

In this video we will be looking at a numerical problem, a simple problem involving the hip joint. This is static analysis of the hip joint.

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Hip Problem
 Consider a situation where a person is standing stationary on the right leg (i.e., in the single-leg stance). Fig A shows the forces that are acting on the leg. F_M is the net force exerted by the abductor muscles, F_J is the joint reaction force acting on the femur by the pelvis, W_1 is the weight of the leg and W is the ground reaction force acting on the foot which is equal to the weight of the body. θ is the angle between the horizontal and line of action of F_M .

Fig B is the mechanical model of the leg with the necessary parameters to define the geometry of the problem. O is a point along the instantaneous axis of rotation of the hip joint, point A is where the hip abductor muscles are attached to the femur, point B is the center of gravity of the leg, and point C is where the ground reaction force is applied on the foot.

a) Draw the FBD for the leg
 b) Determine the force exerted by the hip abductor muscles $F_M = ?$
 c) Determine the joint reaction force at the hip. $F_J = ?$

Problem source: "Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation" 4th edition by Özkaya, Nihat

Let us read this problem. Consider a situation where a person is standing stationary on the right leg that is, this person is standing only on one leg single-leg stance. For the purpose of this problem, let us assume this is true. Figure A shows the forces that are acting on the leg. Capital F suffix M is the net force exerted by the abductor muscles is the force that is exerted by the abductor muscles.

Capital F suffix J is the joint reaction force acting on the femur. It is the force that is exerted by the pelvis on the femur. W_1 is the weight of the leg that is acting from a point B that is shown there W_1 is the weight of the leg. And W is a ground reaction force. This is the force due to the mass of the person. This is the weight of the body. θ is the angle between the horizontal and line of action of F_M .

θ is angle between the horizontal and the line of action of the muscle force, F_M . Figure B that shows a simple mechanical model of this leg, with the corresponding parameters to define the geometry of this problem. O is a point along the instantaneous axis of rotation of the hip joint. O is this point this is the point that is along the instantaneous axis of rotation of the hip joint.

Point A is the point where the muscles are attached to the femur, where the hip abductor muscles are attached to the femur. Point B is the centre of gravity of the leg. That is where the

weight of the leg is acting from the C G of the leg. And point C is where the ground reaction force is acting. Now, the corresponding moment terms are given. Question is rather free body diagram for the leg let us show.

Determine the force exerted by the hip abductor muscle. That is find F M. Determine the joint reaction force at the hip. That is find F J. If this is a problem that we have sourced from a textbook with do thanks to the authors.

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Hip Problem Given (Known): $a, b, c, W_1, W, \theta, \beta$
 Unknown (To find): F_M, F_J

Force	Moment arm	Direction
F_{max}	a_y	5
F_{mg}	a_x	2
W_1	$b_x - a_x$	2
W	$c_x - a_x$	5

$\sum M_A = 0$
 $W(c_x - a_x) + F_{mg}(a_y) - F_{mg}(a_x) - W_1(b_x - a_x) = 0$
 $\sum F_x = 0$ in $\sin \theta$
 $W(c \cos \beta - a \cos \alpha) + F_M \cos \theta (a \sin \alpha) - F_M \sin \theta (a \cos \alpha) - W_1(b \cos \beta - a \cos \alpha) = 0$

$a_x = a \cos \alpha$ ✓
 $a_y = a \sin \alpha$ ✓
 $b_x = b \cos \beta$ ✓
 $c_x = c \cos \beta$ ✓

Problem source: "Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation" 4th edition by Orkaya, Nihal.

Now, let us start with the free bar diagram. This is a free bar diagram. A simple free bar diagram of the situation. F M is acting at this point A and F J is acting at that point O. W 1 is acting at this point B. And W is acting at this point C, capital ABC. What are the known in this? What is known are given? Well, what is known is the distance is A, B etcetera, known. W 1 is known. W is known.

The distances, A, B, C are known that is A that is B and this whole distances these things are. And for convenience the angle that the line A O makes with the horizontal that is that angle is given to be alpha. And that angle is given to be beta that is known. And F M itself is acting at an angle of theta to the horizontal that is also known. Now, can I resolve F M in terms of theta. So, what is known? As you know small a, b, c, W 1, W are unknowns to find.

What is that to find? F M and F J this is what we need to find. Can I write out F M in terms of it is components? The answer is yes, I can do that because theta is given theta is also known by the way theta is also known. So, F Mx that is it not I am interested in knowing that

because this is F_x and that is F_y . F_x then is $F \cos \theta$. And F_y is $F \sin \theta$. Now, if I want to write out the equation of moment about say the point O.

What would be the various forces and the corresponding moment arms that cross this? Now, let us write out the various forces, the corresponding moment arms. And whether that is a clockwise or counter-clockwise movement for each of the force F_x . Force moment arm direction it is useful to come up with a table like that for your own understanding and clarity. F_x the corresponding moment arm is you see, F_x is acting in this direction is it not?

That corresponding moment arm is a_y , that is that distance a_y . That corresponding moment arm is a_y . And that will be causing a counter-clockwise movement. Is it not? Because this is important to determine the sign of this. Then F_y the corresponding moment arm is a_x . So that is the distance for the y component of the force. That is where the y component of the muscle force is acting around.

So that perpendicular distance is this because I am taking the moment about this point O. And that will cause because this will be the force that will cause a clockwise movement. So, the two components of the muscle force cause moments in two different actions, something to keep in mind. W_1 is acting from that point. Is it not from that point? It is a point that is located that is not b_x .

Because I am interested in finding moment about this point O. Remember moment I can take about any point I want in this system. I am interested the way I have drawn that is O actually, technically. So, this is the distance that I am interested in finding out. That distance is $b_x - a_x$. So, there is small distance that is $b_x - a_x$. Remember it is not b_x alone. It is $b_x - a_x$ why? Because I am taking moment about O.

And I am only interested in knowing the perpendicular distance with respect to the point O. That is what I am interested in finding out and that is $b_x - a_x$. That will actually, cause a clockwise movement. Is it not? That will actually, cause a clockwise movement. Then what? Then W is acting at that point C. And that distance is $c_x - a_x$ is it not? And that force is acting in this direction. So, it will be causing a counter-clockwise movement.

Note, I do not know what is a_x , b_x , a_y , b_y , is? I only know a and b the corresponding alphas and betas are given. Now, not only the forces have components moment arms also have components. So, I need to write out a_x , a_y , b_x , b_y , in terms of alpha and beta respectively. This is something that I need to do. Why is that? Because the moment arms have components I need to do that.

So, the component a_x , what is a_x ? That is a_x not that is they said this is a that is the distance between this. The corresponding angle is alpha we said. And I am interested in finding out. What is that distance? That is a_x for me that is, if I know this to be a this is the distance that I need to know and I am interested in knowing what this value is? What is that? That is the adjacent side of this triangle with angle alpha that would be $a \cos \alpha$ is it not?

Check this please spend a few minutes and go back to your resolution of vectors and forces. And check whether I am correct. I know that I am correct. But please do check a_y likewise is that distance. That is the opposite side to angle alpha in the right triangle, this right triangle. That is $a \sin \alpha$ c_x is what? That is $c \cos \beta$. Because I only need a_y , a_x , b_x , a_x and c_x . I do not need b_y and c_y .

So, I need to know c_x , c_x is $c \cos \beta$ and b_x is $b \cos \beta$, check this. Now, I am writing out the equation $\sum M = 0$ counter-clockwise is considered positive. That is the equation I am writing. What would that be? That would be W into $c_x - a_x$. I am writing out the counter-clockwise components. First, plus $F M_x$ into $a_y - F M_y$ $F M_y$ into $a_x - W$ 1 into $b_x - a_x$ that anything else that causes a moment. No, the whole thing is 0.

Substitute for values of $F M_x$ and $F M_y$ in this equation. That is, if I call $F M_x$ and $F M_y$ as 1 and 2, sub 1 and substitute equation 1 and equation 2. In this equation which I am calling as 3. And also substitute for values of a_x , a_y , b_x , c_x in your equation 3. Substitute all these values $F M_x$, $F M_y$, a_x , a_y , b_x , c_x everything I am substituting in one go in this equation 3. When I do that what will I get?

I will get something that is going to look very scary. That is okay. This is where bookkeeping helps, so, you need to know the variables that you are working with. You need to know. What are all the variables that I am working with? I am able to do this because I am having a calm

mind. And I am not having any exam pressure. If you practice doing this several times, you will also develop the calm mind that you can use in your exam critical.

Because in the exam, if you miss one of these variables, you will get the wrong answer. And if you get the wrong answer, you will lose most of the grade most of the points for that question. It is very crucial that you develop the discipline of bookkeeping. What are all the variables that I am working with? What are the corresponding moment arms? What is the direction of the moment?

Because now I am going to directly substitute in one equation all these variables. It is likely that you will make a mistake if you do not keep your books clean. Something to keep in mind, simple tip, pro tip. If I substitute, I will get W times $c_x - a_x$, what is c_x ? c_x is $c \cos \beta - a_x$ is $a \cos \alpha$. Remember that I told that we are going to get something that is going to look scary. That plus $\sin F M_x a_y$.

What is $F M_x$? $F M_x$ is $F M \cos \theta a_y$ is $a \sin \alpha$. Why am I having this θ and α ? I am having an x component of force. And the y component of the moment arm. That is why I am having an α and θ . I am having a sinusoid and I am having a cosine. I am having a \sin of α and I am having a cosine of θ all these things it looks scary.

This is because you are having a_y component of the moment arm that is multiplying that is working with an x component of the force. This is how it will work. Because these are perpendicular. This will always be perpendicular. Because these are components you will always have an α and θ coming into the picture which is why bookkeeping is absolutely crucial in these kind of problems.

Otherwise, you will get lost very easy to make mistakes. Please keep your books clean and practice this problem and problems like this many times before the exam very crucial. Now, $F M \cos \theta$ times $a \sin \alpha$. Then what else $F M_y$ times $a_x - F M_y$ times a_x but what is $F M_y$? $F M_y$ is $F M \sin \theta$ times a_x . What is a_x ? $a_x \cos \alpha$ not be done the answer is no.

There is one more thing minus W times $b_x - a_x - W$ times b_x is $b \cos \beta - a_x$, a_x is $a \cos \alpha$ this whole thing is 0. In this the only unknown in this equation the only unknown is $F M$ everything else is done. I know W , I know C , I know β , I know α , I know θ , I

know a, I know b, I know everything. I know W_1 and I know W . So, I know everything except F_M . And F_M is what I need to find because that is an unknown.

F_M is what I need to find? I know everything else, theta is known, alpha is known, beta is known and everything is known. Now, after some algebra I request you to please do this algebra.

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Hip Problem

$$F_M = \frac{W_1(b \cos \beta - a \cos \alpha) - W(c \cos \beta - a \cos \alpha)}{a(\cos \theta \sin \alpha - \cos \alpha \sin \theta)}$$

$$F_M = \frac{W_1 \cos^2 \beta (b - Wc) + a W \cos^2 \alpha (W - W_1)}{a \sin(\alpha - \theta)}$$

$\sum F_x = 0: F_{Mx} - F_{Jx} = 0$
 $F_{Jx} = F_{Mx} = F_M \cos \theta$
 $F_{Jy} = F_{My} = F_M \sin \theta$

$\sum F_y = 0: W - W_1 + F_{My} - F_{Jy} = 0$
 $F_{Jy} = W + F_{My} - W_1$
 $F_{Jy} = W + F_M \sin \theta - W_1$

$r = \sqrt{F_{Jx}^2 + F_{Jy}^2}$
 $\psi = \tan^{-1} \left(\frac{F_{Jy}}{F_{Jx}} \right)$

Problem source: "Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation" 4th edition by Özkaya, Nihat.

After some simplification, isolate F_M , separate F_M from the rest of the equation. And find the expression for F_M in terms of all of this that would be. What would that be? That would be W_1 times $b \cos \beta - a \cos \alpha - W$ times $c \cos \beta - a \cos \alpha$. The whole thing, divided by a times $\cos \alpha$ sorry $\cos \theta \sin \alpha - \cos \alpha \sin \theta$. I can further simplify this.

That will give me F_M as $\cos \beta$ times $W_1 b - W c + a \cos \alpha$ times $W - W_1$. This whole thing divided by a times \sin of $\alpha - \theta$. That makes sense because $\cos \theta \sin \alpha - \cos \alpha \sin \theta$ looks like \sin of $\alpha - \theta$. We know this from trigonometric identities. So, this is the expression for F_M .

This you will get only after you do a substantial amount of simplification and careful bookkeeping something that you need to develop as a go along. In an NPTEL course like the one that you are now taking the motivation for doing well in the course is left to the students

like you. You need to put the fight, you need to keep the books. You need to learn how to do that?

And I know that you are all self-motivated students which is why you have come this far in the course. Invest a little bit more time and energy into keeping that discipline. Learn the art of bookkeeping and keeping it neat disciplined set of variables that you are working with. For example, like the table that I developed. We developed this table like this. Now, let us proceed we are not done yet because we only found F_M .

We have to find F_J that is more to go. Now, I can say $\sum F_x = 0$. This will mean F_{Mx} minus so that would mean F_{Mx} is in the positive x direction. And F_{Jx} is in the negative x direction. Any other x component, no other x component $F_{Mx} - F_{Jx} = 0$. That is what it means. That would mean that F_{Jx} is F_{Mx} . What is F_{Mx} ? We know what is F_{Mx} ? F_{Mx} is $F_M \cos \theta$ is it not? So, F_{Jx} is $F_M \cos \theta$. We are not done yet.

We have to find F_{Jy} $\sum F_y = 0$ right side going is considered positive. $\sum F_y$ upward force is considered positive. Where $\sum F_y = 0$. What are all the various forces? W is the ground reaction force that is acting in positive y direction W . W_1 is the weight of the leg that is acting downward $W - W_1$ plus what else F_{My} is in positive y direction $+ F_{My} - F_{Jy}$. Any other forces along the y direction and answer is no. This whole thing is 0.

So that would give me F_{Jy} as $W + F_{My} - W_1$ but I know F_{My} . What is F_{My} ? F_{My} is $F_M \sin \theta$ this is F_{Jy} . F_{My} is F_M times $\sin \theta$ remember this is F_M substitute for this value of F_M in your F_{Jx} and F_{Jy} equations. Then you will find F_{Jx} and F_{Jy} . What is F_J ? F_J itself is square root of F_{Jx}^2 plus F_{Jy}^2 is it not? This is the value of F_J .

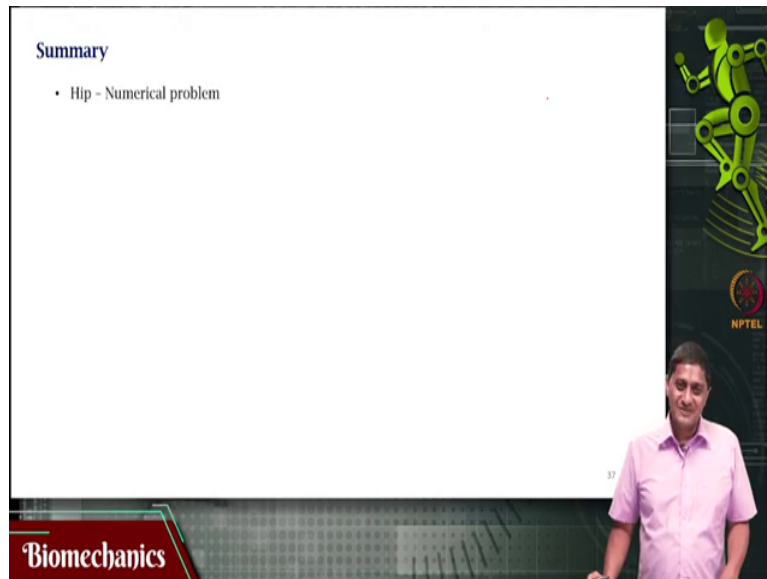
And the direction that it is acting along is some ψ which is \tan^{-1} of F_{Jy} by F_{Jx} . This is the angle that F_J makes with the horizontal ψ , is the angle that angle. Simplify for this and you will find the value of F_{Jx} and F_{Jy} . In the exam, you will be given numerical values of small a , b , c , θ , α and β and weights. You will be asked to find the numerical value of F_M and F_J .

This would, of course, involve quite a bit of computation. So, unless you practice this with dedication and motivation. It will be very challenging for you to solve this in the exam. So,

my request is take some time out and practice this problem. And these types of problem from the same book are other books and get ready for the exam. Preparing for the exam is a semester-long process.

Remember this preparing for the exam is a semester-long process. It is not something that you do the day before the exam remember this.

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So, with this we come to the end of this video. In this video, we saw a simple numerical problem involving the hip or rather a problem in the statics of the hip joint. It was a simple problem but there were many variables involved and it required a lot of bookkeeping. I request you to practice this problem when you have the time and get ready for the exam. Thank you very much for your attention.