

Bioreactor Design and Analysis
Prof. Dr. Smita Srivastava
Department of Biotechnology
Indian Institute of Technology - Madras

Lecture – 17
Design of Continuous Bioreactors Practice Problems - Part 1

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Problem 1

A bacterial culture is grown in a 5-liter bioreactor. The growth kinetics of the bacterium can be expressed as

$$\mu = \mu_{\max} * (1 - e^{-0.01 S}) - k_d$$

Where S is the substrate concentration (mg/l) and μ is the specific growth rate (1/h), The kinetic parameter $\mu_{\max} = 2.0/\text{h}$ and $k_d = 0.05/\text{h}$. The yield parameters are $Y_{x/s} = 0.5 \text{ g cell/g substrate}$ and $Y_{p/x} = 0.1 \text{ mg product /mg of cell}$. The feed substrate concentration is 1000 mg/l.

- I. Determine the exit cell mass concentration at steady state, when the system is operated at a feed flow rate of 2.5 l/h. $\rightarrow F$
- II. What is the volumetric cell mass and product productivity of the system?
- III. What is the washout dilution rate?

Welcome back students. So, today we are going to do some practice problems on continuous reactors. So, let us begin with the first problem. If you can see the problem on the slide the problem states that a bacterial culture is grown in a 5 liter reactor. The growth kinetics of the bacterium can be expressed as the equation given here, which I have underlined. So, this relates how specific growth rate of the culture is changing with the substrate present in the medium.

Where S in this equation stands for the substrate concentration, mu stands for specific growth rate. The kinetic parameters in this equations are mu max which is maximum specific growth rate, the value given is 2 hours inverse, k d has been given as for endogenous metabolism, the constant is 0.05 hour inverse okay. The yield parameters with respect to the substrate. So, Y x by s is given as 0.5 grams cell per gram substrate consumed and Y p by x which is amount of product produced per unit biomass produced is 0.1.

The feed substrate concentration is 1000 milligrams per liter okay. Let us see the first part. Determine the exit cell mass concentration at steady state when the system is operated at a

feed flow rate of 2.5 liter per hours. So, what is given to us is F and we need to determine the X which is the biomass concentration inside the reactor in the CSTR what is exiting out. So, let us first list down whatever is given to us.

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Solution

Given:

$$\mu = \mu_{\max} * (1 - e^{-0.01S}) - k_d ; \quad \textcircled{1}$$

$$\mu_{\max} = 2h^{-1} ; \quad \text{---}$$

$$k_d = 0.05h^{-1} ; \quad \text{---}$$

$$Y_{X/S} = 0.5g \text{ cell/g substrate};$$

$$Y_{P/X} = 0.1mg \text{ product/mg cell};$$

$$S_0 = 1000mg/L; \quad \text{---}$$

$$F = 2.5 \text{ L/h}$$

$$V = 5 \text{ L}$$

$$(i) \quad D = \frac{F}{V} = 0.5 h^{-1}$$

At steady state, $\mu = D$, by substituting for μ in the above equation,

$$0.5 = 2 * (1 - e^{-0.01S}) - 0.05 \quad \textcircled{2}$$

$$S = 32.16 \text{ mg/L}$$

From mass balance equation for biomass in CSTR, we get (after rearrangement)

$$X = (S_i - S) Y_{X/S}$$

$$X = 483.92 \text{ mg/L}$$

Handwritten notes: $\frac{dS}{dt} = DS - DS_i - \frac{\mu X}{Y_{X/S}}$ and $\mu = D \implies (S_i - S) = X$

So, here this is the equation for growth, the specific growth rate model. The constituents of these models are given, yield coefficients are given. The S_0 value which is the feed, S_i stands for the inlet feed substrate concentration which is 1000 mg per liter and the volumetric flow rate is 2.5 liter per hour. So, all this information is what is given in the problem. Now, we need to determine the exit substrate concentration.

Now at steady state we know that for a CSTR when you do a mass balance for the biomass, we get the correlation between the specific growth rate and the dilution rate which is μ becomes equal to D at steady state. So, dilution rate in a CSTR is F by V where F is the volumetric flow rate, V is the volume of the reactor and here we missed out what is given to us is also the volume which is 5 liters.

So, if you do F by V , then it becomes 0.5 hour inverse as your dilution rate. Now at steady state we know $\mu = D$. So, then we put this value of μ in equation 1 and this is what has been done here. If we substitute all the values which are given to us in this equation at steady state, then your substrate concentration at steady state can be obtained which is 32.16 mg per liter. Let us see the second part.

What is the volumetric cell mass and product productivity? So, in order to determine the cell mass productivity and product productivity we need to first determine how much biomass would have been produced at steady state and similarly the product. So, now in order to find the biomass produced at steady state, if you do a mass balance for the substrate and then do rearrangement.

You will find that the biomass produced at steady state can be given as S_i which is the substrate inlet concentration minus S the steady state substrate concentration multiplied by the yield coefficient. So, let us see how do we get it. Let us do a substrate balance at steady state. So, if it is the dilution rate and this is DS_i , this is the input flow rate, this is output, the rate at which the substrate is getting in and this is the rate at which the substrate is coming out.

Let me rewrite again. So, ds by dt will be equal to D is the dilution rate, $DS - DS_0$ which is S_i here minus μX if suppose is the biomass produced, this is the growth rate divided $Y_{p/x}$. So, this becomes the substrate consumption rate. At steady state we know ds by dt is 0 and also $\mu = D$ by biomass balance. Then your $S_i - S$ will become you bring this to the LHS will become equal to X , here $Y_{p/x}$ is multiplied.

So, this is what is given here. So, then if you will substitute the given value of $Y_{p/x}$ by s , the S_i is given, S we have determined from here equation 2. Then we can find the amount of biomass produced at steady state.

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(ii) Biomass productivity = $D \cdot x$ ←

$$= 0.5 * 483.92 \text{ mg L}^{-1}$$

$$= 241.9 \text{ mg L}^{-1}\text{h}^{-1}$$

Product Productivity = $D \cdot p$

$$= D * Y_{p/x} * X$$

$$= 24.195 \text{ mg L}^{-1}\text{h}^{-1}$$

(iii) At washout condition of D , $x=0$, so there will be no consumption of substrate and in the growth equation, S will be S_i

Substituting S_i value in the growth equation, we get,

$$D_{washout} = \mu_{max} * (1 - e^{-0.01 S_i}) - k_d$$

$$= 2 * (1 - e^{-0.01 * 1000}) - 0.05$$

$$= 1.95 \text{ h}^{-1}$$

Handwritten notes:
 $\frac{dP}{dt} = Y_{p/x} \cdot \frac{dx}{dt}$
 $\frac{dP}{dt} = D P_i - D P + Y_{p/x} X$
 $\mu = D$
 $S = S_i$

So, then the biomass productivity in a CSTR can be given as $D X$. So, we know the dilution rate at steady state and the biomass at steady state. So, we can now calculate the biomass productivity. Now in order to find the product productivity it is D times the product formed. So, the amount of product formed given by P by X will be Y_p by x times X assuming sterile feed to this is your product productivity.

Let us do a mass balance for the product and we know that $\frac{dp}{dt} = Y_p$ by x times $\frac{dx}{dt}$. Now if we do a mass balance for the product, the rate of accumulation of product is equal to the rate at which it is coming inside Let us suppose it is P_i minus the rate at which it is going out and also whatever is getting produced. So, this can be given as Y_p by x times $\frac{dx}{dt}$.

Now, this $\frac{dx}{dt}$ can further be written as μ times X and we know at steady state $\mu = D$ and assume sterile feed with no initial product concentration. So, then P_i becomes 0 and at steady state no accumulation. So, $\frac{dp}{dt}$ is also 0. So, then your $\frac{dp}{dt}$ which is the volumetric productivity of the product will be equal to μ times $X Y_p$ by x where μ is nothing but D X is the amount of biomass produced multiplied by Y_p by x , which can give you the product productivity in a steady state CSTR.

Now at washout condition of D , $x = 0$ which means there is no biomass left inside the reactor. So, the third part of the problem is what is the washout dilution rate? So, how will we solve this? We know that at washout there should be no biomass present now inside the reactor. So, in that case when there is no biomass there will be no consumption of substrate. So, your substrate concentration at washout should be equal to the inlet feed concentration of the substrate.

So, then in your specific growth rate model, which was given in equation 1 if you substitute μ when this becomes is the washout dilution rate. So, let us see that. Assume it is D washout at which your S will be substituted as S_i , S_i is given to us as the inlet feed concentration of 1000 mg per liter. Rest of the model constants we substitute in this equation and we will be able to determine the washout dilution rate. So, one has to remember that at washout the specific growth rate at steady state is equals to D and at washout your S will become equal to S_i .