Bioreactor Design and Analysis
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Lecture - 34
Scale-up of Bioreactors – Part 2

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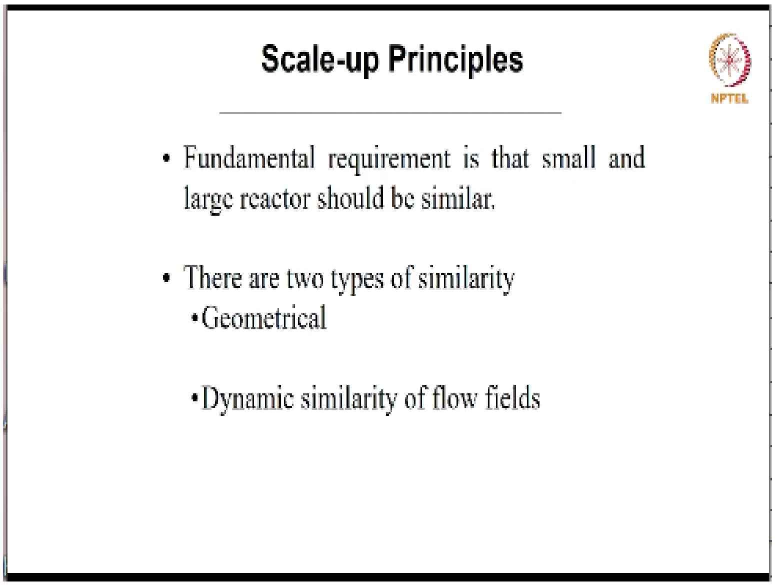

So, what are the fundamental principles? The fundamental requirement would be that at small and large reactor, the geometric should be similar. Now, there are 2 kinds of similarities. One is the geometrical similarity and the other is dynamic similarities of the flow fields. So, these form the principle of scale-up. So, when we are scaling-up from one size to the other, then we need to ensure that the scale-up by 2 different vessels includes geometrical similarity as well as dynamic similarity, which is concerning the similarity between flow fields.

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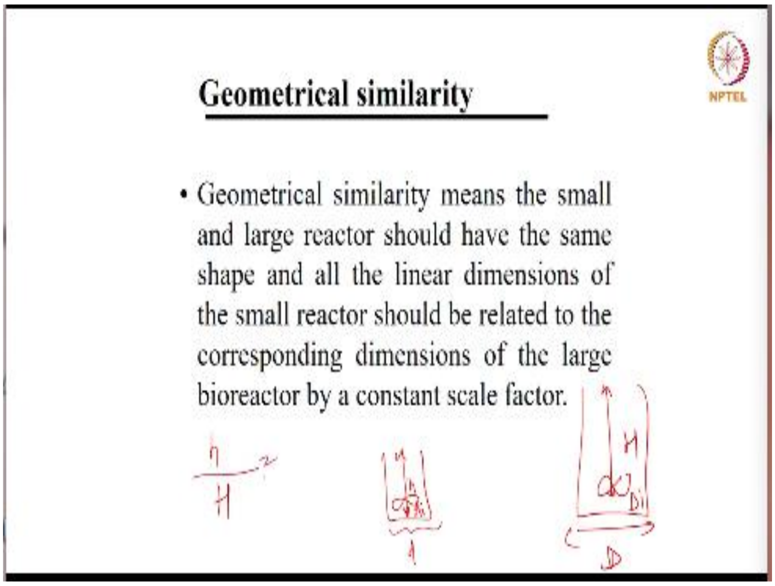

Now, to understand, what is geometrical similarity? Geometrical similarity means, the small and the large reactor should have the same shape and all the linear dimensions of the small reactor should be related to the corresponding dimensions of the large bioreactor by a constant scale factor. Suppose, we have 2 reactors, here, the diameter is small d, the height of the tank is small h, here the diameter is capital D and the height of the tank is capital H.

And suppose, the impeller diameter is D i and here the impeller diameter is capital D i. So, what it means is the linear dimensions in the small scale reactor? They will be related to the corresponding dimensions of the large bioreactor by a constant scale factor which means, H upon h i should be equal to D by small d, should be equal to capital H by small h is equal to capital D by small d is equal to capital D i by small d i.

So, they are all related by a constant factor alpha. So, if you see the 2 schematics, geometrical similarity would mean that if h by capital H is equal to.

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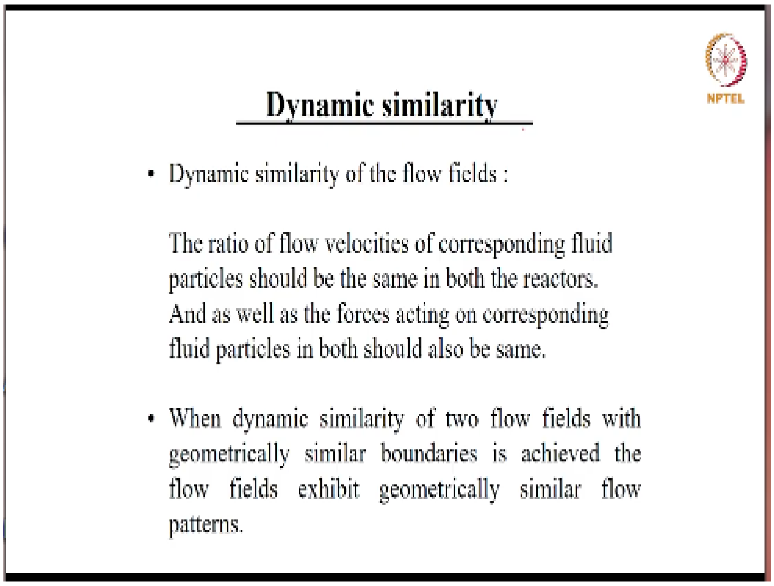

And what does dynamic similarity means? Dynamic similarity is with respect to the flow fields. It means that the ratio of the flow velocities of the corresponding fluid particles should be the same in both the reactors. And as well as the forces acting on the corresponding fluid particles in both should also be same. So, when dynamic similarity of the 2-fold fields with geometrically similar boundaries is achieved, the flow fields exhibit geometrically similar flow patterns.

So, in this case, what it means is that, at a similar location and the 2 scales, the fluid particle should experience same forces which are acting on the fluid particles. And the flow velocities of the fluid particles or the corresponding fluid particles or the 2 scales at a specific location or similar locations should also be the same.

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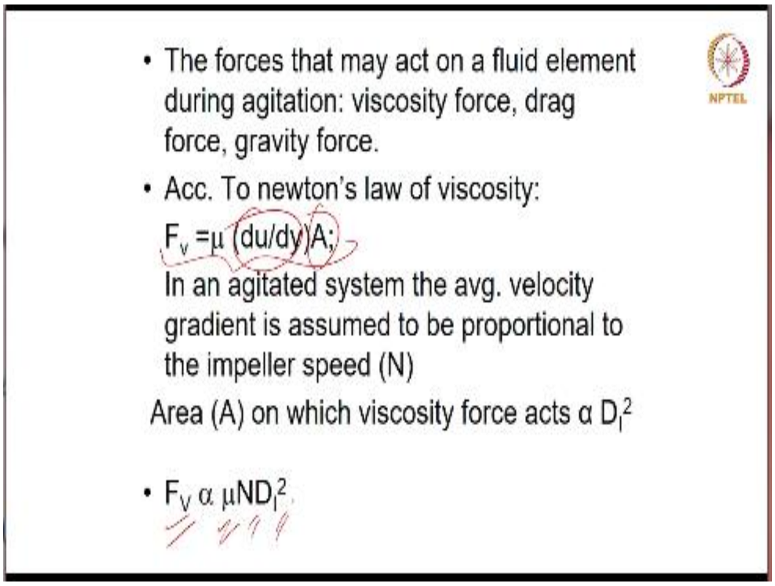

So, the forces that may act on a fluid element during agitation, they include viscous force, drag force, gravity force. Now, according to Newton's law of viscosity, we know viscous force can be given in terms of the stress strain relationship. Now, in an agitated system, the average velocity gradient, this which we have written as du by dy, this can be assumed to be proportional to the impeller speed.

And the area on which the viscous force add which is given here as A is proportional to the square of the impeller diameter. So, then your viscous force can become a function of viscosity, the impeller speed and the diameter of the impeller square.

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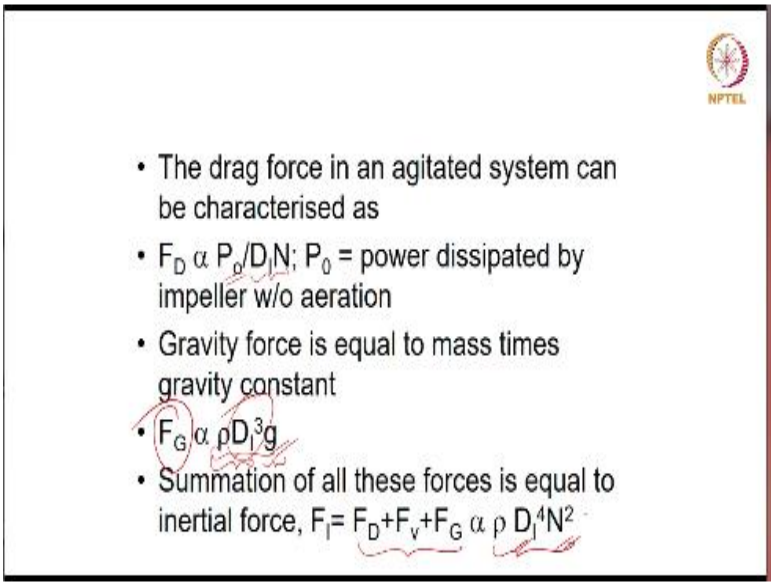

Then the drag force in the agitated system, it can be characterised as given by the power number, where P 0 will be the power dissipated by impeller without aeration. So, here you can say that the drag force will be proportional to the power dissipated divided by the impeller speed and the impeller diameter, which is N D i would be the linear velocity. So, your drag force can be proportional to the power dissipated by the impeller by the linear velocity. Gravity force is equal to mass times the gravitational constant.

So, your mass is density and the volume part has been represented by the cube of the impeller diameter and you have the gravitational constant. So, this becomes the mass and this is the gravitational constant. So, your gravitational force is a function of the impeller diameter and the gravitational constant. Now, summation of all these forces is equal to your inertial force. Now, this is said to be proportional to your diameter of the impeller to the power of 4 and square of the impeller speed.

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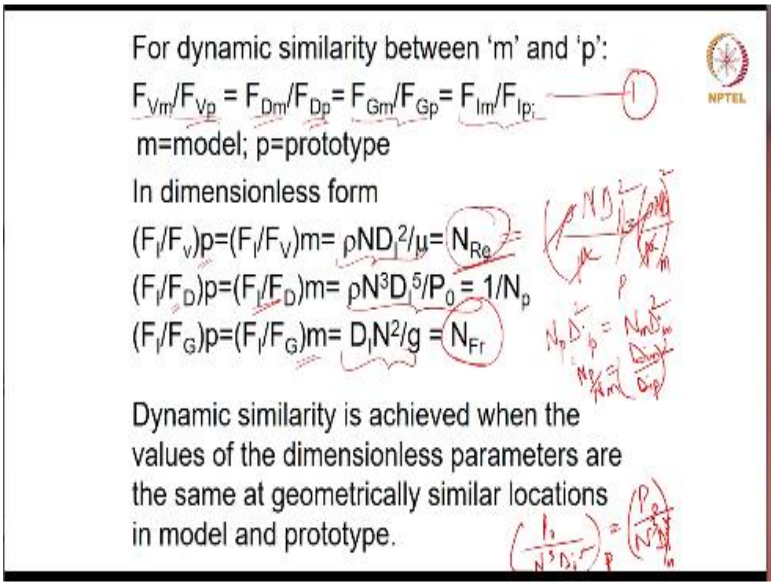

Now, for dynamic similarity between the model and the prototype of the 2 levels, which we will call as m and p. So, dynamic similarity means what? That viscous force at model type to viscous force at the prototype, this ratio is varying can be given by a constant factor and the ratio of the forces acting on the fluid element at the 2 scales are equal. So, your viscous forces model type to viscous force of prototype is equal to drag force at the model type to drag force at the prototype. Similarly, your gravitational force and inertial force.

So, in dimensionless form, you do some rearrangement, your inertial force and viscous force ratio at the prototype can be made equal to the inertial force to viscous force ratio at the model type. Now, the ratio of inertial force to viscous force is what we call as Reynolds number and you can write it as rho N D i square by mu. Similarly, going by the equality given in the line 1.

Your ratio of the inertial force to drag force at the prototype can be made equal to the ratio of the inertial force to drag force at the model type by rearranging the equation 1. And this ratio is nothing but your inverse of power number. Similarly, the ratio of the inertial force to gravitational force at the prototype will be equal to the ratio of the inertial force to gravitational force at the model type. And this ratio is called as Froude's number and can be represented as N squared D i by g.

So, the dynamic similarities achieved only when the values of these dimensionless parameters are the same at the geometrically similar locations in model and the prototype.

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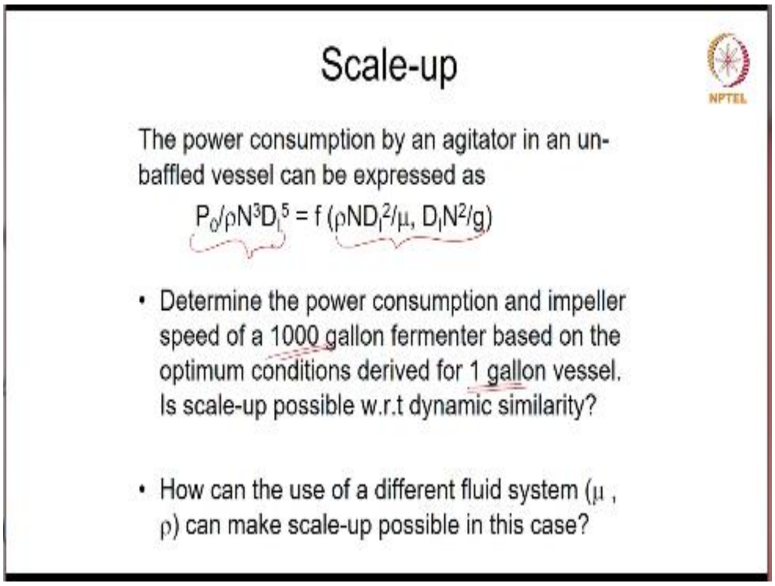

So, let us see by a scale-up is a challenge after understanding that for effective scale-up, what is the principle behind scale-up? So, we have understood now that it is the dynamic similarity which should be met and the geometrical similarity. Now, why it is a challenge to meet the similarities? Let us take this example to understand, the power consumption by an agitator in an unbaffled vessel.

So, your power number is on the LHS, which is set to be a function of your Reynolds number and determine the power consumption and impeller speed of 1000 gallon fermenter based on the optimum conditions derived from 1 gallon vessel. So, our model type volume was 1 gallon and the prototype was 1000 gallon. So, we need to find out first the impeller speed and the power consumption.

And then there is a question which has to be answered, which is a scale-up possible with respect to dynamic similarity.

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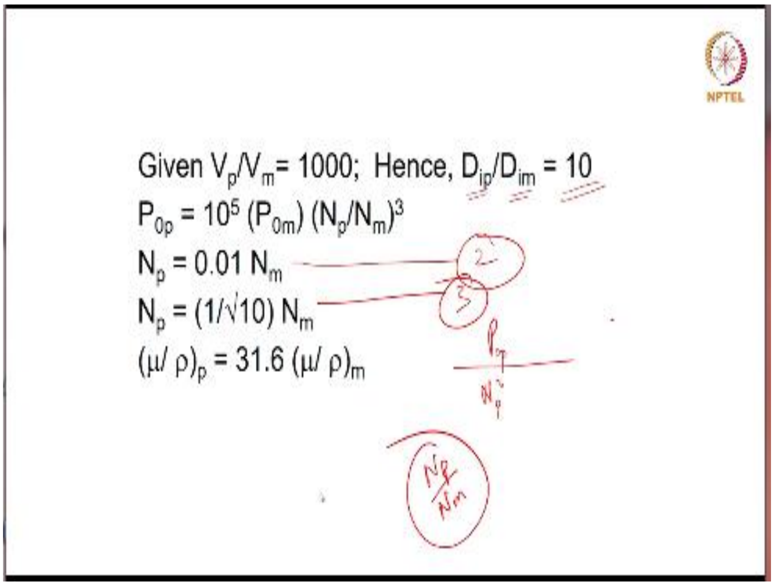

So, let us see. Now, we know that the ratio of the volume or the prototype to model type is given 1000. So, from here, we can find the ratio of the diameter of the impellers of the prototype and the model type which will be your 10. Let us go one by one. Let us take the first requirement, the N Re to be same. So, if N Re is equal at the 2 levels, the correlation between N and D i, because the liquid properties will not change at the 2 levels.

So, let us call this as prototype and the other one as model type. So, the liquid properties can be cancelled, because they will not change. So, then your N p D ip square will be equal to N m D im square. So, your N p by N m ratio is equal to D im by D ip the whole square. Is not it? So, this comes out to be a relationship by taking care of N Re being same at the 2 levels. Now, what about the similarity for the second dimensionless number, which is your power number.

So, if we keep N p same, then your P 0 by N cube D i to the power of 5 at the prototype should be equal to P 0 by N cube D i to the power of 5 and the model type, we know. Now, that P 0 at prototype by P 0 at model type will be equal to. So, here, we know D ip by D im was equal to 10. So, that is what has been substituted here, which will be 10 to the power of 5.

So, if you rearrange this, your P 0 at prototype will become equal to P 0 at model type multiplied by 10 to the power of 5 multiplied by N p by N m the whole cube. And from the previous N Re substitution, we could find that N p by N m was equal to D m by D p the whole square. So, this was one to 10 square, 1 by 100. So, we know that N p by N m is equal to 1 by 100. This is what is being given here.

So, from your first similarity, N p by N m is coming out to be 1 by 100. So, if you see here, Froude's number similarity at the model type should be equal to prototype, should be equal to that other model type. Gravitation constant is the same. So, this is 10. Your N p by N m is 1 by under root 10. But if you remember by the power number similarity, we were finding that N p by N m is equal to 1 by 100.

So, if you see 2 and 3 are 2 different types of correlations for the ratio of N p by N m, which is not possible. So, hence, we can find that keeping dynamic similarities same is very challenging, because generally, what is observed? If you try to keep one criteria met, the other is not possible to be the same. Now, under what circumstances, this can be same only if we manipulate the properties of the fluid, where viscosity, density if they can be made to change.

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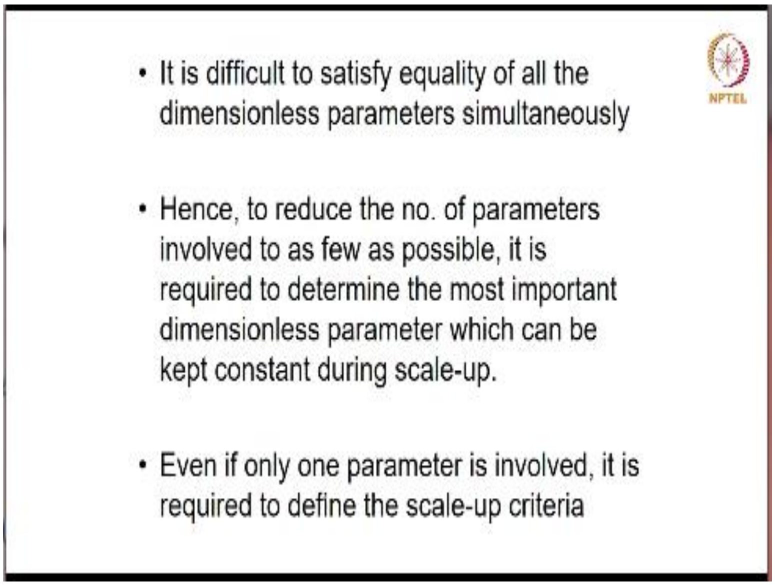

But it is the scale-up of the same system which is to be done. So, therefore, it is difficult to satisfy equality of all dimensionless parameters simultaneously. Hence, to reduce the number of parameters involved as to make them as few as possible, it is required to determine the most important dimensionless parameter, which can be kept constant during the scale-up. So, even if only one parameter is involved, it is required to define the scale-up criteria.

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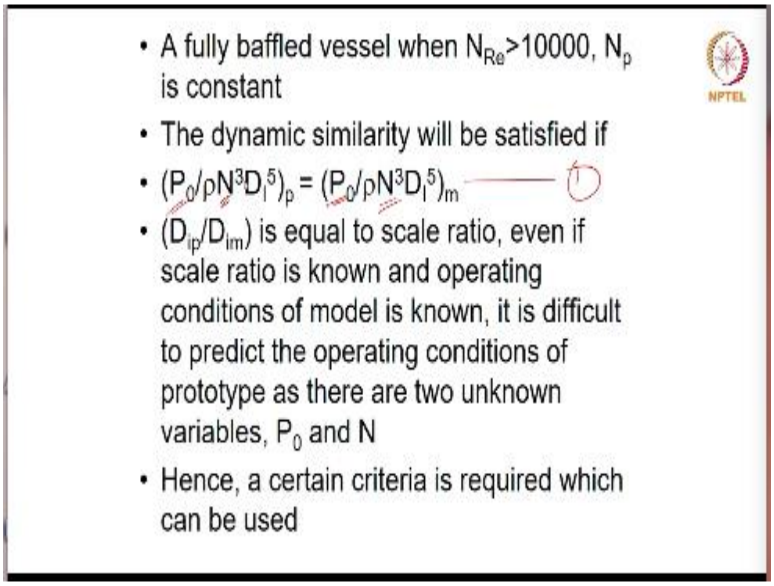

Like for example, for a fully baffled vessel. We can assume that N Re would be greater than 10 to the power of 4, which means it will be in the turbulence region and your power number can be constant. Let us take an example of rustling turbine. So, it can be 10 constant at a value of 6. So, then the dynamic similarity will be satisfied, if the power numbers are same at the prototype and the model type.

So, with this similarity, if we can notice in the equality given here for the power numbers, then your diameter of the impeller ratios are the prototype and the model type is equal to the scale ratio. So, if the scale ratio is known and the operating conditions of the model is known, it is difficult to predict the operating conditions of the prototype still. Why? Because there are 2 unknowns, still P 0 is unknown and the N is also unknown.

So, hence, we then apply a scale-up criteria which can be used to keep this dimensionless number same. So, your ratio of the diameters of the impellers at the prototype and the model type, this will be the scale ratio. So, even if the scale ratio is known and the rest of the operating conditions of the model type is known, it is still very difficult to predict the operating conditions of the prototype. Because there are still 2 variables, the power input and the impeller speed. So, therefore, we again select certain criteria which can be used for scale-up.