Bioreactor Design and Analysis
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Lecture - 35
Scale-up of Bioreactors – Part 3

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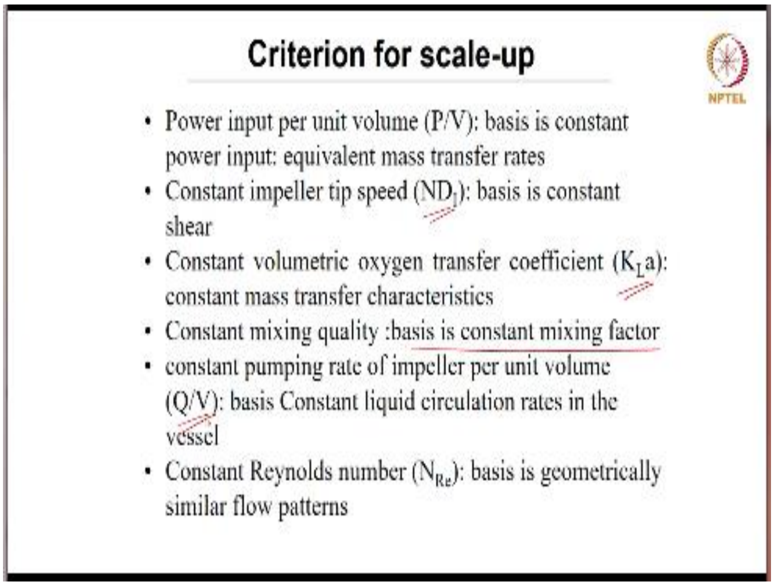

So, let us see, what are the different criteria which are used for scale-up power input per unit volume? Now, the basis of these scale-up criteria is constant power input, which is an equivalent to equivalent mass transfer rate. Constant impeller speed is other criteria, which is here given us ND I. This is the basis for constant shear. Now, the third criteria is constant volumetric oxygen transfer coefficient which is defined as K L a. Now, we already know about this in order to ensure constant mass transfer characteristics in the vessel.

Then if we need to have constant mixing quality, then the basis is constant mixing factor, which is in turn a function of mixing time. Now, if constant pumping rate of the impeller per unit volume is found to be crucial and becomes the basis for scale-up which is given us Q by V. This would ensure constant liquid circulation rates in the vessel. Similarly, constant Reynolds number, this will be the basis for geometrically similar flow patterns, other 2 scales.

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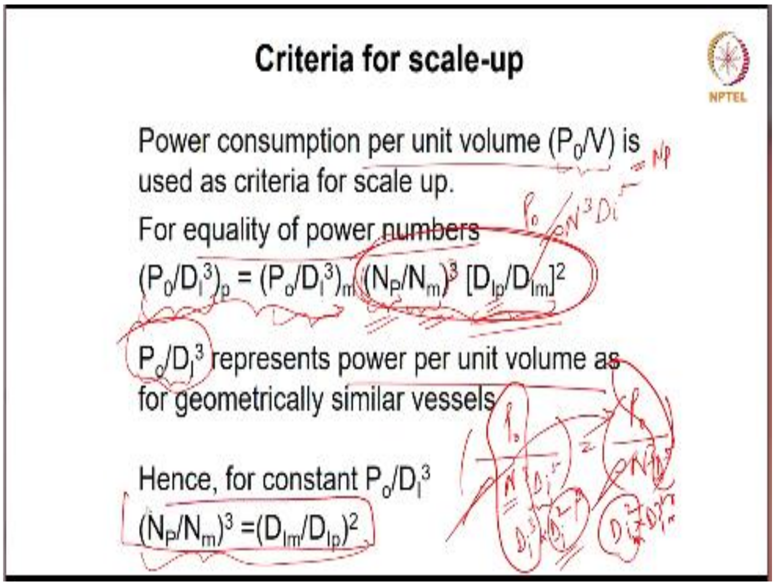

So, let us take these scale-up criteria one by one and try to derive relationships between the different operating parameters are the 2 scales. So, power consumption per unit volume is the first scale-up criteria. If you do equality of power numbers and do the rearrangement, and then your D i to the power of 5, if it can be broken up as D i cube and D i square separately. So, we know that power number is P 0 by N cube D i the power of 5. Is not it? And you have density.

So, this is your power number. So, equality of power number is to be achieved. So, then your P 0 rho N cube D i to the power of 5 and the prototype which means is equal to P 0 rho N cube D i to the power of 5 other model type, density is the same so, it can be neglected, cancelled. So, here, if you further have this as D i cube into D i square and do the rearrangement and make P 0 D i cube at the prototype here, bring it separately on the LHS.

And here you have P 0 D i cube D i square into D i cube of the model type. So, here is the model type P 0 by D i cube, remaining would be D i prototype divided by D i model type to the power of square. So, square is brought together on the RHS and your impeller speed at the prototype, it goes to the RHS to the impeller speed at the model type whole cube. So, this is a rearrangement of the quality of power number.

Now, P 0 by D i cube as I mentioned earlier is a represents your power per unit volume for the geometrically similar vessels. Now, this is the scale-up criteria which means, at the prototype and the model type, we can consider further that these are same. So, then, if we keep this as a scale-up criteria, this term relate the impeller speeds at the 2 levels with the diameter of the impellers other 2 scales.

So, the co-relation between the 2 variables comes out to be N p by N m to the cube is equal to D im by D ip to the square. So, we have rearranged this portion in this form given here as this equation.

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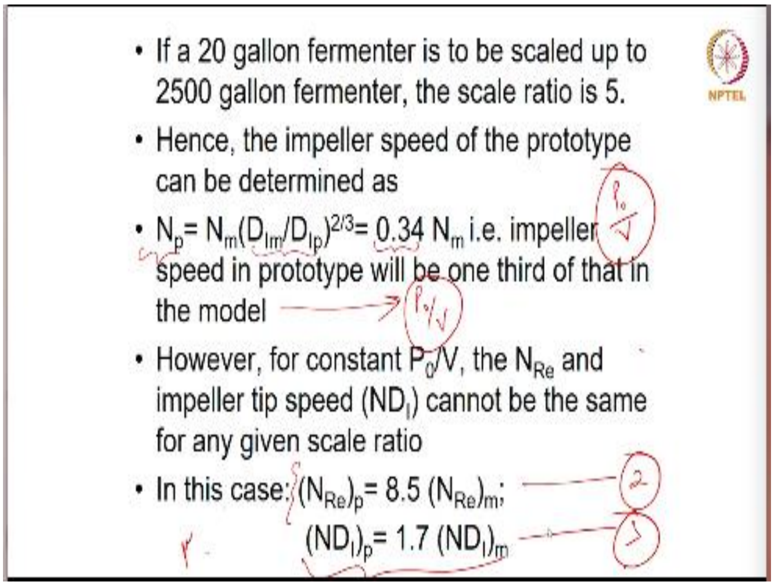

Let us take an example. If a 20 gallon fermenter is to be scaled-up to 2500 gallon fermenter then we know the scale ratio would be 5. The impeller speed of the prototype then can be determined, if P by V is the scale-up criteria. Then in that case, we know now, the diameter ratios can be given as 1 by 5. So, therefore, your impeller speed at the prototype will become 0.34 times of the impeller speed at the model type to keep your power per unit volume constant other 2 levels.

So, if you notice, the impeller speed in the prototype will be one third of that in the model. However, for constant P 0 by V, the N Re and the impeller tip speed cannot be the same for any given scale ratio. So; by keeping your power number equality but compromising on the N Re. So, your N Re will then be related as given here. Similarly, your tip speed which has given as ND i will also be related as shown here in equation 2.

So, your Reynolds number is what? It is rho ND i square by mu, initial inertial force to viscous force. So, this for prototype, you need to find a relationship of rho ND i square by mu other model type. Your N m by N p is equal to 1 by 0.34 and your D im by D ip is equal to 5.

So, if you place this 1 by 5 the whole square and further divided by 0.34, so, for constant P 0 by V the N Re and the impeller tip speed, you will see that they cannot be kept the same.

So, if this relationship holds for constant P 0 by V as shown here between the N p and the N m, then the relationship between N Re at the prototype and at the model type will be shown here as equation 2 and as equation 3 for the tip speed.

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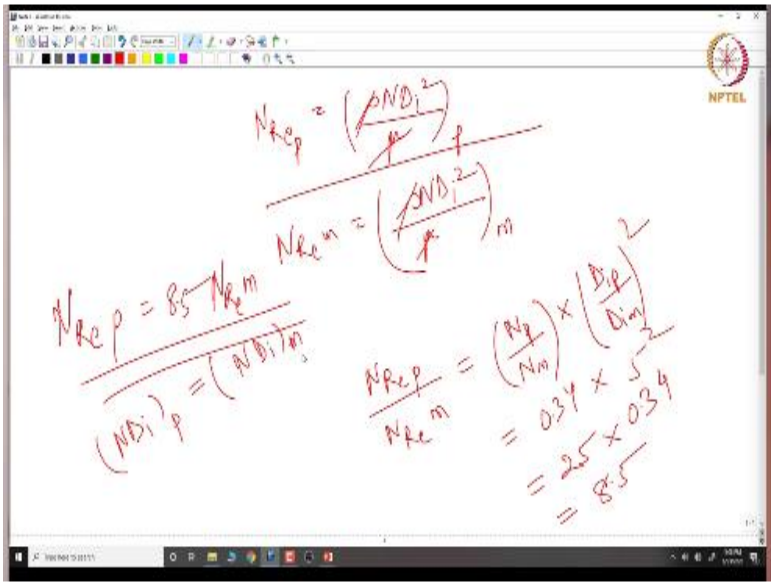

So, N Re is what? Rho ND i square by mu. So, let us call this for prototype and N Re for model type is rho ND i square by mu for the model type. Now, mu and rho are the same. Now, in terms of the prototype, if we divide this, so, N Re at prototype to N Re at model type will become equal to N p by N m multiplied by the D ip by D im the whole square. N p by nm, if you notice N p by N m how are they related for constant P 0 by V was this. N p by N m is equal to 0.34.

And your D ip by D im is related by the scale factor of 5. This is 25 times 0.34 which comes out 8.5. So, your N p, so, we can see that the N Re at the prototype is 8.5 times the N Re at the model type. You can do similarly for the ND i which is called the tip speed at the prototype and here it is the tip speed of the model type.

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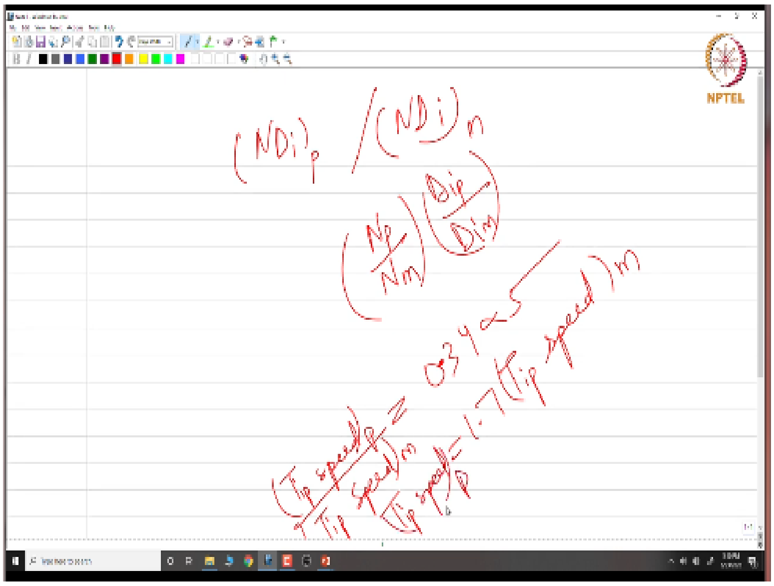

So, tip speed at the prototype by tip speed at the model type, if you do, then we know that N p by N m is 0.34 and D ip by D im is 5. This is the ratio of the tip speed at the prototype and the impeller tip speed at the model type. So, if you need to find the relationship between them, this comes out to be 1.7 times tip speed at the model type will be your impeller tip speed at the prototype. This is what has been shown here in equation 3 and equation 2.

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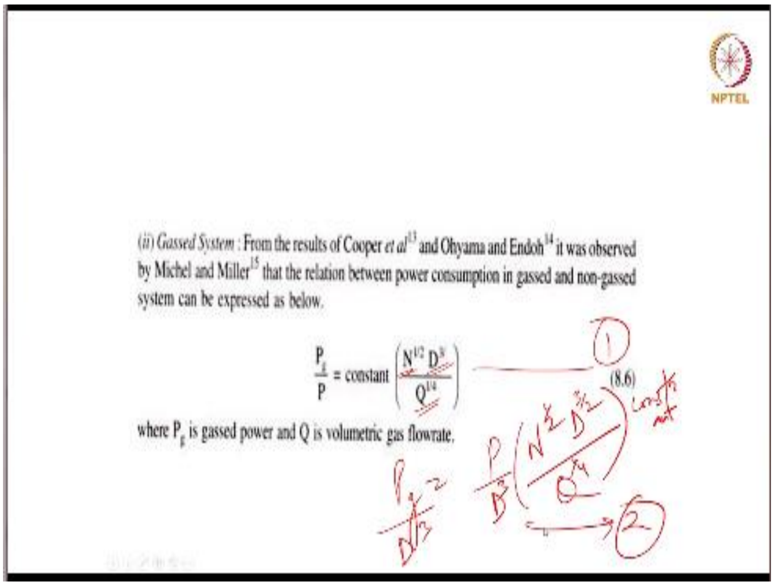

So, if we have a gas system, so, right now, the power per unit volume, the P 0 was the un-gassed power requirement. So, if we have a gas system, will there be a variation in the correlation between the impeller speed and the diameter ratio? So, there is an empirical relationship between the power consumption in gassed and non gas system which is shown here by equation.

Let us call it as equation 1, in terms of the impeller speed, in terms of the diameter of the impeller and the gas flow rate, volumetric gas flow rate, so, the gas power requirement to the un-gas power requirement is set to be proportional to these operating parameters.

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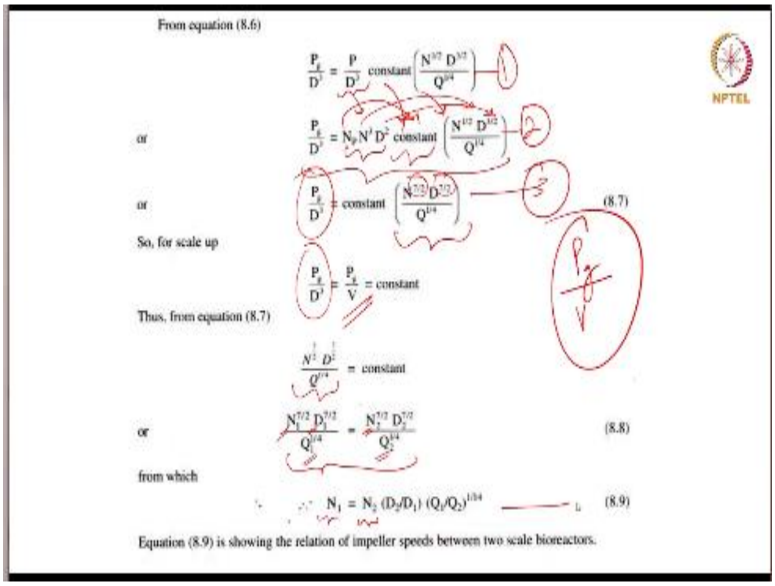

So, if the gas power requirement per unit volume is the scale-up criteria, which means P g. So, the gas power requirement is being called as P g, the un-gas power requirement is P 0 or P here shown on the slide. So, it is said that power per unit volume will be the scale-up criteria and from the previous equation which we called as 1, we can relate the gas power requirement with the un-gas power requirement.

So, your gas power requirement can be given as un-gas power requirement multiplied by this function involving the impeller speed, the diameter of the impeller and the volumetric gas flow rate. Now, if we go by power per unit volume, we can further have; we can divide the situation here both sides by D cube. So, rest is on constant some scale factor, so, constant. So, the same equation 1 can be written as equation 2, as I have written here.

Now, this P by V and this P g by V, we know now, this P by V is can be given as in terms of the power number. So, your power number was what? The power number is P 0 on gas power requirement by N cube D to the power of 5 into a rho. Again, this is a constant. So, then your P 0 by D cube from here can be replaced as N p multiplied by N cube times D square. So, rho will get into the constant, because it is a constant.

And then your this equation can be rewritten in the second line form where we have replaced P 0 by D cube as N p, which is your power number D square and rest of the factor is the same. So, this will become N p times N cube. So, this is what it is. So, power number, this goes up here. So, this is multiplied by the power number N cube D square and we are keeping P 0 by D cube on the RHS.

So, then your equation 1 can be rewritten in the form of equation 2 from the power number correlation. So, now, power number is constant. So, then this can also get into the constant and N then goes inside the brackets and D square also goes inside the brackets, they get added up and the equation 2 can be written as equation 3. So, now, N will become 7 by 2 after addition and D will become 7 by 2, I hope you can understand.

So, it is 3 +1 by 2 to the power of N which is nothing but 7 by 2. Similarly, it has been done for D, 3 by 2 plus squared has been done. So, this becomes again 7 by 2 in same brackets. So, now, if for a gas system power per unit volume is that the scale-up criteria, so, then your gas power requirement per unit volume which was the LHS here is constant. So, which would effectively mean that the; entities in the bracket are nothing but constant at the 2 scales.

So, this will give us a co-relation between the variables the operating parameters, other 2 scales, which involve the impeller speed, the diameter of the impeller and the volumetric gas flow rate. So, this is how the operating parameters at these 2 scales can be related. Linear scale ratio is known. Then, your impeller tips your impeller speeds are the 2 scales can be related by the equation shown here.

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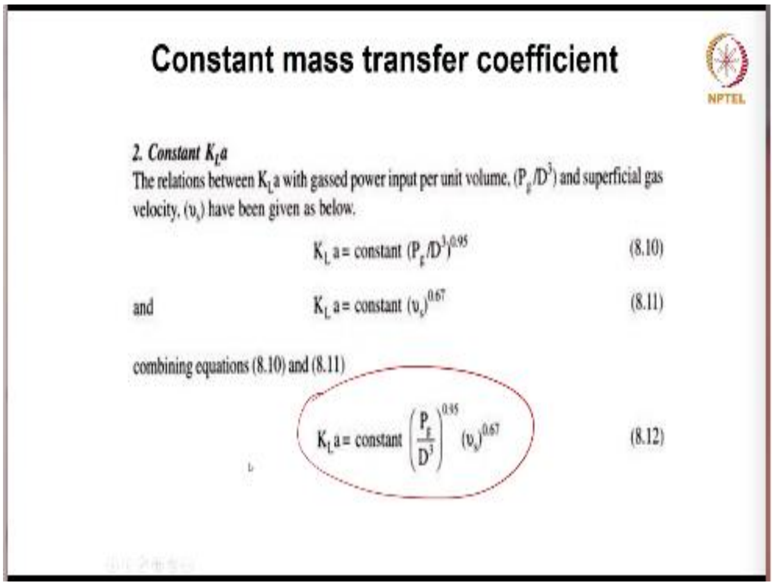

Let us take another scale-up criteria which is volumetric mass transfer coefficient. If suppose, the scale-up criteria being used is constant K L a, then for gassed systems, this volumetric mass transfer coefficient is related to power per unit volume. And your superficial gas velocity in the form of empirical co-relationship shown on the slide where K L a is set to be related to the gas power input per unit volume. And the superficial gas velocity as shown here, this is for us power input system.

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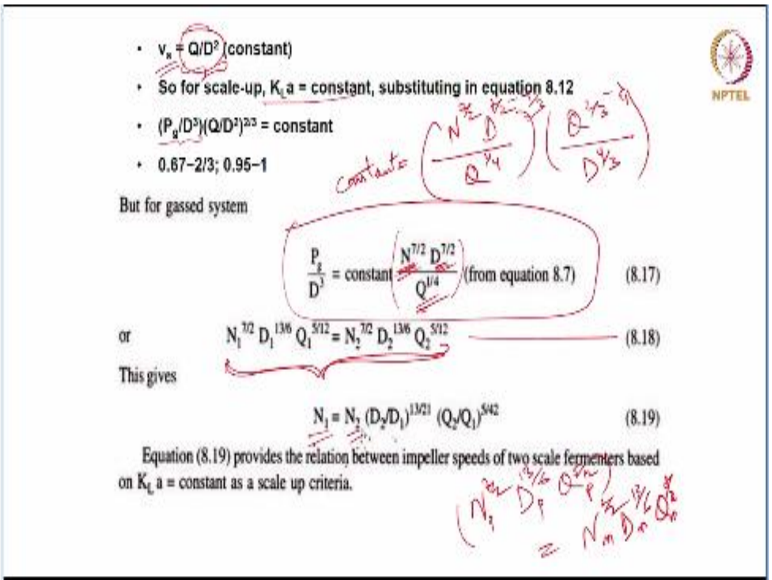

Now, the superficial gas velocity can in turn be written as in the form of the volumetric gas flow rate and the diameter of the impeller square with some constant. So, now again the superficial gas velocity can be written in terms of Q and D as Q by D square. Now, for scale-up, we know that K L a is constant. Now, for making K L a constant, now, this P g by D cube can in turn be written in the form of the operating parameters N, D and the volumetric gas flow rate as shown earlier by this empirical co-relationship as shown here in equation 3.

So, if you see equation 3, we have already derived how P g by D cube can be related to N, D and Q. So, we can replace that co-relation here. So, that the only variables we have now are N, D and Q. So, this turns out to be N 7 by 2 D 7 by 2 by Q 1 by 4 multiplied by Q 2 by 3 by D 4 by 3. So, this is constant which is clearly. So, at the 2 levels prototype and the model type, the co-relationship between all these 3 parameters at the model and the prototype can be given as shown here in this equation.

Where your N 7 by 2 at the prototype, your D it goes minus 4 by 3, if you do 7 by 2 minus 4 by 3, then you become D 13 by 6 and your Q becomes 2 by 3 minus 1 by 4. So, again, it will become 5 by 12; this other prototype is equal to N m 7 by 2 D m 13 by 6 and Q m 5 by 12, this is what has been shown here in this equation, which I have underlined with a curly bracket. So, if now, we know the constant scale factor, then this is how that in the impeller speeds at the 2 scales can be related if K L a is the scale-up criteria.