Biomechanics
Prof. Varadhan SKM
Department of Applied Mechanics
Indian Institute of Technology – Madras

Lecture – 39
Knee Problem

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(FL) Welcome to this video on biomechanics. We have been looking at biomechanics of the knee joint in the last couple of videos. We looked at the movements made by the knee joint and the muscles that are responsible for generating these movements.

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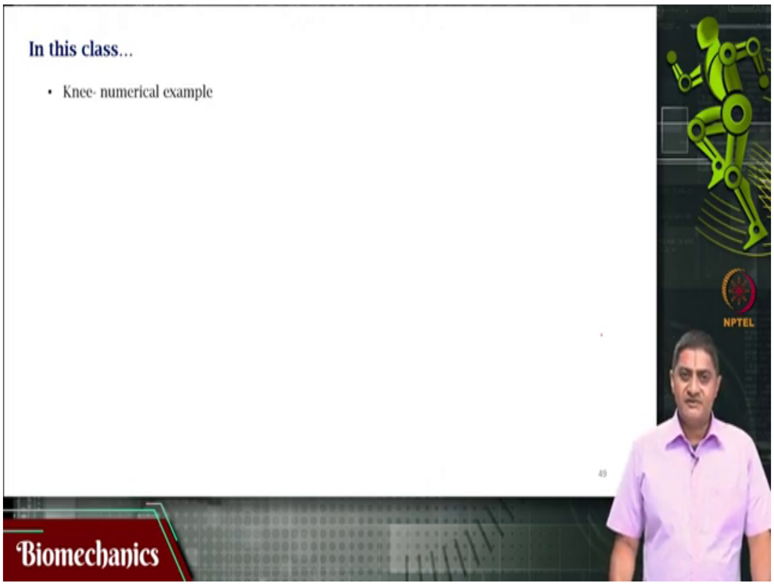

In this video, we will take a simple problem involving the knee joint and solve this using a numerical or symbolic method.

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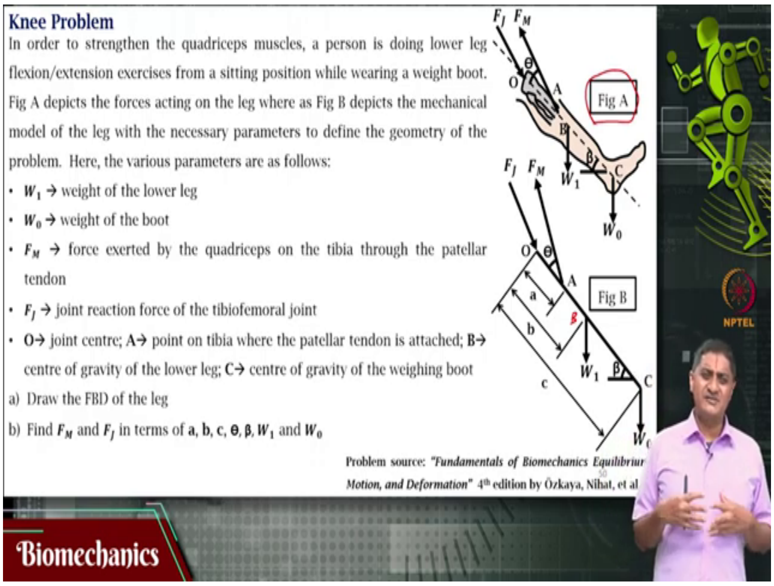

This is the question in order to strengthen the quadriceps muscles, a person is doing lower leg flexion extension exercises from a sitting position while wearing a weight boot. Figure A depicts the this is figure A. Figure A depicts the forces acting on the low acting on the leg, whereas figure B depicts the mechanical model of the leg, with necessary parameters to defend the geometry of the problem. What are these parameters?

W 1 is the weight of the lower leg. W naught is the weight of the boot. F M is the force produced by the quadriceps muscle on the tibia through the patellar tendon and the patellar ligament. F J is a reaction force that is seen at the tibiofemoral joint. O is the point about which the movement is happening or is the joint centre. Capital A is a point on the tibia, where the patellar tendon attaches.

Capital B is the centre of gravity of the lower leg that is that point. Capital C is the centre of gravity of the weighing boot under on the foot. For example, put together where that centre of gravity comes. The question is find the muscle force F M and the joint reaction force F J in terms of the distances a, b, c and theta, beta, W 1 and W naught. A question is, how do theta? Because I know a, b, c.

Somehow let us say that using some model or anthropometric measure, I somehow know where B is, where A is, where C is. For example, I know beta because that is the inclination that the leg is making with respect to the horizontal. Maybe I can measure it using some simple technique like an IMU, for example, how do you know theta? That is the question you actually, do not know not in the specific case that you are discussing but for the purpose of this problem.

Let us assume the theta is somehow given someone knows theta and they have given you theta. And W1 and W naught you can of course, measure or estimate. W1 you can estimate W naught you know because that is the weight of the boot. Some variance of this exercise involves, keeping a dumbbell on the ankle and so on and so, forth. There are many variants of this exercise.

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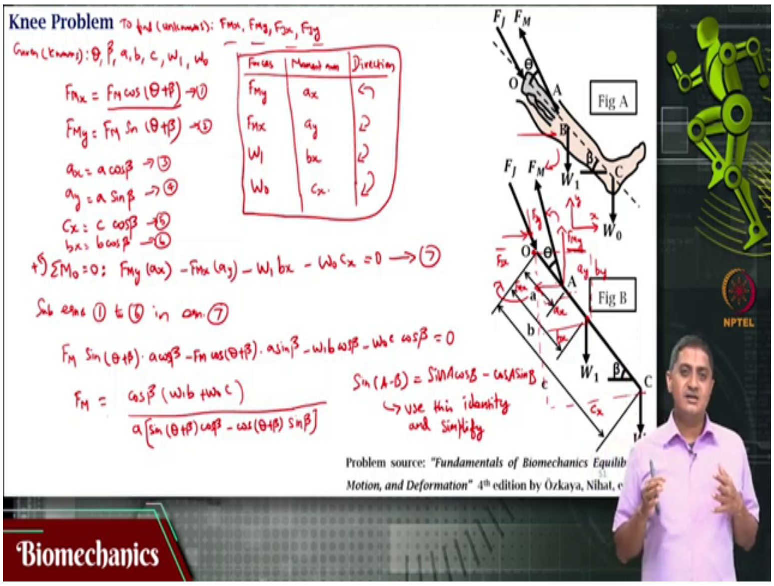

Now let us, write out the knowns, unknowns, the free bird diagram of for this case. I am just going to use the existing picture to draw the free bird diagram I am going to use that x, y axis the vertical distance along the y direction between the origin O and the point A is small ay the horizontal distance is small ax for B that distance is by and that distances bx. Of course, I am interested in finding that distance which is cx.

Now F M itself will have two components, vertical component which I am going to call as F My and it is horizontal component which I am going to call as F Mx. Likewise, F J will have two components, it is horizontal component is F Jx and it is vertical component is F Jy acting in this direction, as shown. To find are the unknowns, are F Mx, F My, F Jx, F Jy. Essentially, the problem asks for the joint reaction force on the muscle force.

So, F M and F J are what is asked we will have to find the individual components and then find the total muscle force and the total joint reaction force. So, essentially, we need to find out F Mx, F My, F Jx, F Jy given the knowns what are the knowns? Theta is known do not ask me how? Let us assume that theta is known, beta is known this we can measure of course, this we know.

Small a, small b, small c W 1 and W naught are knowns, these are known. Now, what is F Mx? F Mx is the horizontal component of the muscle force F M. Remember that the leg itself is inclined at an angle of beta to the horizontal and the muscle force is inclined at the angle of theta to the leg. So, the net angle with respect to the horizontal, is actually, theta + beta is it not? I hope this is clear.

The leg itself is inclined at an angle and the muscle is inclined at an angle to the leg. So, the net angle, with respect to which with respect to the horizontal with which the muscle is acting is theta + beta. So, F Mx is F M cos theta + beta, F My is F M sin theta + beta. Do you know ax and ay? Well, we can find it unlike F Mx and F My, ax and ay are affected only by the inclination of the leg itself which is beta.

So, ax is a cos beta and ay is a sin beta, cx is c cos beta likewise. Now, to solve this, I think it would be convenient to take the moment about the point O here. So, I am just connecting here about that point. Why? Because the giant reaction force F J is acting along O. So, if I take the moment about that point then the components of the moment due to this force will disappear because the moment terms are 0 because there are many unknowns.

It will be useful if I take the moment about one of the points for which I know the moment arms. I know the axis is rotating. For example, so, this is very convenient. So, I am going to write sigma M suffix O counter clockwise considered positive is 0. But before we do that let us write out the individual forces, their moment arms and the sins of the rotation, what are the forces? That act O that can cause a moment about O that not necessarily act at O.

What are the forces that have a tendency to cause a moment about O. F My can cause a moment, forces, moment arms direction. F My can cause a moment and it is moment arm is ax and F My has a tendency to cause a rotation in the counter clockwise direction it is a positive moment arm. How do we know this? Because that is the rotation that this force is causing?

I know this F Mx can cause a moment and it is moment arm is ay that has a tendency to cause a clock wise movement. W 1 can cause a moment in the clockwise direction. Since moment arm is bx, W naught can also cause a moment and it is direction in the clockwise direction this moment arm is cx, is it not cx? Now, we have everything to write out our moment equation. So, let us expand our moment equation which is sigma M O is 0.

That is F My times ax – F Mx times ay – W 1 bx – W naught cx. The whole thing is 0. Now, let us name these equations as 1, 2, 3, 4, 5. Something that we have forgotten is that bx is b cos beta equation 6. Now, substitute for F Mx, F My, ax, ay, bx and cx. That is substitute equations 1 to 6 in this equation which I am going to call as 7, equation 7. And try to simplify after some algebra, please take a few minutes and do the bookkeeping because I am going to skip a couple of steps.

You can substitute F Mx, F My, ax, ay, bx, cx all these 6 into the moment equation and do some adjustments do some algebra? After you do that you will get F M sin of theta + beta times a cos beta – F M cos half theta + beta times a sin beta – W 1 b cos beta – W naught c cos beta is 0 am I correct? Yes, this is correct, is substituting for F My, F Mx and so on and so forth.

Now, take those terms that have F M on one side and W's on the other side, such that I can actually, find the value of F M. Now, simplify such that I can find the value of F M. F M is cos beta times W 1b + W naught c divided by a sin of theta + beta times cos beta – cos of theta + beta times sin beta. This is looking like sin a cos b – cos a sin b. We know that this is sin of a – b.

We know that this is sin of a – b which is sin a cos b – cos a sin b using trigonometric identity sin a – b is sin a cos b – cos a sin b. This is use this identity and simplify. Where do you use this? In the denominator, of course, in the denominator.

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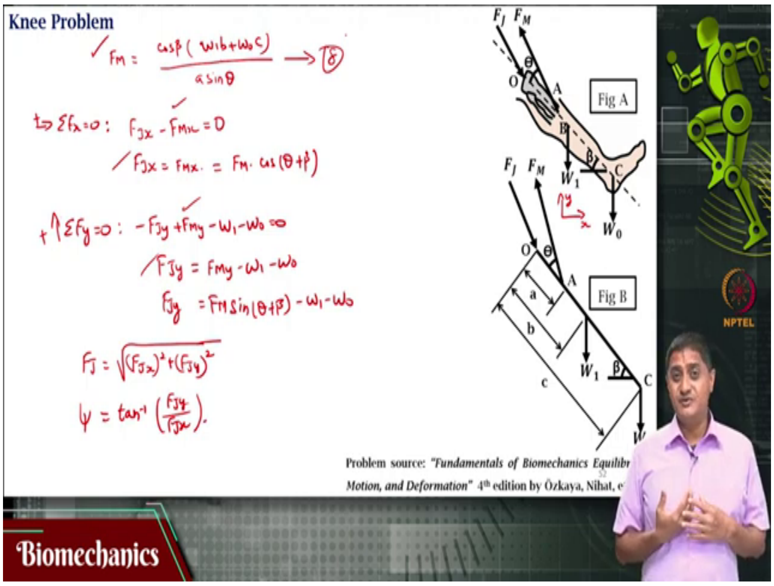

If you use this in the denominator, you will find F M as cos beta times W 1b + W naught c. The whole thing divided by is sin theta. What is this? It is remained ourselves of what this is? We have found something but let us remind ourselves what this is? This is the quadriceps muscle force. Are the force produced by the quadriceps muscle to support the leg at an angle of beta from the horizontal to support the leg at an angle of beta from the horizontal you need this F M force.

Now, how do I find F J well I can use sigma F x = 0, the right side going positive. From the previous slide what are all the forces in the x direction? There are only two forces in the x direction. F Jx and F Mx. W 1, W naught are all in the y direction. So, F Jx and F Mx are the only two forces in the x direction. So that would mean of this F Jx is in the positive x direction because we wanted to use that xy direction not and F Mx is in the negative x direction.

So that means F Jx – F Mx is 0 or rather F Jx is F Mx. But what is F Mx we found from the previous slide. F Mx is F M cos theta + beta is it not? That is if you find F M as per this equation 8. You substitute this value of F M in that equation. So, to find F Jx you substitute this value of F M as found in equation 8 and then multiplied by cos of theta + beta that is F Jx. How do I find F Jy? Same sigma F y = 0 up going is considered positive.

If I use that now there are many others W's will come into the picture that is okay. That is – F Jy + F My – W 1 – W naught is 0. From this I can write F Jy as F My – W 1 – W naught. But what is F My? That is F M sin theta + beta is it not? How do we know this? From the previous slide from my previous work. So, find F M as per equation 8 and substitute in this equation for F Jy so, you can find the component the vertical component of the joint reaction force at point 1. So, I know no F Mx F, My F, Jx and F Jy I know everything.

Now if I am interested in the total joint reaction force F J all I have to do is find the square root of the sum of the squares. And if I want to know the direction at which this reaction force is acting, all I have to do is find the tan inverse of F Jy by F Jx. This is the joint reaction force and the direction at which this is acting. So, we were required to find the F M or the components of F M, F Mx, F My, F Jx and F Jy.

We have found all the things that we were required to find F Mx, F My, F Jx, F Jy are the unknowns or the things that we wanted to find. Now, we have found all those things. It is a little bit of practice with a little bit of practice you will be in a position to master this. Please be very careful with your bookkeeping. What appears like trivial? The most trivial part of this entire video, you would think, is this table that I have made this little table that we have made that one that not.

It turns out that the correctness of the entire problem depends on this why? Because minus plus all those things depend on this. So, you need to know where your forces are what the moment arms are, what the direction of the moments are. Once you are clear with this application of these is a trivial matter. So, please remember this while you are solving problems, please remember this.

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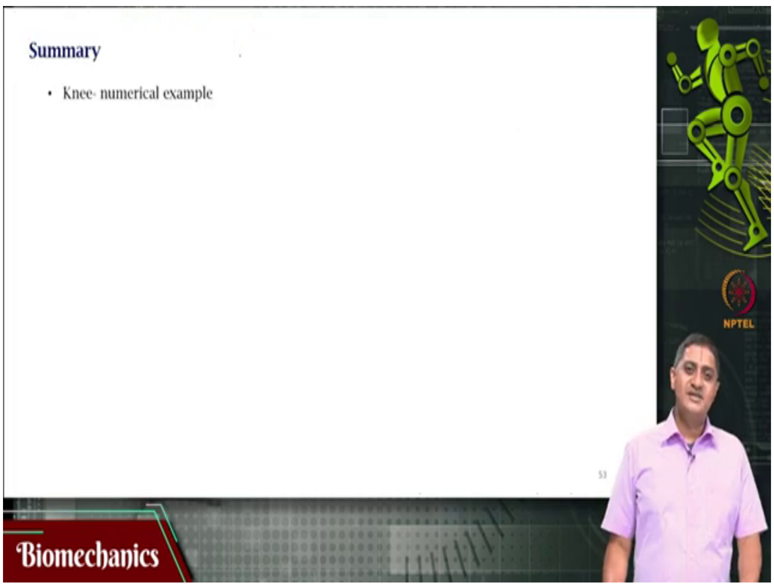

So, with this we come to the end of this video. Thank you very much for your attention.