Biomechanics
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Lecture - 03
Example Problems on FBD and EOE

Welcome to this class on biomechanics. We have been focusing on introductory mechanics statics and dynamics in particular we have been focusing on some simple principles in statics.

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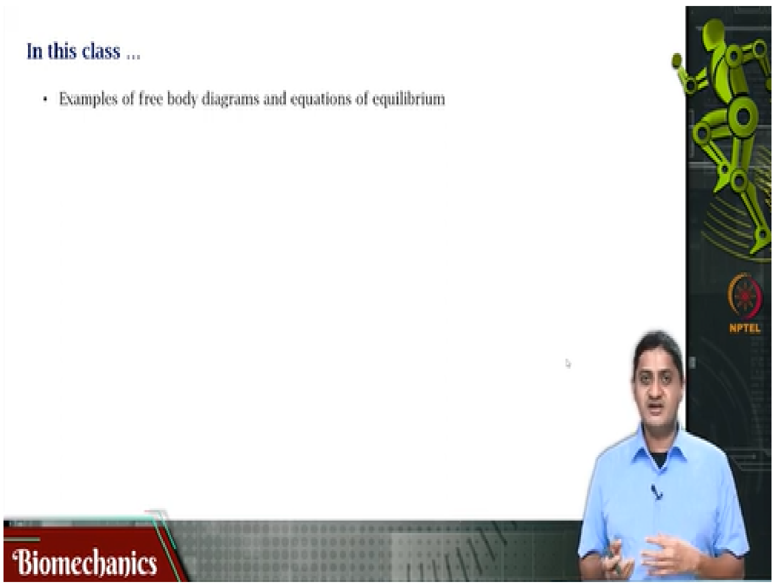

So, in this video we will have a couple of examples of free body diagrams and write out the equations of static equilibrium for those two situations. So, the idea here is that we will build on the knowledge that we have from high school and we will take the learner to a level where they can appreciate research in the field of biomechanics. So, with this idea in mind we start with the bare basics the; with the basics of mechanics itself.

Although this is not 100 percent biomechanics but because we will be focusing a lot on this and we are not assuming prerequisite knowledge. We will be doing these simple problems and mechanics to set up a solid foundation.

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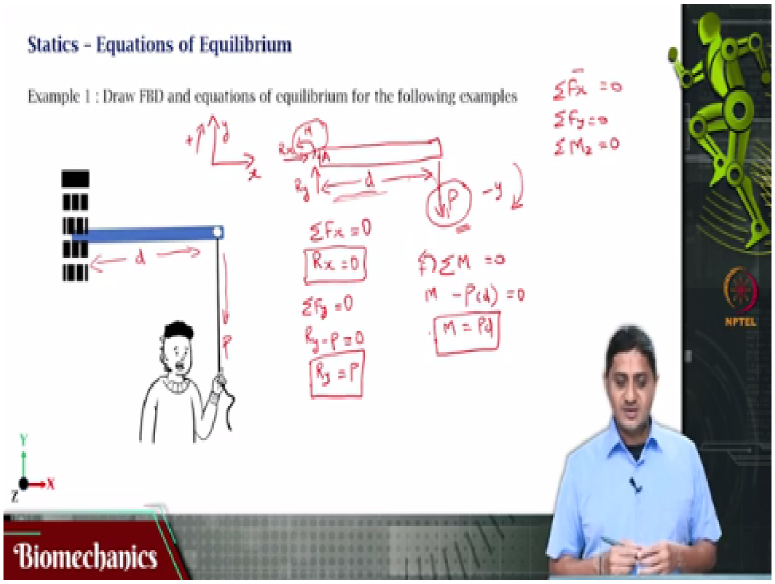

So, these two examples we will start with this. First example, the question is draw the free body diagram and equations of equilibrium for these two this situation. So, that is this person, this person is pulling with a rope with some force pulling with the rope. On this beam that is attached this is a cantilever beam. So, it turns out that it has a specific free body diagram because it cannot move in either this direction or that direction nor can it rotate in the plane of the screen it cannot rotate.

Because of this reason it will have three degrees of freedom as in three equations of equilibrium and so the corresponding free body diagram we will have to draw and this person is pulling at the end of this. Let us assume that the distance that the length of this beam is some d meters so that is the length of this beam d meters. So, if I were to draw the free body diagram what did we say we said an outline of the body is all that will be required.

So, we would like to draw an outline of this body. So, when you say free body diagram sometimes the question is free body diagram of what are you talking about the free body diagram of the wall or the beam or the person's hand. In this case although the question is not stating what it is. I will state it for you the question is draw the free body diagram of this beam, let me redraw this little bit free. So, this is the outline of this beam.

So, now I have removed the beam from its contact. So, when you remove the beam from all the contacts that it is making with other external bodies you will have to replace it with corresponding forces or moments. In this case because the contacted the wall has been removed that will be replaced by one force along the axis of the beam one force that is perpendicular to the long axis of the beam and one moment which I am going to call as M z.

Remember I am using that kind of an xy axis. So, this is the xy axis that I am having. Are we done have we completed this, the answer is no, because there are probably some other forces that are being applied. Did we miss anything? The answer is yes. Because this person is applying a force let us say that this person is applying a force P, I am going to call this as you know P some force some known force P is being applied by this person.

Now are we done? The answer is yes, we are done with this. Also let us remember to mark the dimension the length of this beam as P. This is the free body diagram of the beam given the force that is being applied. Now by using this free body diagram we will be in a position to solve for unknown forces. When some of these forces are given can we find the other forces using this diagram. The answer is yes, we can do that.

How? Let us see, because I know that this beam as a whole is in static equilibrium that means it is not accelerating in any of the three dimensions. In this particular this is a planar case so in any of the two directions it is not accelerating. From that what I know is that I can write the sum of all the forces in the X direction is zero. How do you know this? Because there is no acceleration in the X direction.

Because of this reason I know that the sum of all the forces in the X direction is zero, I know this. What are all the forces in the X direction? Let us write out all the forces in the X direction and sum them one by one. First, I write R x and then I am searching for is there any other force in the X direction. The answer is no, that means what that means R x is zero. How do I know this? Because there is no force other than R x in the X direction.

And the sum of all the forces in the X direction is zero that would mean that R x itself is zero this is how we know this. What I also know is because the object is not this beam is not accelerating in the vertical direction this kind of an acceleration is not happening in the vertical direction. That would mean the sum of all the forces in the vertical direction or in this case the Y direction is also zero. That is sigma F y is zero.

What are all the forces that are there in the y direction? Let us write them out one by one. Remember the positive y axis is the one that is going up, this is the positive y. So, it not what is the force that is along the positive y direction that is the R y force and we will have to sum vectors we will have to sum all the forces in y direction and the sum of all the forces in Y direction is zero.

What other force is there? The force P is there but it is in the negative y direction, is it not? It is in the negative y direction. We know this because how do we know? Because the person is pulling towards the negative y direction. So, I will write this as R y - P and then I start looking is there any other force in the y direction. I am looking there are only three forces one is R x but that is in the x direction, one is R y, the other one is P those two are in y direction I have written them and there is no other force in the y direction.

That means that the expansion of sigma F y = 0 is R y - P is 0. That means because P is a known I can actually write this as R y the value of R y is P this is something that I can write. Are we done? The answer is no, because there is one more equation. This is planar equilibrium that means that there will be three equations of static equilibrium that we will have to write. Let us remember this throughout the course we will be discussing this.

Sigma F x = 0 sigma F y = 0 and sigma M in this case M z I am not you do not have to specify it all the time but we can specify it also sigma M z is 0. So, I can also write the moment that I will take I can write this as sigma M = 0 counter clockwise considered positive. You will have to State the sign convention that you are using. In this case I am considering counter clockwise moment as a positive moment.

What are the counter clockwise moments that are? There is one counter clockwise moment that is there that is M z. There are three forces and each of these three forces may cause a moment depending on the point about which you are taking the moment. They may cause a moment but conveniently I am going to take the moment about this point. Let us call this point as some point A, I am going to call this point as A and I am taking moment about that point A.

When I am taking moment about that point it turns out because moment is a vector product of the force itself and the perpendicular distance between the force and as in the line of action of force and the axis of rotation. And because at A, the perpendicular distance between R y and R x is 0 these two forces do not cause a moment. Are we done? The answer is no, because there is this force that is the P force that could cause a moment. So, let us write this out.

And remember this P force, this force P is acting here at distance d and will it be a counter clockwise moment? The answer is no because it is in the negative y direction. Because of the reason it will cause a tendency to rotate this beam. If it rotates it will rotate this beam in the clockwise direction. Because of this reason this would be minus P times the distance d and this is I am calling this as M is it.

This is actually you know actually this is moment at that point, moment minus P d is 0. This means that because I know the distance and I know P, I can compute that moment that reaction moment at A to be P times d. This is a one simple example of the case of a free body diagram. Let us look at one more example.

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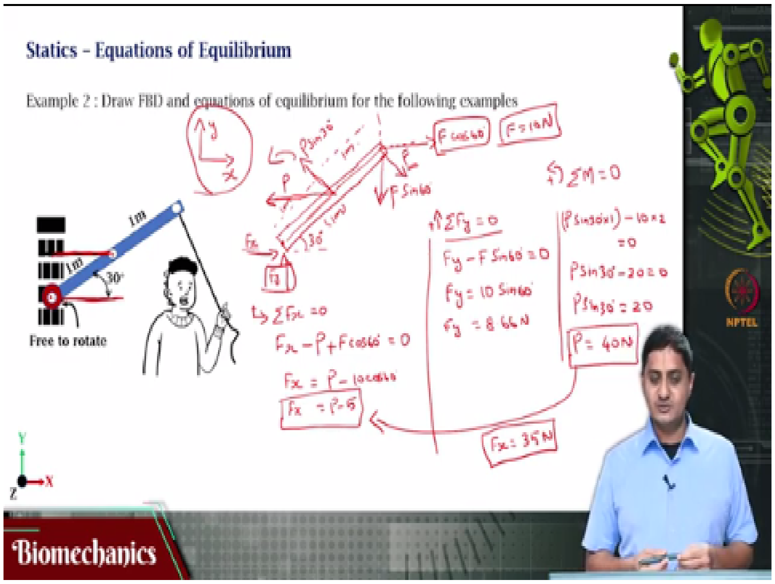

Here is a slightly more complicated situation rather free body diagram and equations of equilibrium of this beam. Consider that at this point A the beam is free to rotate. Also, there are several things the distances are given and there are several other forces that maybe it appears like there are other things that are you know acting here. For example, at this point there is a force that is there is a cable that is attaching this beam to the wall and it is at this end the person is pulling.

And in this case this beam is free to rotate. So, M at that point will not be there, there is no restriction of rotation. Because of this reason there will be no M at that point. So, now let us write out the equations of static equivalent. But first let us draw the free body diagram of the situation. So, this is the beam, I am going to assume a similar in you know inclination, that is the beam. Now let us assume that a force P is being applied by the wall on this beam through this cable and that force is acting in this direction.

And I am going to assume of course I am assuming this xy direction. Let us remember that the beam itself is at an angle of 30 degrees to the horizontal and this person is applying a force that is perpendicular to the beam which that force I am going to call as some force F. Remember that is applied perpendicular to the beam not downwards you know there is a difference. If it is applied downwards this is how it will appear it will be in the negative y direction.

But if it is applied perpendicular to the beam it will have both a horizontal component and a vertical component, we will see that. So, remember that force is applied perpendicular to the beam and I am removing this. Because I have removed this beam from the contact at the wall there are going to be a couple of forces because the beam when it is attached to the wall cannot translate along the x axis and cannot translate along the y axis.

Because these; movements are restricted and only these movements are restricted. It is not restricting the rotation because of this reason there will be two forces because two translations are restricted. This will be the two forces I am going to call this as F y and F x. Now something to remember is that there will there will be a component of P, remember this distance is one meter and that distance is 1 meter.

There will be a component of P acting along the longitudinal axis of the beam and there will be a component of P that will be acting perpendicular to the or transverse to the beam that is something that we will have to find out. Now let us write out the equations of static equilibrium. But before we do that can we check if we have accounted for everything, have we accounted for the wall this, yes. F x, F y and because it can rotate there is no moment.

The person is pulling with a rope that is accounted for with the force that is perpendicular to the beam and the cable is attaching to the wall. Now this is a free body diagram is complete. Now let us write out the equations of static equilibrium. Remember what we said is the equations of static equilibrium sigma F x = 0. How do I know this? Because this object this beam does not have acceleration in the x direction that means that sum of all the forces in the x direction is zero.

At the expense of repeating I am saying this because when you repeat this and when you learn many times you will be able to remember this. So, the sum of all the forces in the x direction is zero. What are all the forces in the x direction? Let us write them out one by one. F x this is the force along the x direction, this F x is different from this F x. So, F x is the reaction force in the x direction that is F x and is the force P along x direction.

It turns out the force P is along the negative X direction. So, I can say it away write it, is that correct? The answer is yes because this force P is applied because this rope is applying the force parallel to the x axis here. So, because of that reason I can straight away write this as negative P. Why is it minus P? Because it is acting in the negative x direction because of this reason. Are we done? can we simply say equal to zero?

The answer is no, why not? Because it turns out that this force that is being applied by this person which I am calling as F not F x simply calling it as F will have both a x component and a Y component, there will be two components to this and right now I do not have a clear idea of what these components are but I can compute this. How can I compute this? Because I know that the angle that this beam is making with the horizontal is 30 degrees.

Because of this reason I know that I can resolve that vector in this xy frame of reference. So, that component would be F cos 60 and this component would be F sin 60 this is how I am resolving this. Now I can write out this horizontal component as the component in the x direction. Is it in the positive x direction? The answer is yes. So, I am writing this as plus F cos 60 = 0. And let us say that the force applied is some force say 10 Newtons is the force that is being applied.

Then I can write this as you know F let us say F = 10 Newton for the sake of discussion. Then I can write this as F x is P - 10 cos 60 but cos 60 is 1 by 2 or 0.5 I can write this as P - 5 and because P is known some known value let us say that is say 50 Newtons then this would be 45 Newtons, F x would then be 45 Newtons. So, I can solve for this, you think are we done? The answer is no, still have to write one more equation of static equilibrium which is sigma F y = 0.

What are all the forces that are in the y direction say that is in the upward is considered positive F x right side is considered positive and so I am writing this. What are all the forces there is this F y here. If it in the positive y direction the answer is yes F y and what else is there? There is F sin 60 that is there but F sin 60 is in the negative y direction, is it not? How do we know this? Because it is acting like this, sine 60 is acting like this I know this.

So, I am going to write this as minus F sin 60. Any other component will P have a component in U direction? The answer is no. Why not? Because P will have a component that is parallel or along the axis of the beam and perpendicular to the long axis of the beam. But for the frame of reference that we considered xy it will only have one component along the x. Because as per the given diagram this frame of reference x axis that I have taken and this cable are parallel to each other.

So, it will only have a x component because of this reason there is no other force that will be acting in the y direction. So, I am writing this as zero and I am substituting F = 10 and substituting for sin 60. You can use a scientific calculator and you will get F y as 10 times sin 60 are another 8.66 Newtons. Are we done? The answer is no. Because this is a problem in 2D a static equilibrium problem in 2D and there will be three equations of equilibrium.

The third equation being sigma M counter clockwise considered positive is zero. Now what are all the forces that will cause this moment? To There will be a moment that will be caused due to this vertical component and that component what is this component this component the value of this will be P sin 30 which is not that will cause a moment and the corresponding moment term will be 1 meter.

So, that I am writing the force is P times sin 30 that is the force the corresponding moment term is one. Is it a clockwise moment or a counter clockwise moment? That will be a counter clockwise moment so that is positive. Then there is something else that is force of 10 units or 10 Newton is acting at a distance of 2 meters 10 times 2. Is that any other force that could cause a moment? Can F x and F y cause a moment?

Answer is no because I am taking the moment about this point. The moment arm for these two cases will be zero and so there is no other force that can cause this moment. So, I can write this as P sin 30 - 20 is 0 or rather P sin 30 let me write out the entire equation zero or another P sin 30 is 20. From this I can get because I am substituting for sin 30 from this, I can get P to be 40 Newtons. Now that I have found out the value of P, I can substitute the value here and find out the value of F x.

F x would then be F x is P - 5 is it not 40 - 5 that would be 35. So, this is another simple example of a case in which we draw a free body diagram and write out the equation of static equilibrium and use that to solve simple problems.

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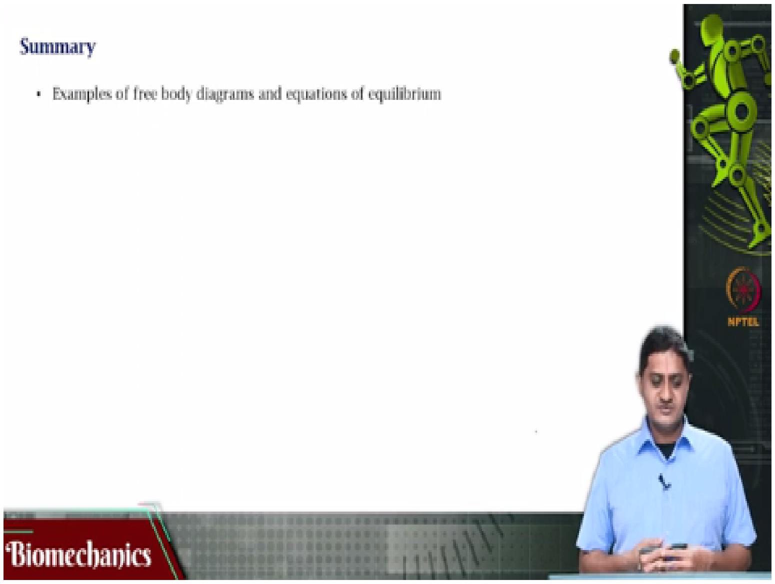

So, in this lecture we saw examples of free body diagrams and some equations of equilibrium using them to solve simple problems in statics. Thank you very much for your attention.