Transport Phenomena in Biological Systems

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Lecture-79
Course Review-Part 2

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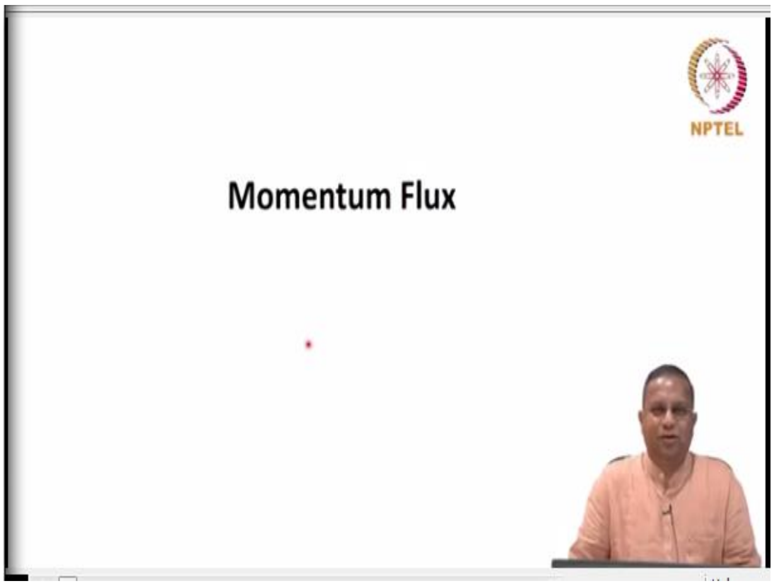

Welcome to the second review lecture. Since the courses consist of a lot of information, a lot of heavy information, I thought of breaking up the review into shorter lectures. That would be much easier for you when you want to look at the overall course just by looking at the review part of it. In this lecture, we would look at momentum flux reasonably in brief we had spent a good amount of time on momentum flux.

In the previous review lecture, we looked at mass conservation as well as mass flux. And I think before I begin this, I should also tell you some aspects of learning, I think you need to take that as a part of you need to internalize that and only then would you be able to see this course in proper light, so let me just quickly go to that.

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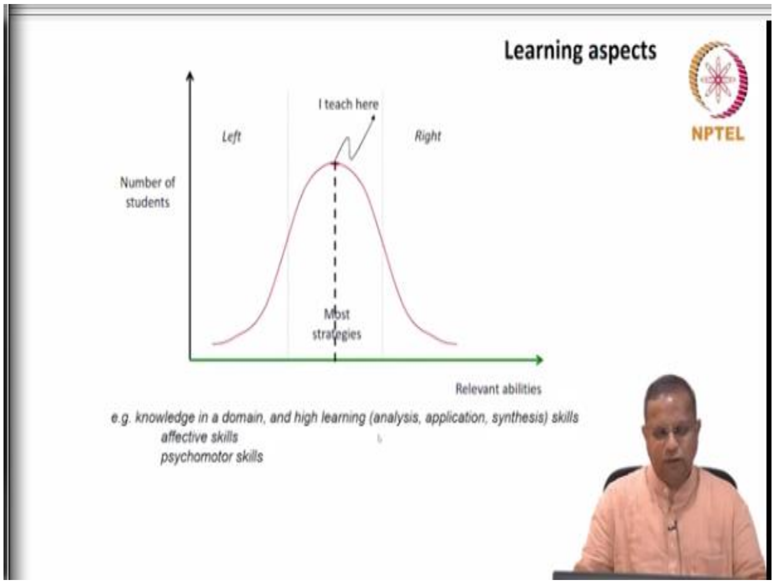

We talked about learning aspects and in a class, different people have different skills and so on so forth. We said that the Gaussian distribution is a typical representation n number of students versus relevant abilities. And this typically holds when the number of students in a class is 15, 20 or more, ok. I have seen this happen time and again the exact nature of the curve could be slightly different with the Gaussian as a very good first approximation.

This we all understand in a class which deserve in a completely heterogeneous class which is this, this I am sure would hold. The relevant abilities include knowledge in a domain and high learning skills, higher learning skills include analysis, application, synthesis. And maybe I think this covers most of it and effective skills and psychomotor skills also as and when needed we could restrict our view to analysis application synthesis as high learning in this for this particular term relevant abilities as well as knowledge in the domain.

The most students would be somewhere here, as indicated by this graph itself. However there are quite a few who would be here and quite a few who would be here, ok. The fraction is small but when the numbers are large, the actual numbers could be large. These people may feel bored with the repetitions because they have gotten it quickly. These people would need repeated exposures just look at the same material a few more times to get there.

Again this is entirely course dependent very specific thing, this distribution maybe rather the person who is here for this particular course, may be here or will be here for something else altogether. I could be somewhere here for this course of this stage whereas probably for singing and dancing I could be somewhere here, ok. So, it is not a judgment of the student, it is just a statement of fact.

And statement of fact more from the point of view of improving the learning what needs to be done to improve, so that we all get to a certain stage, that is the whole idea I am giving you this. And of course I said that most strategies, most learning strategies, address students who are in the average domain here, the average students as they call them. Whereas I could also address these people may not be to the extent that I can do in a regular class.

But I have attempted some of these things here, there have been a lot of strategies that have been oven into this course, it is just not giving you information alone, the way it has given the way problems are placed, the way the problems are solved and so on so forth.

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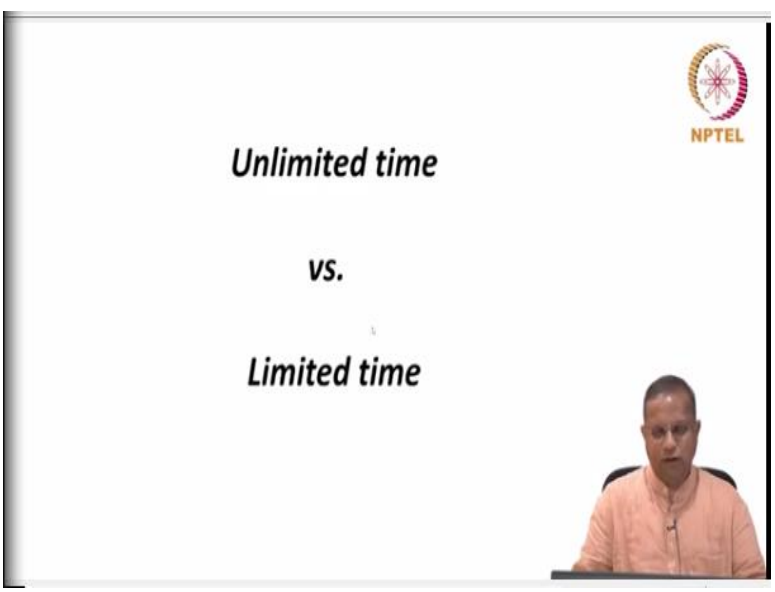

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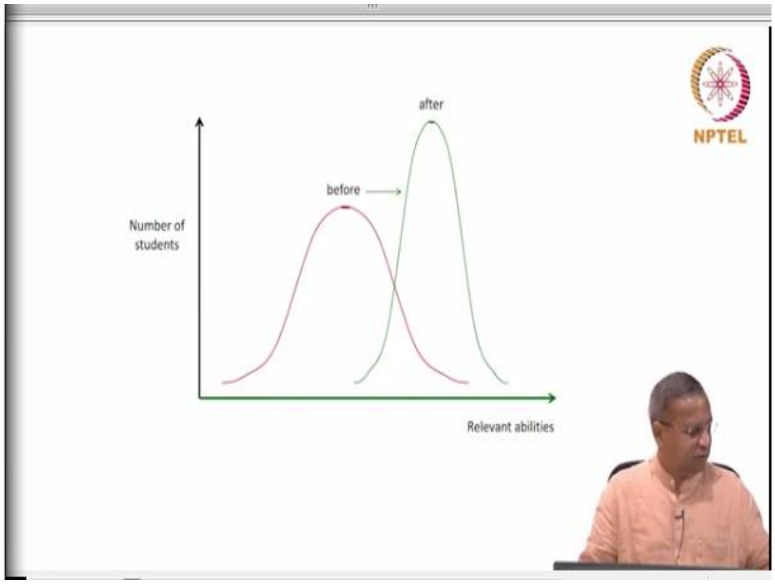

All go into improving the learning of students, so that whatever the distribution is or was at the beginning of the course becomes this at the end of the course, that is the whole idea or this is the central themes central basis on which I work, I have no other interest just to move the average to the right as well as narrow the distribution, ok. I think this needs to be kept in mind, so that the way you approach suppose if we find a little uncomfortable just redo that.

If you are comfortable, if you are too comfortable with it just look for bigger challenges it is a class and we need to take the entire class together, I teach the average level and so they should be fine, which is most of the students, ok. Now let us look at momentum flux aspects.

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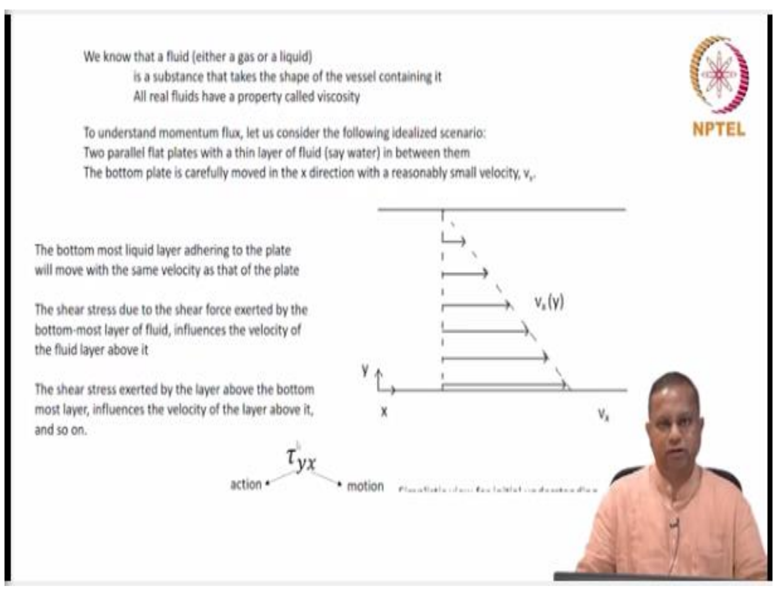

We talked about fluids and we said we looked at the case where a thin layer of fluid is placed between 2 parallel plates, I mean finite. And then the lower plate is moved with a velocity very low velocity in the X direction positive X direction. The upper plate is held constant and with this scenario, we could introduce the concept of shear stress. Because the layer that is closest to the lower plate will move with the velocity of the plate.

And that influences the layer above it and that will start moving with a slightly slower velocity. The second layer influences the third layer through the shear stress and that will start moving at a slightly slow velocity compared to the second and so on and so forth. To result in this kind of linear velocity profile, we also talked about our terminology here τ_{yx} the first subscript is for the direction of action for first approximation.

The second is for the direction of motion, this for initial understanding later you it gets a little complex as you already know and this is the shear stress.

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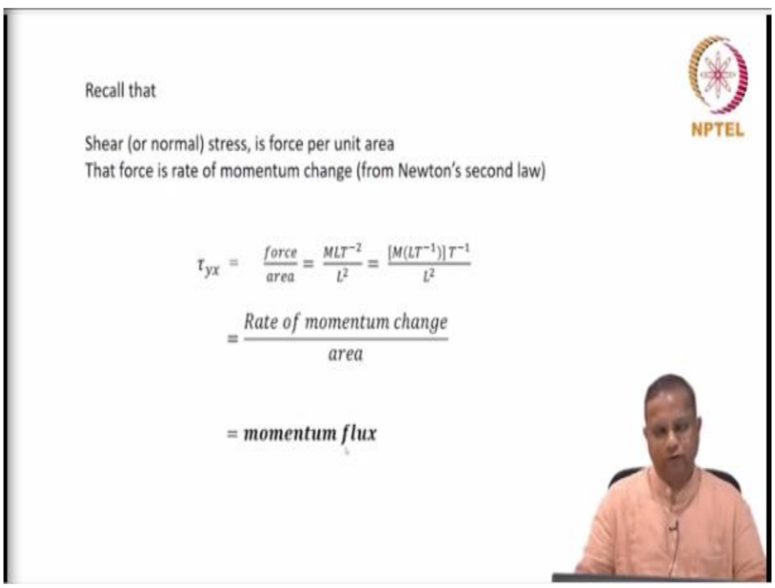

Then we saw how the shear stress is can be interpreted as the rate of momentum change per unit area or the momentum of flux.

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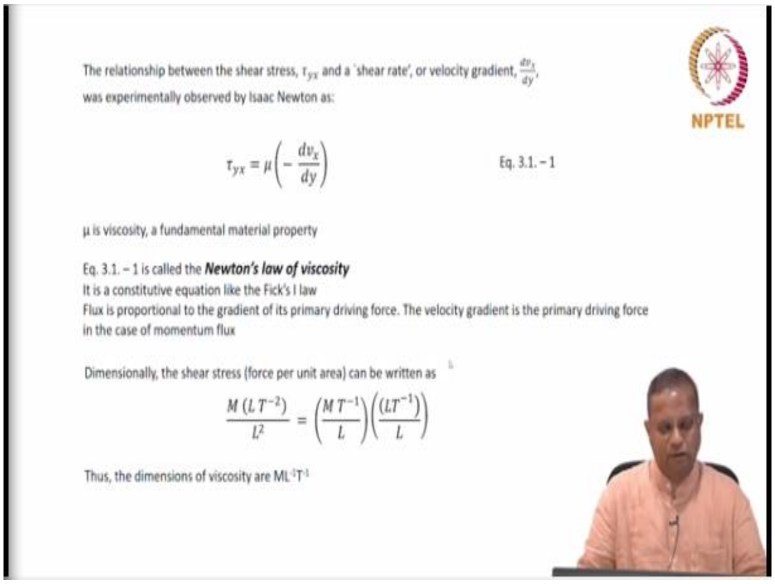

And then we started looking at some properties of fluids themselves. We said that there class of a large class of substances which follow the Newton's law of viscosity. Newton's law of viscosity is a relationship between shear stress and the velocity gradient or the shear strain as it is called. And then it is a linear relationship between the 2 with the line passing through the origin and such a fluid is called a Newtonian fluid.

And also this is a constitutive relationship similar to the Fick's first law of , flux the momentum flux is directly proportional to a velocity gradient and the constant of proportionality happens to be the viscosity here, that is what he said. Viscosity is the fundamental material property and you could back out the units for viscosity as $ML^{-1}\ T^{-1}$ using the dimensions of these various other terms here.

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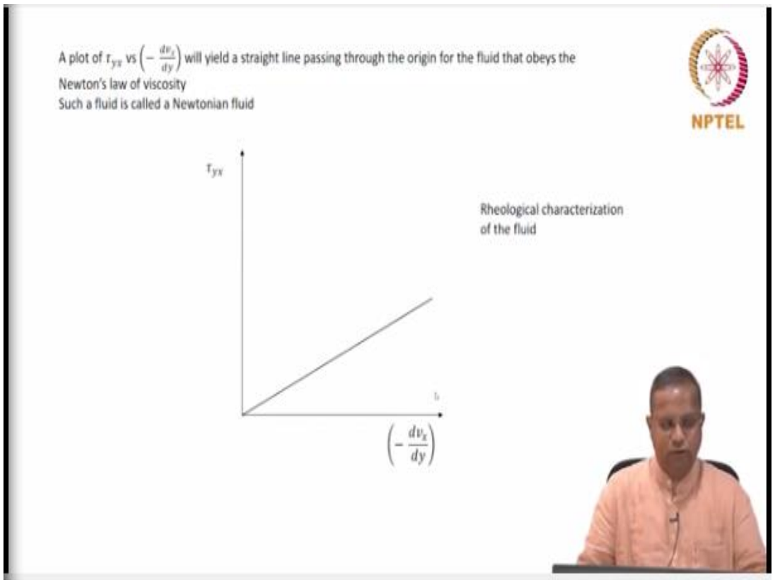

So, this is our Newtonian fluid in the rheological characterization which is a relationship between the shear stress and the shear rate, Newtonian fluid.

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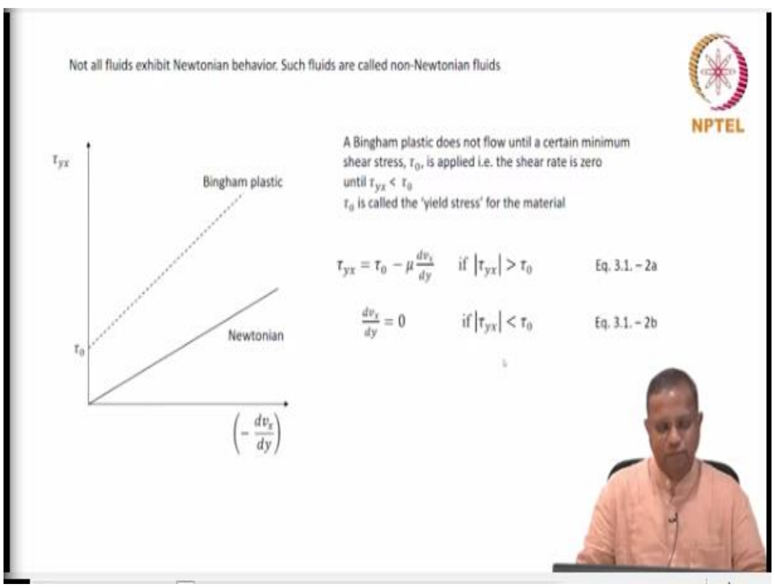

And then we saw another kind of fluid are being a plastic which needs a particular threshold shear stress to manifest a velocity gradient in other words to move, when it moves there is a velocity gradient. That will happen only when a certain threshold shear stress is reached or not, that is called the Bingham plastic and this is the expression for the Bingham plastic $\tau_{yx} = \tau_0 - \mu(dv_x/dy)$ if τ_{yx} is greater than τ_0. $(dv_x/dy) = 0$ if the shear stress is less than a threshold shear stress. So, this is the mathematical representation of a rheological representation of the Bingham plastic in a mathematical form.

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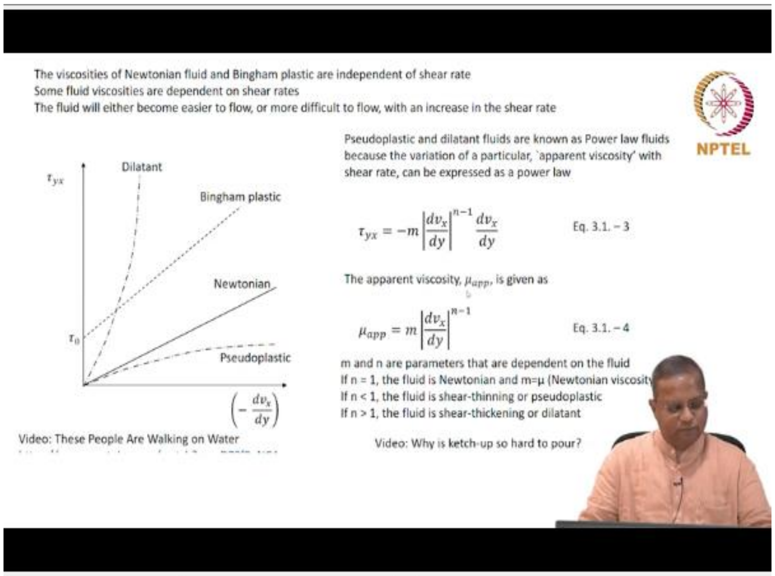

Then we looked at the other types of fluids that are there, one is the pseudoplastic fluid, the other one is the dilatant fluid. The pseudoplastic fluid and the dilatant fluids can be described by using a power law their rheological relationship is given by a power law. That is something like this $\tau_{yx}= - m\left|\frac{dv_x}{dy}\right|^{n-1} \frac{dv_x}{dy}$, all these can be considered together times of course the velocity gradient.

Of course you could take the minus along with this $\frac{dv_x}{dy}$, ok. Here this term $m\left|\frac{dv_x}{dy}\right|^{n-1}$ is called the apparent viscosity. And if n equals 1 this term disappears and m equals μ then return into viscosity. If n is less than 1 then you get the pseudoplastic behavior, if n is greater than 1 you get the shear thickening or the dilatant behavior. So, these are the kind of fluids.

$\mu_{app} = m\left|\frac{dv_x}{dy}\right|^{n-1}$

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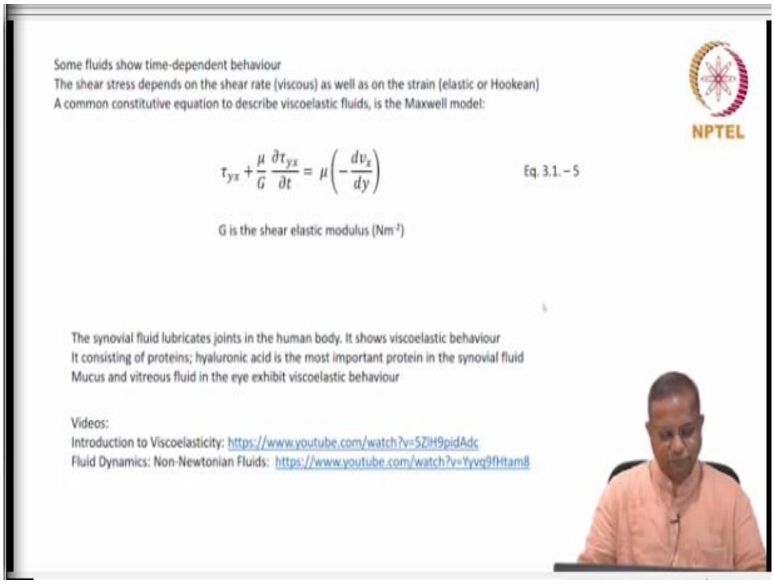

In addition you have something called those are some of the videos. In addition there are fluids where the shear stress depends on the shear rate as well as the strain and the Maxwell's model that is given here $\tau_{yx} + \frac{\mu}{G}\frac{\partial \tau_{yx}}{\partial t} = \mu(-\frac{dv_x}{dy},)$, this is the one that describes a viscoelastic fluid, ok, this is called the viscoelastic fluid. Examples of synovial fluid and hyaluronic acid and so on so forth mucus, ok.

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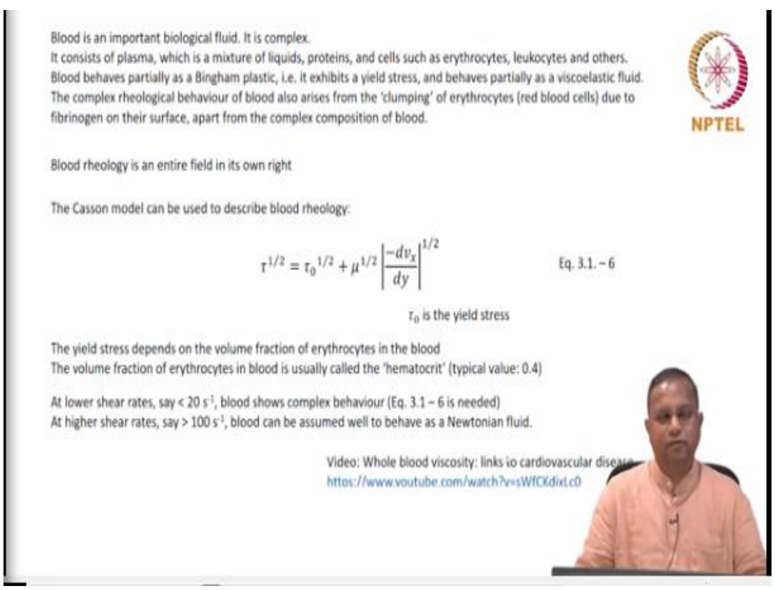

Then of course we talked a little bit about blood being a very complex fluid . Also interestingly, if the shear rate is less than 20 times per centimeter with a shear stress is less than 20 yeah no shear rate is less than 20 second inverse. Then it behaves as a complex fluid, if it is greater than 100 second inverse then a Newtonian fluid is a very good approximation to the behavior of blood ok, then video is given.

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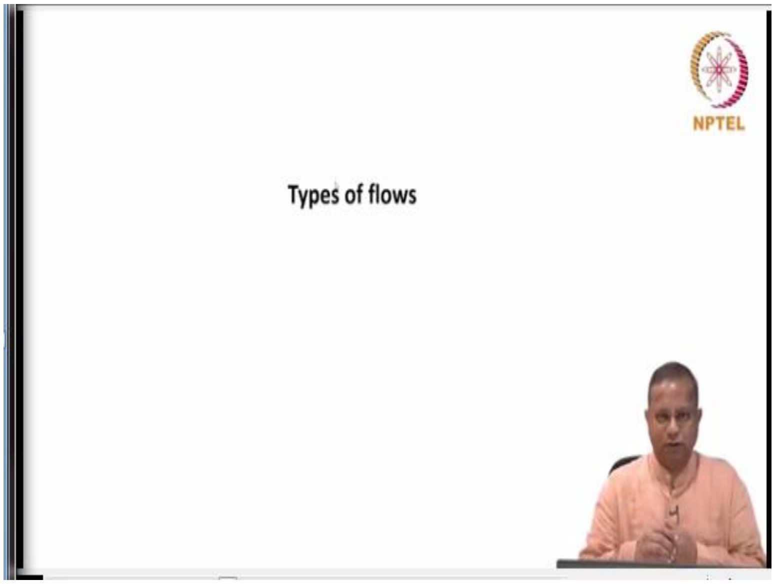

Also we said that the type of flow is important for the characterization for our understanding of the fluid behavior and then design an operation and so on so forth.

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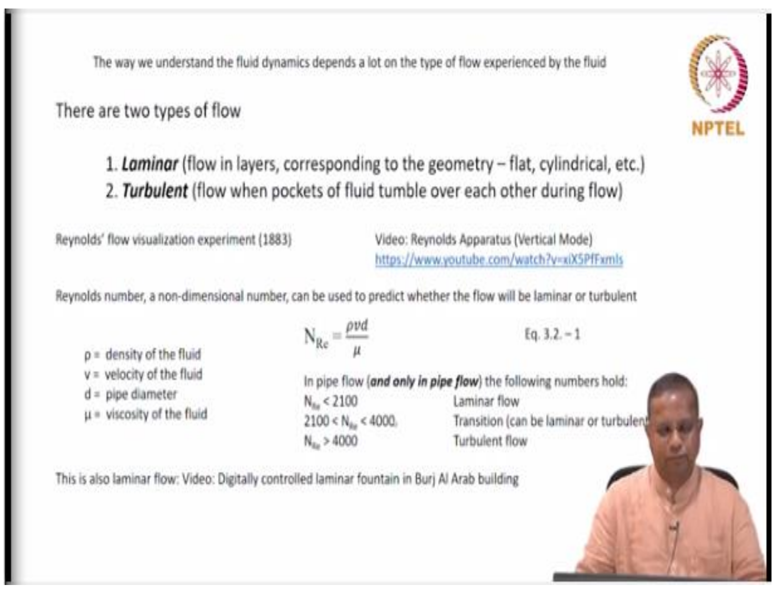

So, there are 2 major types of fluids one is called laminar which is flow in layers depending on the geometry of the system, the layers could be either flat layers or they could be cylindrical layers as in a tube or it could depend on the geometry of flow. Essentially layers or it could be turbulent when pockets of fluid tumble over each other during the flow ok. So, these are the 2 types of flows, we look predominantly at laminar because it lends itself to a certain understanding.

A turbulent flow we saw how to approach it and then we had to resort to not so rigorous approach to make use of or to attempt to design with turbulent flow and so on so forth. Let me briefly get there of course, we talked about the Reynolds number which is nothing but the ratio of $\rho vd/\mu$, ρ and μ density and viscosity of the fluid, velocity of flow and the diameter of or characteristic dimension it could be the diameter of the tube the distance from the starting point for a plate whatever it could be ok.

This can also be interpreted as the ratio of inertial forces to viscous forces in a fluid, ok. And only in a pipe flow if the Reynolds number is less than 2100 you will have laminar flow, if the Reynolds number is between 2100 and 4000 we are not too sure, we called it the transition regime. If it is greater than 4000 then it is an turbulent flow, this is been found, ok.

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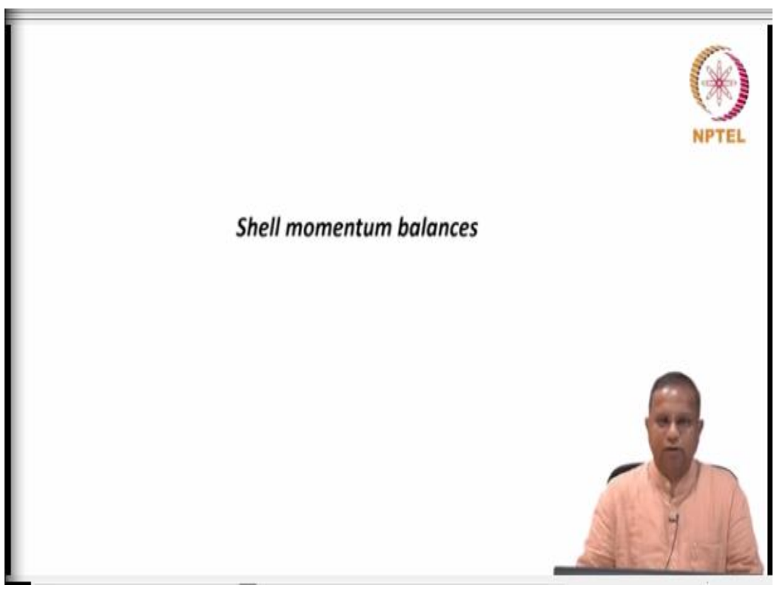

Then, we started looking at solving problems, essentially we are interested in getting velocity profiles and shear stress profiles. That is the big insight that we get from these analysis and they are very helpful, useful for design an operation. There are 2 major approaches as we have seen earlier in the case of mass flux, one is balances over representative shell or shell momentum balances in this case.

The second one is application of the conservation equation, in this case application of the equation of motion which is the equation of the conservation of momentum, Newton's second law whichever way you want to call it, ok.

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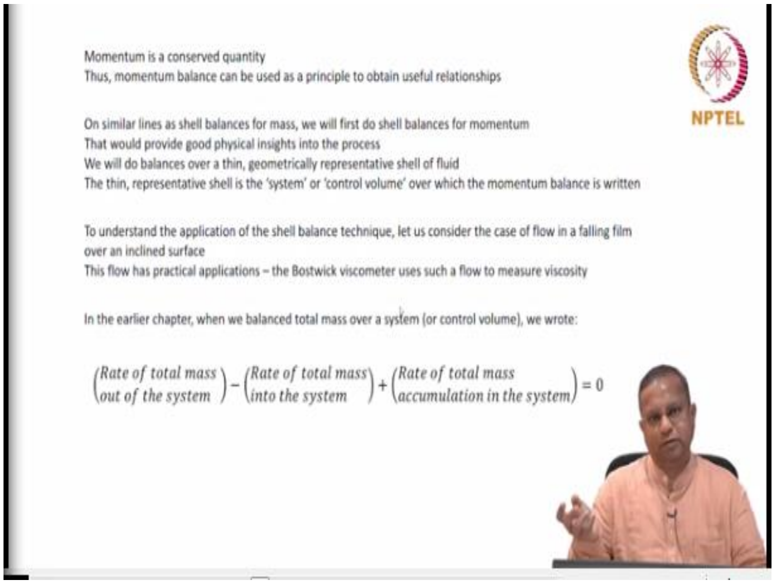

So, when we applied the shell balances and gave us as expected a very good physical feel we could visualize the forces, how they related to each other and so on so forth. Only thing is that it is the approach becomes cumbersome especially when we have the cylindrical coordinate system and the spherical coordinate system, ok.

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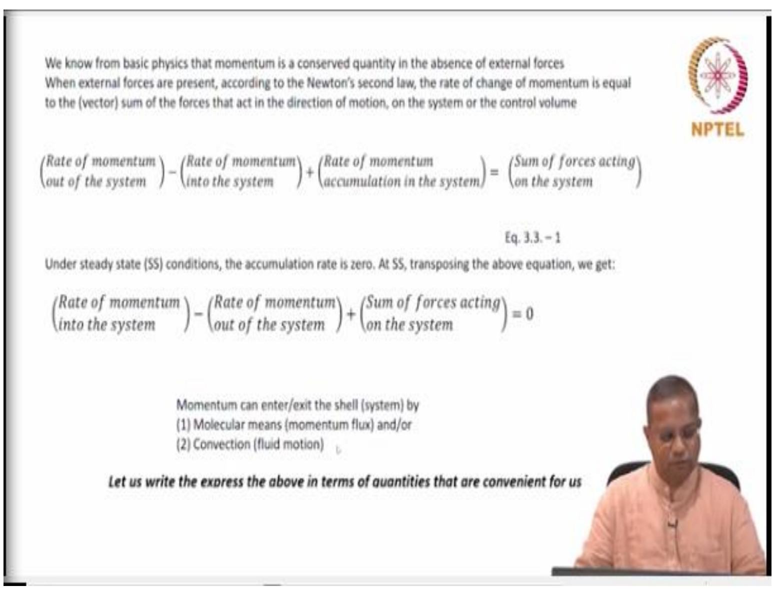

So, this is the momentum balance equation, a useful form of that rate of momentum out of the system minus rate of momentum into the system plus the rate of momentum accumulation in the system equals to sum of forces acting on the system, ok. And this we had applied to the shell, in this case a cuboidal shell. And we said that the no sorry this was shell balances, right yeah.

It is cuboidal shell but this was let me see which flow I applied it to, yeah it was flow over a flat inclined plane, Bostwick viscometer right. We said that there are 2 major means by which you can account from momentum. One is the molecular means, the other one is convection, ok which is a bulk flow which is a velocity there is mass associated with there is a momentum. And in the case of molecular means, we have already seen that the shear stress is nothing but the momentum rate , momentum flux. So, there are 2 different contributions, major contributions.

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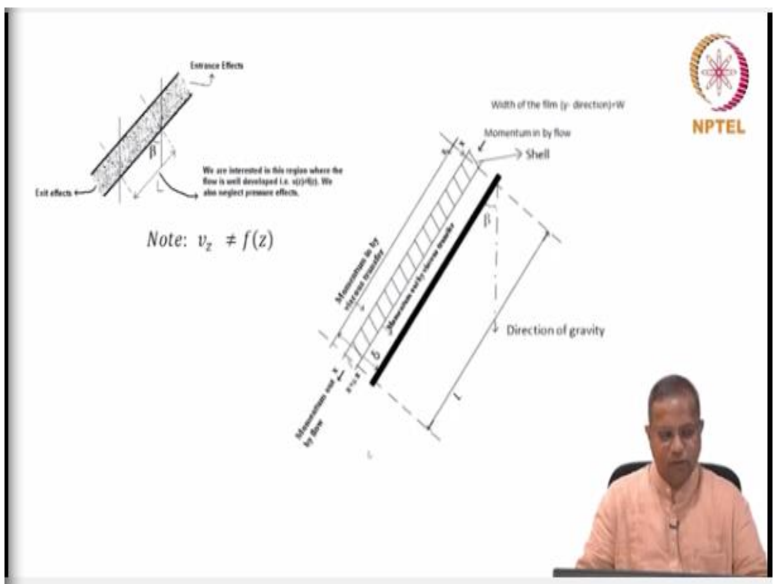

And we looked at the flow over a flat plate, well developed flow over a flat plate, the thickness of the flow layer is a small delta, this is the direction of gravity, this is the direction of flow z. And there would be changes in velocity and shear stress in the X direction, direction that corresponds to the thickness of the liquid layer.

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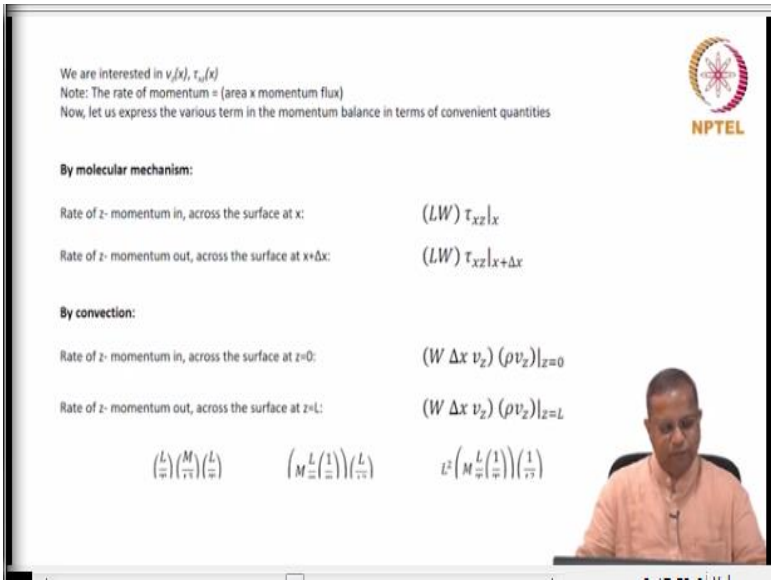

Then we went systematically wrote down the terms molecular mechanism terms you know area times the shear stress that should give us the shear the momentum rate. And for convection we had used the ρ v_x , v_x as the momentum flux times the corresponding area and that would give us the momentum rate. However we wrote it, we combined it slightly differently for it to make some sense later.

So, we wrote it like this and I had shown you that you get the you know this is momentum M L/T is momentum mass into velocity, momentum rate and momentum rate, this is you take all these 3 together it is momentum of flux. Therefore you need to multiply it by the area to get your momentum rate, ok.

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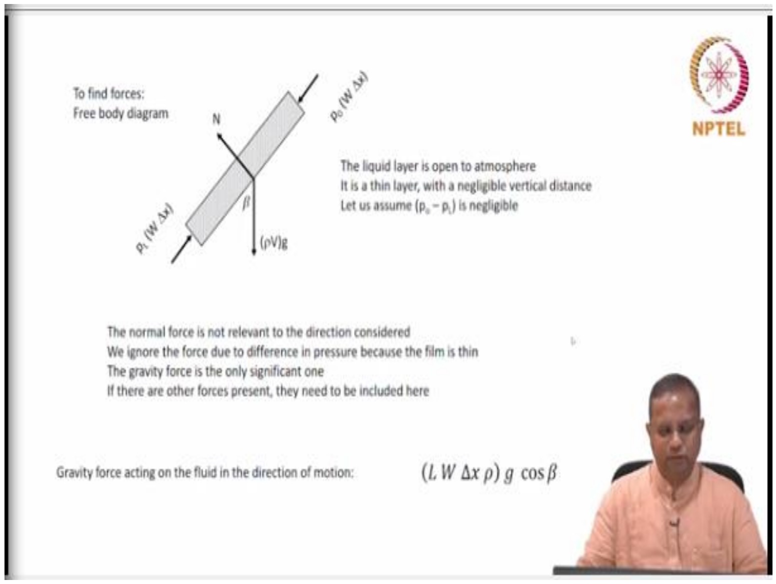

So, then we looked at a free body diagram to work out the forces that are active. Then in this situation we saw that the pressure forces will cancel when if we consider the thickness of the film to be very small, right. And so the only main force that acts in the direction of motion is a component of the gravity. So, this is the only force that is relevant here for our situation.

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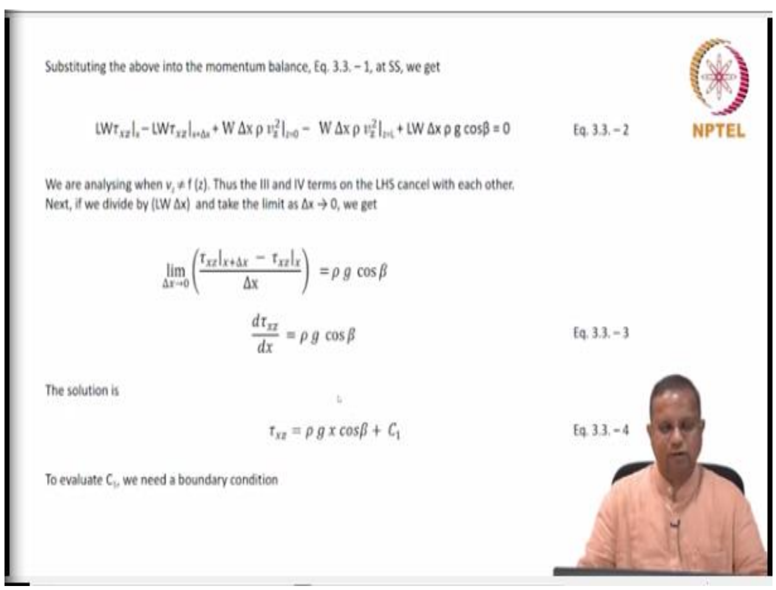

Then, we wrote the balance and then got an expression for the shear stress.

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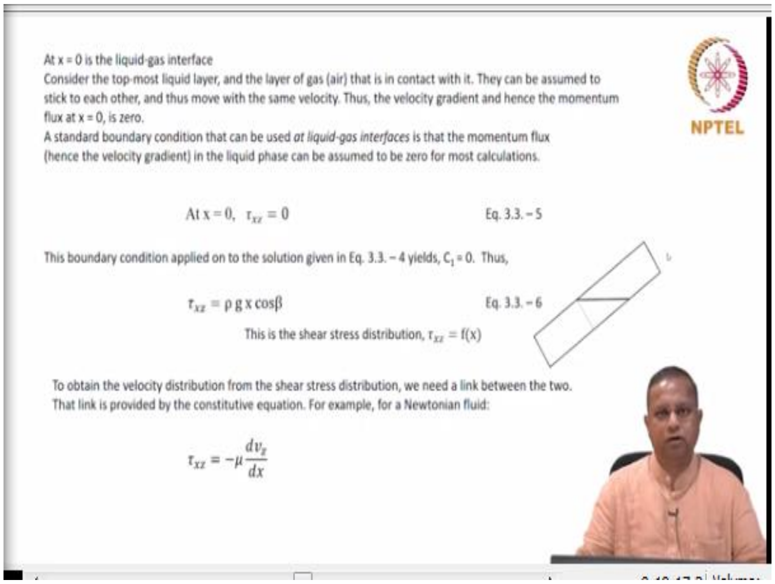

As this the complete expression was after we got the constant of integration. That was $\tau_{xz}=\rho g x \cos\beta$, right. And then we said that we also are interested in the velocity profiles, we have the shear stress profile. Therefore you need a relationship between the shear stress profile, shear stress and velocity to get the velocity profiles. And that relationship is directly with the Newton's law of viscosity because we have considered a Newtonian fluid.

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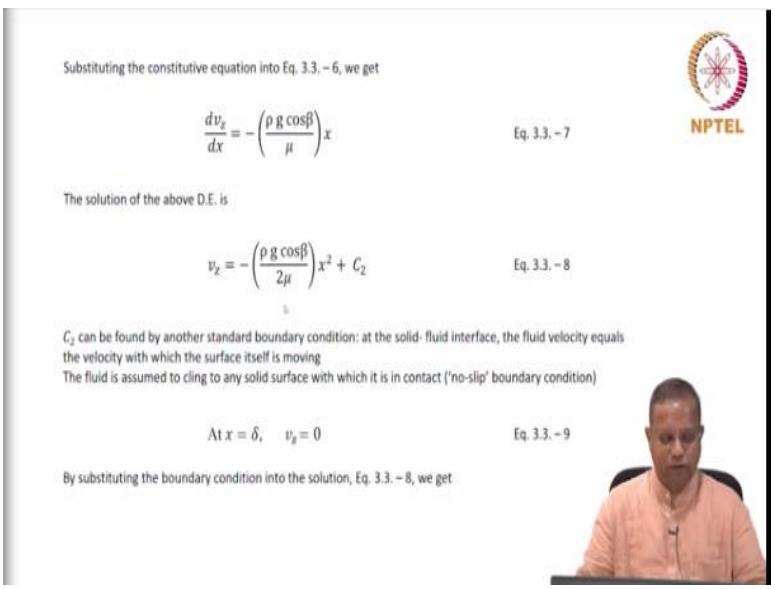

And when we substituted that we got an expression for the velocity profile, ok.

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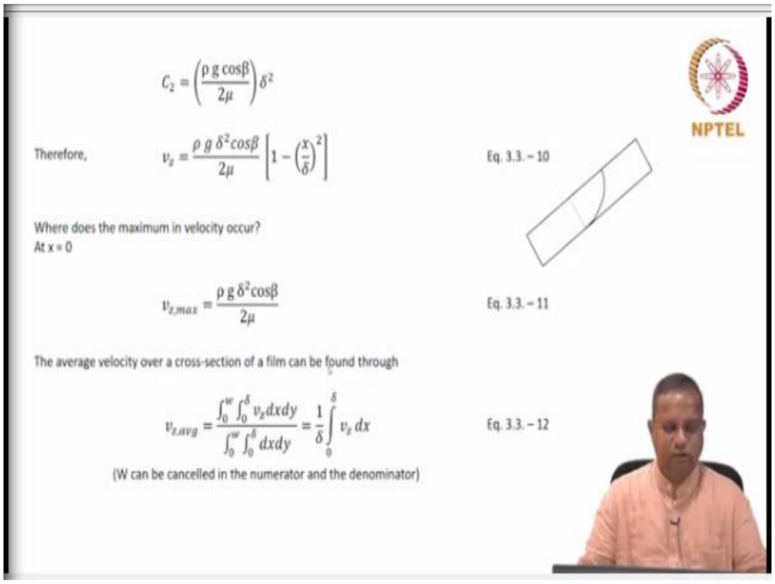

The velocity profile was a parabolic velocity profile and then we backed out the maximum velocity as well as the average velocity and the flow rate, right.

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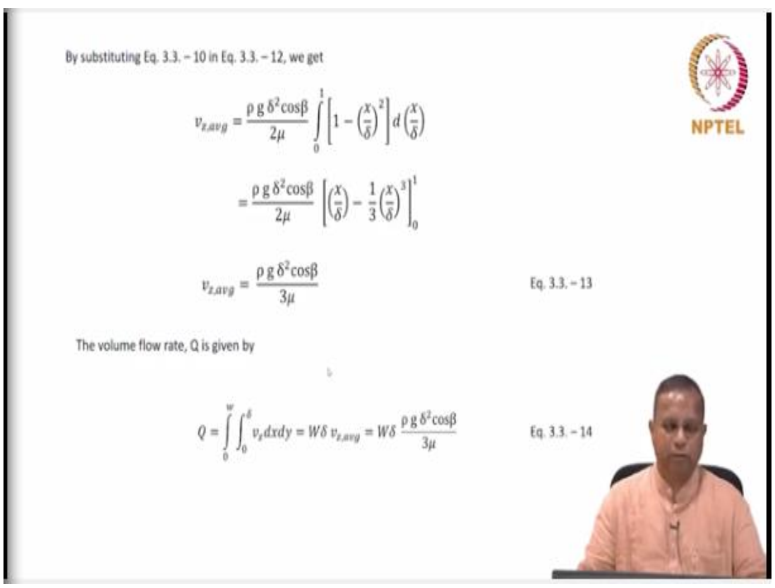

So, that was through shell momentum balances.

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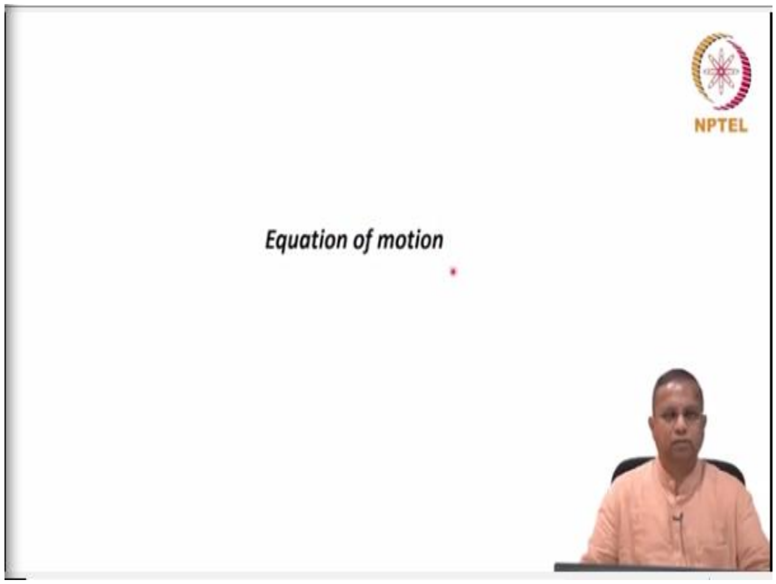

And then in the next chapter or the next lecture, next sub chapter, we derived the equation of motion.

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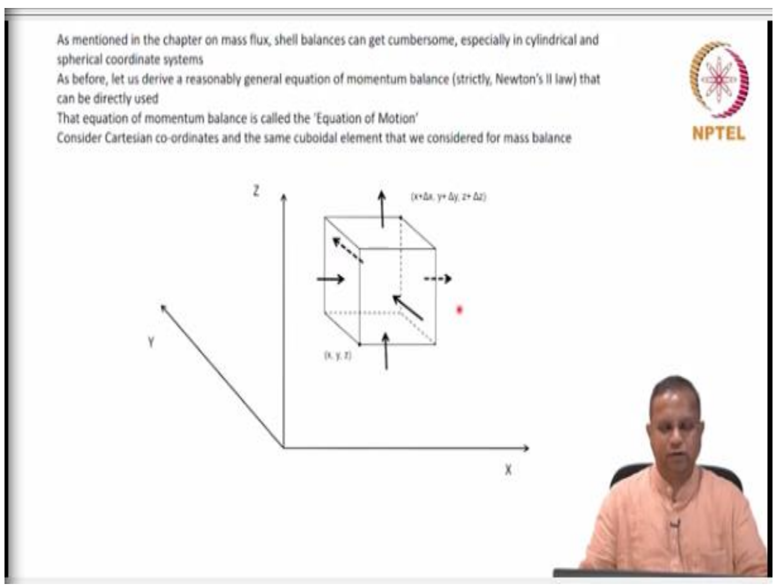

We derive the equation of motion by considering a generic situation, just this the same way that we did for mass balances or the equation of continuity. And then the these are the direction of flows, the flow in the x direction entry exit, the flow in the y direction entry exit and the flow the z direction entry exit, so very general situation.

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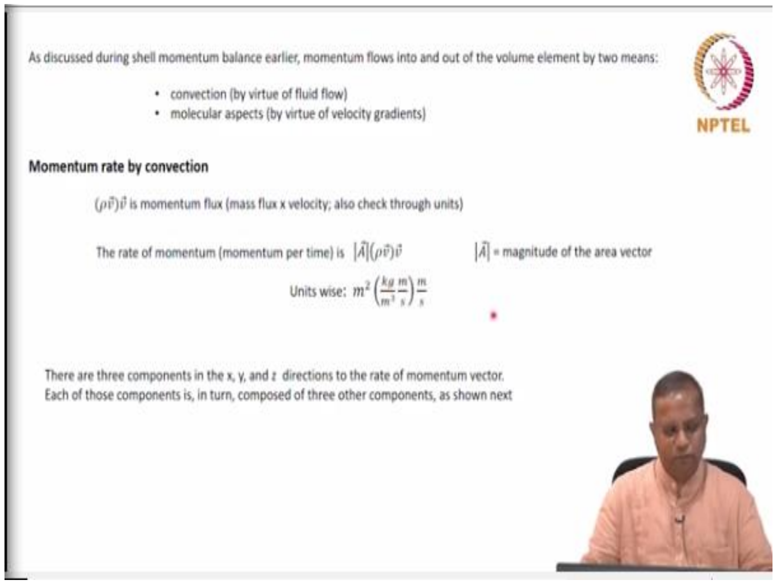

And then we wrote the contributions of momentum flux due to convection and molecular aspects in terms of the variables that we can measure we are comfortable with and so on and so forth.

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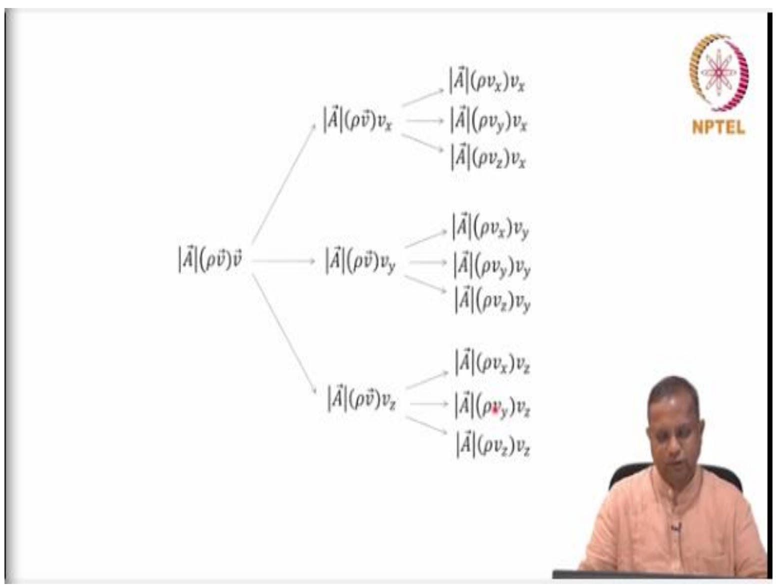

Then we did this, we saw that this term the convective term for the momentum of flux has 9 different components. So, this is the x momentum rate, y momentum rate, z momentum rate. The x momentum rate itself has 3 components, y momentum rate has 3 other components and so on, alright.

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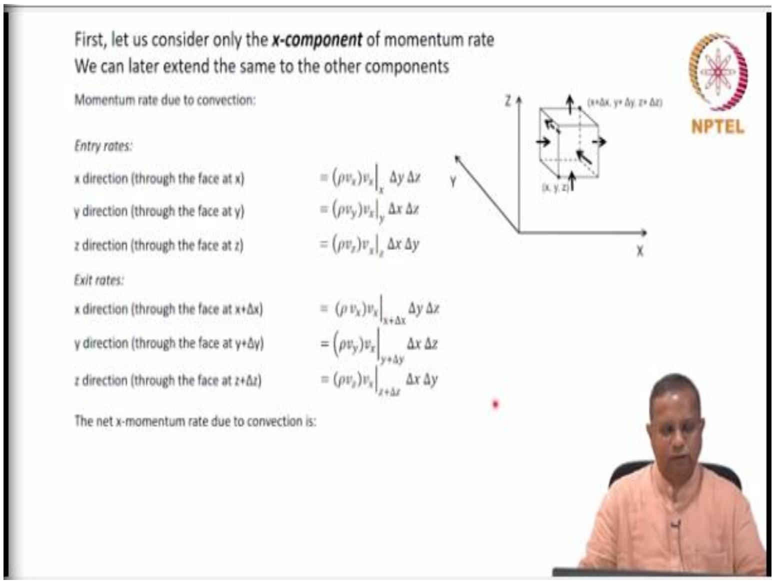

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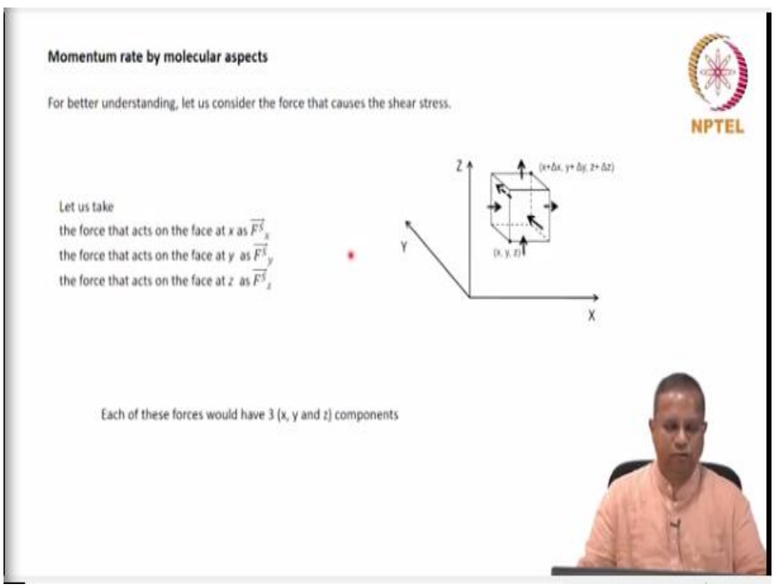

Similarly we looked at the momentum rate by molecular aspects for that we looked at the force.

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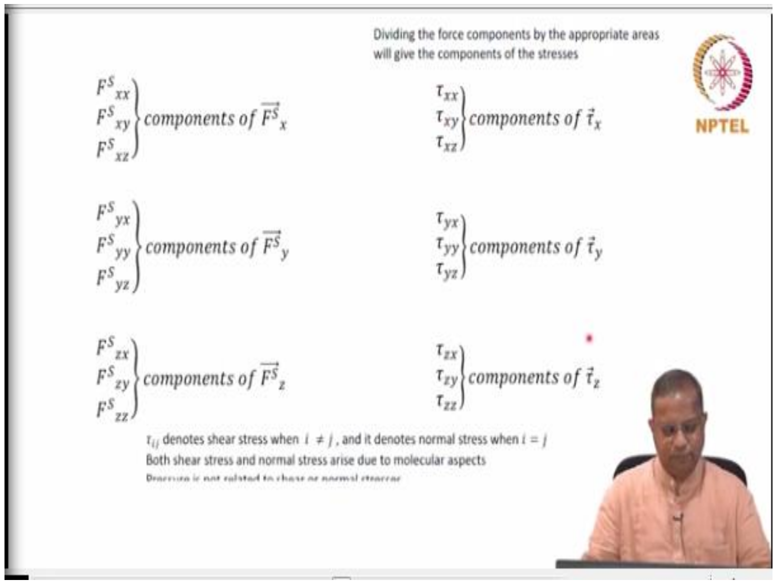

The surface force that causes the shear stress or the momentum of flux, sorry. The components we saw, then if we divided by the area and we got the stresses both normal stresses and shear stresses. Also it was pointed out that the normal stress is a different quantity, different physical quantity. It is different from pressure although they could be added for many calculations and so on so forth, they are 2 different physical quantities.

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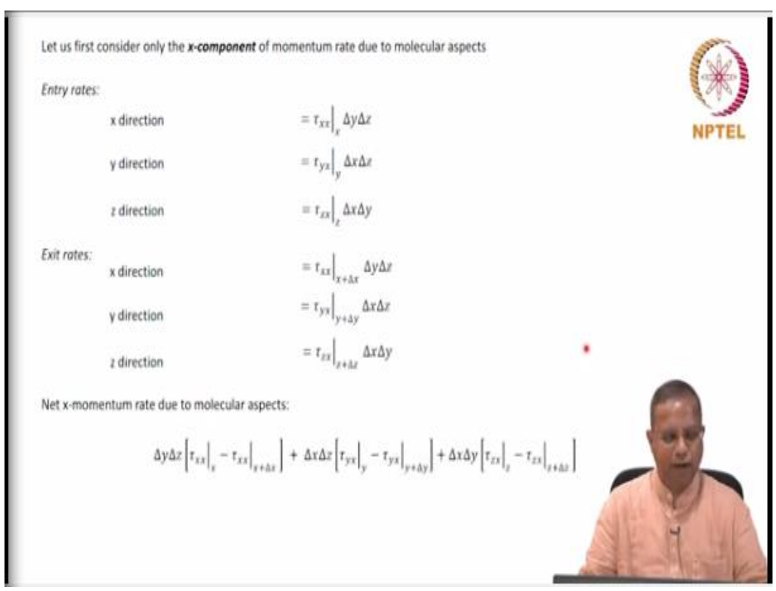

Then after a balance.

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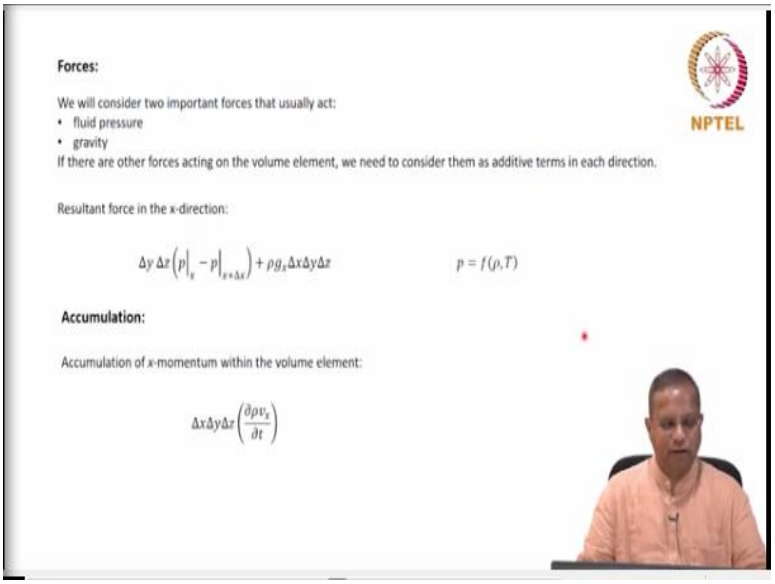

And.

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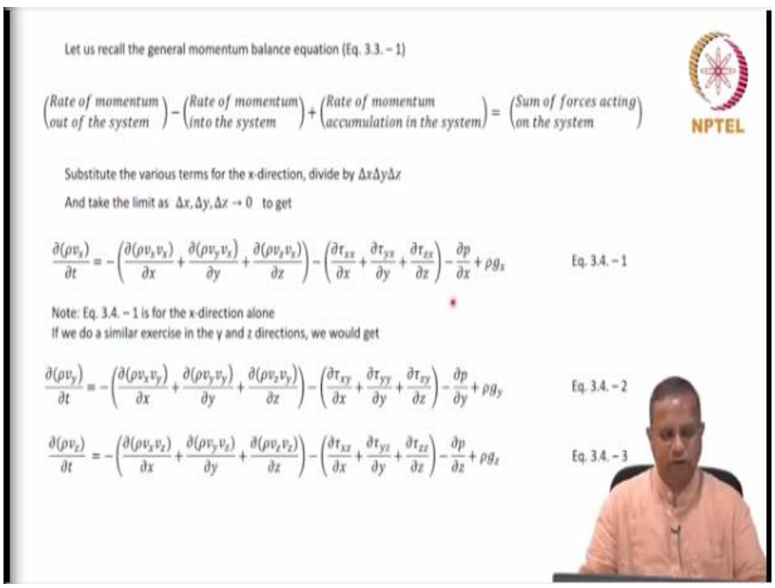

By a lot of simplification using various different physical relationships such as the equation of continuity as well as mathematical representations.

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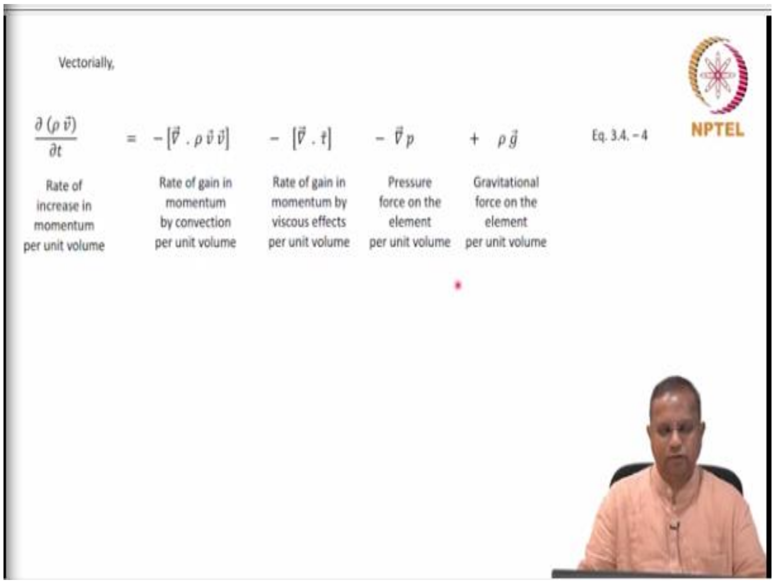

In compact, vectorial notation

$$\frac{\partial(\rho\vec{v})}{\partial t} = -[\vec{\nabla}.\rho\vec{v}\vec{v}] \quad -[\vec{\nabla}.\tilde{\tau}] \quad -\vec{\nabla}p \quad +\rho\vec{g} \tag{3.4-4}$$

Rate of increase in momentum per unit volume	Rate of gain in momentum by convection per unit volume	Rate of gain in momentum by viscous effects per unit volume	Pressure force on the element per unit volume	Gravitational force on the element per unit volume

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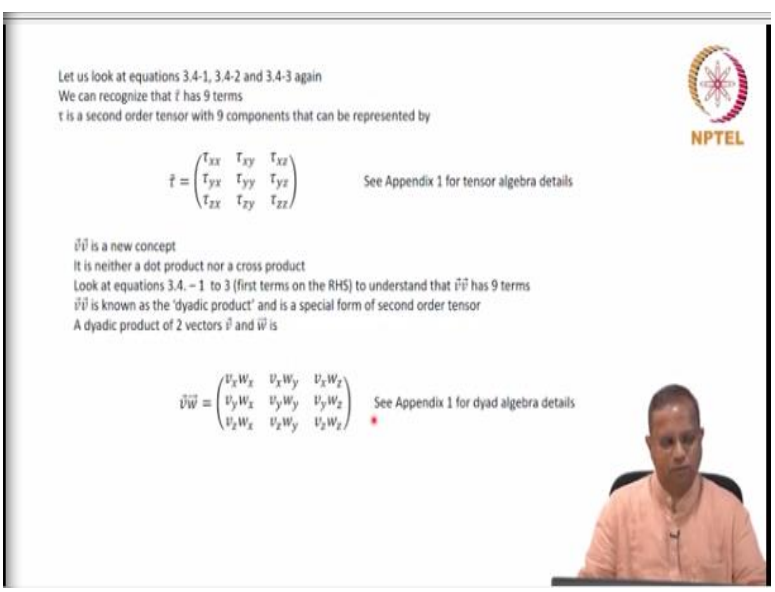

Then I think we looked at these terms which are different ρ vv and τ which are actually second order tensors which have 9 components, we saw some aspects of those. And then if you want to know the tensor algebra, we were directed to the appendix of the textbook.

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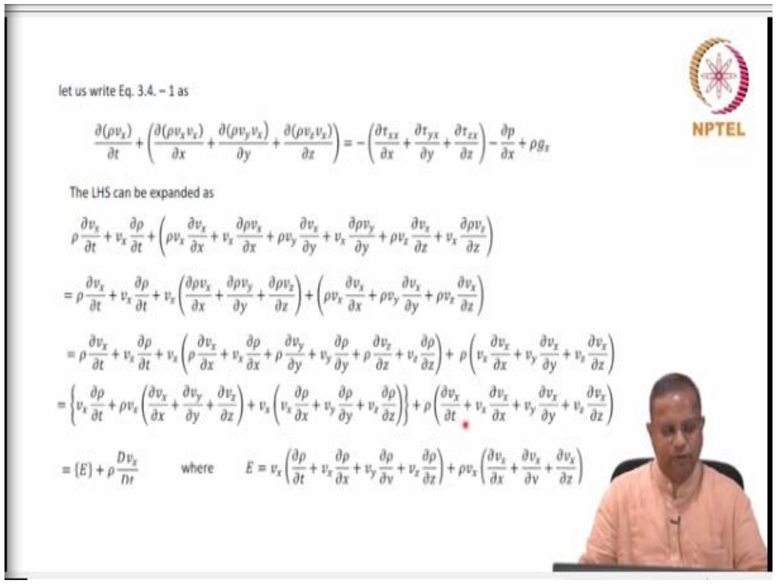

So, this is the dyadic product, the product between 2 vectors, ok, it is very different from either a dot product or a cross product that you are familiar with. This is a dyadic product which results in 9 components. Then, we went through simplifications as I mentioned.

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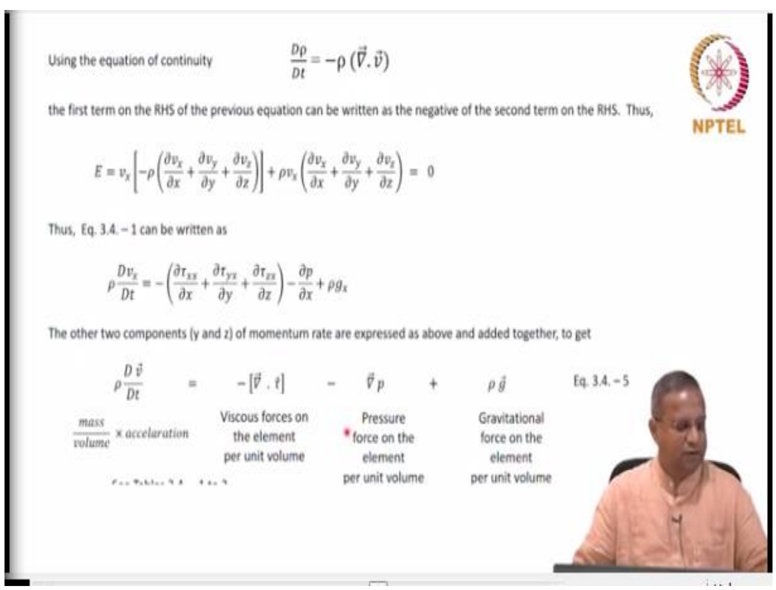

$$\rho \frac{D\vec{v}}{Dt} = -[\vec{\nabla} \cdot \tilde{\tau}] - \vec{\nabla} p + \rho \vec{g} \qquad (3.4\text{-}5)$$

$\frac{\text{Mass}}{\text{Volume}} \times$ Acceleration	Viscous forces on the element per unit volume	Pressure force on the element per unit volume	Gravitational force on the element per unit volume

So, this has been written in it is various components or in various coordinate systems and all those are available as tables, table 3.4.1 to 3, you are asked to make a copy of those tables and keep it for your reference. Because as you saw we refer to them very often whenever we are looking at velocity profiles or shear stress profiles and so on and so forth, we need to refer to them very often.

Because essentially we have brought down the equation of motion or that principle the momentum balance principle onto these equations reasonably general in applicability. And all we need to do is go to the table, pick up the relevant equation, cancel the irrelevant terms and you directly have a governing equation well grounded in the principles that would lead to a robust analysis, so that is the advantage here.

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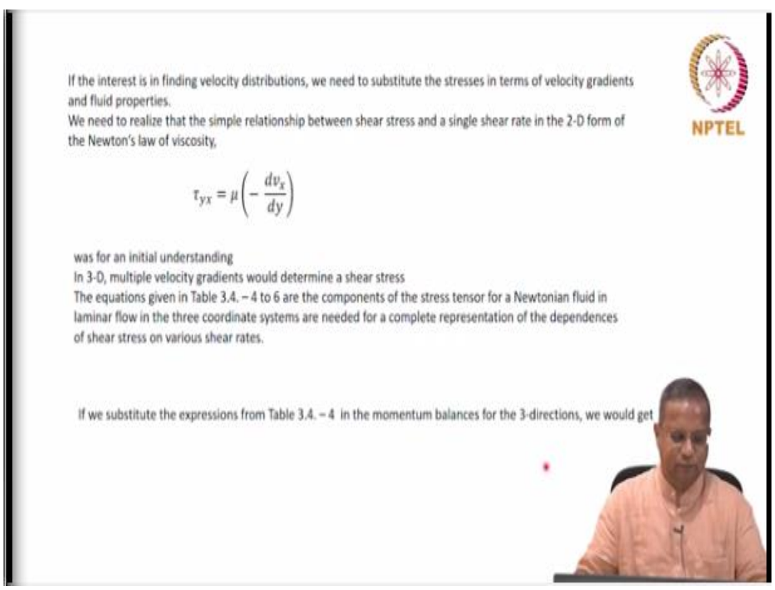

So, I had shown you how this can be applied to the case that we have already seen which is the flow over a flat plate to get relationships in a useful fashion without expanding too much effort you could do that.

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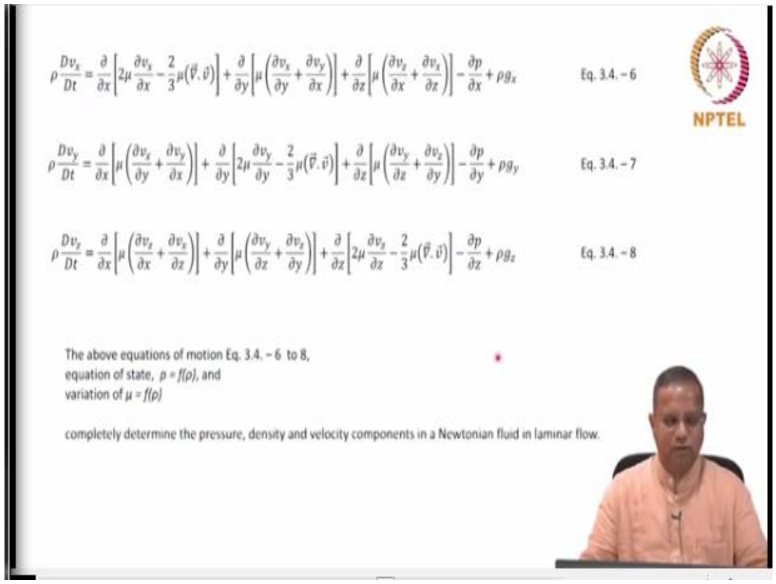

Which may not be the case if you use shell balances. Then we looked at the simplifications for a Newtonian fluid in laminar flow.

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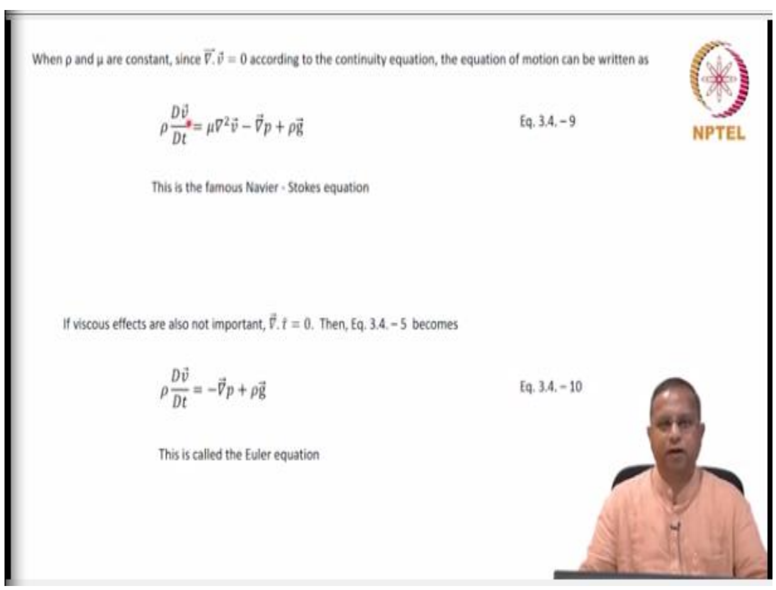

And that is when we get to an Navier-Stokes equation, if ρ and μ are constant. Also if the shear effects are also not important then you get to the Euler equation, right. Navier-Stokes equation is very popular and that is a special case of the momentum balance equation, ok. Then I showed you the various different applications of this equation of motion very relevant, highly useful situations we saw. This is what I have shown earlier.

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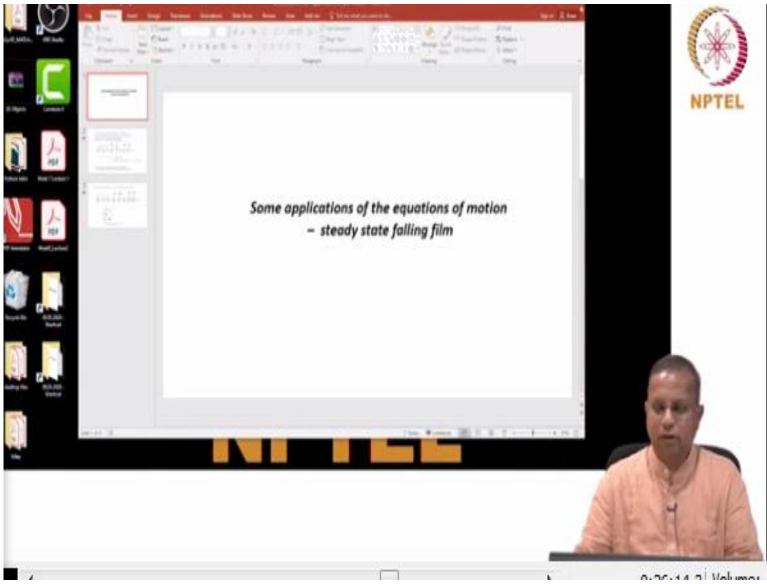

Yeah, this I had already discussed for in film.

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We had looked at flow in a cylindrical pipe got very useful relationships, this flow in a cylindrical pipe you could apply it to a wide range of situations right from flows in the body to flows in the industry and so on so forth.

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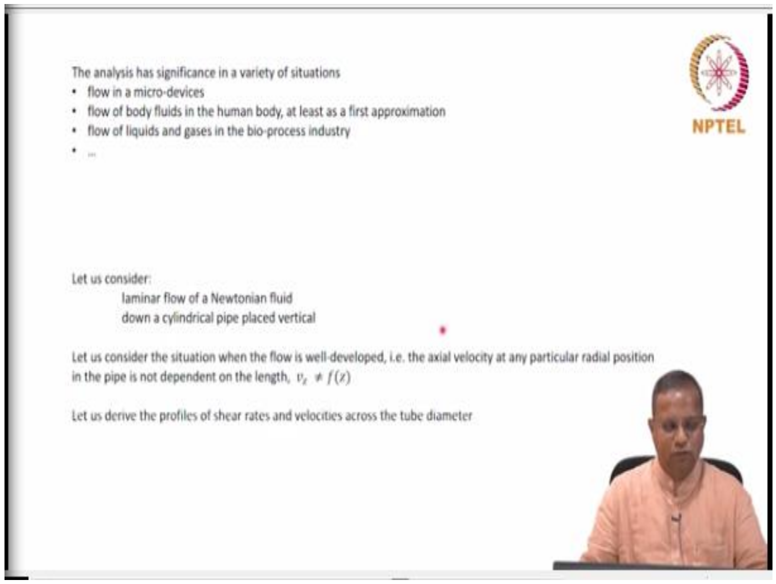

We used the equation of motion.

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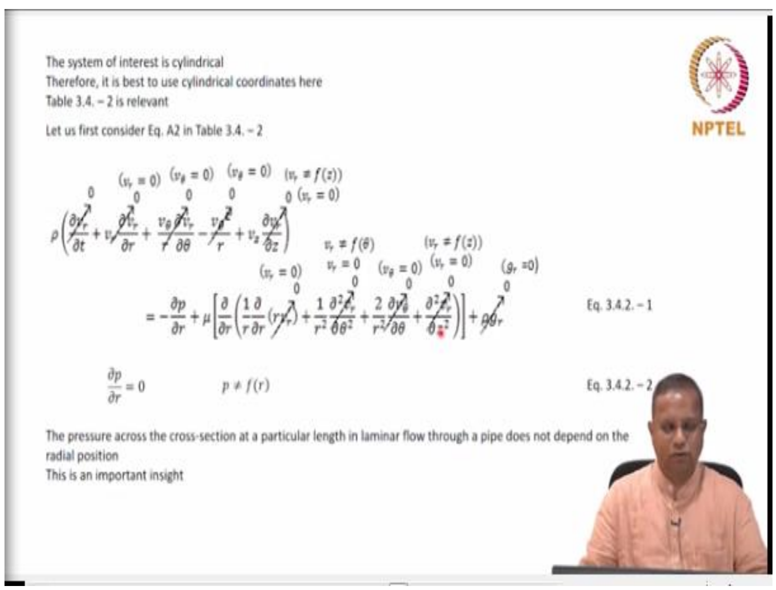

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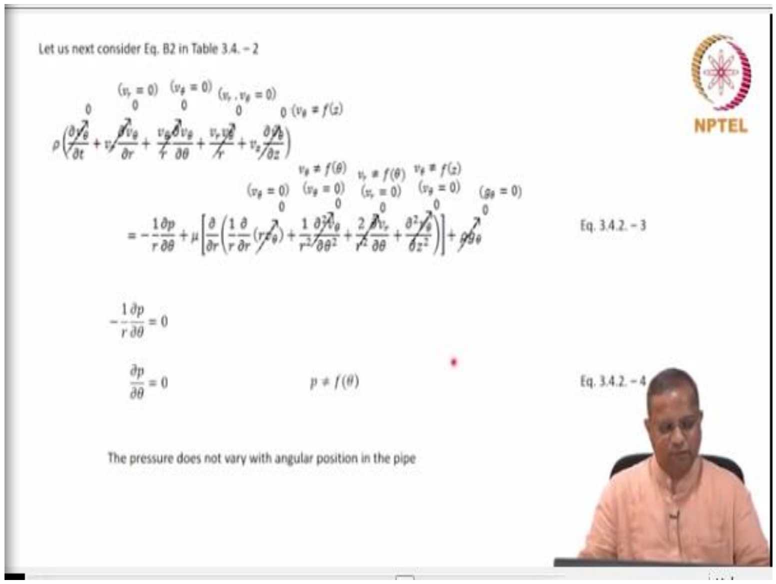

To come up with the fact that the pressure across a cross section is the same.

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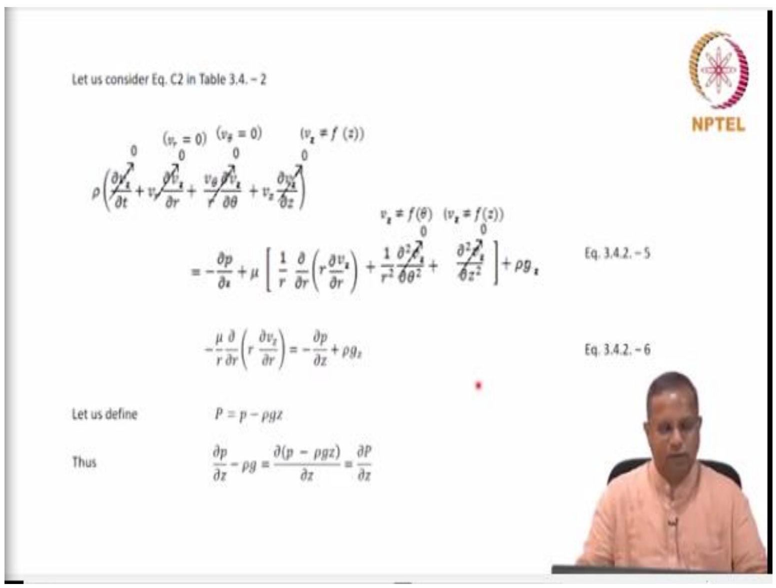

The pressure across 2 different cross sections could be different, pressure across a particular cross section is the same, it does not vary with radius or with your angle.

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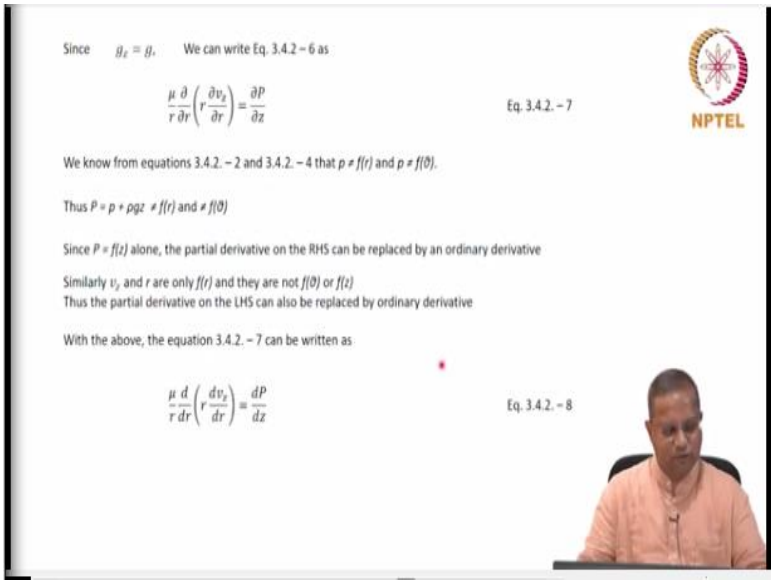

And then we got the velocity profile I think.

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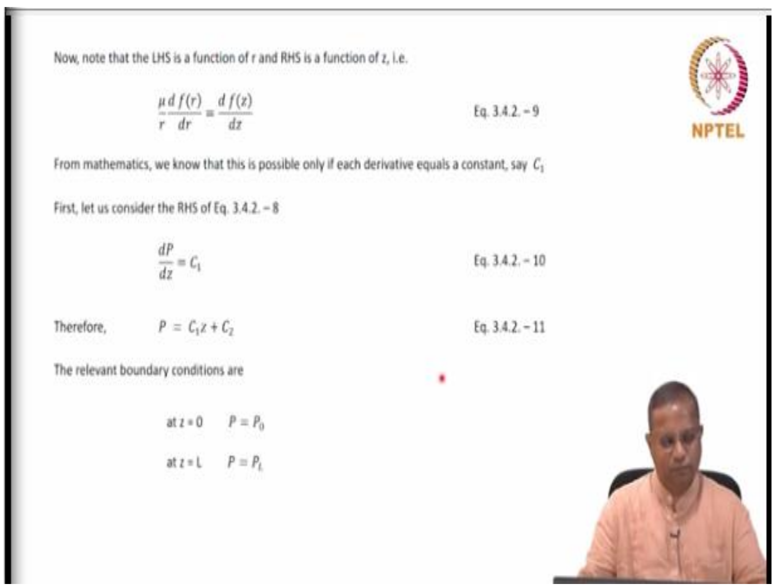

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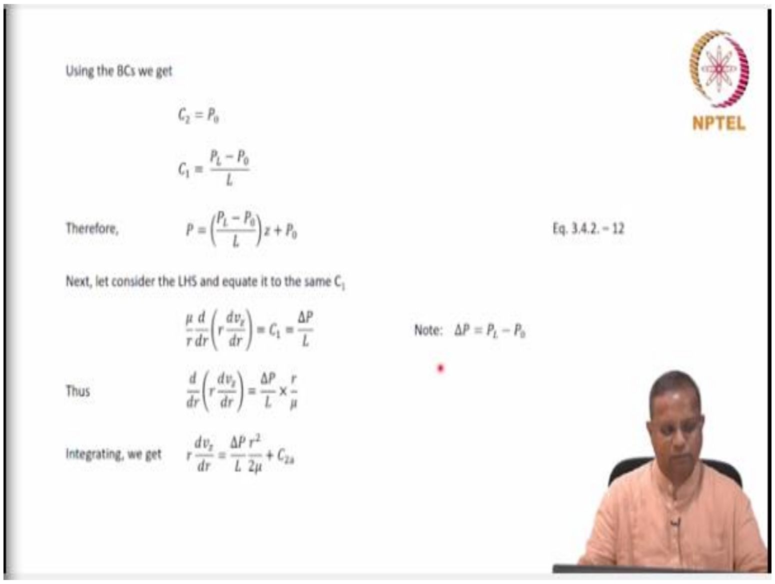

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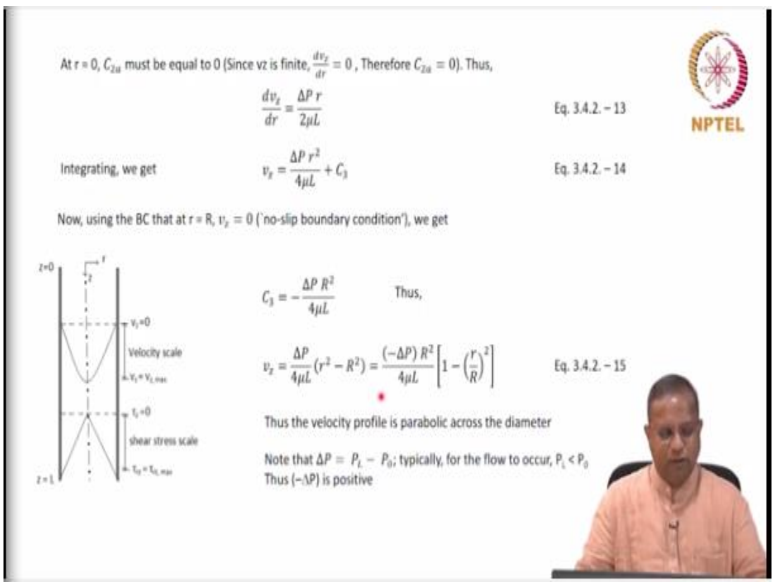

Thus

$$v_z = \frac{\Delta P}{4\mu L}(r^2 - R^2) = \frac{(-\Delta P)R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (3.4.2\text{-}15)$$

That is the typical parabolic velocity profile in laminar flow in a pipe we got an expression for that. So, this is the sorry parabolic velocity profile in laminar flow that I have shown. And then we looked at the shear stress profile which turned out to be a linear shear stress profile, ok, is a highly relevant.

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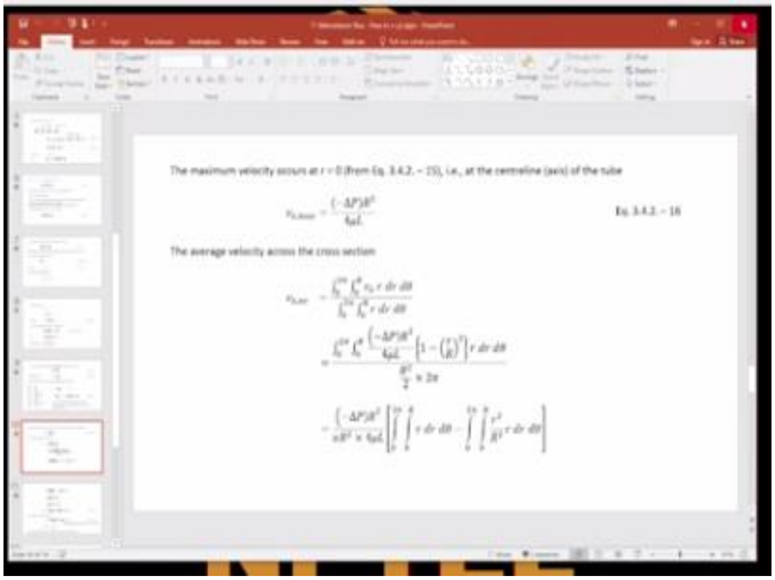

And then I think that is good enough for this. I think **I** we of course backed out the expression for maximum velocity, flow rate, average velocity and so on. And also looked at the Poiseuille equation which relates pressure drop and volumetric flow rate. We said that if you double the diameter then the flow rate increases 16 fold and so, ok, those were the salient points there.

Then there were applications to capillary flow, cuvette flow which is flow between 2 cylinders in this case, the example of cuvette flow. Then you were introduced to a dimensional analysis which involves non dimensional numbers. There we saw that even without knowing much about the system just by doing an analysis on the dimensions of the system using the Buckingham pi theorem and a certain procedure that seems to work. We can get very good insights of the kind of relationship between variables even if we know nothing about the system.

We have not done any experiments and so on and so forth. Then probably a few experiments can be done to fix the constants and so on so forth, ok. Then we got into unsteady state flow where once you bring in the unsteady state $\frac{\partial}{\partial t}$ term, then it just complicates the mathematics quite a bit, ok. And again to emphasize this is not a course that tests your proficiency in mathematics, you need to know how to solve something that is it, how complex it is and so on so forth.

We are not really bothered at this undergraduate level, ok they are of course important that can probably be taken up with the graduate level. Therefore the idea here is to show you that there are solutions that exist. We do not expect you to become experts in the kind of solutions, kind of heavily involved mathematical solutions in this course, basal level, yes. That is based on your the information that you picked up in the engineering mathematics courses.

After the solving differential equations, knowing some methods for solving partial differential equations, some methods for solving ordinary differential equations and so on so forth. Any of the other things are of course specialized and that was given to you as information, ok, the expertise in that is not expected as a deliverable in this course at all. Then we spent a good amount of time or before even that I should say an example of unsteady state flows pulse the time flow. We saw how to handle that and then we also saw how to handle turbulent flow.

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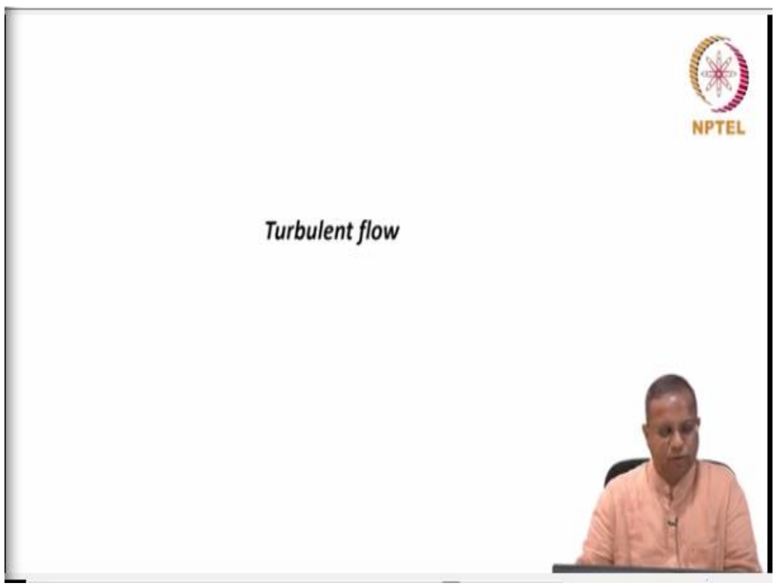

Probably we should spend a little bit of time there in the review.

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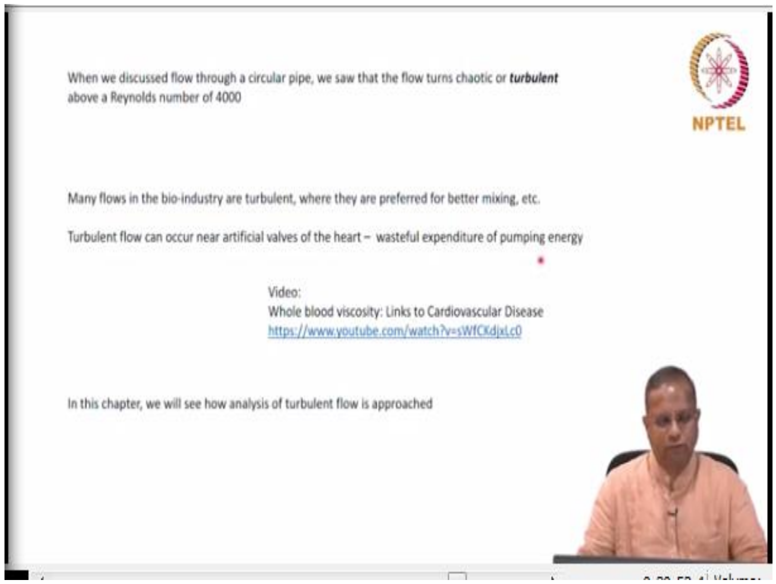

We said that the physical principles should be applicable to the flow irrespective of the type of flow ok, because those are physical realities.

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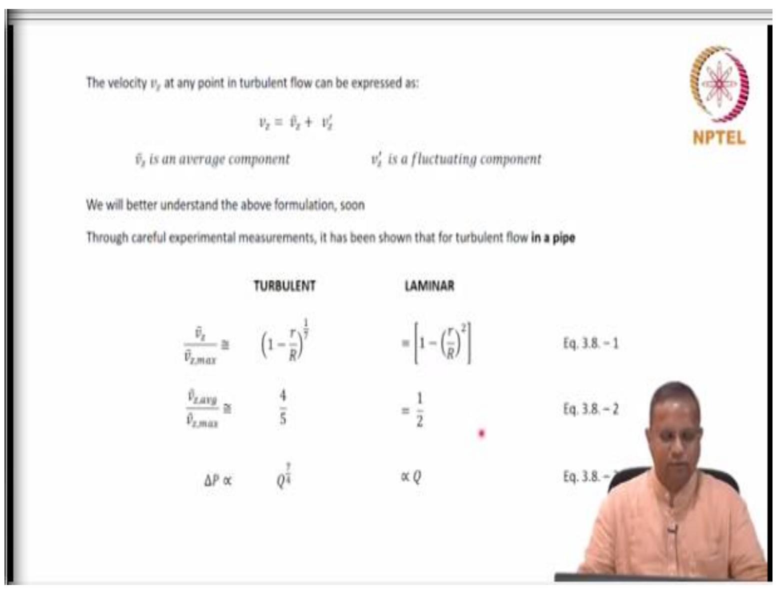

Therefore the equation of motion, equation of continuity would certainly be applicable. And we said if you can express your velocity as a sum of an average component and a fluctuating component. The pressure as the sum of an average component and a fluctuating component, the same equations that we had can be used. They reduce to simpler forms.

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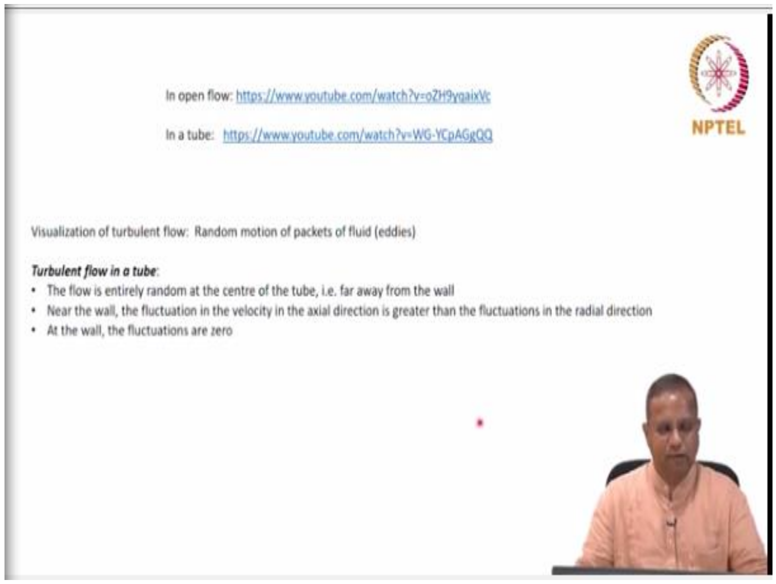

And we could use the simpler forms if it becomes necessary to use them.

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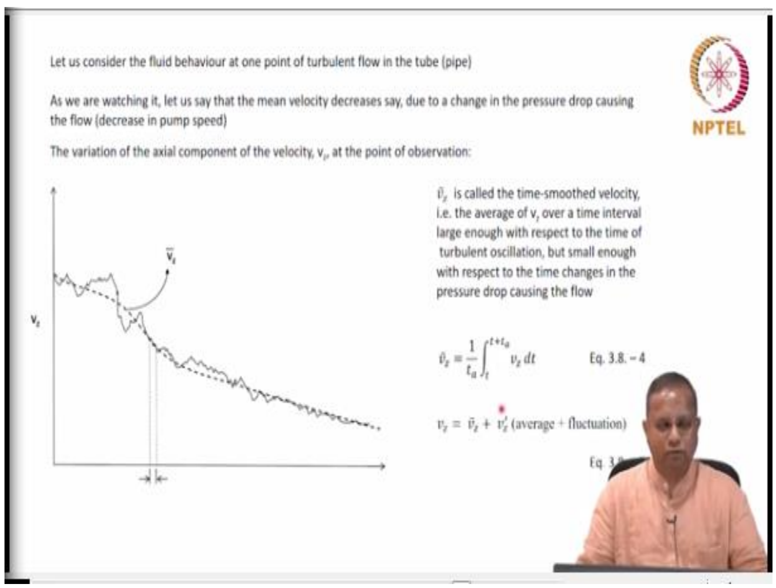

Then we slowly weird of into an empirical way of approaching turbulent flow.

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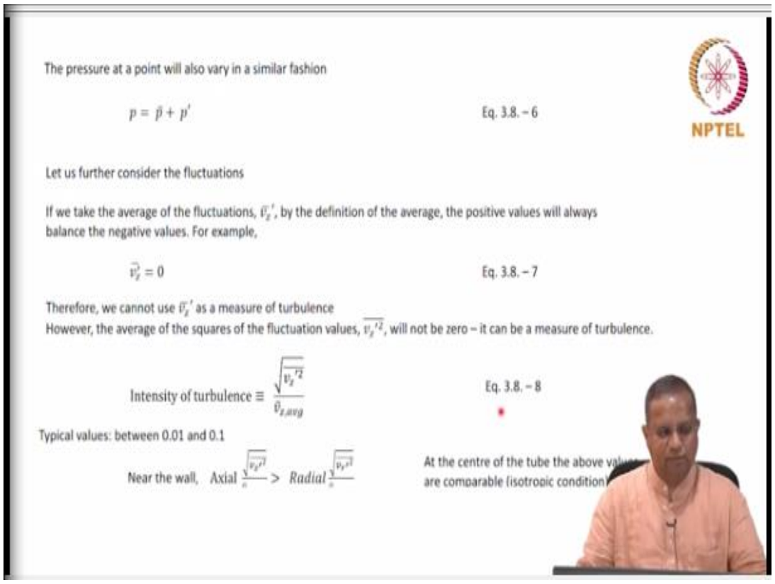

Let me just get there.

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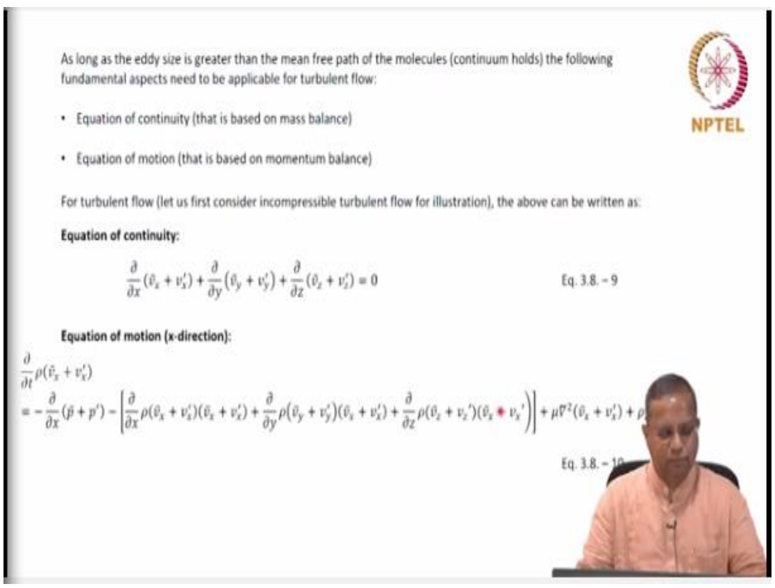

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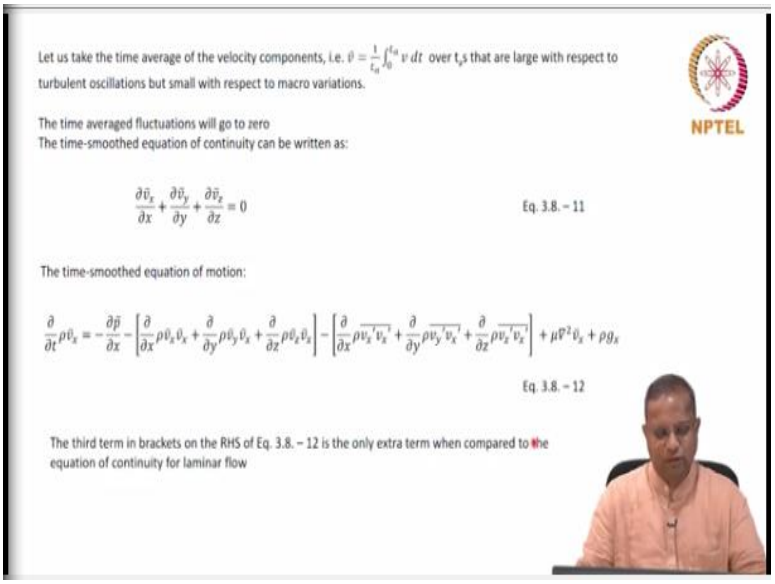

So, once you did the time smoothing, this becomes your equation of motion which is essentially the same except that you are using essentially the same as laminar case except that you are using the average components of the velocities here. The time smoothed equation of motion is almost the same except for this additional term with Reynolds stresses, ok.

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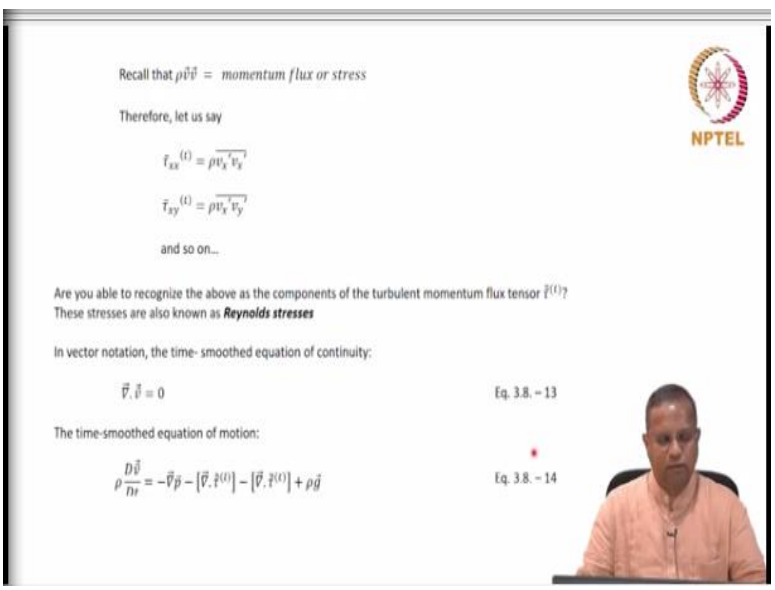

Then.

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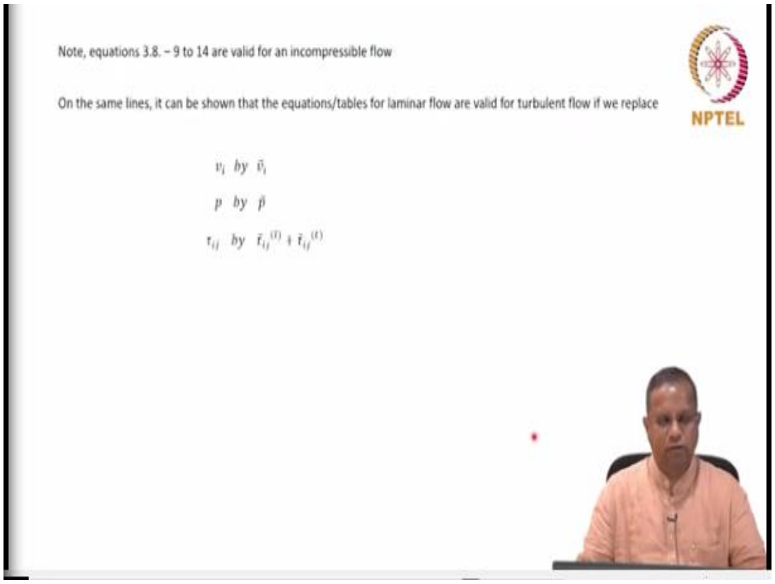

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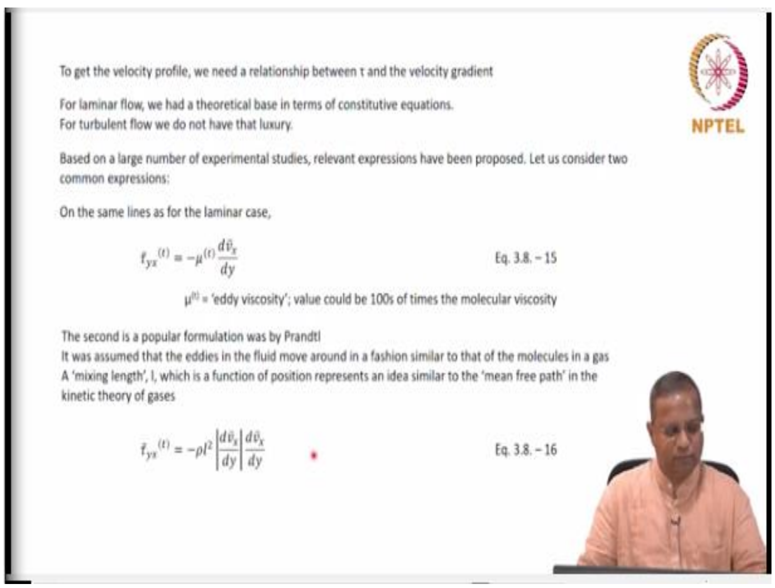

$$v_i \text{ by } \bar{v}_i$$

$$p \text{ by } \bar{p}$$

and

$$\tau_{ij} \text{ by } \bar{\tau}_{ij}^{(l)} + \bar{\tau}_{ij}^{(t)}$$

Replacing by a laminar component and a turbulent component, the shear stress, a laminar component and a turbulent component. Then you could use the same equations as earlier, the velocity profiles we saw, we said that this is although in form it is the same as for a Newtonian fluid in laminar flow and so on and so forth, sorry yeah.

You could use that for turbulent flow also with the recognition that this is not the molecular viscosity, right, that is what we said.

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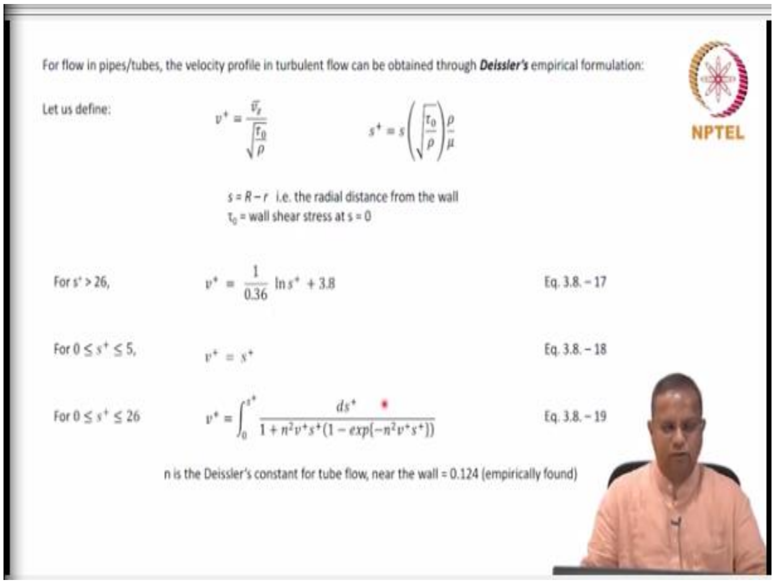

Then we saw a Deissler's empirical formulation, so far it was based on the fundamental equations and now it is an empirical method. This is a useful method to get velocity profiles based on experiments and therefore they are limited to the ranges over which they have been found to be applicable. The other ranges we do not have any confidence in applying them, good. So, then we got into the macroscopic aspects of fluid flow of momentum balance and so on and so forth.

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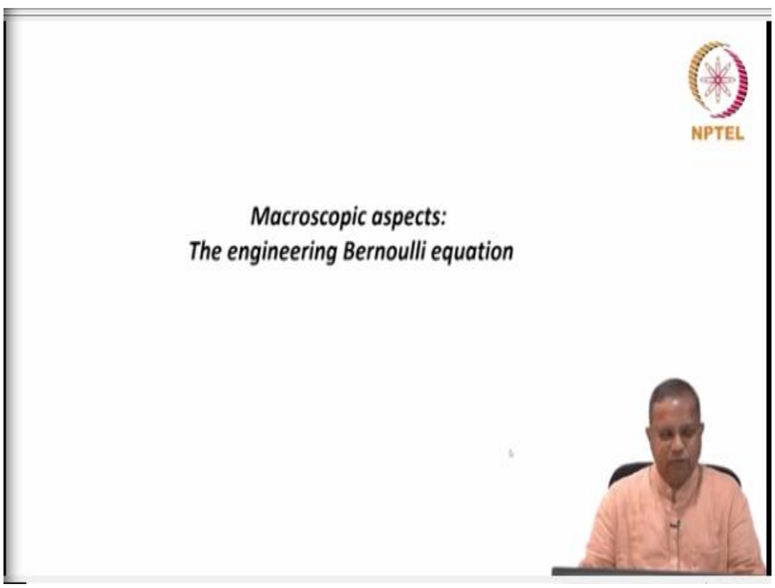

We looked at the engineering Bernoulli equation although we did not derive it in detail because of the reasons that I mentioned. We looked at the various applications of the engineering Bernoulli equation. And we said that if you can define a friction factor for each situation, then the friction factor approach can be taken to gain good insights also and definitely use it for design an operation, alright.

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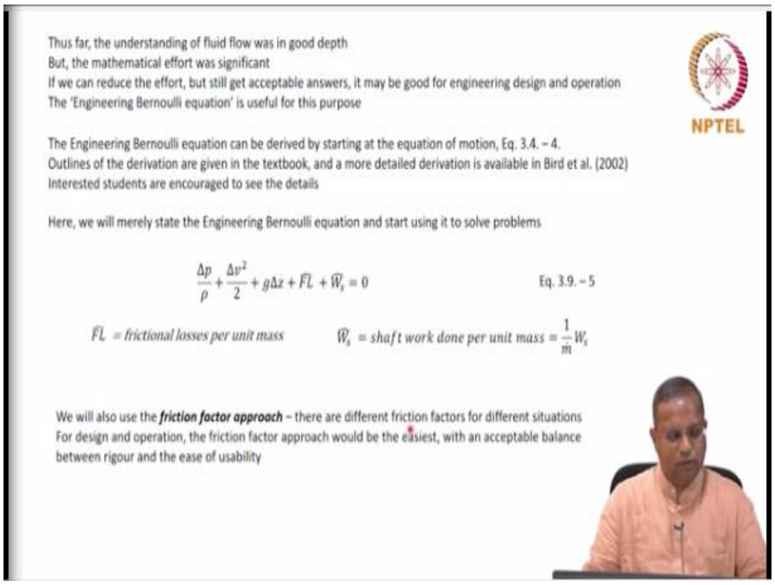

$$\frac{\Delta p}{\rho} + \frac{\Delta v^2}{2} + g\Delta x + \widehat{FL} + \widehat{W_s} = 0 \qquad (3.9\text{-}5)$$

where

$$\widehat{FL} = -\frac{1}{\dot{m}}\int (\tilde{\tau} : \vec{\nabla}\vec{v})\, dV$$

$$\widehat{W_s} = \frac{1}{\dot{m}} W_s$$

Equation 3.9-5 is a useful form of the engineering Bernoulli equation.

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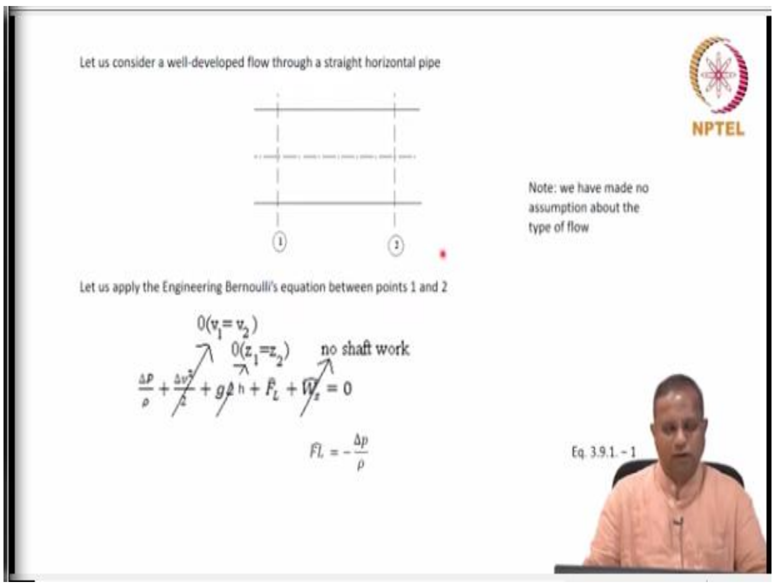

And the main idea the underline theme for this part of the course was, what are the friction factors for various situations. In this case friction factor for flow through a straight horizontal pipe.

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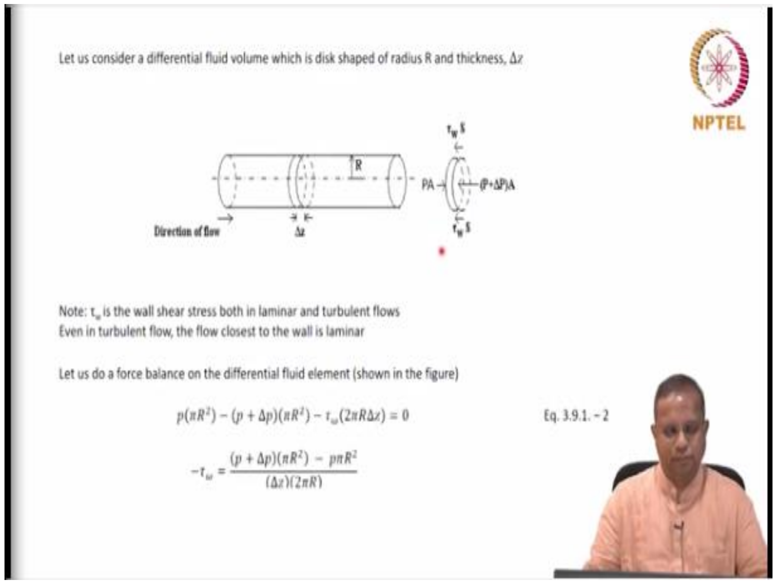

We got as**.**

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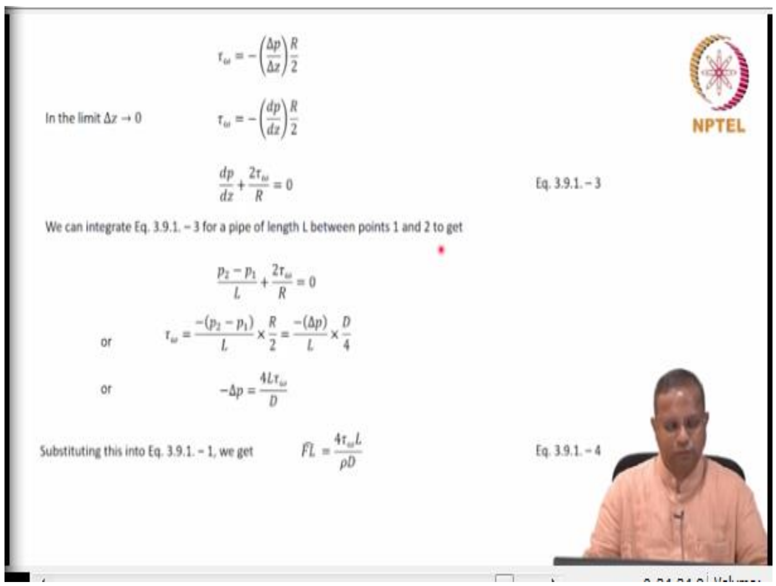

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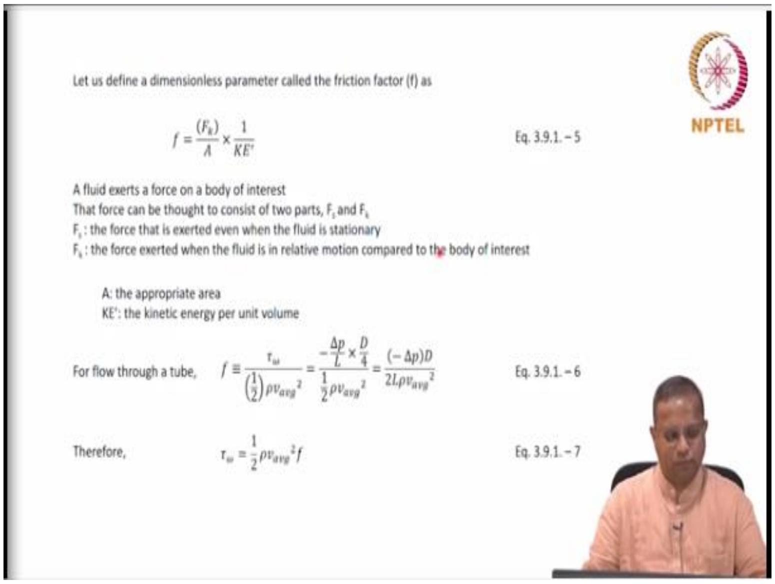

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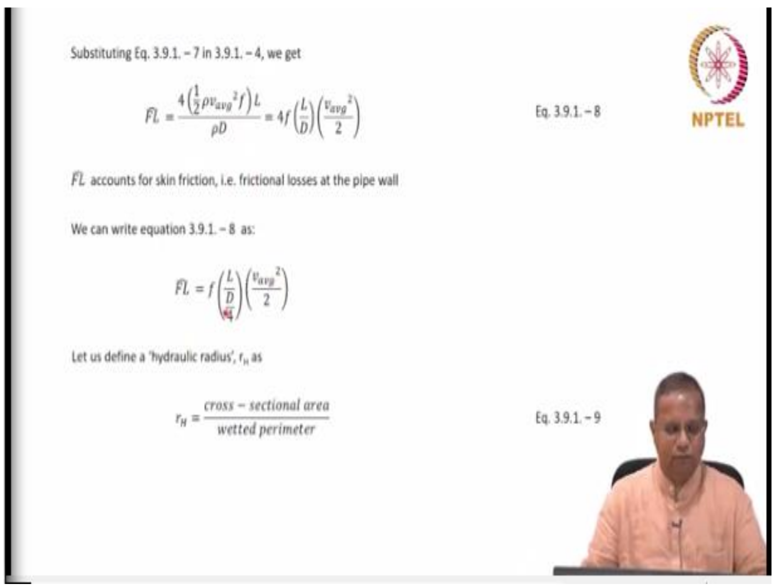

$$\widehat{FL} = f\left(\frac{L}{\frac{D}{4}}\right)\left(\frac{v_{\text{avg}}^2}{2}\right)$$

and you could also write it in terms of the hydraulic diameter and thereby make it applicable to a non circular cross section. Then we said since this does not take into account the type of flow, this is applicable for laminar flow as well as to turbulent flow. The details would of course be different but they form a certainly applicable.

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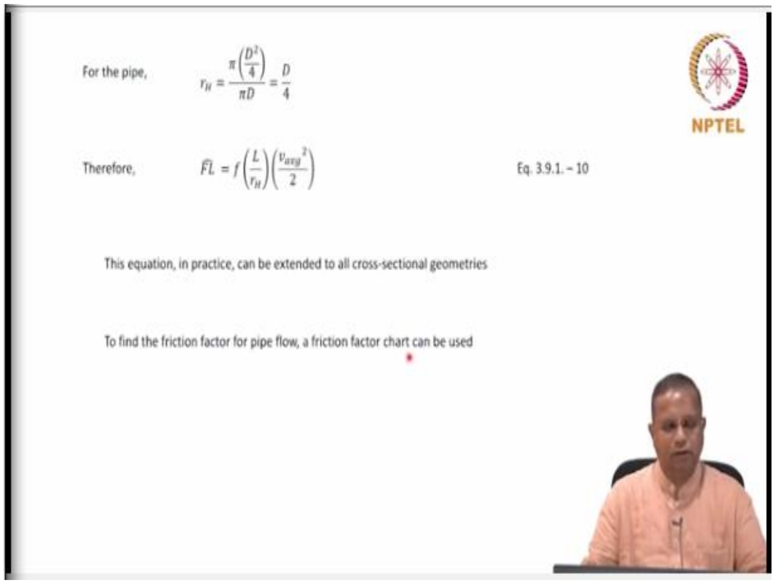

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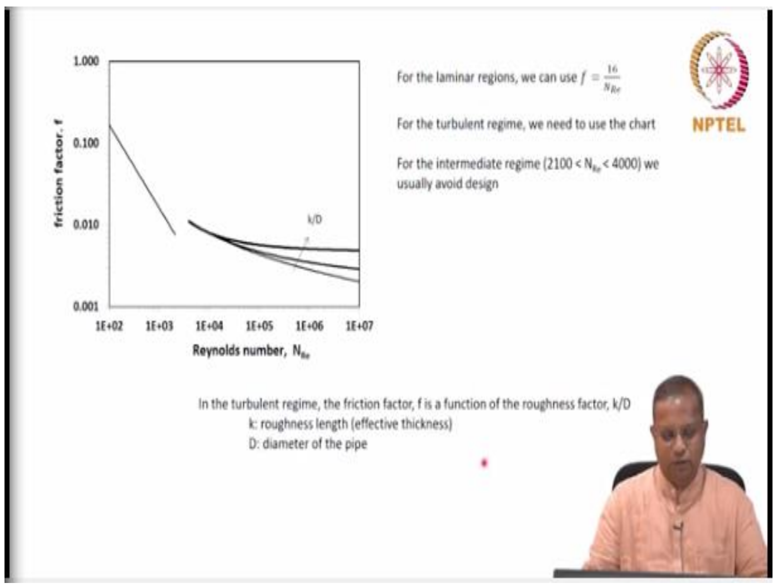

Then yeah, this is the friction factor chart, defining friction factor chart, friction factor versus the Reynolds number. So, this is 16 /N_{Re}, this is you have to read it from the chart for various values of pipe roughness. Then we saw the application to different situations one was stenosis in the artery to get to an very interesting condition of the cavitations effects when do they become important and so on.

Then we saw application to relative motion between a solid and a liquid then to packed beds and so on so forth, ok. So, large number of applications for the exact expressions, please go back to your notes and take a look at them. I am trying to give you an overall picture, that is the reason why I am not showing you the exact equations except when there is been if I felt a need for them, ok.

So, this being a review, I am not going through each and every single aspect of it. Please go through your notes and fill in the details whenever you need to, ok. Forgetting is very normal and that is a reason why we have textbook, notes and so on so forth. So, right ok, I think we need to stop here, we have been at it for quite a while, it is best to break it up, when we meet next, we will do the next part of the review and probably one more and finish up with that, see you, bye.