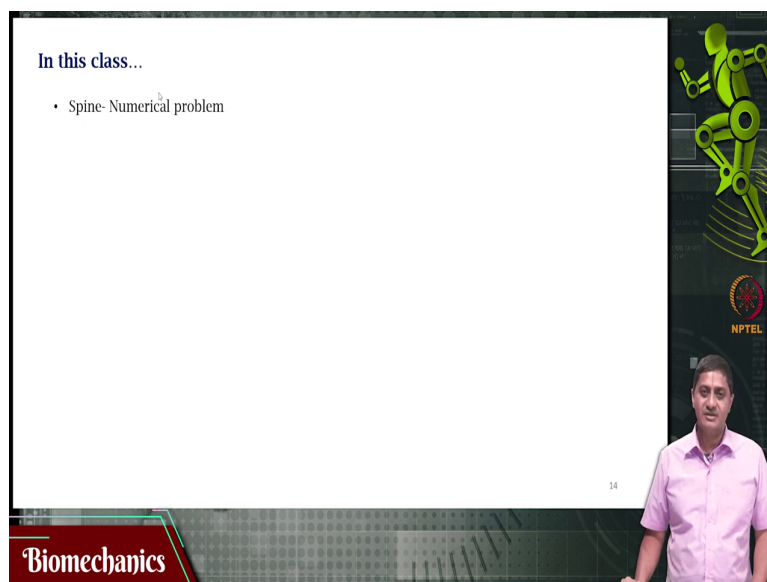


**Biomechanics**  
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**Lecture – 33**  
**Spine Problem**

Welcome to this video on biomechanics we have been looking at the spine how the vertebral column is organized and how it plays its important role of supporting the head and allowing for articulation in various directions they sound. We also saw the muscles that support movements from the level of the neck to the level of the spine and the various muscles are various sub muscles of the broad group of muscles called as erector spine this is what we saw in the previous videos.

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In this video we will be solving a simple numerical problem involving the spine. This is statics of spine.

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**Spine Problem**  
 Fig A depicts a person lifting a weight  $W_0$ .  
 Fig B depicts the forces acting on the lower part of the person's body.  
 Fig C depicts a mechanical model of the person's lower body (legs + pelvis). The various parameters in Fig B and Fig C are as follows:

- $\theta$  → flexion angle of the trunk from the vertical position.
- $W$  → weight of the person;  $W_1$  → weight of the legs + pelvis;  $W + W_0$  → total ground reaction force applied through the feet at point C.
- $F_M$  → Force applied by the erector spinae muscles to support the trunk.
- $F_J$  → compressive force at point O which is the union point of the fifth lumbar vertebrae and sacrum.
- B → center of gravity of the legs + the pelvis; a → length of lever arm of  $F_M$  relative to O.

Find  $F_M$  and  $F_J$  in terms of a, b, c,  $\theta$ ,  $W_0$ ,  $W_1$  and  $W$ .

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Consider this problem you know this problem looks like a busy slide it looks like a busy slide it is not all the details that are there are required to solve this problem. Now figure a depicts a person lifting a weight of  $W$  naught,  $W$  naught is the weight that this person is lifting. Figure B shows the forces acting under lower limb lower part of that person's body. Figure C here it shows a mechanical model of the person's lower body which is essentially the leg plus the pelvis.

The parameters marked in figure B and C are given described here Theta is this angle that the muzzle Force  $F_M$  is making with respect to the vertical that is the flexion angle of the trunk from the vertical position this is the flexion angle of the trunk from the vertical position.  $W$  is the weight of the person  $W_1$  is the weight of the leg which is acting from point B assume that only one leg is supporting because the person's weight is  $W$  and the person is carrying an external weight of  $W$  naught.

The reaction was and because the entire system is in static equilibrium the reaction force from the ground on the leg will be  $W$  plus  $W$  naught that is here. The total ground reaction force that is applied at Point C.  $F_M$  is the force that is produced by the erector spinae muscle to support the trunk  $f_j$  is the force that is felt at point O right which is the point of the fifth lumbar vertebra and the sacrum are the so-called L5 S1 joint are the lumbosacral joint LSJ the lumbosacral joint.

Point B is the center of gravity of the legs plus the pelvis okay that is the point from where the weight of the legs plus pelvis is acting from  $W_1$ . A is the momentum of muscle  $F_M$

relative to O that is this is point O and this is point A right. Remember the point at which the compressive force is felt the reaction force is felt is L5 S1 and the point at which the muscle is applying the force is a point that is different this L5 S1 is O muscle forces separated A and there is a distance between this.

That perpendicular distance is A that is that perpendicular distance as I am marking on the screen now that perpendicular distance. The question is if small a small b small b Theta W naught W 1 and W are given to you find the value of F M and F J in terms of these nodes okay that is the problem. I need to find an expression for F M and F J in terms of small a small b small c Theta W naught, W 1, W this is what I need to do.

Now this is a problem that we have taken from a book thanks to the authors for allowing us to use this problem.

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**Spine Problem** Known (given):  $\theta, a, b, c, W, W_1, W_0$   
To find (unknown):  $F_M, F_J$

Force	Known	Direction
$F_M$	a	2
$F_J$	0	-
$W_1$	b	2
$W+W_0$	c	S

$\sum \tau M_O = 0:$   
 $-F_M(a) - W_1(cb) + (W+W_0)c = 0$   
 $F_M(a) = (W+W_0)c - W_1(cb)$   
 $F_M = \frac{(W+W_0)c - W_1(cb)}{a}$  ✓  
 $F_{Mx} = F_M \sin \theta$   
 $F_{My} = F_M \cos \theta$  ✓  
 $\sum F_x = 0: F_{Mx} - F_{Jx} = 0$   
 $F_{Jx} = F_{Mx} = F_M \sin \theta$  ✓  
 $\sum F_y = 0: F_{My} - F_{Jy} - W_1 + (W+W_0) = 0$   
 $F_{Jy} = F_{My} - W_1 + W+W_0$   
 $F_{Jy} = F_M \cos \theta - W_1 + W+W_0$  ✓

Problem source: "Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation" 4th edition by Özkaya, Nihat, et al.

Now let us list the known's in this what is known to us or what is given to us Theta is given to us small a is given to us small b is the distance between the Point O and point B what is point O that is the L5 S1 joint that distance along the x axis perpendicular distance between the force W 1 remember Force W 1 is acting vertically downwards why is that because that is the weight of the leg and the pelvis that will always act downwards.

So, that perpendicular distance is b and W + W naught is the ground reaction force that is always acting upwards why will it average act upwards? Because it is a ground reaction force it will always act upwards that corresponding momentum is small c. Theta is known Theta

remember is the angle between the trunk and the vertical not the angle between the trunk and the horizontal something to keep in mind.

It is the angle with the vertical see here that angle that angle is theta remember this. So, Theta is known a, b, c or no let us say  $W$  is known  $W_{naught}$  is known and  $W_1$  is known. What is  $W_{naught}$ ? That is the external weight that the person is carrying. What is  $W_1$ ? That is the weight of the leg and the pelvis  $W$  is the weight of the person the whole weight of the person.

So, the weight the total weight of the person along with the external weight is  $W$  plus  $W_{naught}$ . Now I am interested in finding an expression for  $F_M$  and  $F_J$  that is the question what is not known to find this  $F_M$ ,  $F_J$  in terms of all these above things right this is what I need to find. Well one thing I can do is I can write out the moment at Point O if I do that  $F_J$  will disappear then all the moments will be in terms of  $W_{naught}$ ,  $W_1$ ,  $W$  and  $F_M$ .

And then I can maybe try to simplify that and try to find the expression for  $F_M$  maybe that would be a good idea. So, I am going to say is counter clockwise considered positive if I say this but before I say this let us try to list out all the forces that are there and their corresponding momentums. What are the forces and whether that moment is going to be a clockwise moment or a counter clockwise moment.

Force, momentum, direction,  $F_M$  is the force  $F_M$  the momentum is a is it not  $F_M$  is acting in this direction the momentum because I am taking perpendicular distance from this point O is it not. This is the perpendicular distance between the line of action of force and the pivot about which the rotation is happening is it not. So, that is that distance which is small a and that will cause a clockwise moment.

Now then what else  $F_J$  will not cause any more moment because its momentum is zero that is no because  $F_J$  is acting along O. So, it will not cause any but anyway for correct completeness we will also write it out. Then  $W_1$  is the weight that is acting at that point and that moment arm is small b and that is a clockwise moment is it not clockwise moment then  $W$  plus  $W_{naught}$  is acting at with the momentum of small c.

$W$  plus  $W_{naught}$  what is this? This is the weight of the person plus the weight that he is holding. Its momentum is small c and that is that will cost because this is where that is where

the pivot point is the axis of rotation is that is going to be a counter clockwise moment. Let us go back once and check if there are any other forces that we have missed out this is the free body diagram this is the diagram that we have right.

$F_M$  we have accounted for  $F_J$  we have accounted for although the momentum is 0,  $W_1$  we have accounted for and  $W$  plus  $W_{naught}$  we have accounted for now I can write out the equation  $\Sigma M$  about  $O$  is zero that would be minus  $F_M$  times  $a$  why minus because that is a clockwise moment minus  $W_1$  times  $b$  why minus again because it is a clockwise moment Plus  $W_1$  plus  $W_{naught}$  times  $c$  why plus because this is a counter clockwise moment this is zero.

This is a single equation in one variable in this equation the only unknown is  $F_M$  everything else is known. So, all I need to do is simplify and find the value of  $F_M$  let us try to do that. So, that would give me  $F_M$  a  $F_M$  times  $a$  is you know  $W_1$  plus  $W$  sorry this is  $W + W_{naught}$  okay  $W + W_{naught}$  now  $W_1 + W_{naught}$   $W$  plus  $W_{naught}$  times  $c$  minus  $W_1$  or rather minus  $W_1$  times  $b$ .

Now  $F_M$  would then be  $W + W_{naught}$  times  $c$  minus  $W_1$  times  $b$  the whole thing divided by small  $a$ . This is the expression for  $F_M$  the force  $F_M$ . Now because I am also interested in finding the force  $F_J$  can I try to express  $F_M$   $X$  and  $F_M$   $Y$  are the  $X$  and  $Y$  components of  $F_M$  because it seems like I might need that to compute  $F_J$  because  $F_M$  is not equal to  $F_J$  likely  $F_M$  is not equal to  $F_J$  that is very likely how do you know.

Because there are the  $X$  component might be the same or we will have to check that we will have to check that. The  $Y$  component will definitely not be the same because there will be many other things  $W_1$ ,  $W + W_{naught}$  many other things that come into the picture it is better to Express  $F_M$   $X$  and  $F_M$   $Y$  in terms of  $F_M$  and  $\Theta$  can we try to do that if we go back to our previous videos on resolving vectors and resolving forces.

I requested to take a few minutes and refresh this in case you are not able to follow. Remember one thing that  $\Theta$  is the angle that  $F_M$  is making with respect to the vertical the usual resolution of forces and vectors we will say  $\Theta$  is angle that it makes with the horizontal. Here it is the angle that it is making with the vertical remember that something to keep in mind. So, because of that the  $X$  component of  $F_M$  the  $X$  component of  $F_M$  will be

that that is the X component of  $F_M$  that would be  $F_M$  times  $\sin \theta$  because that is opposite to  $\theta$ .

Because that is opposite to  $\theta$  that would be  $F_M$  times  $\sin \theta$  and the Y component of  $F_M$  would be that is the Y component of  $F_M$  and that is an adjacent side to  $\theta$  is it not. So, because of that I know that this is going to be  $F_M$  times  $\cos \theta$ . This is different from the previous formulations because  $\theta$  here is the angle that is measured with respect to the vertical.

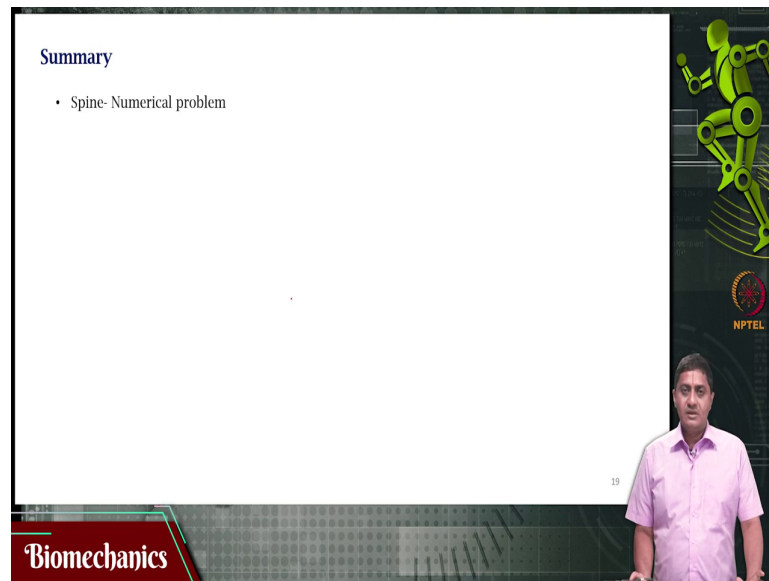
Please check why this is the case. Please check and review previous videos on how to resolve vectors and forces.  $F_{Mx}$  is  $F_M \sin \theta$  and  $F_{My}$  is  $F_M \cos \theta$  now I can write other equations of static equilibrium which is  $\sum F_x = 0$  right side going is considered positive right that is what are all the various forces that are acting along the X direction there are only two forces one is  $F_{Mx}$  that is acting along the positive X Direction and  $F_{Jx}$  that is acting along the negative X Direction the whole thing is zero there is nothing else.

So, this means  $F_{Mx}$  is equal to  $F_{Jx}$  or rather  $F_{Jx}$  is the unknown in this that is equal to  $F_{Mx}$  and what is its value from our previous computation  $F_M \sin \theta$  and because I have already computed  $F_M$  this is  $F_M \sin \theta$  right this is  $F_{Jx}$ . Now I can say  $\sum F_y = 0$  I am going forces considered positive say if I say that what are all the forces that are there in the Y direction?

Now there are many things there is plus  $F_{My} - F_{Jy} - W_1 + W + W_{naught}$  that anything else  $F_{My} - F_{Jy} - W_1 + W + W_{naught}$  that is no other force in the Y direction and this whole thing is zero in this I know  $W_1$ ,  $W$ ,  $W_{naught}$  the only unknown in this is  $F_{Jy}$  I take that to the right hand side and then I rewrite that equation slightly I will get  $F_{Jy}$  is  $F_{My} - W_1 + W + W_{naught}$ . In this I know  $F_{My}$  as that  $F_M \cos \theta$   $F_M \cos \theta - W_1 + W + W_{naught}$ .

In this I know the value of  $F_M$  to be this I know this substitute for this value of  $F_M$   $\theta$  is already given to you  $W_1$ ,  $W$  and  $w_{naught}$  are already given to you. This is the value of  $F_{Jy}$  and this is the value of  $F_{Jx}$ . What was asked to find  $F_M$  and  $F_J$  in terms of  $\theta$ ,  $W_1$ ,  $W_{naught}$ ,  $W$  and  $abc$  is it not that is what we have done.

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Summary

- Spine- Numerical problem

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So, with this we come to the end of this video. In this video we saw a simple numerical problem involving spine. In the exam you might actually be required to solve for this using specific values of  $a$ ,  $b$ ,  $c$ ,  $W$ ,  $W$  naught,  $W_1$  etcetera something that you might want to keep in mind that because this course will most likely have exams that are of descriptive type not just multiple choice questions.

So, you might be required to solve problems. So, it will make sense for you to attempt this problem and this derivation on your own at least once or twice after watching this video that will help you to do well in your NPTEL course itself. Thank you very much for your attention.