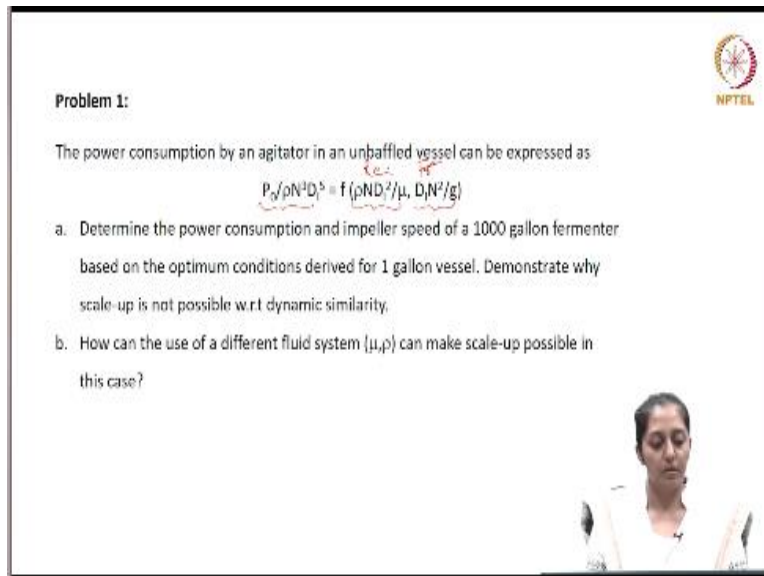


Bioreactor Design and Analysis
Dr. Smita Srivastava
Department of Biotechnology
Indian Institute of Technology – Madras

Lecture - 37
Scale-up of Bioprocesses – Practice Problems

Welcome back students. So, today, we will do some practice problems on scale-up of bioprocesses. So, let us begin with the first problem.

(Refer Slide Time: 00:26)



The slide contains the following text:

Problem 1:

The power consumption by an agitator in an unbaffled vessel can be expressed as

$$P_g/\rho N^3 D^5 = f(\rho N D^2/\mu, D_i N^2/g)$$

a. Determine the power consumption and impeller speed of a 1000 gallon fermenter based on the optimum conditions derived for 1 gallon vessel. Demonstrate why scale-up is not possible w.r.t dynamic similarity.

b. How can the use of a different fluid system (μ, ρ) can make scale-up possible in this case?

The slide also features the NPTEL logo in the top right corner and a small video inset of a woman in the bottom right corner.

See if you can see on the slide, the problem here is about power consumption by an agitator in an un-baffled vessel. So, we know that this is how this expression which is given the power number is a function of your Froude's number and impellers Reynolds number. So, your LHS stands for the power number and inside the 2 terms, they stand for the impeller Reynolds number and your Froude's number.

So, based on this correlation, we need to determine the power consumption and impeller speed of 1000 gallon fermenter based on the optimum conditions derived using 1 gallon vessel. So, we also need to demonstrate that whether scale-up is possible or not with respect to dynamic similarity.

(Refer Slide Time: 01:36)

p = prototype, m = model

Given, $\frac{V_p}{V_m} = 1000$ → 1

Scale ratio, $\frac{(D)_p}{(D)_m} = (1000)^{1/3} = 10$ → 1

To achieve dynamic similarity,



$$\left(\frac{P_p}{\rho N^3 D^5}\right)_p = \left(\frac{P_m}{\rho N^3 D^5}\right)_m \quad \rightarrow 2$$

$$\left(\frac{N D^2}{A}\right)_p = \left(\frac{N D^2}{A}\right)_m \quad \rightarrow 3$$

$$\left(\frac{N^2 D}{g}\right)_p = \left(\frac{N^2 D}{g}\right)_m \quad \rightarrow 4$$

If we use the same fluid for the model and the prototype, $\rho_p = \rho_m$ and $\mu_p = \mu_m$

From equations 1 and 2,

$$(P_p)_p = 10^5 (P_m)_m \left(\frac{N_p}{N_m}\right)^3$$



So, let us first look at the first part of the problem. So, we will mark p with the prototype and we will have model marked as m. So, your volume of the prototype to model type, it is increasing by 1000 volts from the model type. So, this is one correlation between the volumes of the 2 level. So, we know now the scale ratio, which is the ratio of the linear dimensions at the model and the prototype would be taking the cube root of this, which will be a factor of 10.

So, for the linear dimensions, the ratio is 10, the scale-up ratio. Now, in order to achieve dynamic similarity, your dimensionless numbers should be equal at the 2 scales, which is the model and the prototype and we are using the same fluid so, obviously, the properties of the fluid will not change are the 2 scales so, we can negate those terms. So, by the equality of these dimensionless numbers for dynamic similarity, you can find a correlation between the power consumed and the impeller speed at the 2 levels where we know the linear dimensions are changing by a factor of 10.

So, using the equation 2 given here, we will get this correlation which will correlate your power consumed with the impeller speed at the 2 levels.

(Refer Slide Time: 03:20)

Equality of Reynolds number requires,

$$N_p = 0.01 N_m$$

Equality of Froude number requires,

$$N_p = \frac{1}{\sqrt{10}} N_m$$



Since the equalities are conflicting, it is impossible to satisfy the requirement of dynamic similarity unless we use different fluid systems

If $\rho_p \neq \rho_m$ and $\mu_p \neq \mu_m$, to satisfy equations 3 and 4, the following relationship must hold

$$\left(\frac{\mu}{\rho}\right)_p = \frac{1}{31.6} \left(\frac{\mu}{\rho}\right)_m$$

Therefore, if the kinematic viscosity of the prototype is similar to that of water, the kinematic viscosity of the fluid, which needs to be employed for the model, should be 1/31.6 of the kinematic viscosity of water. It is impossible to find the fluid whose kinematic viscosity is that small.

Hence, if all three dimensionless groups are important, it is impossible to satisfy the dynamic similarity.

While, if you use Reynolds number equality as shown here in 3, equation 3, then the liquid properties I said can be neglected, the linear dimensions are related by a factor of 10 at the 2 levels. So, your impeller speeds at the 2 levels are correlated as given here. We call this as equation 5. So, this will call it as equation 5; this is equation 6. So, if you see equation 5 where the equality of power number was assumed, we saw that this is how power consumed and impeller speed can be related.

And equality of Reynolds number gave you the correlation between the impeller speeds at the 2 levels as shown here. And if you do the quality of Froude's number given in equation 4, then the linear dimension factors are known. The gravitation constant value is same, linear dimensions are related by a factor of 10. So, your impeller speeds at prototype and model type can be related as and under root of 10.

So, if you see the equality of these dimensionless numbers, it is very clear that it is very difficult to keep all the dimensionless numbers same at the 2 levels. Because, then the equality between the operating parameters here, it is impeller speed changes. So, since the qualities are conflicting, one can make out that it is impossible to satisfy the requirement of dynamic similarity until unless we change the fluid properties.

So, if at the model and the prototype, the density and the viscosity are unequal, then equation 3 and 4 where the quality of a knowledge number and we were looking at the quality of Froude's number, so, here gravitation constant is the same. So, we will get an equality that N_p is 1 by under root 10 of N_m . If this can be substituted here so, we can find, how the fluid

properties, the μ by ρ at the prototype will be related to μ by ρ of the model type, which comes out to be changing by a factor of 31.6.

So, if the kinematic viscosity, so, μ by ρ is nothing but the kinematic viscosity is similar to that of water, let us assume of the prototype, it is similar to that of water, then the kinematic viscosity of fluid which needs to be employed for the model should be 1 by 31.6 of the kinematic viscosity of water. It is impossible to find the fluid whose kinematic viscosity is that small. Hence, all 3 dimensionless groups, they play an important role.

However, keeping all of them equal at the model and the prototype to satisfy the dynamic similarity is very difficult. So, one has to find depending on which characteristics are important whether heat transfer characteristics, mass transfer characteristics or shear sensitivity or oxygen mass transfer is more crucial, we use the scale-up criteria.

(Refer Slide Time: 07:43)

Problem 2:

A pilot-scale fermenter of diameter and liquid height 0.5 m is fitted with four baffles of width one-tenth the tank diameter. Stirring is provided using a Scaba 6SRGT curved-blade disc turbine with diameter one-third the tank diameter. The density of the culture broth is 1000 kg/m^3 and the viscosity is 5 cP. Optimum culture conditions are provided in the pilot-scale fermenter when the stirrer speed is 185 rpm. Following completion of the pilot studies, a larger production-scale fermenter is constructed. The large fermenter has a capacity of 6 m^3 , is geometrically similar to the pilot-scale vessel, and is also equipped with a Scaba 6SRGT impeller of diameter one-third the tank diameter.

- What is the power consumption in the pilot-scale fermenter?
- If the production-scale fermenter is operated so that the power consumed per unit volume is the same as in the pilot-scale vessel, what is the power requirement after scale-up?

Let us see the problem 2. So, a pilot scale fermenter of diameter and liquid height, so, diameter and liquid height both are 0.5 metres, is fitted with 4 baffles of width one tenth of the tank diameter. Stirring is provided using Scaba curved blade disc turbine which is one third of the tank diameter. So, the impeller diameter is of this impeller is one third of the tank diameter. The density of the culture broth is given as 1000 kg per metre cube, the viscosity is 5 centipoise.

So, optimum culture conditions are provided in the pilot scale fermenter, where the impeller speed is 185 rpm. Now, following completion of the pilot studies, a larger production scale

fermenter is to be constructed. The larger fermenter has a capacity of 6 metre cube and is geometrically similar to the pilot scale vessel. So, geometrical similarity has been met and it is also equipped with a impeller of diameter one third of the tank diameter.

So, again the diameter of the impeller is one third of the tank diameter at the production scale. So, the linear dimensions are the ratios are possibly same. What is the power consumption in the pilot scale fermenter? So, we need to find the power consumption at the pilot scale.

(Refer Slide Time: 09:30)

Given:

Pilot-scale fermenter

$D=H=0.5\text{ m}$
 four baffles, $W/D = 1/10$
 Stirring - Scaba 6SRGT curved-blade disc turbine with Diameter = $1/3 D$
 $D_i = 0.167\text{ m}$
 $\rho = 1000\text{ kg/m}^3$
 $\mu = 5\text{ cP} = 5 \times 10^{-3}\text{ kg/ms}$
 $N = 185\text{ rpm} = 3.08\text{ s}^{-1}$
 $V = 0.098\text{ m}^3$

Large fermenter

$V = 6\text{ m}^3$
 $D=H$
 Impeller diameter = $1/3 D$
 Geometrically similar to the pilot-scale vessel,
 $D_o = 1.96\text{ m}$
 $D_i = 0.65\text{ m}$

Handwritten calculations:
 $0.098 = \frac{10^3}{4}$
 $D = \frac{\sqrt{24.8}}{\sqrt{22}}$

So, let us first find out the linear dimensions. Let us list down the information which is provided to us. So, the diameter of the tank and the height of the tank, the liquid height in the tank are the same, which is 0.5. 4 baffles are there and width to diameter ratio of these baffles, it is 1 by 10. And the turbine has the diameter; the impeller has the diameter one third of the tank diameter.

So, impeller diameter can be calculated 0.5 divided by 3 which is 0.167; density is given; viscosity is also given and the impeller speed optimum impeller speed has been provided at the pilot scale. So, volume can be calculated for the tank assuming cylindrical. So, working volume can be calculated as, volume of the tank would be $\pi r^2 H$. So, we can calculate. So, it comes out to be 0.098 metre cube using the diameter of the tank and the height.

Now, large fermenter, the volume is given as 6 metre cube and the diameter is the same as the height in this tank. And again the impeller diameter is also one third that of the tank diameter. So, it is a geometrically similar vessel. Using the volume, we can find out the diameter of the tank, because D is equals to H. So, πD^2 by 4 into H, so, D and H are equal, this becomes cube is equal to 6.

So, we can calculate the value of D and find it cube root. So, once we know the tank diameter, we can find out the impeller diameter, which is one third of the tank diameter.

(Refer Slide Time: 12:10)

a. For the pilot scale reactor :-

$$Re = \frac{N D^2 \rho}{\mu}$$

$$Re = 17179.6$$

Re is > 10000

For turbulent flow, Power number is constant = 6.2

$$P = N_p \rho N^3 D^5$$

$$P = 23.53W$$

b. Power consumed per unit volume is constant

$$\frac{P_1/V_1}{P_2/V_2} = 1$$

$$P = 1440.6 W = 1.4 KW$$

Now, for the pilot scale reactor, the Reynolds number is given by the expression $N D^2 \rho / \mu$. Now, viscosity and density remain the same of the fluid, the impeller diameter is known and the impeller speed at the pilot scale level is also known, which was 185 rpm. So, we can find the Reynolds number at the pilot scale reactor. Now, this is greater than 10,000 values which means it is in the turbulent region.

Now, for turbulent region, the power number is constant and for the kind of impeller 6 blade rustling turbines Scaba, the power number has a value of 6.2. So, using the power number expression, we can find the power consumed at pilot scale by the value of the power number and fluid density. Impeller speed is known and diameter of the impeller also at the pilot scale is known.

So, we will find that the power consumed at the pilot scale is 23.5 watt; this is the power consumed in the pilot scale fermenter. Let us see part B. Part B says that if the production

scale fermenter is operated, so, that the power consumed per unit volume is the same. So, our scale-up criteria is power per unit volume at the pilot scale level. What is the power requirement after scale-up?

So, we need to find at the production scale, how will the power consumed will change? So, if P by V is the scale of criteria, then we can find the power consumed at the production scale which comes out to be 1.4 kilowatt. Because, now the volume is known at the 2 scales and the power consumed at the pilot scale is known. So, we can determine the power consumed at the production scale.

(Refer Slide Time: 14:28)

Problem 3:

Consider the scale up of a fermentation from a 10 L-10000 L vessel. The small fermenter has a height to diameter ratio of 3. The impeller diameter is 30% of the tank diameter and air is sparged at 1 vvm. Agitator speed is 500 rpm and three Rushton impellers are used. Determine the dimensions of the large fermentor and agitator speed for:

a. Constant P/V
 b. Constant impeller tip speed

Handwritten notes:
 $H/D = 3$
 $\frac{D_i}{D} = 0.3$
 $\frac{Q}{V} = 1 \text{ vvm}$
 $N = 500 \text{ rpm}$

The slide also features the NPTEL logo in the top right corner and a video feed of a presenter in the bottom right corner.

Let us see problem 3. Consider the scale-up of a fermentation. The small scale fermenter has a height to diameter ratio of 3. So, H by the linear dimension ratio at the small scale is given H by D ratio which is 3, the impeller diameter is 30% of the tank diameter. So, D_i by D is also given as 0.3. So, Q by V is given to us as 1 VVM so, where Q was the volumetric gas flow rate and V is the volume.

So, the air sparging rate is 1 VVM. Agitator speed is 500 rpm, Agitator speed, so, N is given to us as 500 and free rustling turbine impellers are being used. So, we need to determine the dimension of the large fermenter and agitator speed for the scale-up criteria as constant P by V .

(Refer Slide Time: 15:55)

Model:



$V = 10\text{L}$
 $H/D = 3$
 $D = 1.62\text{m}, H = 4.86\text{m}$
 Agitator speed, $N = 500\text{ rpm}$
 Impeller Dia = $0.3 D$
 Flow rate of air sparging = 1 vvm

Prototype:

$V = 10000\text{L}$
 Scale up factor = 10

Dimensions of the prototype calculated from scale up factor as follows

$D_p = 16.2\text{m}, H_p = 48.6\text{m}$

So, we know that for constant P by V criteria, how impeller speed and diameter, they can be related. So, let us see that for us (()) (16:09) system. So, volume at the model type is given. H by D ratio is given. So, the tank diameter and the height can be calculated based on the volume which is given. Agitator speed is 500 rpm and impeller is 0.3 times the tank diameters. So, now, we can come to know about the impeller diameter as well.

So, at the prototype, the volume is 10,000. So, we can see that the scale factor for the linear dimensions is 10, which means the diameter ratios at both the scales. So, your height at the production scale would be 3 times the diameter at the production scale. Assuming the linear dimension ratios are the same, because it is having geometrical similarity before scale-up. So, your tank diameter at the prototype and your height at the prototype level is now known.

(Refer Slide Time: 17:26)

Agitation speed :

a) Constant P/V



$N^3 D^5 = \text{constant}$ → ①

$N_p = N_m \left(\frac{D_m}{D_p} \right)^{3/5} = 107.75\text{rpm}$

b) Constant impeller tip speed

$N D = \text{constant}$
 $N_p = 50\text{ rpm}$

$N_p D_p = N_m D_m$
 $N_p = \frac{N_m D_m}{D_p}$
 $= \frac{500 \times 1.62}{16.2} = 50$

For constant P by V, if you remember, the correlation between the impeller speed and the diameter of the impeller is shown here $N^3 D^5$ turns out to be constant. So, if this is constant, then for the prototype, if the impeller speed is to be calculated, we know the ratio of the diameters of the impellers, which is the linear dimension, linear ratios of the tank remains the same.

So, there is a factor of 10 and the impeller speed is 500 rpm. So, we can calculate the impeller speed at the prototype. Now, constant impeller tip speed, if that is the scale-up criteria, then the correlation between the impeller speed and the diameter of the impeller changes as $N D$ is equal to constant. So, if this is constant, we know the scale factor as 10 and the impeller speed as 500 rpm.

So, let us assume, this is prototype and that is model type, m stands for model type. So, if we need to find out the rpm at the prototype with the constant impeller tip speed, then it can be calculated as 1 by 10. The linear ratio was 10; scale factor was of 10. So, the impeller speed at the prototype will be equal to 50 rpm with constant impeller tip speed as the scale-up criteria.