

**Bioreactor Design and Analysis**  
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**Lecture – 27**  
**Heterogenous Reactions in Bioreactors – Practice Problems**

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**Problem 1**

Mushroom tyrosinase is immobilised in 2-mm spherical beads for conversion of tyrosine to DOPA in a continuous, well-mixed bubble column. The Michaelis constant for the immobilised enzyme is  $2 \text{ gmol/m}^3$ . A solution containing  $15 \text{ gmol/m}^3$  tyrosine is fed into the reactor; because of the high cost of the substrate, the desired conversion is 99%. The reactor is loaded with beads at a density of  $0.25 \text{ m}^3/\text{m}^3$ ; all enzyme is retained within the reactor. The intrinsic  $V_{\max}$  for the immobilized enzyme is  $1.5 \times 10^{-2} \text{ gmol/s per m}^3$  beads. The effective diffusivity of tyrosine in the beads is  $7 \times 10^{-10} \text{ m}^2/\text{s}$ ; external mass-transfer effects are negligible. Immobilisation stabilises the enzyme so that deactivation is minimal over the operating period. Determine the reactor volume needed to treat  $18 \text{ m}^3$  tyrosine solution per day.

Welcome students. We will do some practice problems now on heterogeneous reactions in fermentations taking examples from immobilized enzyme systems or immobilized biomass. Let us take the first problem. Mushroom tyrosinase this is an enzyme which has been immobilized in 2 millimeter big spherical beads for conversion of tyrosine to DOPA in a continuous well-mixed bubble column.

The Michaelis-Menten constant for the immobilized enzyme is given as 2 gram moles per meter cube. So, this is our  $K_m$  value in Michaelis-Menten kinetics. A solution containing 15 gram mole per meter cube tyrosine, so this is our initial substrate concentration of the feed, is fed into the reactor because of the high cost of the substrate the desired conversion is 99%. So, if you can think this can give us our final substrate concentration desired because the conversion is given to us as 99%.


The reactor is loaded with beads at a density of 0.25 meter cube per meter cubed, so this is some void fraction. All enzyme is retained within the reactor. The intrinsic  $V_{\max}$  for the immobilized enzyme is  $1.5 \times 10^{-2}$  gram molar per second per meter cube

of the beads. So, the  $V_{max}$  value is given to us. The effective diffusivity of the tyrosine in the beads is also given. External mass transfer effects are negligible.

So, it is assumed that there are no external mass transfer limitations. So, which means at the surface of the bead the concentration of the substrate is  $S_0$ . Immobilization stabilizes the enzyme so that the deactivation is minimal over the operating period. So, the enzyme concentration remains the same, so there is no deactivation of the enzyme taking place. Determine the reactor volume needed to treat 18 meter cube tyrosine solution per day. So, this is nothing but the volumetric flow rate of the substrate.

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Solution

$$\begin{aligned}
 K_m &= 2 \text{ gmol/m}^3 \\
 V_{max} &= 1.5 \times 10^{-2} \text{ gmol/s per m}^3 \\
 R &= 10^{-3} \text{ m} \\
 D_{AE} &= 7 \times 10^{-10} \text{ m}^2/\text{s} \\
 S_0 &= 15 \text{ gmol/m}^3 \\
 F &= 18 \text{ m}^3/\text{day} = 2.08 \times 10^{-4} \text{ m}^3/\text{s}
 \end{aligned}$$


For 99% conversion,

$$S = 0.01S_0 = 0.15 \text{ gmol/m}^3$$

$S \ll K_m$ , so we can assume first order kinetics

$$K_1 = V_{max} / K_m = 7.5 \times 10^{-3} / \text{s}$$

Thiele modulus for spherical catalysts (first-order kinetics),

$$\begin{aligned}
 \Phi &= \frac{R}{3} \sqrt{\frac{K_1}{D_{AE}}} \\
 \Phi &= 1.09
 \end{aligned}$$

Now, let us list down the information which has been provided. The Michaelis-Menten constant for the enzymatic system is given. The maximum reaction rate is also given. The radius of the bead can be calculated from the diameter which was given as 2 millimeters spherical bead. So, the radius of the bead can be calculated. Diffusivity is also given. Initial substrate concentration let us call it as  $S_0$  as shown here.

So, the final which is 99% converted, so the residual substrate will become 0.15 gram mole per meter cube. The volumetric flow rate of the feed is given. Now if you notice your substrate concentration in the reactor is very less in comparison to your Michaelis-Menten constant value. So, we can assume it to be the first-order kinetics as  $V_{max} S / K_m$  is equal to our  $V$ .

Now, if you remember the mass balance for the substrate in an immobilized system with external mass transfer absent and only internal mass transfer present as the substrate diffused in and we created a sphere, spherical shell and we did a mass balance for the accumulation of the substrate in this spherical shell. Now, what do we need to find? We need to find the reactor volume needed to treat this much solution per day.

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Effectiveness factor-

$$\eta_{ii} = \frac{1}{3\phi^2} (3\phi_1 \coth 3\phi_1 - 1) = 0.64$$

$\eta_e = 1$  (external mass transfer is negligible)

$$\eta_T = \eta_e \eta_{ii} = 0.64$$

$$D(S_i - S) = \frac{V_{max} S}{K_M + S}$$

② = Actual  
 $\rightarrow$  rxn  
 $= f(\phi)$   
 $= D(S_i - S)$

For immobilized enzymes, the reaction can be modified to

$$D(S_i - S) = \frac{\eta_T V_{max} S}{K_M + S}$$

$$D = 4.51 \times 10^{-5} / s$$

$$V = \frac{F}{D}$$

$$V = 4.6 \text{ m}^3$$

$\left( \frac{V_{max} S}{S + K_M} \right) \eta = \text{Actual}$   
 0.64

Now, we need to find first the effect of the mass transfer limitation. Now because external mass transfer limitation is absent for the first-order reaction kinetics, we determine how the efficiency of the reaction or how much substrate would be actually available for the reaction can be obtained with the effectiveness factor. And our effectiveness factor was nothing but the actual reaction rate to the reaction rate in the absence of any diffusion.

So, if you remember this effectiveness factor was a function of Thiele's modulus and in turn the Thiele's modulus could be calculated as the ratio of the reaction rate diffusivity as shown here in this equation 1 for first-order kinetics where R is our radius of the pellet, enzymatic bead. So, here when this is expanded, the effectiveness factor can be given as the function of Thiele's modulus and the value comes out to be 0.64.

Now, this means that the actual reaction rate would be reaction rate in the absence of diffusion. So, if there is no diffusion limitation, it will become  $V_{max} S / (S + K_M)$  multiplied by effectiveness factor in the presence of diffusion what will be the actual reaction rate? So, effectiveness factor value we will put as 0.64 and we can find the value. Now, this is the actual rate at which the substrate will be getting consumed in the reaction.

Now because there is no accumulation take place, so this should be equal to the rate at which it is getting transferred inside the reactor, now which is equal to the rate of input minus rate of output in the immobilized enzyme, so which is  $D S_i$  minus  $DS$ . So if you substitute all the values, the dilution rate comes out to be this because here rest all is known.

Now once we calculate the dilution rate for this continuous reactor, we know dilution rate is  $F$  by  $V$ . Volumetric flow rate is given to us which is 18 meter cube per day. So, we can calculate the volume which comes out to be 4.6 meter cube.