Transport Phenomena in Biological Systems
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Lecture-80
Course Review-Part 3

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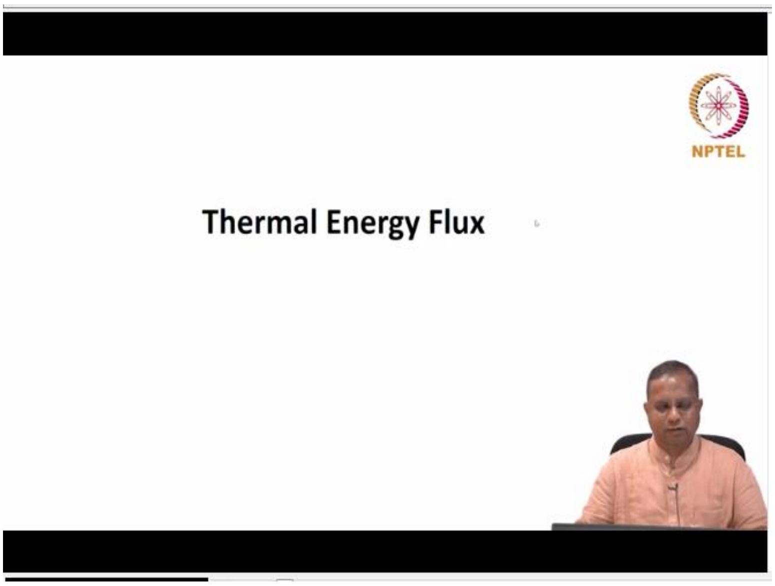

Welcome to the third review lecture. In this lecture we look at thermal energy flux as well as the charge flux. The thermal energy flux is a thermal heat transport is more relevant to us. However, the total energy is conserved, thermal energy alone is not conserved. Therefore, we looked at total energy and then backed out thermal energy part of it on one side and that is how we could write this expression.

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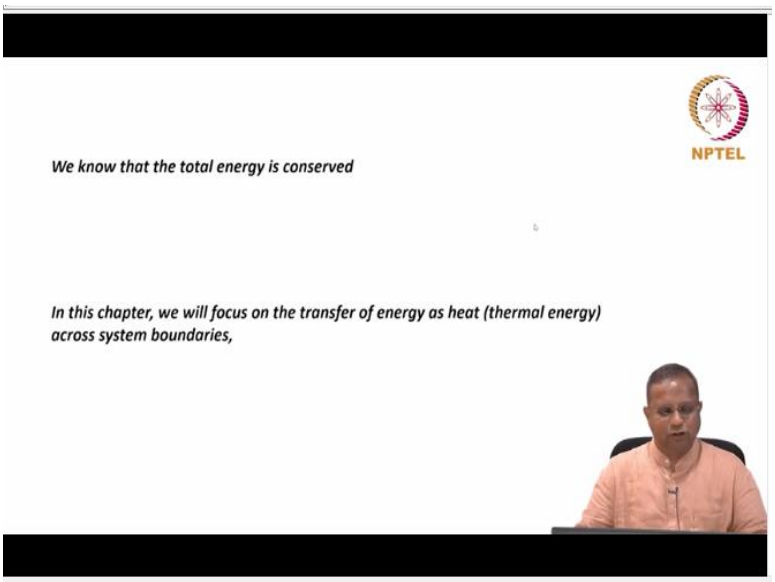

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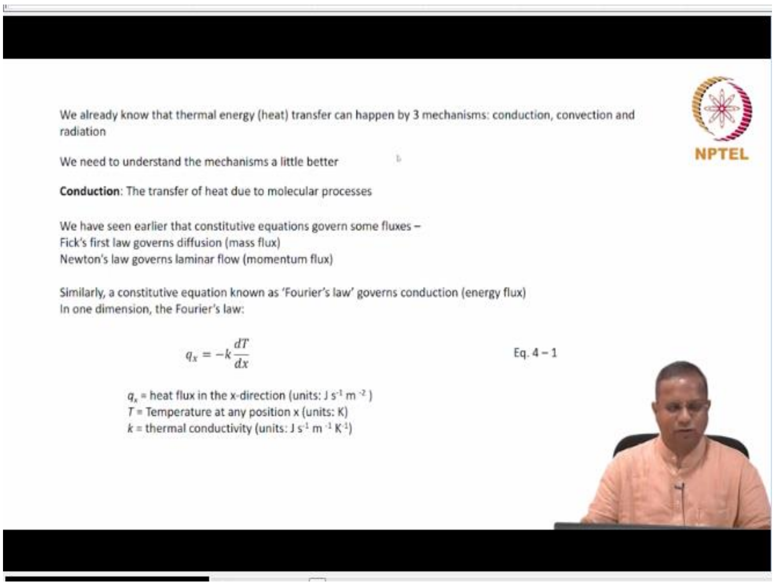

So, we also saw before attempting an equation of heat energy, that heat is transferred by 3 major mechanisms, one is conduction due to molecular processes. The other one is convection due to movement of the medium bulk flow. The third is radiation, which is through electromagnetic waves and so on so forth. It can happen even in free space.

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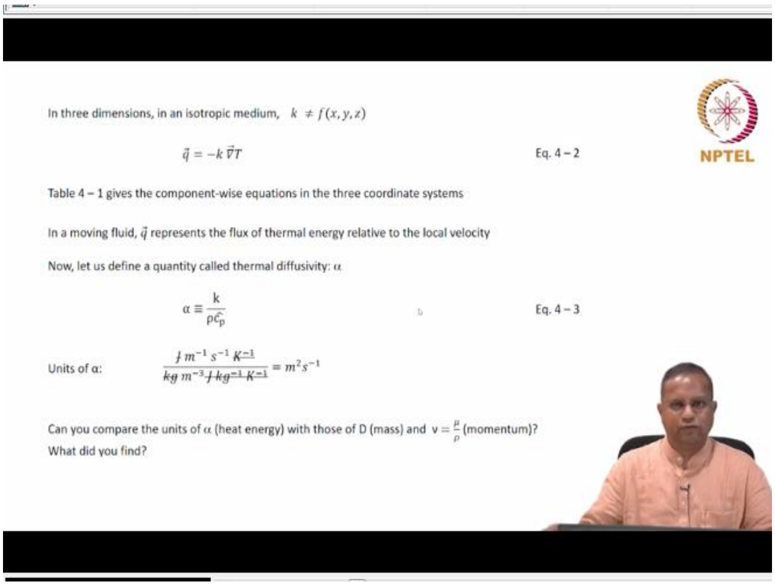

The first we saw the equation for conduction alone the Fourier law q = - k∇T okay dT/dx in 1 dimension or ∇T in 3 dimensions, then α, the thermal diffusivity k/ρ C_p has the same units as that of mass diffusivity, as well as intrinsic viscosity μ/ρ okay. So, μ/ρ has same units,m^2s^{-1}. So, they are all equivalent physical quantities is what we can say for different transports.

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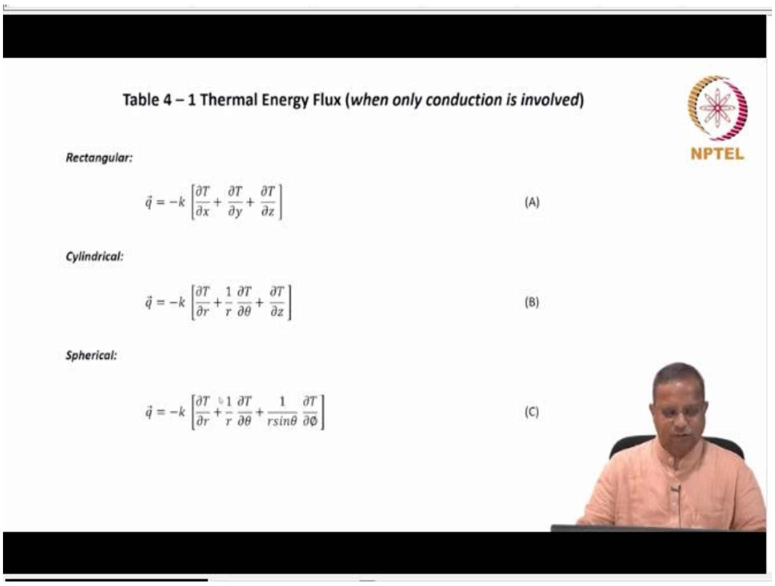

Then this table gives you the thermal energy flux when only one only conduction is involved, you are asked to make a copy of it and keep it as a part of your notes, for these 3 different coordinate systems okay.

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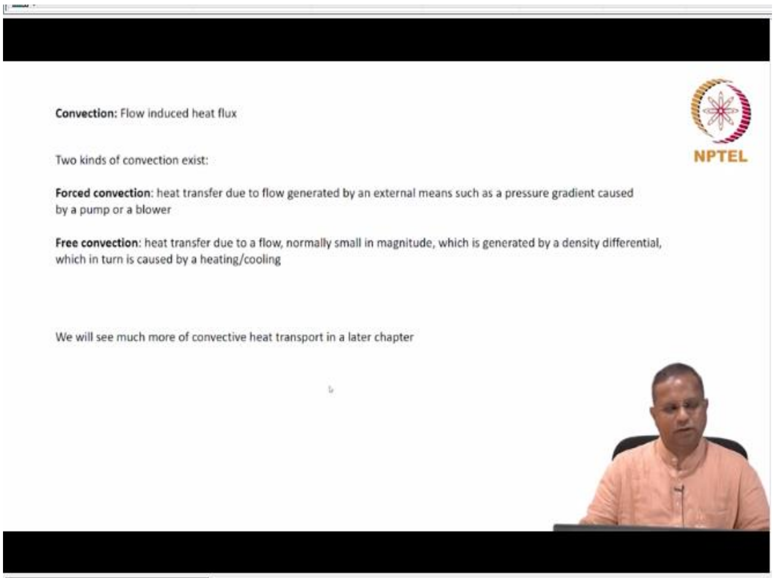

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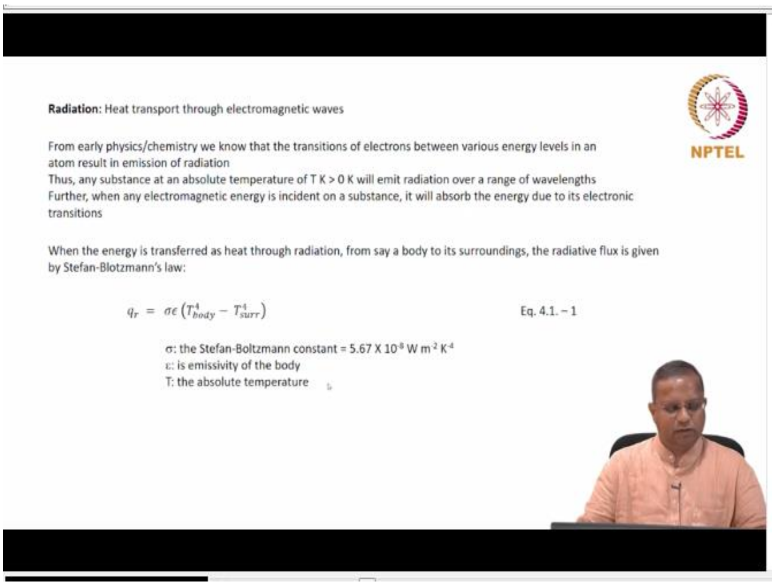

Then we had looked at the equation of energy.

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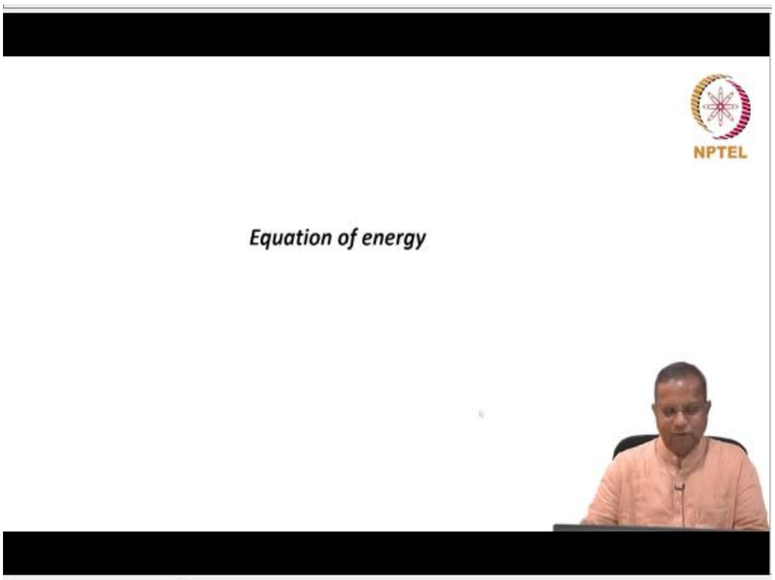

We did not spend as much time with the equation of energy.

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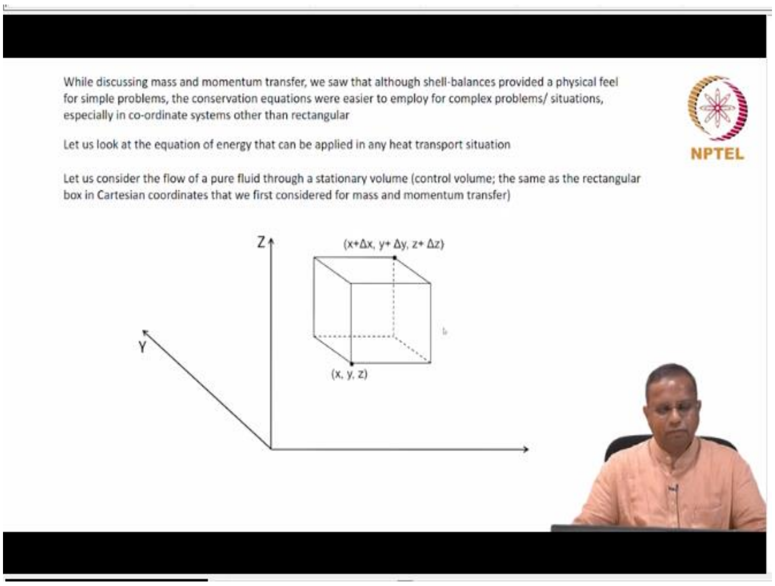

However, the equation of energy, the same way, before that let me say this, again that you could do 2 approaches here, one is shell balances, the other one is derive the conservation equation, write it in a form that would be useful and use it directly for more situations. I did not explicitly show you, shell balances here, I went directly to the equation approach.

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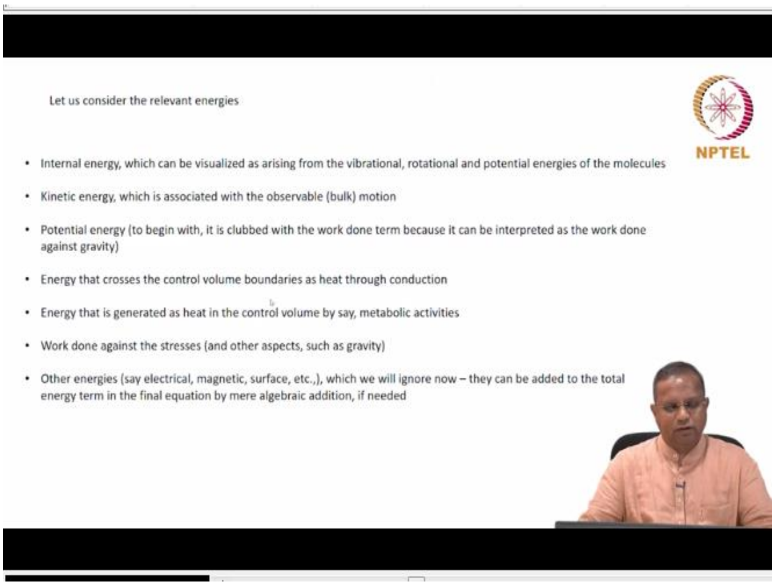

And that was the outline of how we got.

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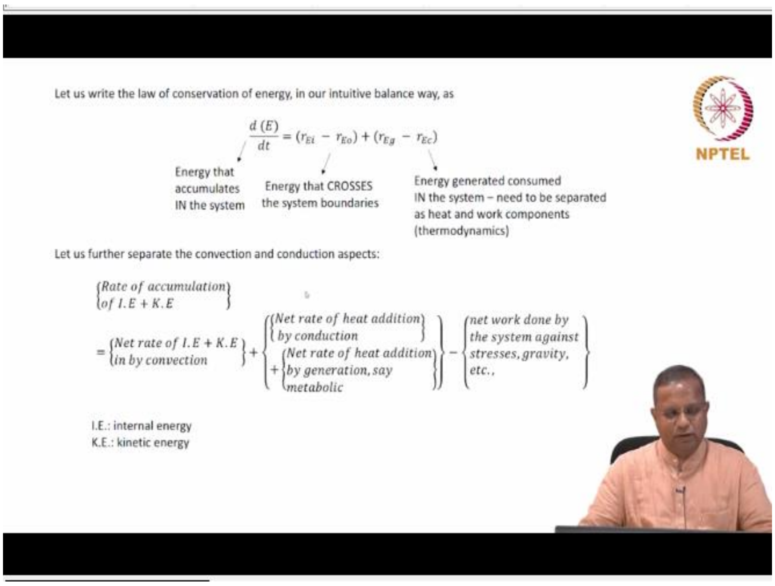

There was discussed in some detail.

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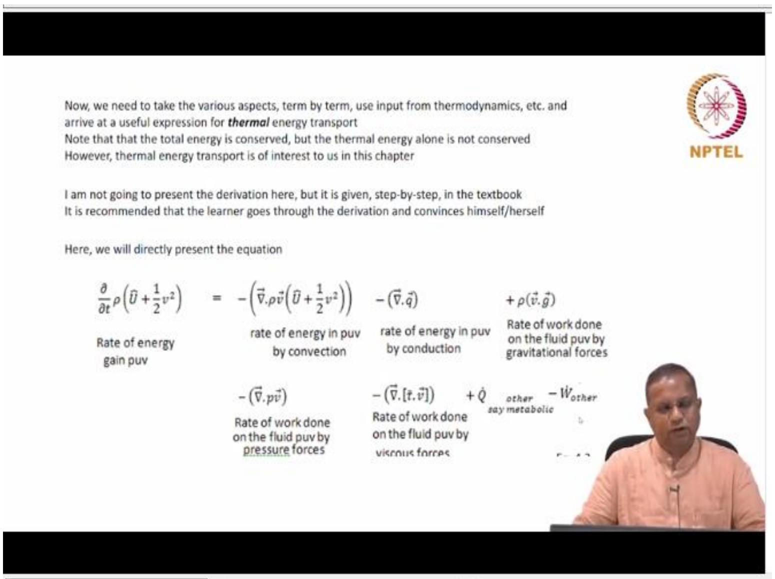

They put everything together. We got the equation of energy as $\frac{\partial}{\partial t}\rho(\hat{U} + 1/2v^2)$. Therefore rate of energy gain per unit volume of the control volume equals $- \nabla \cdot \rho v(\hat{U} + 1/2v^2)$. This is rate of energy in per unit volume by convection, $(- \nabla \cdot q)$ rate of energy in per unit volume conduction, $(+ \rho v.g)$, the rate of work done on the fluid per unit volume by gravitational forces $- \nabla.pv$.

The rate of work done on the fluid per unit volume by pressure forces, $- \nabla .(\tau.v)$ rate of work done on the fluid per unit volume by viscous forces $+ \dot{Q}$ (other it could be a metabolic) $- \dot{W}_{other}$ okay, we saw other shop work and so on okay.

In vector notation

$$\frac{\partial}{\partial t}\rho\left(\hat{U}+\frac{1}{2}v^2\right) = -\left(\vec{\nabla}.\rho\vec{v}\left(\hat{U}+\frac{1}{2}v^2\right)\right) \quad -(\vec{\nabla}.\vec{q}) \quad +\rho(\vec{v}.\vec{g})$$

$\frac{\partial}{\partial t}\rho\left(\hat{U}+\frac{1}{2}v^2\right)$	$-\left(\vec{\nabla}.\rho\vec{v}\left(\hat{U}+\frac{1}{2}v^2\right)\right)$	$-(\vec{\nabla}.\vec{q})$	$+\rho(\vec{v}.\vec{g})$
Rate of energy gain puv	Rate of energy in, puv by convection	Rate of energy in, puv by conduction	Rate of work done on the fluid puv by gravitational forces

$-(\vec{\nabla}.p\vec{v})$	$-(\vec{\nabla}.[\tilde{\tau}.\vec{v}])$
Rate of work done on the fluid puv by pressure forces	Rate of work done on the fluid puv by viscous forces

$$+\dot{Q}_{\text{say, other like metabolic heat}} - \dot{W}_{\text{other}} \qquad (4.2\text{-}3)$$

where puv is per unit volume.

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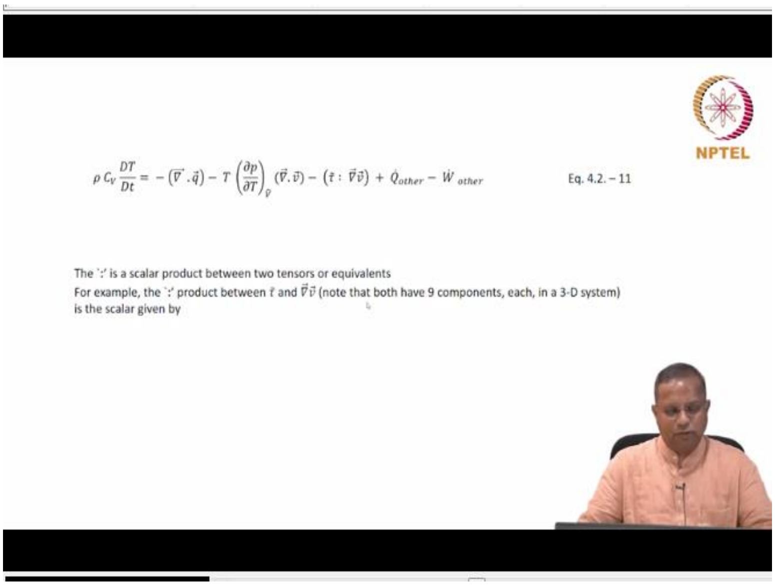

Other work and so on. So, that we wrote in a useful form, we looked at the double dot product which is a scalar product between 2 tensors or equivalents.

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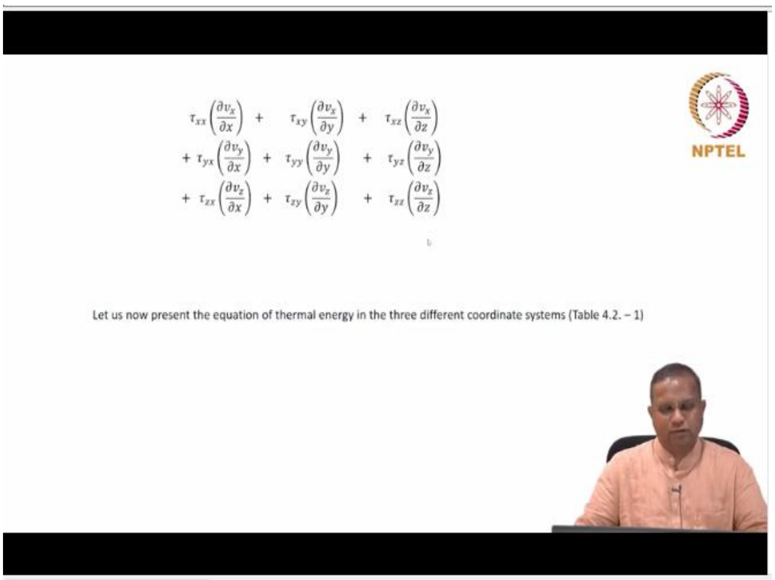

Which also has 9 components.

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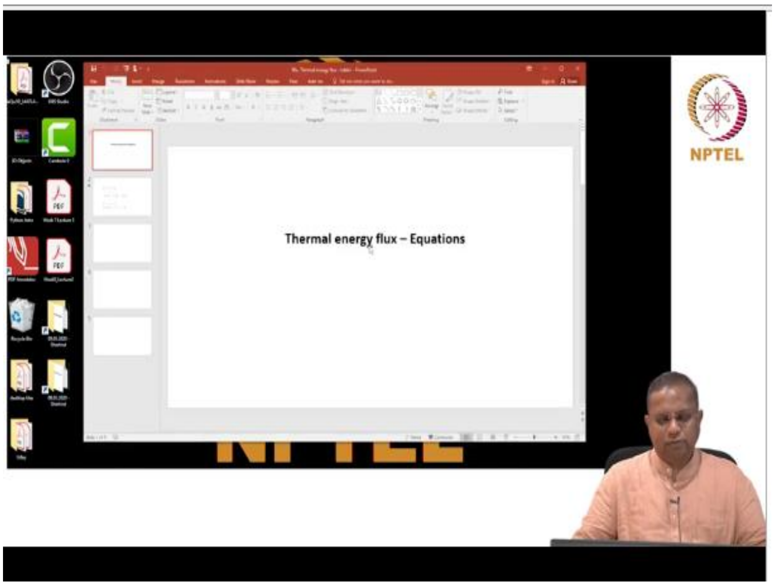

And then I showed you the application of course they were tables of the energy equation, which I would asked you to make a copy off and keep as a part of your notes.

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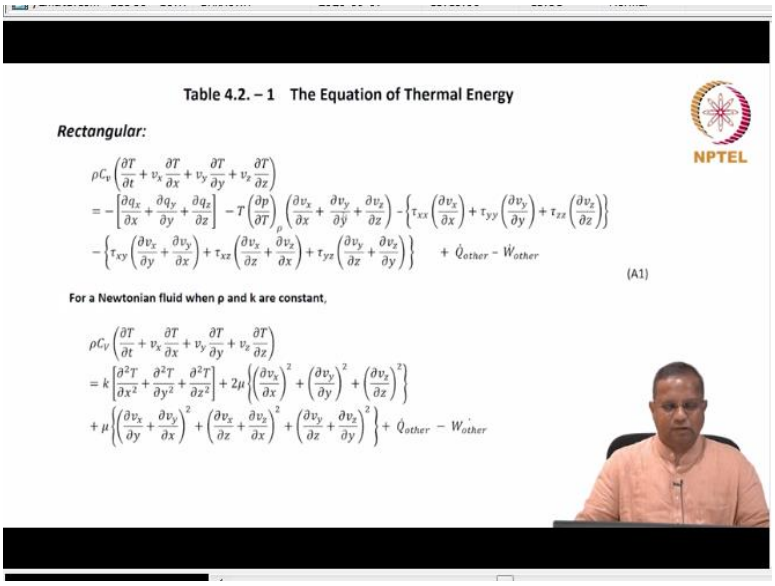

Again, this flashing this here I am not going to discuss this here. Then I showed you how we discussed.

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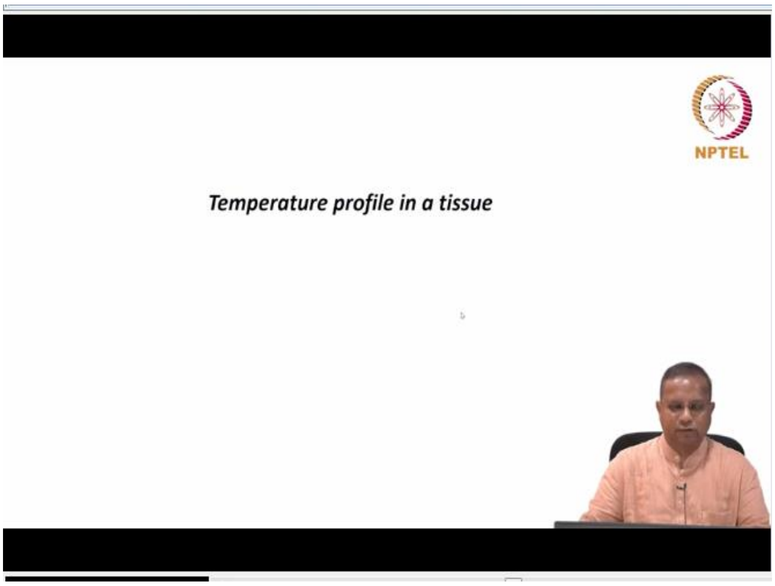

How you could use the equation of thermal energy to get to the temperature profile in a tissue, this was the first example.

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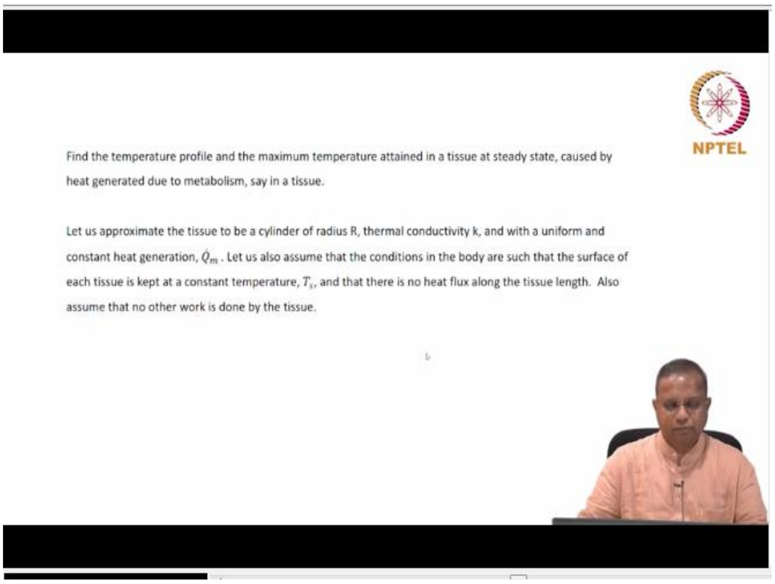

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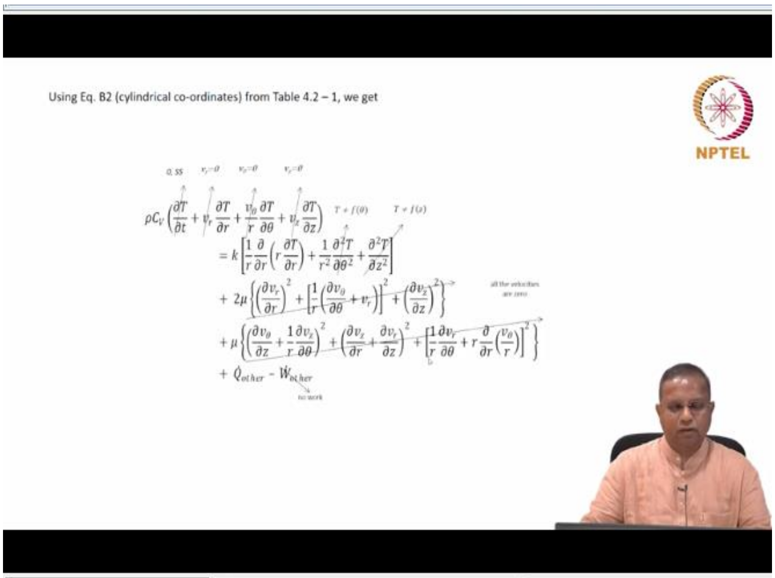

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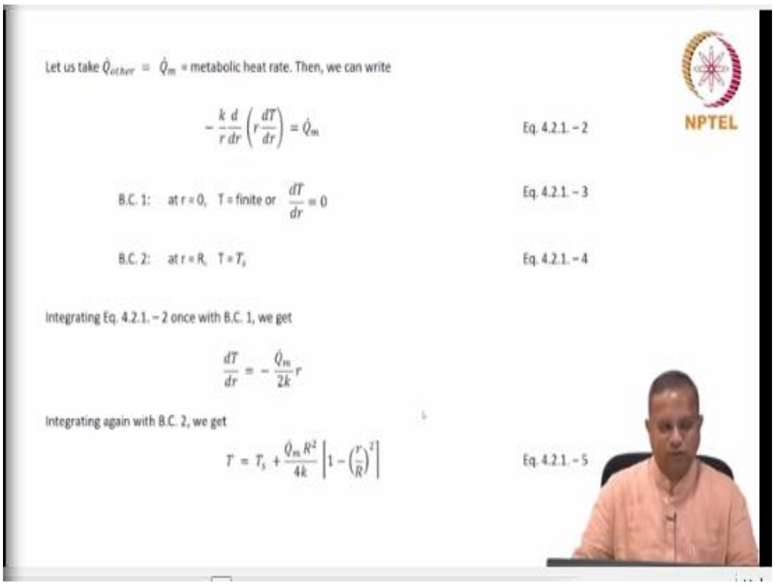

Then by cancelling the irrelevant terms we could get to the governing equation.

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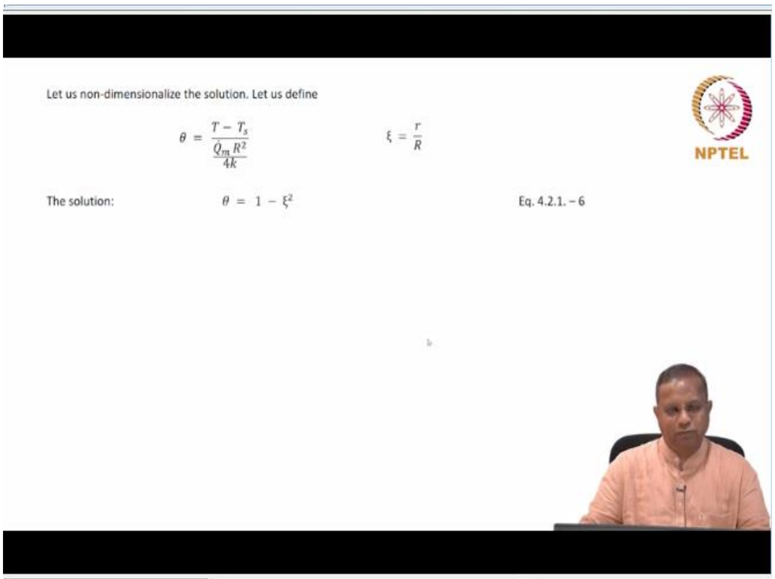

Which we can solve in a fashion similar to what we did for laminar flow through a pipe.

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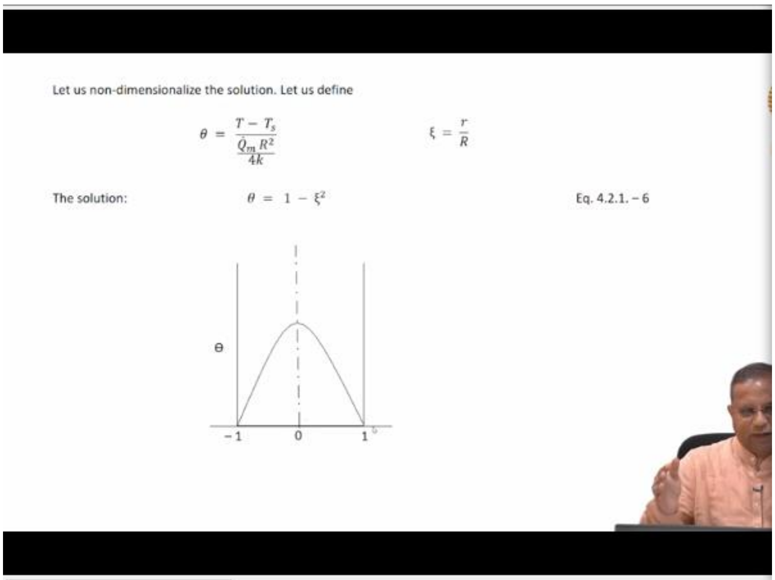

To get a parabolic profile of non dimensional temperature verses non dimensional dimension.

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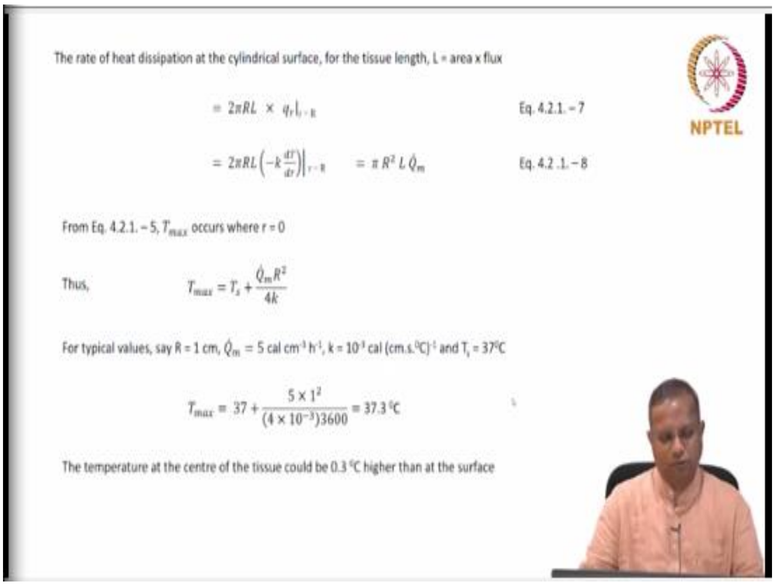

And also we look at the rate of heat dissipation at the cylindrical surface. Then we plugged in some numbers to see that the maximum temperature is about 0.3 degrees higher or could be about 0.3 degrees higher compared to this surface temperature, ok. Then we saw an example with unsteady state right, this is been the pattern we looked at steady state cases and then unsteady state. Unsteady state just to give you a flavour of the complexities involved and what does unsteady state really mean where all does it become relevant and so on so forth.

The way you should start looking for unsteady state, ok despite the mathematical complexity that is.

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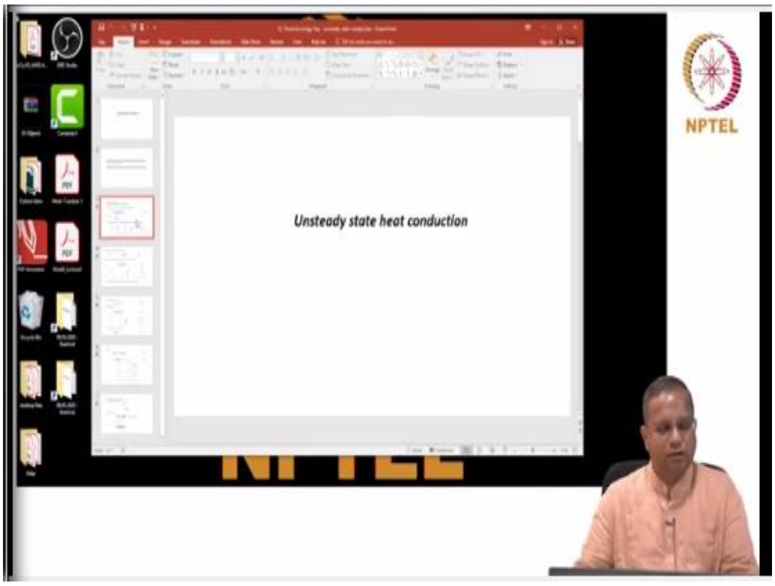

And an example was given that complicated with the complications.

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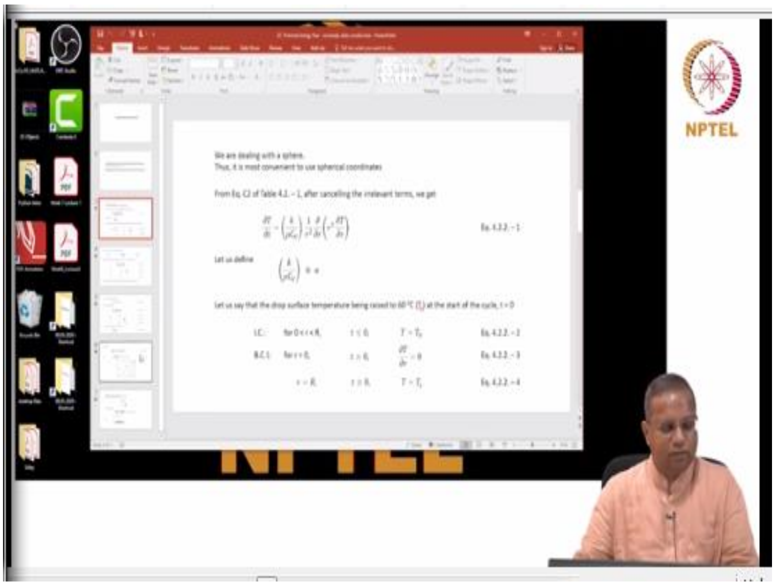

In the mathematical part of it.

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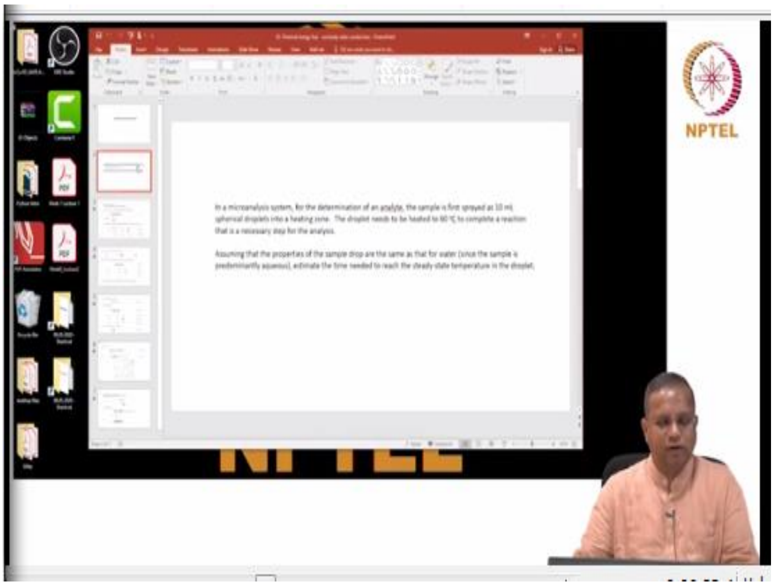

And the it is a very relevant situation, ok, we looked at a micro analysis system the time that it takes for the entire droplet to reach a certain temperature which is necessary for a certain reaction to occur which could be one of the key reactions in the step of processes, that are relevant for that particular system.

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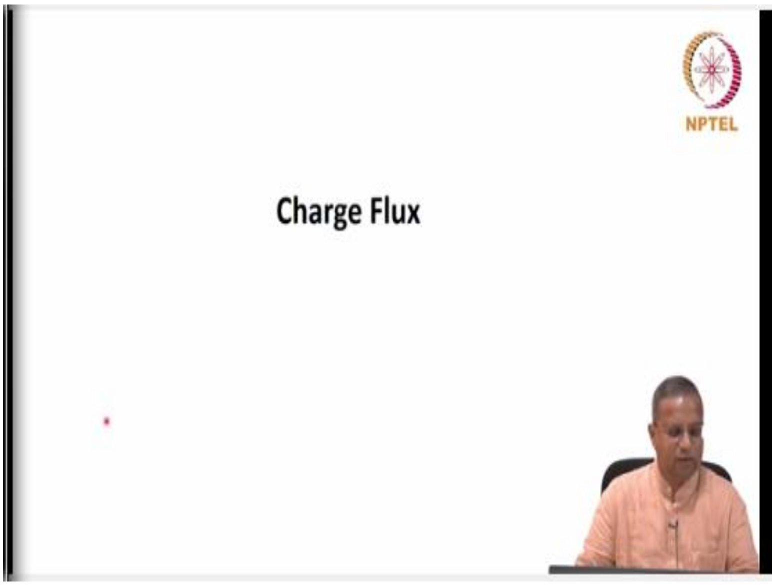

Then we looked at charge flux, ok. We said we biological engineers need to look at charge flux in some detail.

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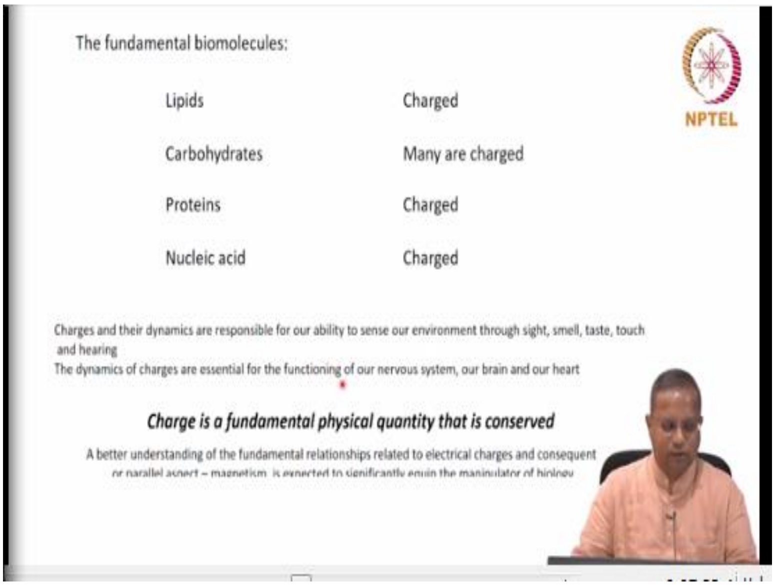

Because the fundamental biomolecules could all be charged, ok, lipids are always charged, carbohydrates are many of them are charged, proteins are charged, nucleic acids are charged. And therefore we are dealing with charged particles has the fundamental units of all our systems and therefore we need to look at charge flux is what was mentioned.

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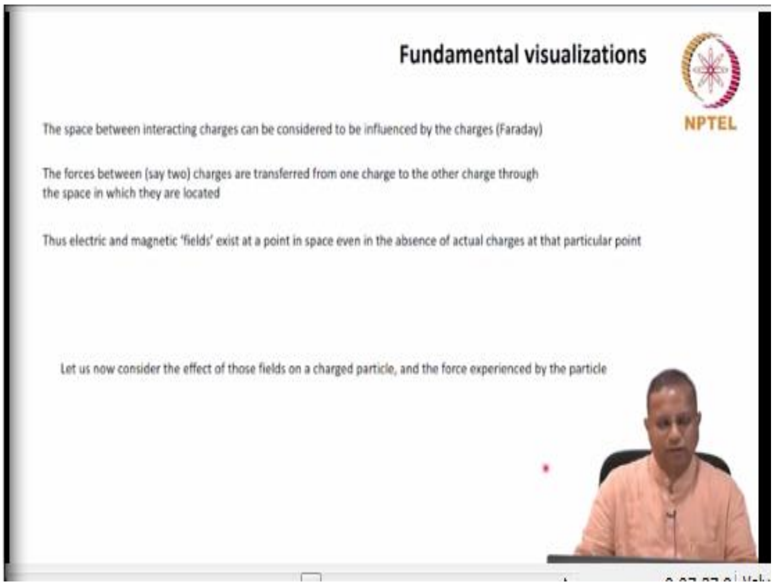

And we looked at some background you know the we derived the charge balance equation in the same way that we derive the other balance equations.

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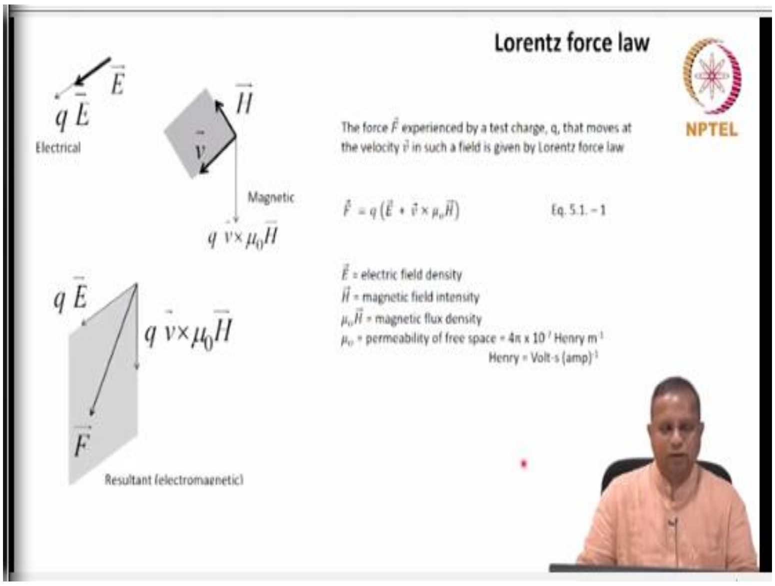

Before that the force that is experience by a particle that is moving in a electromagnetic field is given by Lorentz force law, F = q (E + v $\times \mu_0$ H), this is we reviewed this.

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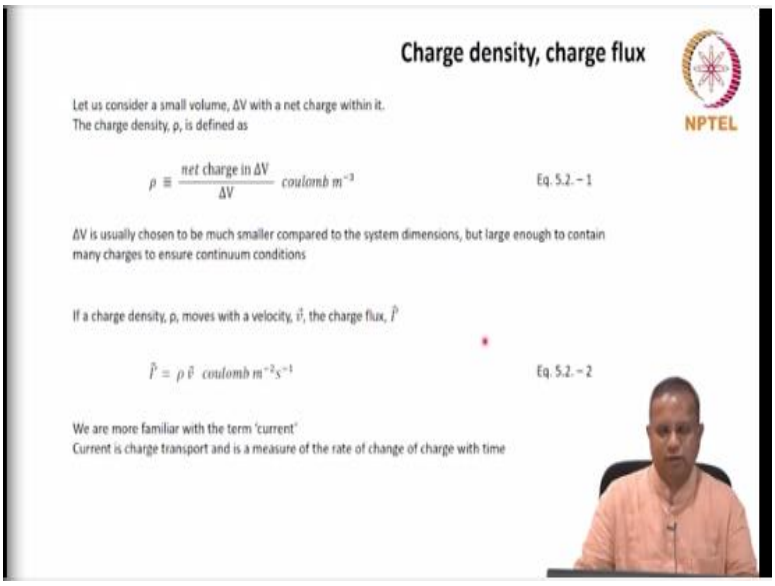

And then we define charge density, charge flux.

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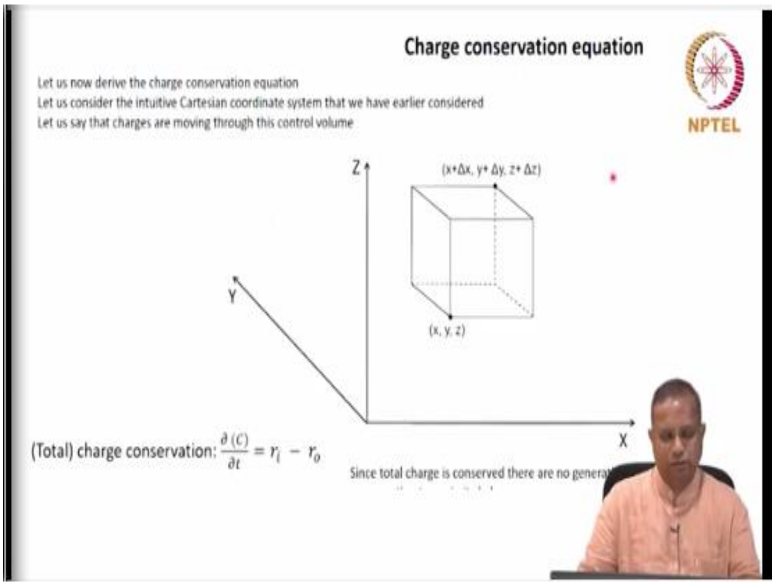

And then based on those quantities we derive the charge conservation equation.

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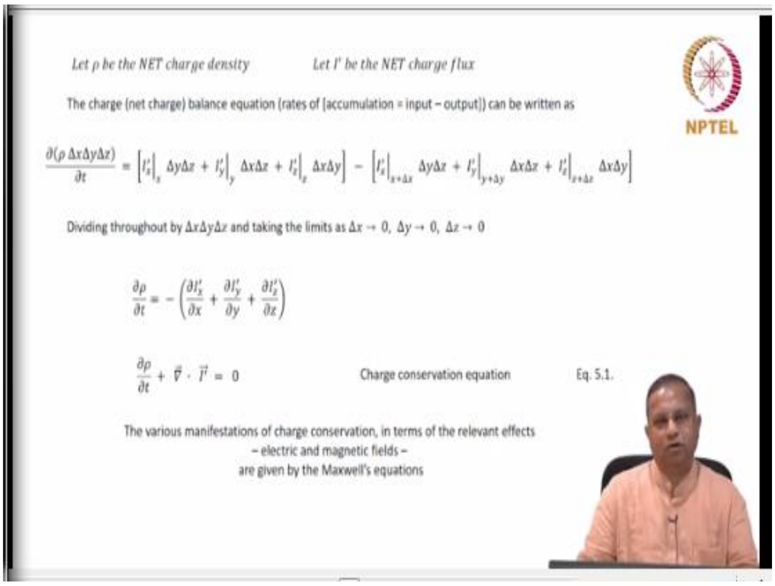

Rather simple derivation compared to the other things and we could get $\frac{\partial \rho}{\partial t} + \nabla . I' = 0$, the charge flux here equals 0, the differential form of the charge conservation equation. Then you are introduced to the Maxwell's relations, Maxwell's equations which are the fundamental equations for all electromagnetism.

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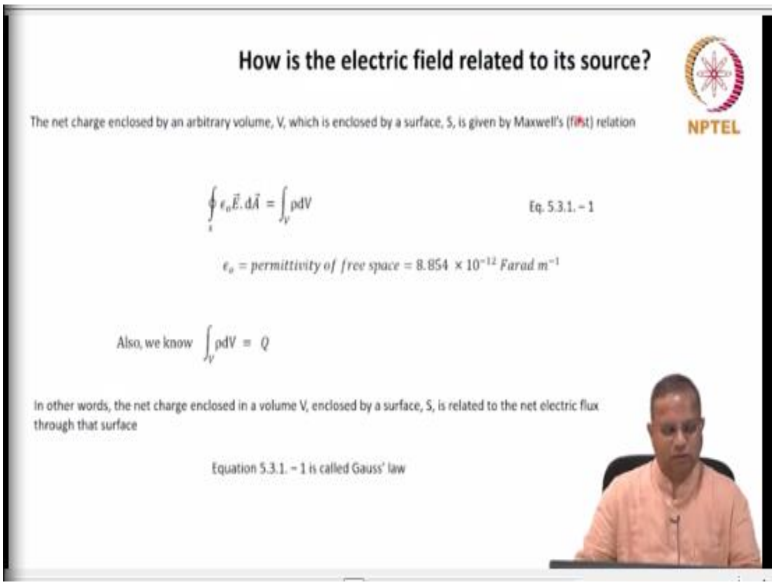

They answer these questions, first one answers how is the electric field related to it is source, the Gauss' law.

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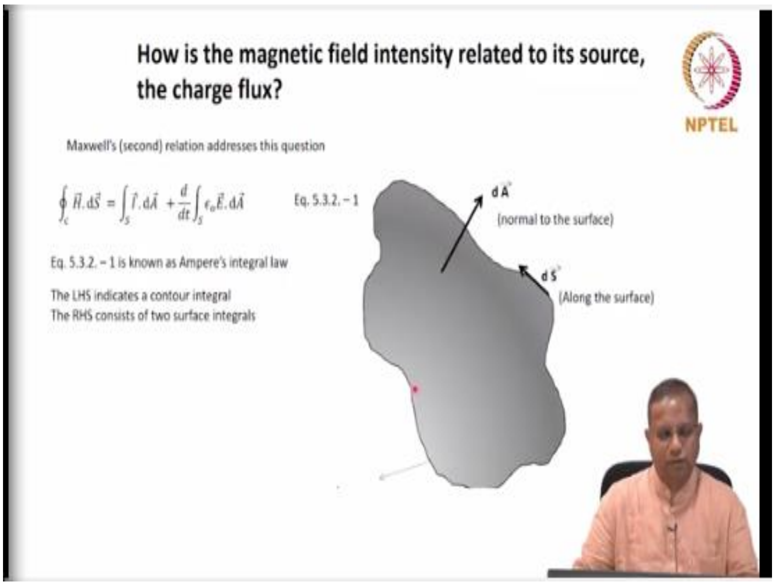

Second one answers how is the magnetic field intensity related to it is source, the charge flux, that results in Ampere's integral law.

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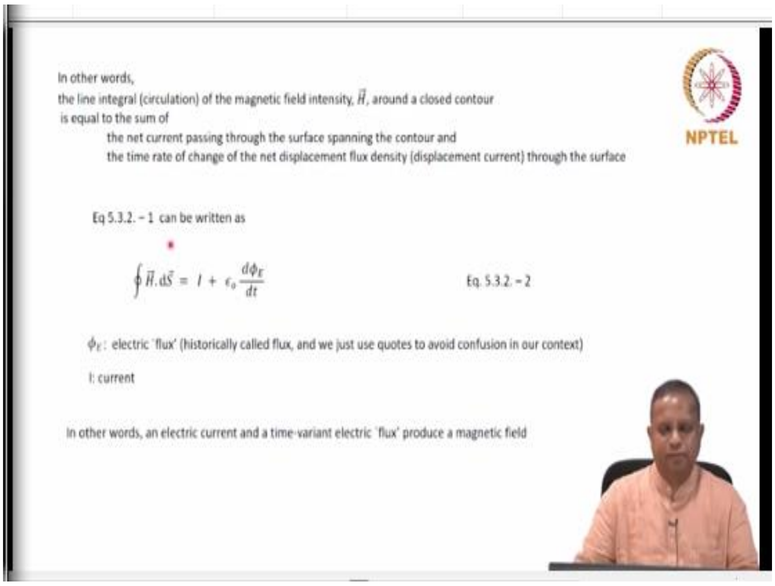

Then the.

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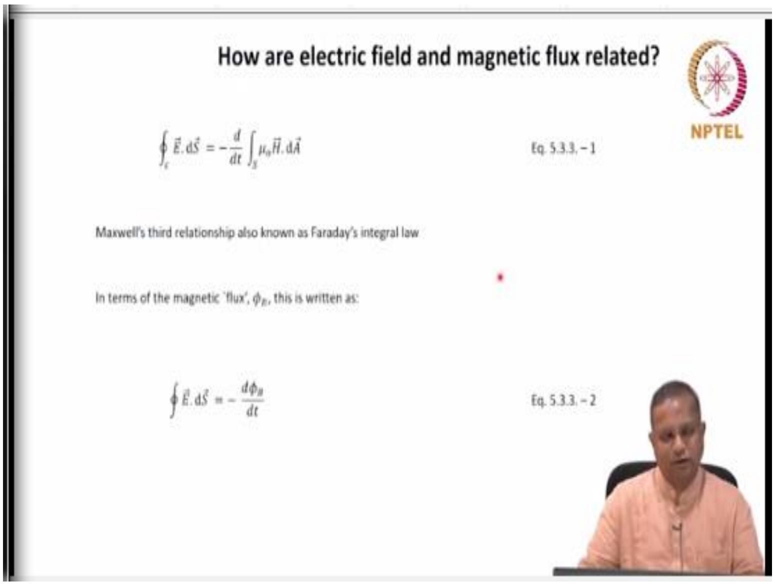

Third Maxwell's equation describes how are electric field and magnetic flux related, ok that is the Faraday's integral law.

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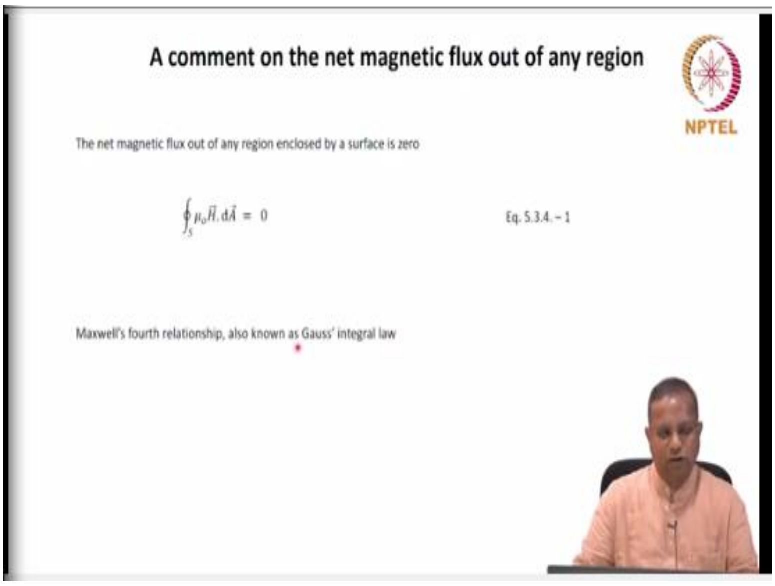

Then a comment on the net magnetic flux out of any region was given by Maxwell's fourth equation which is also called the Gauss' integral law.

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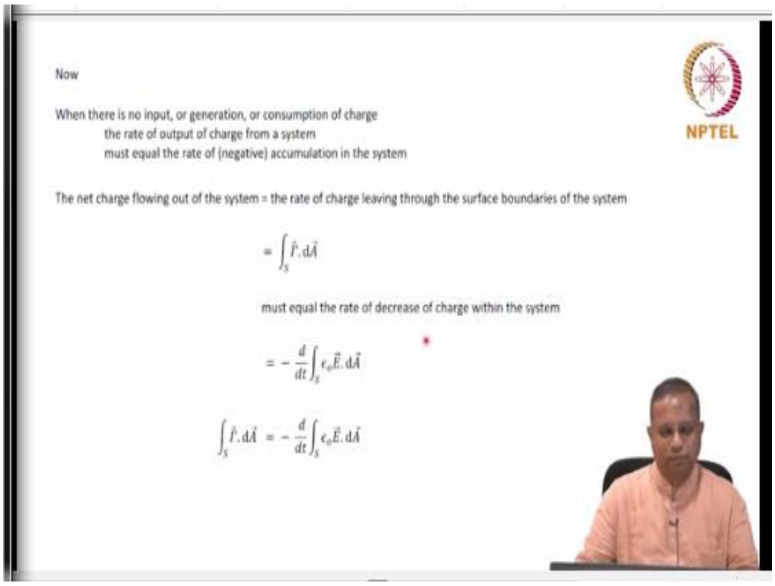

Then we saw.

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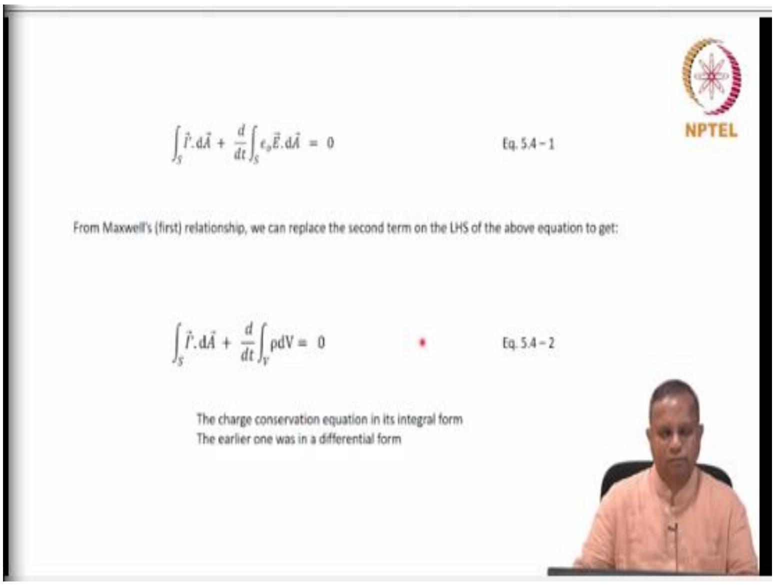

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A few simplifications that make our life easier especially when we deal with electromagnetism in biological systems.

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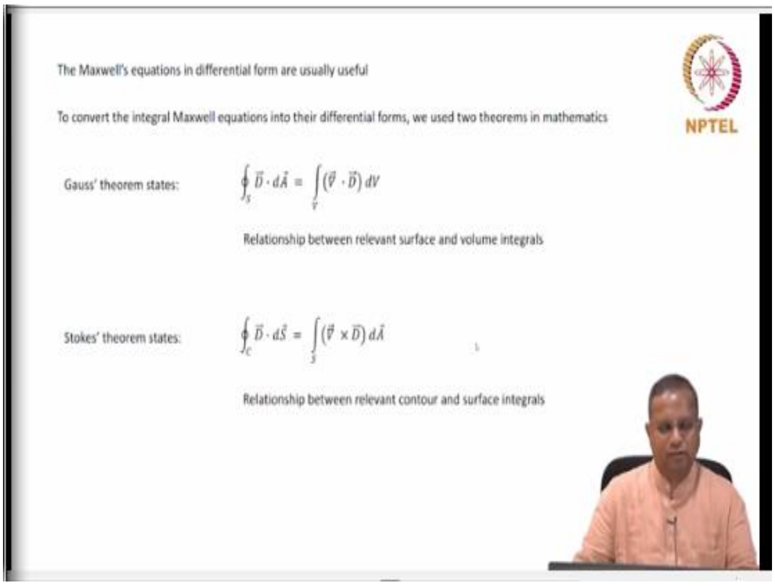

Such as.

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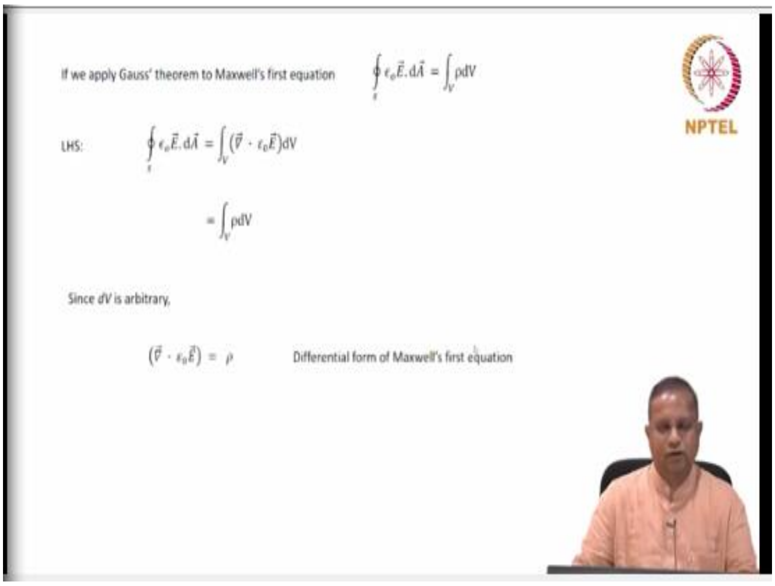

The differential forms.

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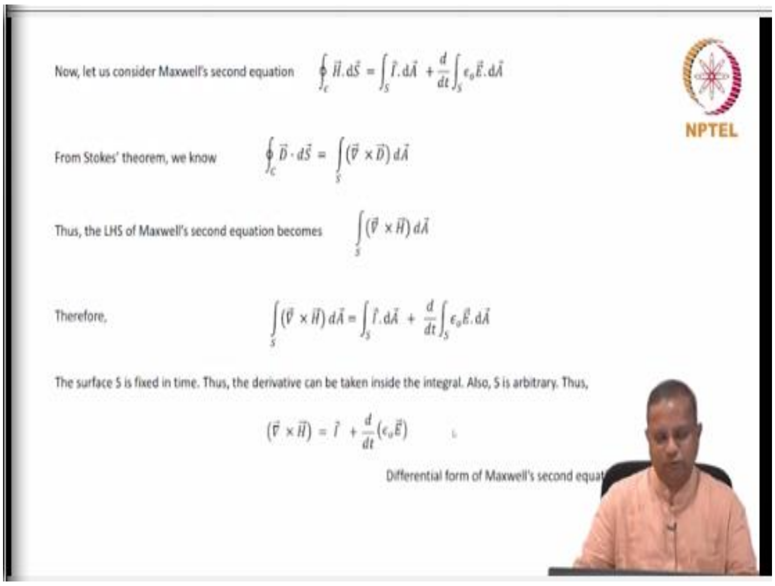

How to convert the integral forms and the differential forms and the differential forms we saw by using the 2 theorems the Gauss's theorem, the Stokes theorem in mathematics.

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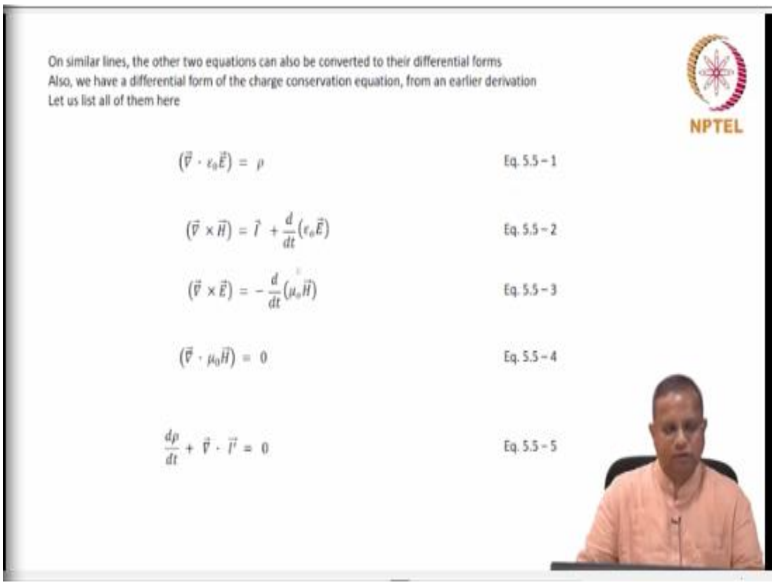

And then these were the differential forms, on top of that we said that these are fine for free space, for biological systems, you do not have free space, you have a medium. And therefore you cannot use the permittivity of free space.

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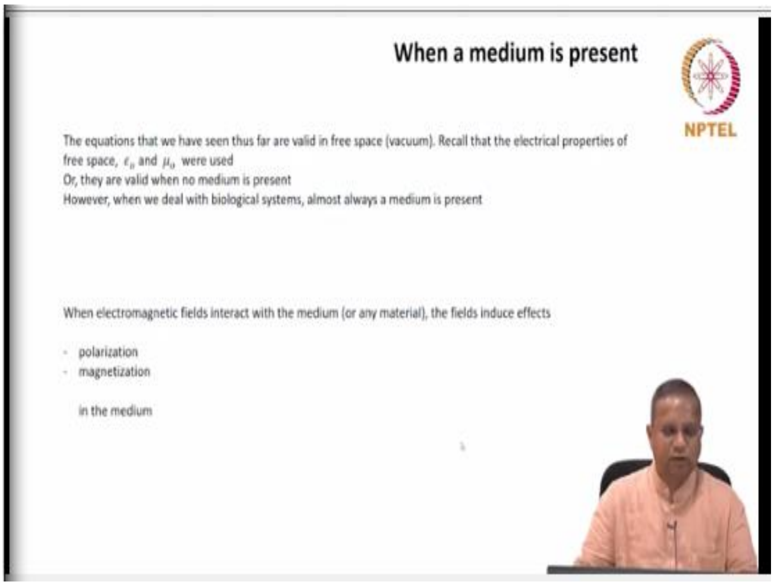

You will have to use the permittivity of the medium, however the permeability of free space is does not change in these equations. So, the same equations that we had earlier can be used just by changing ε_0 to ε. The effects that we discussed because of the presence of the medium are polarization and magnetisation.

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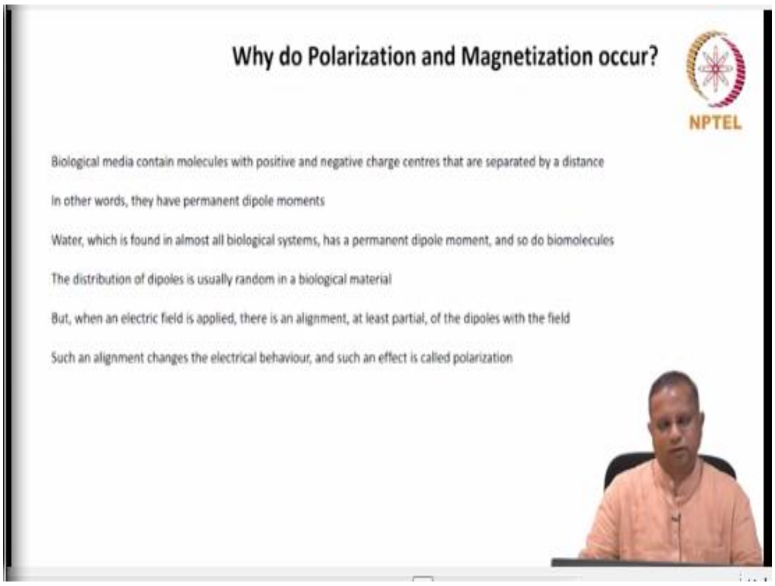

Alright.

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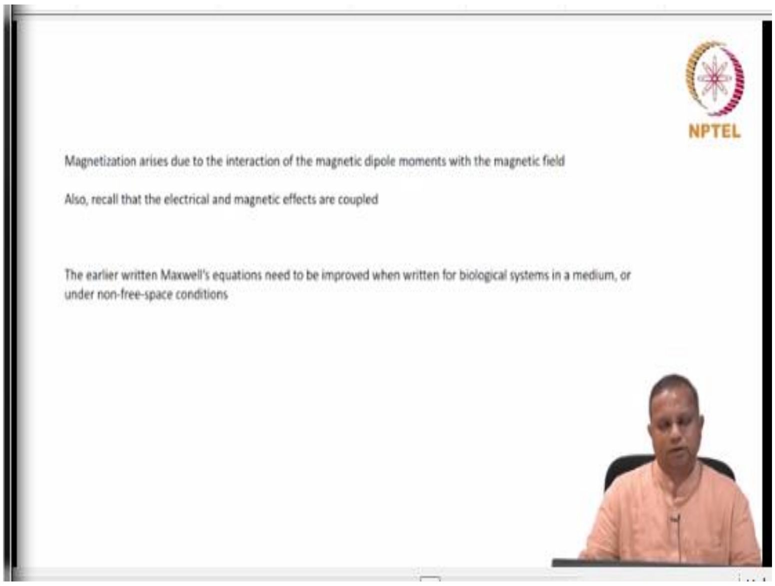

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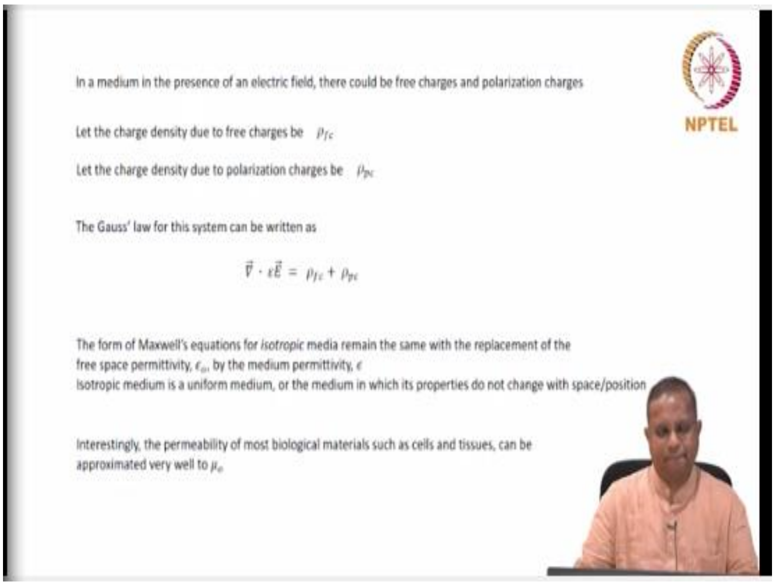

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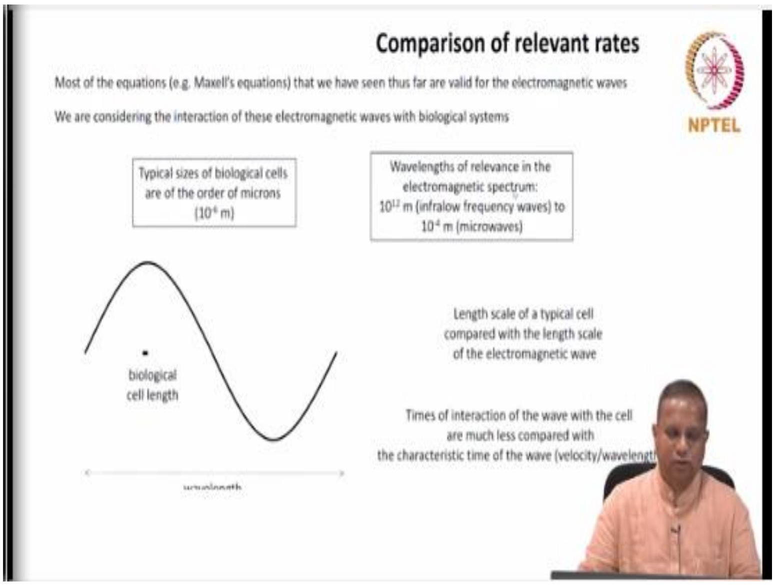

Then we also saw the.

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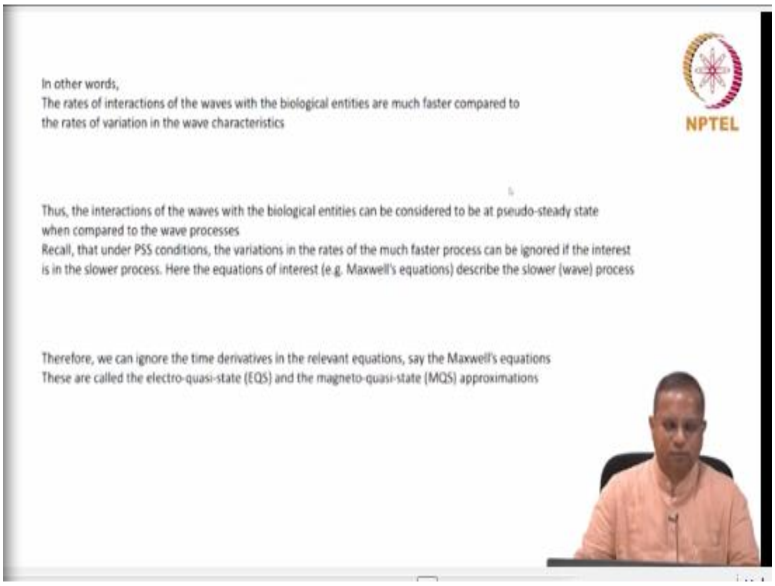

Electro-quasi state approximation and the magneto-quasi state approximation which effectively delete the time variations and the time derivatives in the Maxwell's equations.

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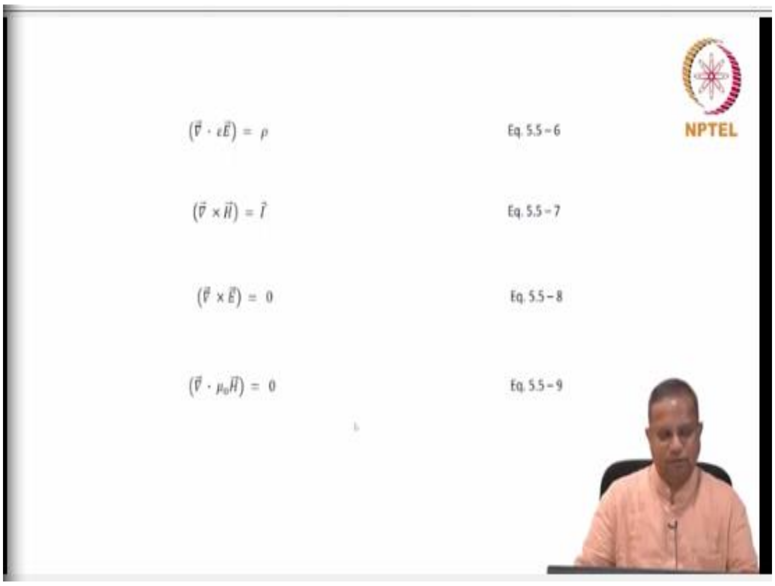

And they become simpler for us to use for interactions with biological situation, ok and biological systems that is, so these are the equations of relevance. We also found that, we also saw that these equations can be used for any situation, we saw how you could use this to get a capacitor equation and the bile lipid bile layer membrane can be viewed as a capacitor.

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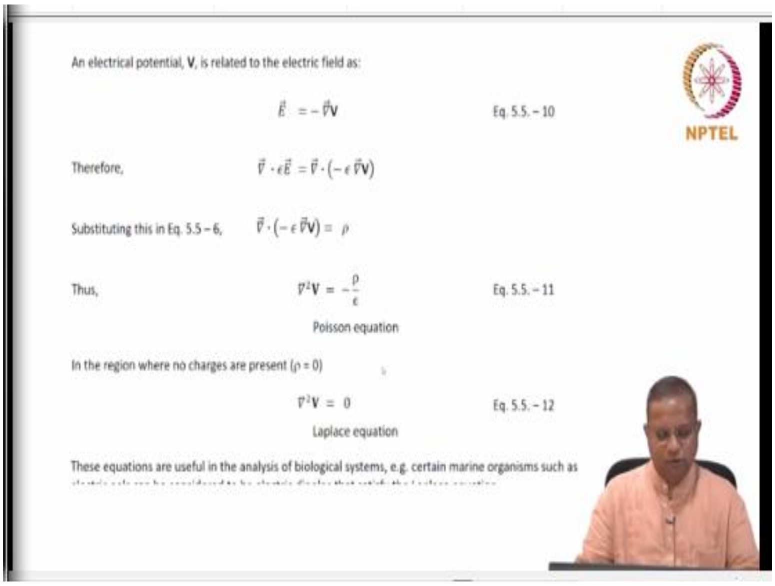

Right, then we saw the Poisson equation, Laplace equation which are useful equations for analysis.

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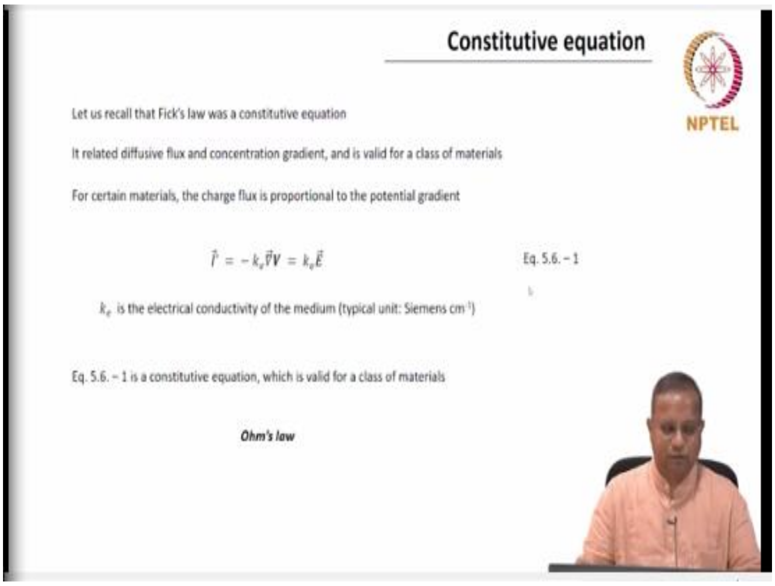

Then we saw Ohm's law as a constitutive equation, ok that is I think that is where we finished up there. And then we saw some Maxwell application of Maxwell's equation as I Just mentioned. And we also said that the fundamental equations on which EEG is based, this also derived from Maxwell's equation. For that matter anything is fundamentally derived from Maxwell's equation. Then we saw the aspects of charges and solution 3 different important aspects.

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Related to charges in solution.

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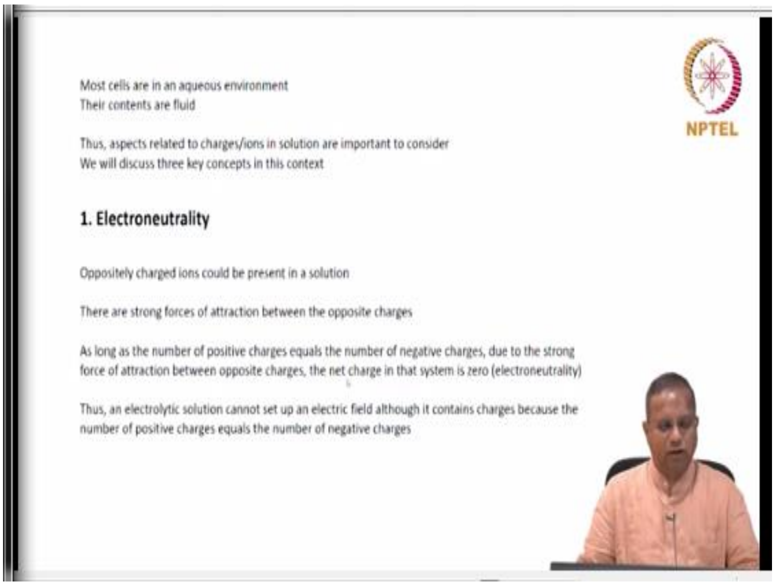

Or ions in solution, first one was electroneutrality.

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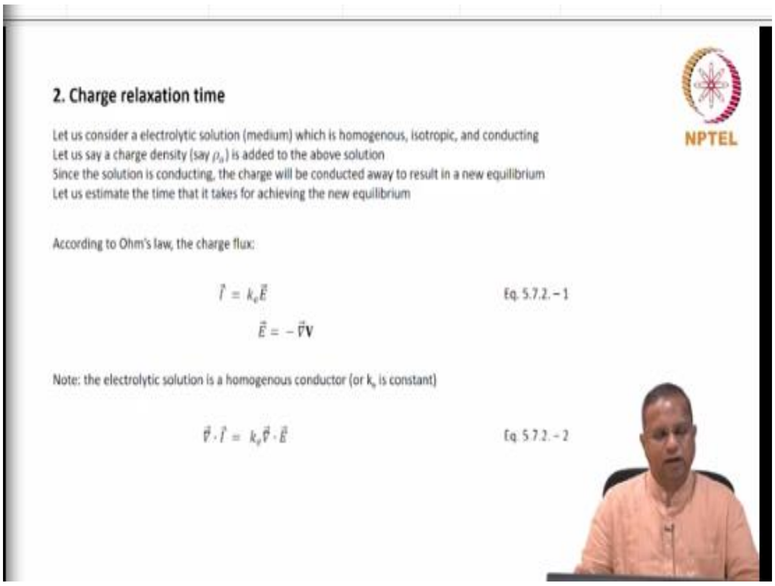

The second one was charge relaxation time and the third aspect.

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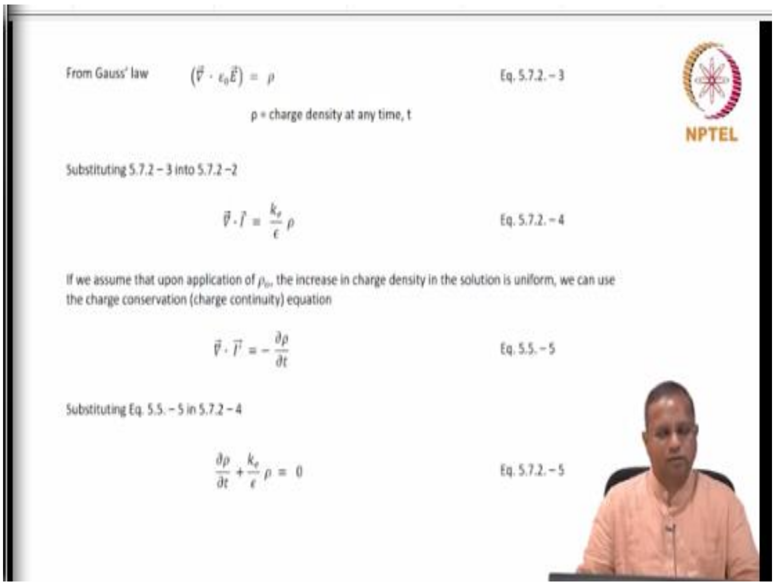

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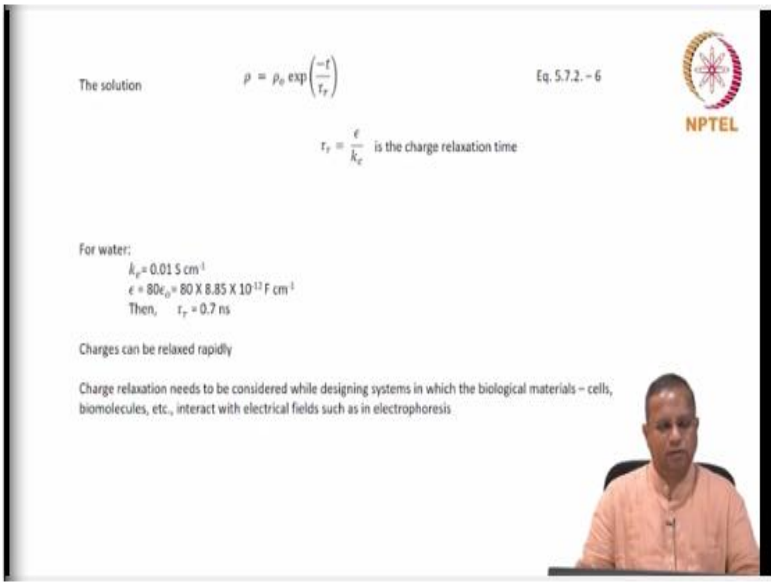

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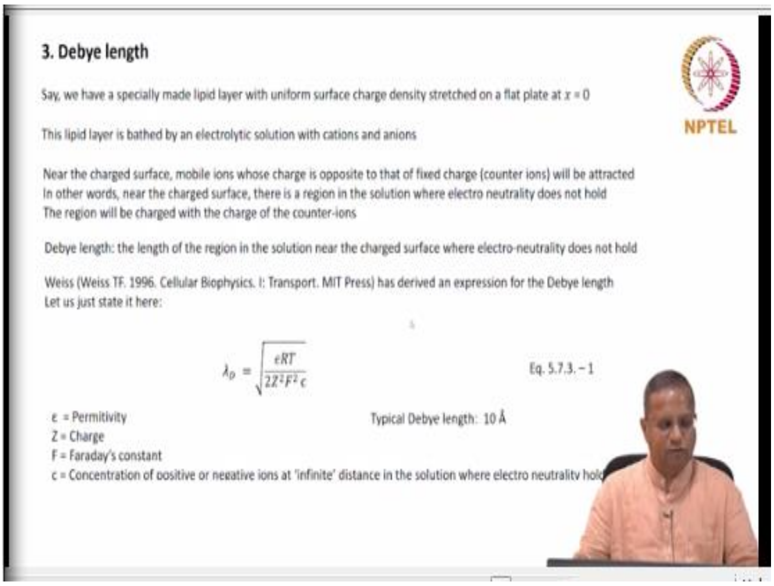

Was the Debye length, ok. The region where the electroneutrality does not hold especially when you have a surface of when you have a charge surface in solution, ok. These are all relevant aspects for analysis, I think this is what we saw.

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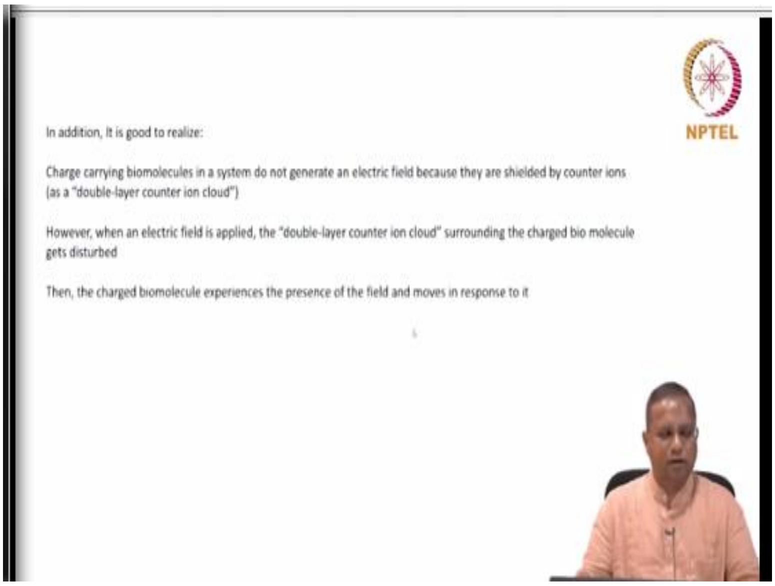

Till this chapter, we also saw that it is good to realize charge carrying biomolecules in a system, do not generate an electric field because they are shielded by counter ions as a double layer counter ion cloud. However when the electric field is applied the double layer counter ion cloud surrounding the charged biomolecules get disturbed. And then the charged biomolecule experiences the presence of the field and moves in response to that ok.

Ok, we will take a break here, I think it is time then when we meet for the next lecture the last review lecture you would look at multiple driving forces being responsible for a flux or multiple fluxes, ok. Predominantly a flux that is the review of the last chapter, see you then.