

Bioreactor Design and Analysis
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Lecture - 40
Non-Ideal Reactors: Design and Analysis – Practice Problem

So, welcome students. Now, we are going to do some practice problems on non-ideal reactors.

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Problem 1

Dispersed non-coalescing droplets ($C_{A0} = 2$ mol/liter) react ($A \rightarrow R$) as they pass through a reactor where $-r_A = kC_A^2$ and $k = 0.5$ liter/mol. min.

Find the average concentration of A remaining in the droplets leaving the reactor if their RTD is given by the curve in Fig. 1

Fig. 1

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Let us see the first problem. Here, if you can see the C pulse curve is given, the problem states that the dispersed non-coalescing droplets with the reactant concentration given as 2 moles per litre, reacts to give the product as it passes through a reactor. Now, this reactor, the reaction kinetics this follows a second order reaction kinetics, whether reaction rate constant value is given to us.

Now, the average concentration of A which is remaining in the droplets, we need to determine if the RTD is shown here. So, we know now, they have used C pulse tracer experiment, but if you notice, the unit of this is minute inverse. This indicates that this C pulse is equal to the E curve. So, the area in the shaded region is equal to 1. So, therefore, your E value at this point from between 1 to 3 years nothing but we can calculate. So, this will come out to be 0.5.

So, we need to find out, what will be the residual concentration of A after the reaction, after it comes out of the contactor. So, the reaction rate constant is given; it is a second order reaction rate.

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Solution

$C_{A0} = 2 \text{ mol/liter}$
 $k = 0.5 \text{ liter/mol. min}$
 $-r_A = kC_A^2$

For a second order equation,

$$\frac{C_A}{C_{A0}} = \frac{1}{1 + kC_{A0}t}$$

$$\frac{C_A}{C_{A0}} = \frac{1}{1 + (0.5)2t} = \frac{1}{1+t}$$

$$t = \frac{C_{A0} - C_A}{kC_A^2} = 0.5$$

$$\frac{C_A}{C_{A0}} = \int_0^t \frac{C_A}{C_{A0}} E dt = \int_0^t \frac{1}{1+t} \cdot (0.5) dt = 0.5 \ln 2 = 0.347$$

$C_A = 1.312$
 $X_A = 0.347$
 $X_A = 1 - 0.347 = 0.653$

$\frac{C_A}{C_{A0}} = \frac{1}{1+t}$

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So, the initial concentration is known; it is a second order reaction. So, dC_A by dt is what is $K C_A$ square. It is an reactants so, the negative sign. Now, we can integrate this to C . How C_A is related to will change with time? So, if we integrate this putting the limits of C_A C_{A0} between 0 to t , then we will get C_A by C_{A0} . Now, if we substitute the value C_{A0} and K given here, it will become 1 by $1 + t$ which is nothing but the fraction of C_A by C_{A0} .

Now, E curve, we know is C pulse divided by the area under the curve and by convolution integrals if you remember, this fraction will be so, this is actual and this is theoretical. Now, we know that C_A by C_{A0} between 1 to 3, this can be given by the function 1 by $1 + t$, the E value in this time is 0.5. And we can the integral limits will be between 1 and 3. So, if we solve this, we will find that your actual C_A by C_{A0} fraction comes out to be 0.347 and not just as it would have been with 1 by $1 + t$.

So, the actual conversion can be given us $1 - C_A$ by C_{A0} conversion fraction, so, which will be equal to 0.653 and in percentage it becomes nearly 65% conversion due to the non-ideality.

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Problem 2

A large tank (860 liters) is used as a gas-liquid contactor. Gas bubbles up through the vessel and out the top, liquid flows in at one part and out the other at 5 liters/s. To get an idea of the flow pattern of liquid in this tank a pulse of tracer ($M = 150$ gm) is injected at the liquid inlet and measured at the outlet, as shown in Fig. 2

- Is this a properly done experiment?
- If so, find the liquid fraction in the vessel.
- Determine the E curve for the liquid.
- Qualitatively what do you think is happening in the vessel?

Fig. 2

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Let us see the problem 2. A large tank 860 litres, this is used as a gas-liquid contactor. Gas bubbles, they rise up through the vessel and they move out from the top the liquid flows in at one part and it leaves from the other at a flow rate of 5 litres per second. Now, in order to get an ideal flow pattern of the liquid in the tank, a pulse of the tracer, the amount of the tracer is given as 150 grams has been injected at the liquid inlet.

And the concentration of the tracer is being measured. When it was measured, the concentration looks like this as shown in the figure 2. Now, what is being asked to check is if this is a properly done experiment. Now, in order to check that, this is a properly done experiment. We can do a material balance over the tracer. We know the total amount of tracer is 150 grams.

So, the concentration curve is given to us, the sum of area of all the C curve which is given here will be equal to capital M by small v. Small v was the volumetric flow rate, which is 150 by 5 litres per second. So, this value comes out to be 30 grams second per litre. So, if we want to find, then let us do the sum, which is $A_1 + A_1 \text{ by } 4 + A_1 \text{ by } 16, A_1 \text{ by } 64$ and so on.

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Solution



(a)

$$\text{Area} = \frac{M}{v} = \frac{150 \text{ gm}}{5 \text{ liters/s}} = 30 \frac{\text{gm} \cdot \text{s}}{\text{liter}} = 0.5 \frac{\text{gm} \cdot \text{min}}{\text{liter}}$$

From the tracer curve,

$$\text{Area} = A_1 \left(1 + \frac{1}{4} + \frac{1}{16} + \dots \right) = 0.375 \left(\frac{4}{3} \right) = 0.5 \frac{\text{gm} \cdot \text{min}}{\text{liter}}$$

The values agree and so the results are consistent.

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So, if we take A 1 common, the series will become like this. So, this is nothing but seems to be a geometrical series with an R value of 1 by 4. So, R infinite geometrical series if you sum up with an R value of 1 by 4, you will get the value of 4 by 3. And you substitute the value of A 1 here, which can be obtained from the plot and thus, we get the total area as 0.5. This is grams minute per litre please note, as the time is given here is in minutes.

So, we will have to convert this 30 grams litre seconds into minutes. So, this will be dividing by 60, so, it comes out to be 0.5. So, now, we see that the theoretical area is equal to the area under the C curve obtained. So, this shows that the experiment has been conducted properly. Now, we also need to find out the liquid fraction in the vessel.

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(b) Mean residence time of liquid =



$$\bar{t}_i = \frac{\int rC dt}{\int C dt} = \frac{1}{0.5} \left[2A_1 + 4 \times \frac{A_1}{4} + 6 \times \frac{A_1}{16} + 8 \times \frac{A_1}{64} + \dots \right] = 2.67 \text{ min}$$

Liquid volume in vessel can be given by

$$V_l = \bar{t}_i v_i = 2.67(5 \times 60) = 800 \text{ liters}$$

Volume fraction is given as follows

$$\text{Fraction of liquid} = \frac{800}{860} = 93\%$$

$$\text{Fraction of gas} = 7\%$$



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For that, we need to first find how much is the volume of the liquid in the vessel? Now, in order to find the volume of the liquid in the vessel, we will have to, we know the volumetric flow rate. So, mean residence time is V by F . So, your volume can be given, if the volumetric flow rate can be multiplied by the mean residence time. So, mean residence time knowing the C curve can be calculated using discrete time intervals.

So, if you do this which is summation of $t C \Delta t$ by summation $C \Delta t$ i. So, if we do this, what is t ? 2 then 4, 6 and so on, after every 2 minutes. So, this is what and then it is the area $C \Delta t$. So, which is being multiplied here A_1 then A_1 by 4, A_1 by 16 and so on. And (\int) (09:26) is divided by the total area under the curve which we have already calculated as 0.5.

So, if we do this, this is again if you take up $2 A_1$ common, this will end up in Taylor series and infinite series and we can sum up and give you the value of 2.67. So, you need to have a little bit of revision of your geometrical series, arithmetic series, summations or Taylor series, expansions for infinite series. So, then you will be able to reach here. So, for that, you take up $2 A_1$ common here, so, it will make get converted to a Taylor series.

So, once we know this mean residence time, we can calculate the volume knowing the volumetric flow rate. So, volumetric flow rate was given as 5 litres per second has been converted to minutes, because this is in minutes. So, therefore, we get the total volume of the liquid which is 800 litres. So, now, we need to calculate the volume fraction. The total reactor is the volume 860 litres. So, volume fraction in the vessel would be 93%. So, the remaining would be gas fraction which comes out to be 7%.

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(c) We find E curve as follows

$$E = \frac{C_{\text{pulse}}}{A} = \frac{C_{\text{pulse}}}{(0.5)} = 2 C_{\text{pulse}}$$

E curve for the liquid is as shown in Fig. 3

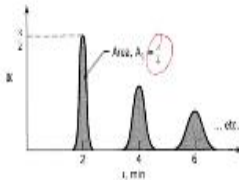


Fig. 3

(d) The vessel has strong recirculation of liquid probably induced by the rising bubbles.

Ocane Lovamika, Chemical Reaction Engineering, IIT Bombay

Now, the third part of the problem is to find the E curve. So, we know that C pulse curve can be converted to E curve by dividing the C values by the total area and under the C pulse curve. Area under the curve, we know is 0.5. So, the entire curve can be made 2-folds, the values, the corresponding C values can be converted 2-folds to give the corresponding E values and this is what has been done here.

So, the area for the first plot will become 3 by 4 twice and so on, for the rest of these spikes. Now, the fourth part says qualitatively, what do you think is happening in the vessel? So, if you remember, spikes coming out at small intervals, regular intervals with decreasing area or the peak is decreasing demonstrates that there is some internal recirculation happening.