


**Bioreactor Design and Analysis**  
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**Lecture - 36**  
**Scale-up of Bioreactors – Part 4**

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### Dimensionless mixing factor

- It is dependent on many factors, impeller speed, height of liquid, diameter of the tank, impeller diameter, properties of the liquid
- It is constant under turbulent regime where  $N_p$  is constant

So, 
$$\phi = \frac{t \cdot (ND)^{2/3} \cdot g^{1/6} \cdot D_i^{1/2}}{H_i^{1/2} D^{3/2}} = \text{constant} \quad (8.20)$$

in which  $H_i$  is liquid height = 1.2 D,  $D_i$  is impeller diameter = 1/3 D,  $t$  is mixing time and  $g$  is gravitational factor. Putting these values in equation (8.20) one has



So, welcome back students. Yesterday, we were discussing about the scale-up criteria, which are to be used to keep dynamic similarity at the 2 scales during scale-up. In order to reproduce the results, what we get at the lab scale to get reproduced at large scale level. So, in that we were discussing that there are various scale-up criterias where yesterday we discussed in detail, the power per unit volume scale-up criteria, which is equivalent to considering equivalent mass transfer characteristics and both the scales.

And then volumetric mass transfer coefficient, which will be demonstrating equalent oxygen mass transfer characteristics at both the scales. So, now, we use these scale-up criterias and if you remember, we were able to devise, how with this scale-up criteria? The different operating parameters like the impeller speed, the volumetric gas flow rate in case of a sparged system and diameter of the impellers are the 2 scales, the model and the prototype which we named it, it will be correlated.

So, we had devised various correlations between them. So, now, the third criteria will be the dimensionless mixing factor. Now, this is dependent on many factors like the impeller speed,

height of the liquid in the vessel, diameter of the tank, impeller diameter and the properties of the liquid. So, it is to be constant under the turbulent regime where the power number is constant.

So, dimensionless mixing factor can be considered as constant in the turbulence regime, where we know the power number is also constant. So, if you see the empirical correlation here given on the slide, the dimensionless factor is a function of the mixing time, the impeller speed, the diameter of the impeller, height of the liquid in the tank, diameter of the tank and the gravitational constant.

Now, as the liquid height being a linear dimension can be related to the diameter of the tank by thumb rules. So, the liquid height can be represented as a function of the diameter of the tank. And further the impeller diameter again can also be related by thumb rules to diameter of the tank. So, effectively this entire correlation can be made in the terms of the impeller speed and the diameter of the impeller and mixing time.

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$$\frac{t \cdot (ND_1)^{2.7} \cdot g^{0.45} \left(\frac{L}{D}\right)^{0.2}}{(LD)^{0.2} D^{0.2}} = \text{constant} \quad (8.21)$$

Simplifying equation (8.21)

$$t \cdot N^{2.7} D^{-3.9} = \text{constant} \quad (8.22)$$

Also, from equation (8.3)  $N_1$  constant, power per unit volume constant

$$N^2 D^5 = \text{constant} \quad (8.23)$$

Taking 18th power of equation (8.22)

$$t^{18} N^{48.6} D^{-70.2} = \text{constant} \quad (8.23)$$

and taking 4th power of equation (8.3)

$$N^{12} D^2 = \text{constant} \quad (8.24)$$

Dividing equation (8.23) by equation (8.24)

$$t^{18} D^{-11} = \text{constant} \quad (8.25)$$

or

$$t_1^{18} D_1^{-11} = t_2 D_2^{-11} \quad (8.26)$$

from which

$$t_2 = t_1 \left(\frac{D_2}{D_1}\right)^{11/18} \quad (8.27)$$

Equation (8.27) provides the relationship between mixing time in two scales fermenters when constant mixing quality is the basis of scale up.

So, if you put all these correlations of height in terms of the diameter of the tank and the impeller diameter in terms of diameter of the tank, then we can reduce that same correlation in the form given here. So, if you see the, I will name it as equation, on this screen, it has equation 8.22; this is directly a screenshot from the book. So, let us call this as equation 1. So, t is the mixing time and N is the impeller speed and D is the diameter of the tank.

Now, we also know that the power number is constant. So, if power number is constant and power per unit volume is also constant. Then the correlation which holds true for impeller speed and diameter can be given as  $N^3 D^2$ , which is given here. So, now, if you take 18th power of equation 1 shown by me, then you will end up in the equation shown here as 8.2.3 which I will call it as 2 further, if the power per unit volume constant in the turbulence region.

Where power number is constant, then  $N^3 D^2$  is also a constant. If we take the 4th power of it, then this is also constant. So, now, we can substitute this in equation 2 and the only 2 variables remain here in the correlation will be the mixing time and the diameter. So, now, the mixing times at the 2 scales can be related with the ratio of the diameters of the tank and the 2 scales.

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- $P \propto N^3 D_i^5$
- $V \propto D_i^3$
- $Q \propto N D_i^3$
- $P/V \propto N^3 D_i^2$
- $Q/V \propto N$
- Thus, fixing  $N$  and  $D_i$  fixes all these quantities

So, effectively, this is how power input is related to impeller speed and diameter of the impeller. This is how volume of the tank is related to the diameter of the impeller. Knowing the diameter of the impeller is proportional to the diameter of the tank, some scale ratio. And the volumetric gas flow rate is also related to the impeller speed and the diameter of the impeller, power per unit volume can also be related now.



So, your equation 1 can then change to let us call this as equation 3, 2 and this will be 4. So, if you apply one divide equation 1 by  $D_i Q$ , then you will end up in equation the correlation 4. Similarly, your third correlation, so, where this is  $Q$  by  $V$  criteria, thus volumetric gas flow

rate per unit volume will give you the 5th correlation, where this is just a function of the impeller speed.

So, why we have written these correlations? If you notice, this means that, that most of these quantities power, volume, volumetric gas flow rate or power per unit volume or volumetric gas flow rate per unit volume which may be useful in scale-up are related to impeller speed and the diameter of the impeller. This means that fixing the diameter of the impeller or the impeller speed can fix all these said quantities.

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Property	Pilot scale	Plant scale 10000L			
	80 L	Constant P/V	Constant N	Constant ND	Constant NRe
P	1	125	3125	25	0.2
P/VOL	1	1	25	0.2	0.0016
N	1	0.34	1	0.2	0.04
D (impeller Diameter)	1	5	5	5	5
Q (Fluid Pumping rate of the impeller)	1	42.5	125	25	5
Q/VOL	1	0.34	1	0.2	0.04
Max. impeller speed (ND)	1	1.7	5	1	0.2
Reynolds number (ND <sup>2</sup> v/u)	1	8.5	25	5	1

Let us take some examples here. If you see the table here, the pilot scale is at which we may call as model is 80 litre, the prototype or the plant scale is 10,000 litres as shown here. So, your scale will become will be the volume ratio. So, your diameter linear diameter scale will be the scale factor is 5. So, pilot scale if we consider the ratio, then at constant P by V, the power will be changing.

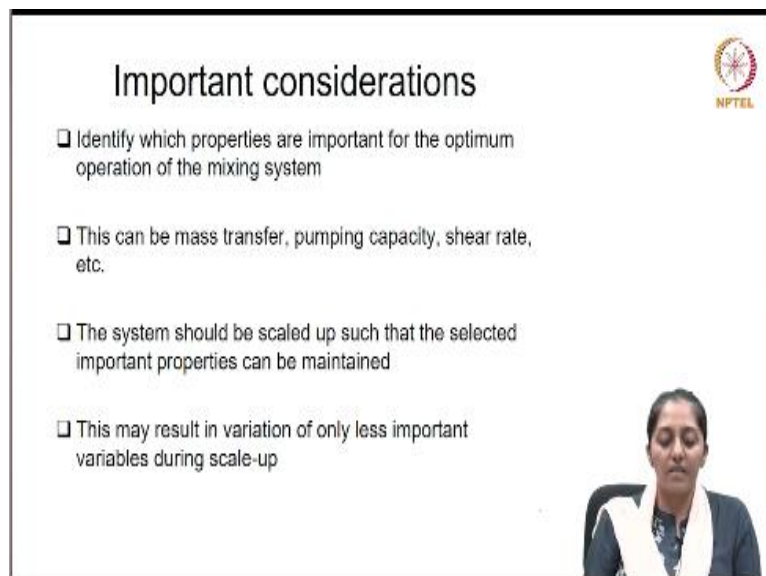
If you remember, how power was related? The power was related as  $N^3 Q D^5$  to the power of 5 and your diameter scale is 5 times. So, your constant P by V if it is the criteria, then it is  $D^5$  square N cube. So, they are writing here that the P by V, your power will change 125 times add the plant scale, the power will change by a factor of 3125 if P is 1 there. Then at constant impeller tip speed which is  $ND$ , the power will change by a factor of 25 and at constant N Re, then your power is getting reduced by 0.2.

Then the property of power per unit volume, if at the pilot scale which is model type, the power per unit volume at constant  $P$  by  $V$ , obviously, it will remain the same. With constant  $N$  which is impeller speed, the power per unit volume changes by 25 volts. At constant impeller tip speed, it will change by 0.2 volts and at constant energy it will change by 0.0016 volts. So, if you see similar information as given here, we have highlighted few things in red colour.

Ones which are showing as one which means that there is no change in that property, because it was held constant are the 2 scales. The which of the property when held constant is bringing about minimum change in the properties on the left hand side, first column. So, let us take for example, the numerals which are being highlighted in red here at constant  $P$  by  $V$ , it demonstrates how various properties shown in the first column left hand side will be changing at the plant scale level.

The power will be changing by a factor of 125,  $P$  by  $V$  was constant. So, it remains the same, the impeller speed at 0.34, the diameter of the impeller 5 volts, the fluid pumping rate of the impeller which is  $Q$ , it is 42.5 and fluid pumping rate per unit volume affects the suspension. So, this is 0.34. Maximum impeller speed will change by 1.7 and Reynolds number will change by 8.5 volts. So, in this case, I find that when we keep constant  $N Re$ , the volt change is minimum in most of the properties.

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**Important considerations**

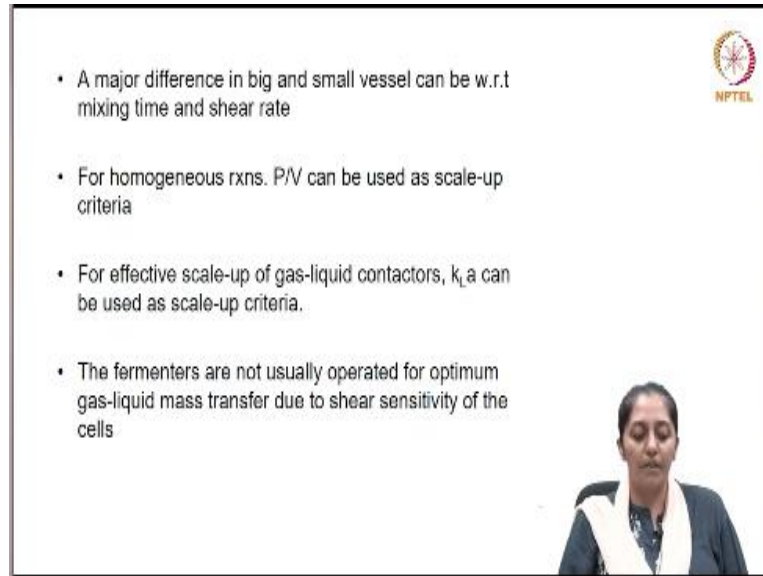
- Identify which properties are important for the optimum operation of the mixing system
- This can be mass transfer, pumping capacity, shear rate, etc.
- The system should be scaled up such that the selected important properties can be maintained
- This may result in variation of only less important variables during scale-up

The slide also features the NPTEL logo in the top right corner and a video inset in the bottom right corner showing a woman speaking.

So, what are the important considerations? First, we need to identify which properties are important for the optimum operation of the mixing system. Now, this can be mass transfer or

pumping capacity or shear rate or any other such factors. Now, the system should be scaled up such that the selected important property can be maintained. So, as to bring about reproducibility. Although, this may result in variation of less important properties during scale-up.

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- A major difference in big and small vessel can be w.r.t mixing time and shear rate
- For homogeneous rxns.  $P/V$  can be used as scale-up criteria
- For effective scale-up of gas-liquid contactors,  $k_L a$  can be used as scale-up criteria.
- The fermenters are not usually operated for optimum gas-liquid mass transfer due to shear sensitivity of the cells

So, the major difference in the big and the small vessel can be with respect to the mixing time and the shear rates. Now, for homogeneous reactions, power per unit volume can be used as a scale-up criteria, cells are uniformly suspended, it can be called as a homogeneous mixture. For effective scale-up of gas-liquid contactors, where 2 phase interactions are important gas-liquid interactions then  $K_L a$  can be used as a scale-up criteria.

The fermenters that are not usually operated for optimum gas-liquid mass transfer due to shear sensitivity of the cells.

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1. Constant power per unit volume input  


$$N_2 = N_1 (D_1/D_2)^{2/3}$$

2. Constant tip velocity  

$$N_2 = N_1 (D_1/D_2)$$


3. Constant dimensionless mixing factor  

$$\frac{t_{M,2}}{t_{M,1}} = \left( \frac{N_1 D_1}{N_2 D_2} \right)^{1.6}$$




So, let us see the correlation. Once again for constant power per unit volume, the correlation between the impeller speeds and the diameters is shown here. Then for constant tip velocity which was  $N D$ , this will be the correlation which is very simple. Then constant dimensionless mixing factor will give as the correlation here. If you remember, equation 1 shown here was a correlation for dimensionless mixing factor.

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If constant power input ( $P/V$ ) is maintained in both scales of operation

$$\frac{t_{M,2}}{t_{M,1}} = \left( \frac{D_2}{D_1} \right)^{11/18}$$



So, if apart from the constant mixing factor, if we can imply constant power input per unit volume at both the scales, then further your correlation 3 can be reduced to only correlation relating the mixing time with the diameters.

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**Steps to be followed in scale-up**



- Investigate the process in the laboratory (selecting the strain, suitable raw materials and their composition)
- Optimize the process condition (pH, temperature etc.)
- Measure Q, N, yields, fermentor dimensions.
- Vary Q,N and study its effect.




So, what are the basic steps which one must follow during scale-up? First, investigate the process in the laboratory, select the strain, the suitable raw materials and their composition. Optimise the process conditions like pH, temperature, etcetera. Measure the properties like volumetric gas flow rate, the impeller speed, various yield, parameters, yield of biomass with respect to substrate, yield of oxygen biomass with respect to oxygen consumed, yield of product with respect to biomass.


Or yield of product with respect to substrate consumed to various yield, parameters and the dimensions of the fermenter. Now, vary Q which is volumetric gas flow rate or the pumping rate, impeller speed and study its effect on the performance of the reactor.

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**Steps to be followed in scale-up**



- Determine broth properties (density, viscosity, diffusivity etc.)
- Calculate  $VVM = Q/V$ , aeration number ( $Q/ND_i^3$ ), shear rate and Reynolds number.
- Predict power input, power input per unit volume, volumetric mass transfer coefficient etc.






Further determine the broth properties, the liquid properties, density, viscosity, diffusivity and other parameters of the liquid. Calculate the VVM which is  $Q$  by  $V$ , the volumetric gas flow rate per unit volume. So, aeration number which is nothing but  $Q$  by  $N D^3$ , shear rate and Reynolds number. So, with the help of the properties measured, we can calculate the gas flow rate in terms of VVM, aeration number, shear rate and Reynolds number. Now, predict power input, power input per unit volume, volumetric mass transfer coefficient.

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### Steps to be followed in scale-up

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- Determine which parameter influences the yield and productivity. This parameter will be kept constant on both scales
- Go to the plant scale bioreactor calculations.
- Choose volume required based on yield and required capacity
- Calculate the dimensions based on geometrical similarity.




Now, determine which parameter influences the yield and productivity most. This parameter should then be kept constant on both scales. Now, going to the plant scale reactor calculations, choose the volume required based on the yield and required capacity, calculate the dimensions based on geometrical similarity.

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### Steps to be followed in scale-up

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- Based on the chosen scale-up criterion, determine airflow rate, rotational speed of the impeller.
- Calculate the power consumption and other parameters.



Based on the chosen scale-up criteria, determine the airflow rate, the rotational speed of the impeller at the large scale of the prototype. And calculate the power consumption and other parameters.