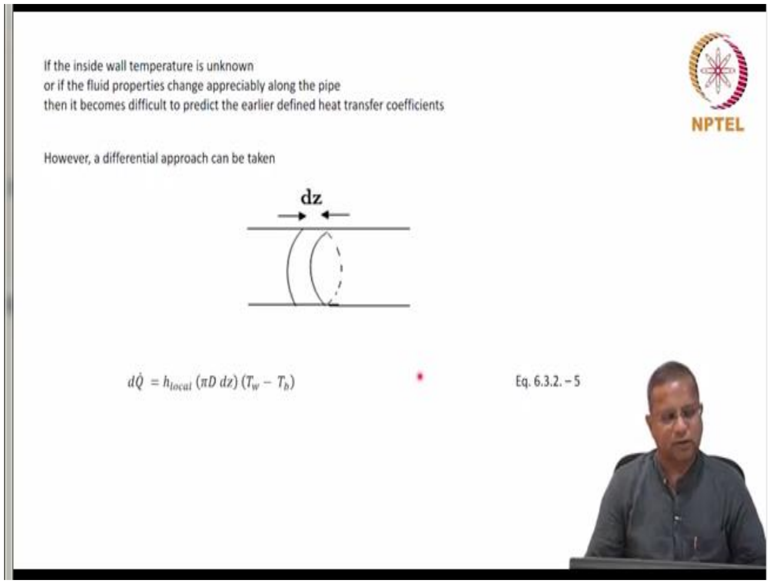

If the inside wall temperature is unknown, you need to place them a couple there which could be difficult if you already have an existing system and so on so forth or if the fluid properties change appreciably along the pipe then what do you do ok. Their conditions are changing your relations have varied only for a differential section ok. So, you write your relations or a differential section, where you know it is valid and then you integrate over the length ok, simple usual trick.

So, this is the case here then it becomes difficult to predict the earlier defined heat transfer coefficients. And therefore we use a differential approach, we take a differential element thin element, cylindrical element here of thickness dz. Otherwise at the slab here, the radius is the same as that of the pipe only the thickness is much, much thinner compared to the length of the pipe dz.

And therefore in this case our differential heat that is transferred is h_{local} at the local point times π D Δz which is the surface area of this element here times T_w - T_1 alright which is the temperature difference here.

Under these circumstances, a differential approach is taken wherein a force balance is performed on a slice of the fluid (Fig. 6.3.2-4).

$$dQ = h_{local}\ (\pi D dz)\ (T_w - T_b) \qquad (6.3.2\text{-}5)$$

So, you do not have to worry about 1 and 2 here this is a differential element you have this and you have reasonably complete description here and if you integrate you will get the overall picture. This is the equation 6.3.to - 5 or we doing in terms of time ok.

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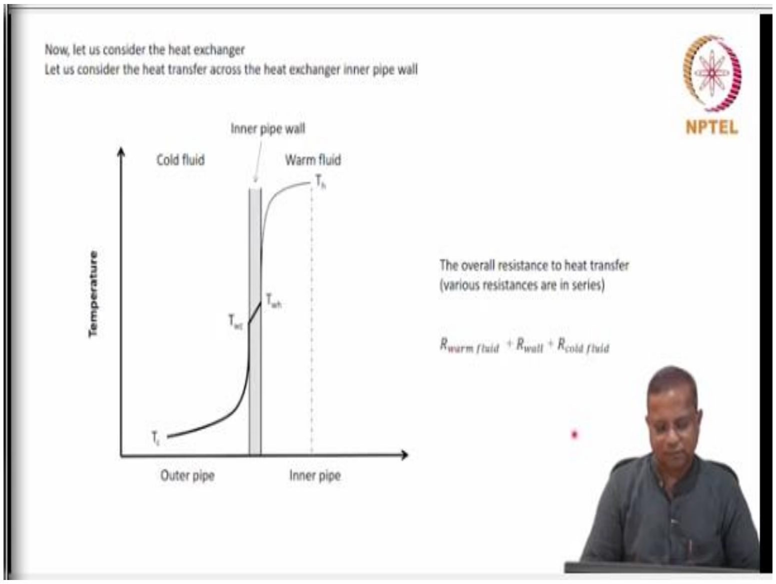

Let me see what we have ok, I will probably mentioned this and then we will take a break and then come back. Let us consider the heat exchanger now, we going back to the heat exchanger after giving you some basal aspects of how things are done, general aspects. Let us consider the heat transfer across the heat exchanger inner pipe wall which is where the relevant heat transfer takes place.

So, I have represented the inner pipe wall here, the warm fluid is flowing here and the cold fluid is flowing here. This is the outer pipe, this is the inner pipe ok representation outer pipe inner pipe. So, inner pipe of course there is a center point here and so on ok, so outer pipe is cold, warm fluid is flowing here. So, there is heat transfer taking place from the warm fluid to the cold fluid, also I have shown the temperature on this axis but the corresponding distance as shown here.

So, the warm fluid is let us say at a bulk temperature of T_h and the temperature profile if you see. There is a region close to the wall ok it is film region again right where the temperature drops significantly, it goes from somewhere close to T_h to T_{wh}. And there is conduction in across the solid wall here inner pipe wall. So, conduction converts it from T_{wh} to takes it from T_{wh} T_{wc}.

There is another film that is here, ok they are both of flowing fluids, opposite directions maybe, so T_{wc} and T_c is the bulk temperature of the cold fluid. So, the overall resistance to heat transfer is the sum of the various resistances which are in seas. So, you can view the heat transfer having to overcome maybe some resistance here ok, maybe some resistance at the film, resistance to conduct across this inner pipe wall resistance to overcome this.

And maybe a small resistance here, ok, we do not have to worry about the resistance in the bulk, definitely there is this, there is this, there is this ok. Let us say how we are going to handle these aspects. So, the heat transfer problem has been brought down to a conductance across a series of resistances. So, we are converting it into an equivalent problem because we can add resistances and so on so forth, we can use those principles.

So, the main resistance is here are the resistance of due to the warm fluid overall I have taken, resistance of the wall and the resistance of the cold fluid taken in series and therefore you could add the resistances.

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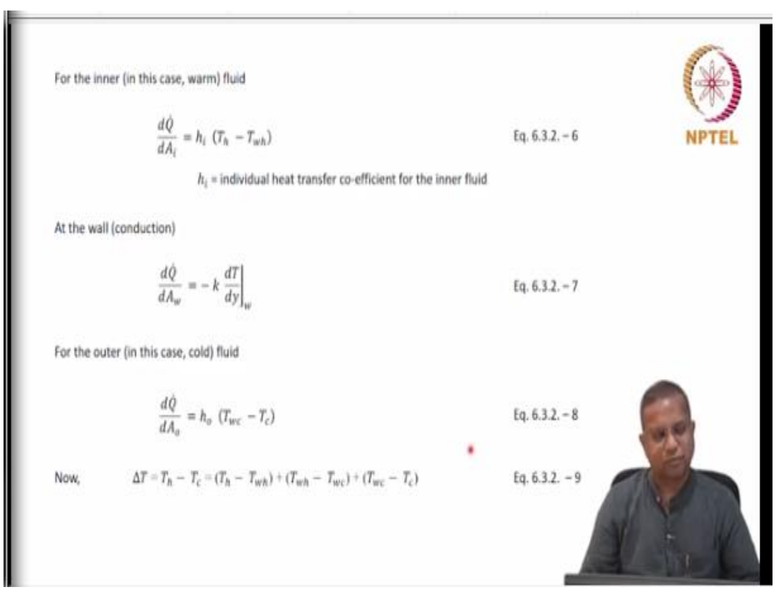

I think let us take a break now, ok, we have been at it for some time. I think it is best to take a break then come back and go for a certain length of time, see you in the next class. Welcome back, we are looking at heat exchanger design, we are in our third lecture, we are taking it in small chunks so that we can appreciate it much better. We looked at the heat exchanger as consisting of the heat exchanger situation consisting of resistances to heat transfer.

That are located in series, the resistance in the warm fluid plus the resistance due to the wall and the resistance due to the cold fluid. They are all in series with each other for the transfer of heat from the warm fluid to the cold fluid. So, this is the view that we are taking to heat transfer. For the inner in this case warm fluid we can write the heat flux as a certain heat transfer rate times the temperature difference of relevance which is T_h the bulk - T_{wh} ok.

T_h is here, T_{wh} is here that is the temperature difference and we could write the heat fluxes h_i in a fluid T_h - T_{wh} in a differential section. That is going to change across the length therefore we are looking at a differential section, h_i is the individual heat transfer coefficient for the inner fluid, equation 6.3.2 - 6. Then at the wall of course it is conduction, we know the Fourier's law, you can directly use that.

Now, let us consider the heat transfer across the heat exchanger inner pipe wall as detailed in Fig. 6.3.2-5.

The overall resistance to heat transfer (various resistances are in series)

$$R_{\text{warm fluid}} + R_{\text{wall}} + R_{\text{cold fluid}}$$

For the inner (in this case, warm) fluid

$$\frac{d\dot{Q}}{dA_i} = h_i (T_h - T_{wh}) \qquad (6.3.2\text{-}6)$$

where h_i is individual heat transfer coefficient for the inner fluid.

At the wall (conduction)

$$\frac{d\dot{Q}}{dA_w} = -k\left.\frac{dT}{dy}\right|_w \qquad (6.3.2\text{-}7)$$

For the outer (in this case, cold) fluid

$$\frac{d\dot{Q}}{dA_o} = h_o(T_{wc} - T_c) \qquad (6.3.2\text{-}8)$$

Now

$$\Delta T = T_h - T_c = (T_h - T_{wh}) + (T_{wh} - T_{wc}) + (T_{wc} - T_c) \qquad (6.3.2\text{-}9)$$

Now this is the beauty of using the resistance and series approach, the overall temperature difference ΔT which is T_h the bulk hot flow temperature minus the bulk cold fluid temperature T_h - T_c that is what the overall driving forces is nothing but the sum of the individual temperature differences, individual driving forces (6.3.2-9).

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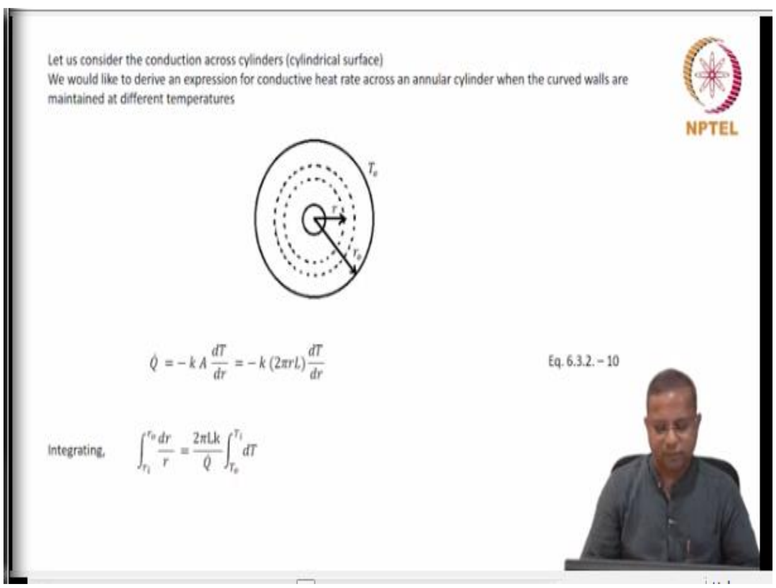

Now we will have to consider the conduction across cylinders, we need to get an expression for that we have not done that as yet ok. Let us see how to get an expression for conduction, so that we can take care of this term appropriately and then bring in these terms for an overall design. So, conduction across cylinders it is a cylindrical surface, we would like to derive an expression for conductive heat rate across an annular cylinder when the curved walls are maintained at different temperatures.

That is the situation here, that is the wall of the inner tube that we are looking at as a annular cylinder here. So, you have something like this and you have this is the inner radius, this is the lumen of course they have blown it up, typically speaking the wall thickness is going to be small and so on. But here to understand this is the inner diameter, this is the outer diameter, the shell at any distance in the wall, the wall is from here to here, ok, that is what I mean.

The shell we are going to do balances over the shell maybe, so this is the resistance of R the outer radius is r_0 which is maintained at T_0 ok. The heat transfer is happening in the reverse direction in this case, no reverse the it is from the hot flow to the cold flow, the inner fluid is hot it is in this direction it is from the inside to the outside itself.

$$\dot{Q} = -k\,A\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr} \qquad (6.3.2\text{-}10)$$

Upon integration

$$\int_{r_c}^{r_o}\frac{dr}{r} = \frac{2\pi Lk}{\dot{Q}}\int_{T_o}^{T_i} dT$$

$$\dot{Q} = \frac{k(2\pi L)(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (6.3.2\text{-}11)$$

$$= \frac{k\overline{A_L}(T_i - T_o)}{r_o - r_i} \qquad (6.3.2\text{-}12)$$

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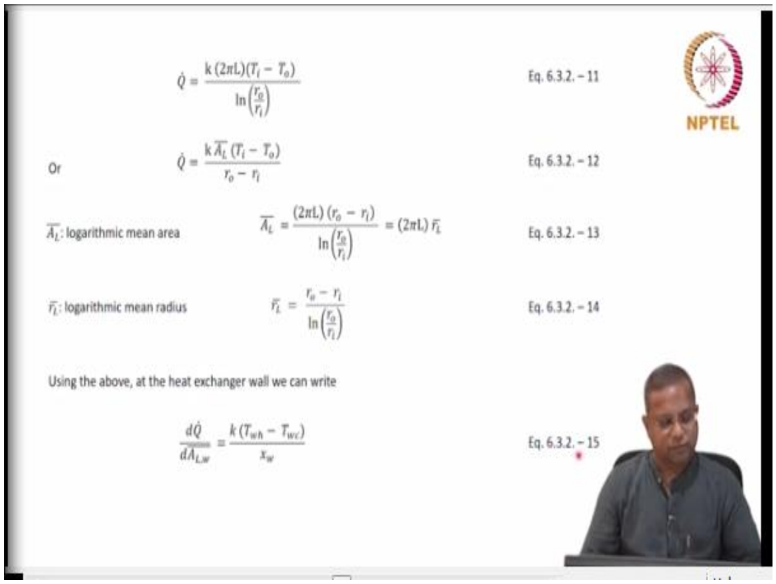

$$\overline{A_L} = \frac{(2\pi L)(r_o - r_i)}{\ln\left(\frac{r_o}{r_i}\right)} = 2\pi L\,\overline{r_L} \qquad (6.3.2\text{-}13)$$

$\overline{A_L}$ is the logarithmic mean area, and $\overline{r_L}$ is the logarithmic mean radius, which is defined as

$$\overline{r_L} = \frac{r_o - r_i}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (6.3.2\text{-}14)$$

Using the above for the heat exchanger wall

$$\frac{d\dot{Q}}{d\,\overline{A_{L,w}}} = \frac{k(T_{wh} - T_{wc})}{x_w} \qquad (6.3.2\text{-}15)$$

So, you substitute all this back you will get this into, you differentiate this. You have an expression for $\dot{Q}$ written in terms of logarithmic mean radius and then you substituted back here, why are we doing that after integration you will see very quickly the other 2 forms are still in the differential forms ok. So, you have this, equation 6.3.2 - 15.

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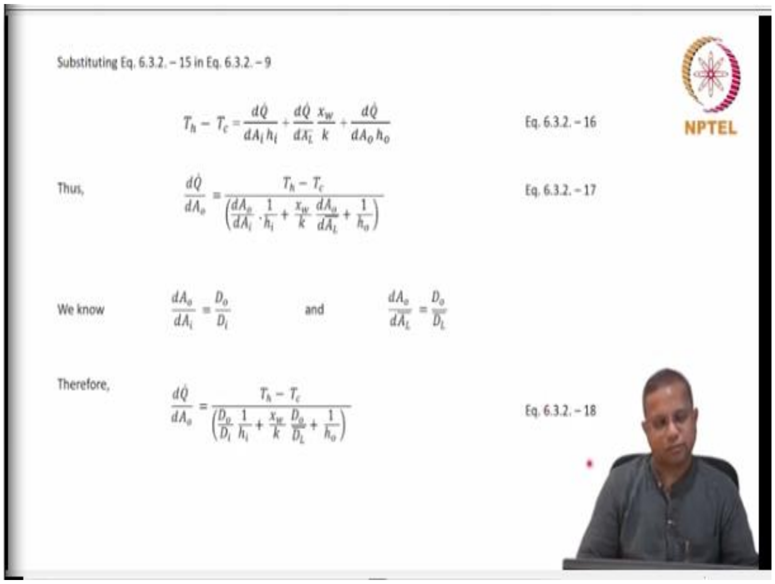

So, if you substitute this back to the overall expression $T_h - T_c$ is one temperature difference and the inside plus the temperature difference at the wall plus the temperature difference at the outside right, that written in terms of the rate and the heat transfer coefficient is this.

Substituting Eq. 6.3.2-15 in Eq. 6.3.2-9, we get

$$T_h - T_c = \frac{d\dot{Q}}{dA_i h_i} + \frac{d\dot{Q}}{d\overline{A_L}}\frac{x_w}{k} + \frac{d\dot{Q}}{dA_o h_o} \qquad (6.3.2\text{-}16)$$

Therefore

$$\frac{d\dot{Q}}{dA_o} = \frac{T_h - T_c}{\left(\frac{dA_o}{dA_i}.\frac{1}{h_i} + \frac{x_w}{k}\frac{dA_o}{d\overline{A_L}} + \frac{1}{h_o}\right)} \qquad (6.3.2\text{-}17)$$

We also know that

$$\frac{dA_o}{dA_i} = \frac{D_o}{D_i}$$

and

$$\frac{dA_o}{d\,A_L} = \frac{D_o}{D_L}$$

Therefore

$$\frac{d\dot{Q}}{dA_o} = \frac{T_h - T_c}{\left(\frac{D_o}{D_i}\frac{1}{h_i} + \frac{x_w}{k}\frac{D_o}{D_L} + \frac{1}{h_o}\right)} \qquad (6.3.2\text{-}18)$$

We know that

$$\text{Flux} = \frac{\text{Driving force}}{\text{Resistance}}$$

or

$$= \text{Conductance} \times \text{Driving force}$$

Thus, the conductance

$$U_o = \frac{1}{\left(\frac{D_o}{D_i}\frac{1}{h_i}\right) + \left(\frac{D_o}{D_L}\frac{x_w}{k}\right) + \left(\frac{1}{h_o}\right)} \qquad (6.3.2\text{-}19)$$

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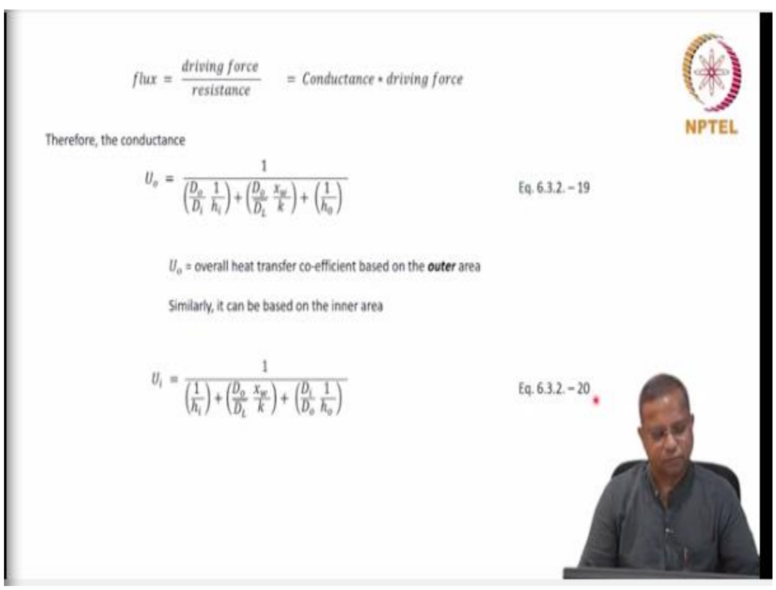

Now we know that flux in general is a driving force by resistance ok you can express flux is the driving force by resistance. One by resistance is nothing but the conductance and therefore you take 1 by resistance is conductance, conductance times the driving force, it is flux. Therefore the conductance is nothing but, in this case the flux is this the conductance is 1 by of this times the driving force is $T_h - T_c$. So, 6.3.2-19, this is where the outer area of the inner pipe inner area of the inner pipe, why. Because you have a pipe, this is a physical pipe, a physical pipe has a certain thickness.

So, there is an inner diameter and there is an outer diameter for the inner pipe alone. So, you can have an area based on the inner diameter of the inner pipe and an area based on the outer diameter of the inner pipe, ok. So, this overall heat transfer coefficient U o is based on the outer area of the inner pipe because we have normalized with respect to the outer area. Similarly, it can be based on the inner area of the inner pipe. As if you pull out the corresponding different areas you could either pull out you know we pulled out this area here.

U_o is overall heat transfer coefficient based on the *outer* area. Similarly, it can be based on the inner area.

$$U_i = \frac{1}{\left(\frac{1}{h_i}\right) + \left(\frac{D_o}{\overline{D_L}}\frac{x_w}{k}\right) + \left(\frac{D_i}{D_o}\frac{1}{h_o}\right)} \qquad (6.3.2\text{-}20)$$

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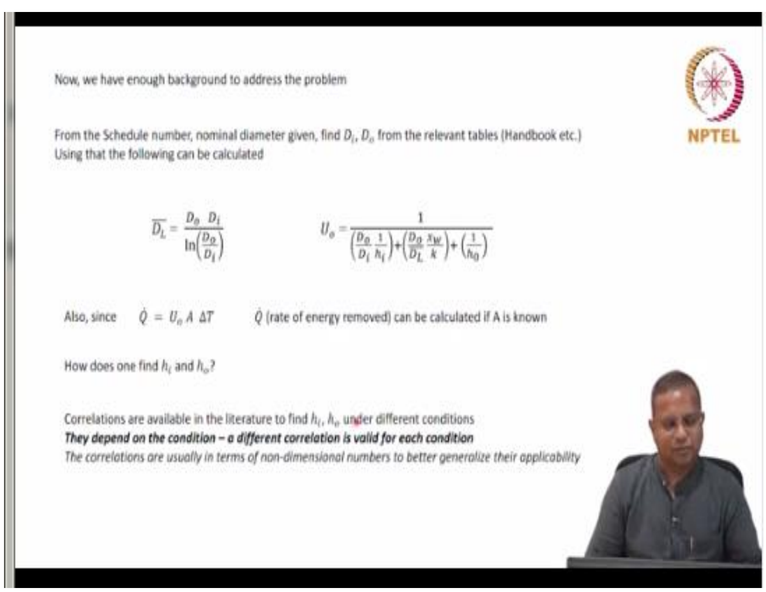

Now, we have good enough background to address the problem from the schedule number that is given the nominal diameter given you can find D_i and D_o from the relevant tables.

From the schedule number and the nominal diameter given, we can find D_i, D_o using the relevant tables (as mentioned in Chapter 3). The following can be calculated

$$\overline{D_L} = \frac{D_o - D_i}{\ln\left(\frac{D_o}{D_i}\right)}$$

$$U_o = \frac{1}{\left(\frac{D_o}{D_i}\frac{1}{h_i}\right) + \left(\frac{D_o}{\overline{D_L}}\frac{x_w}{k}\right) + \left(\frac{1}{h_o}\right)}$$

and with A, $\dot{Q}$ (rate of energy removed) can be calculated.

But, how does one find h_i and h_o? Correlations are available in literature (e.g. the correlation developed by Sieder and Tate).

Correlation are available in the literature to find out h_i and h_0 under specific different conditions. Because they depend on the condition and a different correlation is valid for each condition ok, this is what makes it a little difficult to use, whatever correlation we use needs to be applied to that exact condition. And we do have no confidence whether that correlation is going to work for another condition.

The correlations are usually written in terms of non dimensional variables of course to better generalize their applicability this is to be expected.

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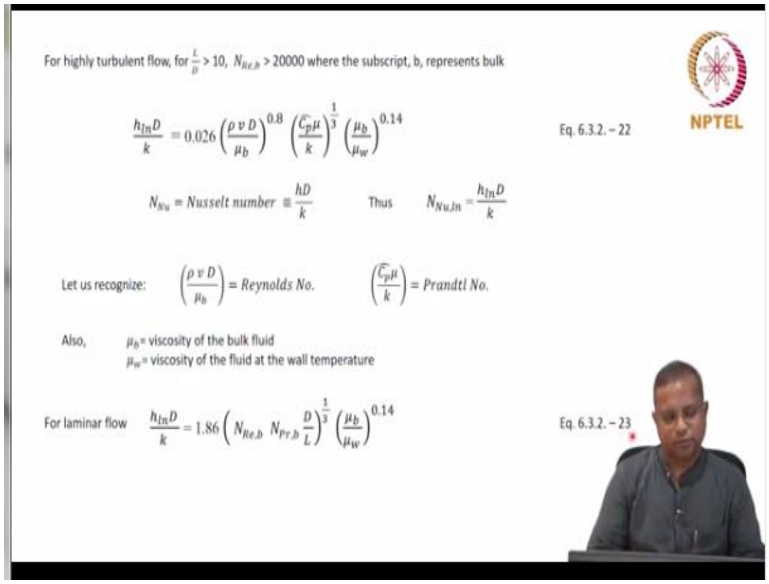

Let me give you a examples of a few correlations for highly turbulent flow ok, first the flow is highly turbulent Reynolds number is very high. And for the L by D ratio the length by the diameter ratio that has to be greater than 10 under these conditions. And if the bulk Reynolds number is greater than 20,000 then ok if these conditions are met if the bulk Reynolds numbers is greater than 20,000, if L by D is greater than 10 then this correlation works.

This correlation has been found through number of experiments and then you know in some insights and so on and so forth to come up with this in terms of a non-dimensional quantities

First, let us consider a useful non-dimensional number.

$$\text{Nusselt number, } N_{\text{Nu}} = \frac{hD}{k} \qquad (6.3.2\text{-}21)$$

Thus

$$N_{\text{Nu,ln}} = \frac{h_{\text{ln}} D}{k}$$

Some approximate equations that can be used to find Nusselt number are given below.

For highly turbulent flow, i.e. for $\frac{L}{D} > 10$, $N_{\text{Re},b} > 20000$ where the subscript b represents bulk.

$$\frac{h_{\text{ln}} D}{k} = 0.026 \left(\frac{DG}{\mu_b}\right)^{0.8} \left(\frac{C_p \mu}{k}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad (6.3.2\text{-}22)$$

where G is mass velocity = ρv, μ_b is viscosity of the bulk fluid and μ_w is viscosity of the fluid at the wall temperature.

For laminar flow

$$\frac{h_{\text{ln}} D}{k} = 1.86 \left(N_{\text{Re},b}\, N_{\text{Pr},b} \frac{D}{L}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad (6.3.2\text{-}23)$$

Therefore you need that difference raised to the power of 0.14 ok, this correlation works for this condition, equation 6.3.2 - 22. This (h_{ln} D/k) is called the Nusselt number which one comes across very frequently when you have heat transfer situations with flow. Thus Nusselt number the logarithmic mean Nusselt number is (h_{ln} D/k) which is based on the logarithmic heat transfer coefficient h_{ln}, ok does it let us recognize that this is the Reynolds number Reynolds number for of 0.8 as I had already mentioned.

This is the Prandtl number as I had already mentioned also viscosity at the wall temperature. And for laminar flow this we said for turbulent flow bulk Reynolds number gradient is 20,000. For laminar flow a different correlation works it is h_{ln} D/k equals instead of 0.026 you have 1.86, the product of Reynolds number, Prandtl number and D/L raised to the power of 1 by 3 everything raised to the power of 1 by 3 the product raised to the power of 1 by 3. This is bulk Reynolds number, this is bulk Prandtl number D/L raised to the power of (1/3) and (μ_b/μ_w) raised to the power of 0.14 this is for laminar flow, when the flow changes your correlation changes, equation 6.3.2 - 23, ok.

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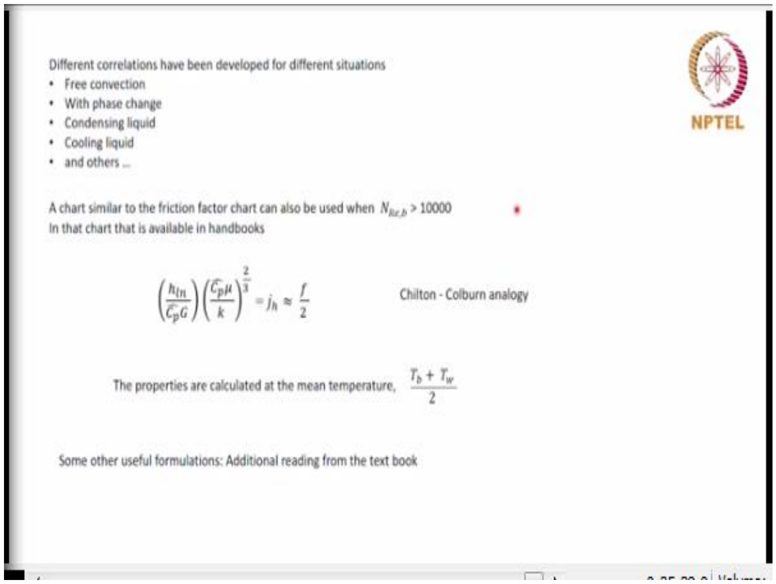

So, you use different correlations that have been developed for different conditions in the literature. You have correlations that are very different for free convection, when temperature differences in the fluid set up density differences. And therefore the flow occurs lower density fluid would rises the higher density fluid drops and therefore a flow occurs ok that is free convection, there could be phase changes that occur in these cases.

When a phase change occurs a totally different correlation is valid even in the earlier case. If you have a condensing liquid then a different correlation, if you have a cooling liquid then a totally different correlation and so on and so forth. So, you need to know the conditions, you need to know the operating conditions, you need to know the physical conditions of the fluids. And then you need to choose a corresponding correlation, hopefully that correlation is already available.

If not you will have to develop the correlation which is not an easy thing to do. A chart similar to the friction factor chart can be used when the Reynolds number is greater than 10,000. And in that chart that is available on handbooks this is the Nusselt number and the Prandtl number raised to the power (2/3), this product happens to be related to the friction factor, f/2 ok. So, this is called the j_h factor, which is approximately equal to the friction factor, f/2.

A chart similar to the Moody's chart can also be used when $N_{\mathrm{Re},b} > 10000$. In that chart available in handbooks

$$\left(\frac{h_{\mathrm{ln}}}{C_p G}\right)\left(\frac{C_p \mu}{k}\right)^{\frac{2}{3}} = j_h \approx \frac{f}{2}$$

which is known as the Chilton-Colburn analogy.

The properties are calculated at the mean temperature $\frac{T_b + T_w}{2}$.

So, this correlation can be used which simplifies various calculations, this is called the Chilton Colburn analogy. And the properties are calculated at the mean temperature of the bulk and the wall taken ok, $(T_b + T_w)/2$ i.e. (the average of the 2). And there are other useful formulations you can read the textbook your textbook for some formulations. You can read other books on heat transfer, mass transfer for many other formulations.

The heat transfer, mass transfer books are filled with correlations of different kinds for different situations. I think this what I have now for you , with this we have looked at the essence of heat transfer exchanger design. It is a operating piece of equipment again there is right there. The way to design it we have seen the intricacies of designing that are the principles that go into design that we have seen.

And of course, if something is going wrong you can go back and start looking at it from these points of view to get a handle on how to operate it better, ok. I think we have completed the course with this lecture. In the next lecture, let us start with a review. I think we will do the review over a few lectures that will be useful for you the review will help you see things in perspective.

You have done each one with a focus on that aspect alone, there are beautiful insights to be drawn when you look at them together. We are going to draw some insights as well as we are going to review the entire course in the next few lectures, see you there bye.