Biomechanics
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Lecture – 42
Ankle Problem

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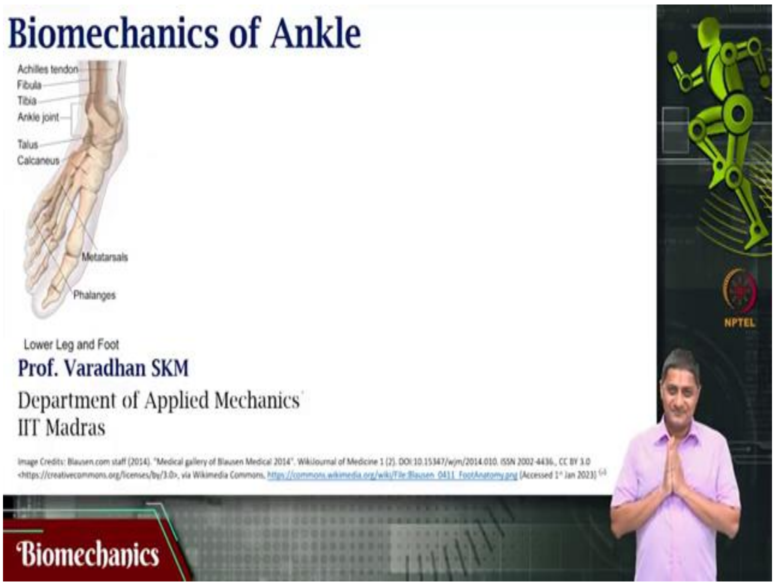

(FL) Welcome to this video on biomechanics. We have been looking at biomechanics of the lower limb. We looked at the hip and we followed it up with a numerical problem on the hip and then we looked at the knee. The movements in the knee and the muscles of the knee and then followed it up with the numerical problem in the knee. In the last couple of videos, we looked at the anatomy of the ankle joint and the movements that are possible in the ankle joint.

And then we looked at the muscles that are responsible for causing these movements at the ankle joint.

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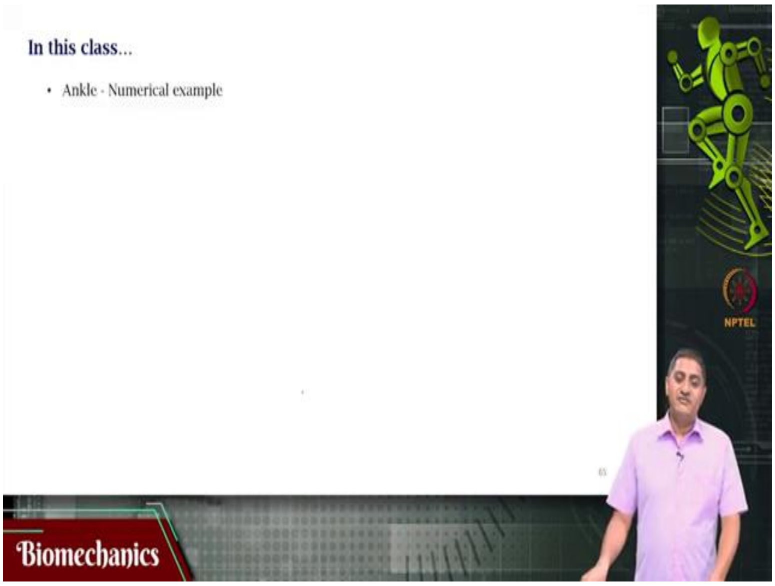

In this video, we will be looking at a numerical problem or at a problem involving the statics of the ankle joint.

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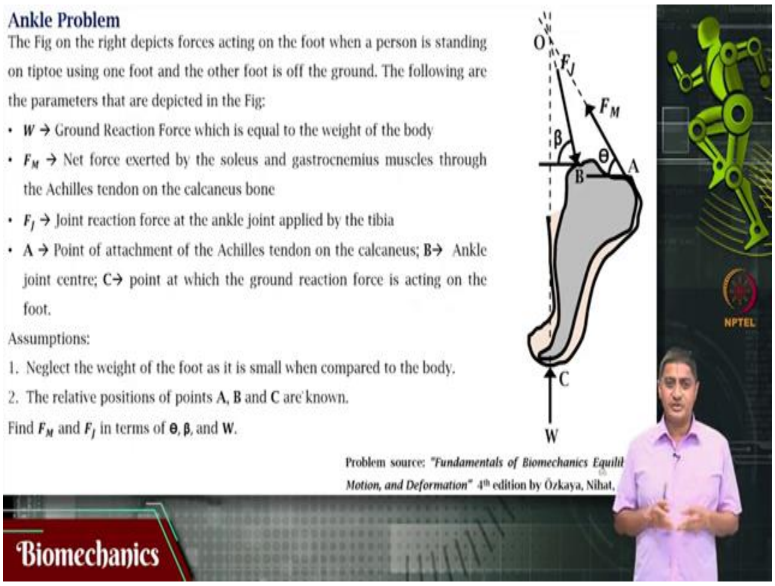

So, let us focus on this. The figure on the right depicts forces acting on the foot when a person is standing on tip toe using one foot and the other foot is off the ground. So that means that the person is using a single foot stance is having only he is using only one foot. The following are the parameters that are depicted in the figure. W is a ground reaction force. This is essentially the weight of the body.

The entire weight of the body is acting at this point tip toe 1 point**.** F M is the muscle force which is the force exerted by the soleus and gastrocnemius muscles through the achilles tendon, on the calcaneus bone. F J is a reaction force filtered the ankle joint, the talocrural joint that is applied by the tibia on this joint capital A is a point of attachment of the achilles tendon to the calcaneus bone.

Capital B is the point at which the talocrural joint is located or the joint centre. Capital C is the point at which the ground is contacting the foot or the footage contacting the ground. The point at which ground reaction force is acting on the foot. Some assumptions are also given for us neglect the weight of the foot because it is very small when compared with the weight of the body.

The relative positions of the points, a, b, c are all already known. The question is find the muscle force F M and the giant reaction force F J in terms of theta beta and weight? Beta is the angle that the joint reaction force is making with respect to the horizontal and theta is the angle that the muscle force is making with respect to the horizontal.

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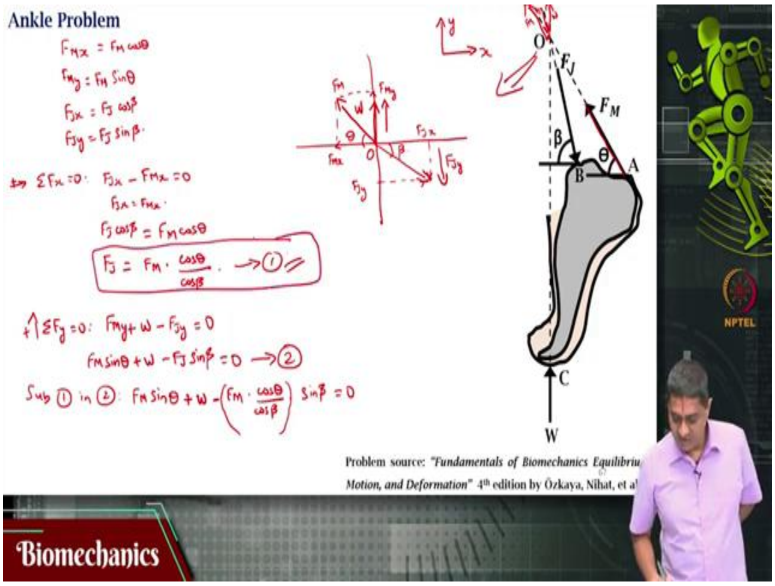

First of all, let us know that this problem is slightly different from the problems that we have seen previously. That does not mean that this problem is harder. It is only slightly different. Now, let us look at what is special about this program. It turns out that the weight of the entire body is applied. At that point, the joint reaction force is acting at that point. Along this line of action, the weight is acting along this line of action and the muscle force is acting along this line of action.

Now, we are looking at all this at a plane all these are happening in a plane. Remember in one of our first classes, we said that we can move a force along it is line of action without altering it is effect. This principle, of course, is the so-called principle of transmissibility. Remember if you do not remember, I request you to pass this video and go back to the first week in which we discussed the principle of transmissibility of forces that principle states for your review one more time.

That principle is, I can move a force along its line of action without causing any effect on the and how the force is acting or without altering its effects? These three forces form a three force system and I can use the principle of transmissibility to move these forces such that they all have a common point of intersection. Now, you might wonder, what is this common point of intersection? Is that an anatomical construct? Is it present on the body?

Remember that I this principle is abstract I can move this force along the line of action without altering its effect. That does not mean that it has to be a point on the physical body that is not necessary. It is a principle it is an abstract concept, so, I can move this and proceed with the analysis. I know some of you are a little uncomfortable with this but please bear with me and then you will slowly realize after we solve the problem and then you will be convinced.

So, this is these three forces F J, F M and W form a three force system that are intersecting at that point O. Now, what I am going to do? I am going to move this point O and this three force system around point O separately I am going to draw that three force system separately as a figure and then do the rest of the analysis. Now, let us draw that now, W, W is acting like this W is so that is the x, y axis.

Note that F M and F J are inclined to both the x axis and the y axis. But W is purely along the y axis in this construction. So that is W for me that is W. Then I have F M that is acting from O at an angle of theta to the horizontal. You are saying hey but this is not acting at O. It is not but I have moved it to point O. So, essentially, I have moved this to this point such that it is acting like that. W is acting like that F J is acting like that.

That is F J this is F M and this is W and I am redrawing the whole figure separately here. So, this is what has been zoomed out for you, so that is F M for me. And F J is acting from point O like that F J is making an angle beta to the horizontal. Technically, this beta is not exactly the same beta that F J is making. because if you see this is point O and this is point B where this is beta.

Not this is how it is given that is that point, whereas what I am doing is, I am moving FJ to this point and then I am marking this angle. That is like I have moved to this point and then I am marking that angle which is actually, the opposite angle. I am using some geometry which is actually, the opposite angle and because it is an opposite angle that angle is beta. I hope that is clear for you, because of that reason this is beta that is F J.

Now, this F M will have an x component and a y component. So that will be F My and this will be F Mx and this will be. This will be F Jx and this will be F Jy. This is the construction of the situation. Now, I can analyse this problem. And because I can transfer all these forces to point O. These forces form a concurrent force system, force is concurrent at a point in a plane.

Now I can find F M and F J that is the question that is asked for us. The question is find F M and F J? Let us go back and verify, what is the question find F M and F J? That is the question. So, I can find F M and F J in terms of theta beta and other things and W etcetera W is not. Now, I can use my knowledge of resolving vectors to write F Mx as F M cos theta and F My as F M sin theta and F Jx as F J cos beta and F Jy as F J sin beta.

Now, I can say sigma Fx = 0 going forces is considered positive. Now, if you say sigma Fx = 0, what do we get? F Jx is going toward the right and F Mx is going towards the left –F Mx because this is F Mx and this is F My and this is this is F Jx sorry, F J1 and this is F Jx. F Jx – F Mx is 0 or F Jx is F Mx substitute for F Jx and F Mx. What is F Jx? F Jx is F J cos beta is F M cos theta.

So that means FJ is F M times cos theta divided by cos beta is not? This equation I am going to call as a equation 1. Now, let us write sigma Fy = 0 upward going is considered positive. What are all the forces in the y direction F My is in the positive y direction that is that force is not? +W is a ground reaction force this is the force that is applied by the ground on the toe.

This is not the weight of the body, numerically, it is equal to the weight of the body but it is acting from the ground to the body. In other words, it is acting in the positive y direction, not the negative y direction. The weight is acting the weight of the body is acting in the negative y direction. The ground reaction force because the whole system is in static equilibrium. The ground reaction force is acting in the positive y direction.

Because of which W is +W – F Jy that is that force is it not? F Jy force the direction is negative. Now, I can write out F My and F Jy, F My is F M sin theta + W – F J sin theta sorry sin beta – F J sin beta. This whole thing is 0 and I am going to call that equation as equation 2. Now, substitute equation 1 in equation 2. When I do that what do I get? F M sin theta + W – F M, F J is F M cos theta by cos beta, is it not?

That is what I am substituting F M times cos theta by cos beta times sin beta. The term in the parenthesis, the term in the big bracket is what I have substituted from equation 1. This whole thing is 0. Have I missed anything, the answer is no, I have done it correctly. Now, in this equation theta and beta are known and W is known. Let me rewrite this equation in the next slide and then we will proceed.

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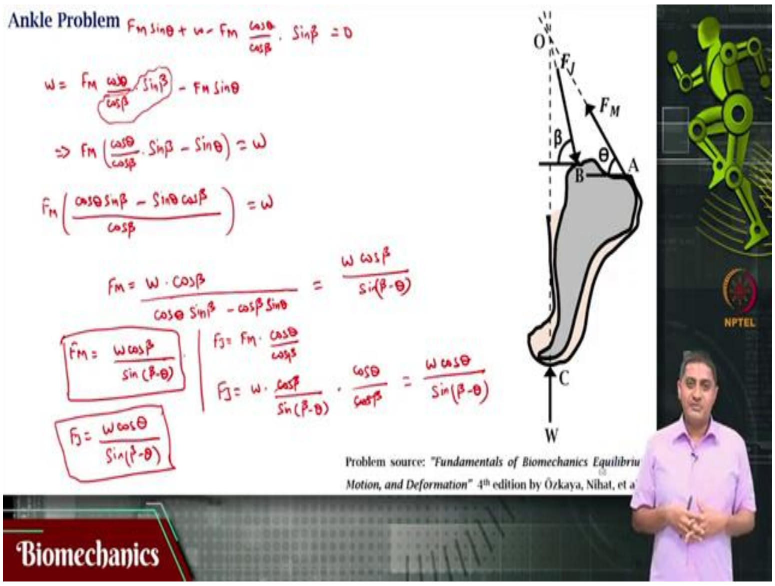

That is F M sin theta + W – F M cos theta by cos beta times sin beta is 0. Now, I am rewriting this equation, as W is F M cos theta by cos beta times sin beta – F M sin theta. Now, I can take out F M on the right hand, side and say that this is cos theta by cos beta times sin beta – sin theta or rather and that is equal to W just exchanging LHS and RHS. Now, after some more adjustments, for example, I can take LCM cos theta sin beta – sin theta cos beta the whole thing divided by cos beta is W in this.

I know W and I know theta and beta, so, I am taking F M on one side. So, F M = W times cos beta divided by cos, theta sin beta – cos beta sin theta, is it not. This is sin of beta – theta. The denominator is sin of remember from trigonometric identity is this is sin of beta – theta. This is F M, F M is W cos beta divided by sin of beta – theta. Now, substitute this F M in the expression for F J. What is that?

Remember from the previous slide F J is F M times cos theta divided by cos beta that is equation 1. This is that equation F J is F M times cos theta divided by cos beta. Substitute this F M in this equation for F J. That would mean F J is W times cos beta divided by sin of beta – theta times cos theta divided by cos beta, cos beta and cos beta I can cancel. So, that will give me F J as W cos theta divided by sin of beta – theta.

So, F M is W times cos beta divided by sin of beta – theta and F J is W times cos theta divided by sin of beta – theta. See when I was solving this problem there was this constant temptation to mark this sin beta by cos beta and tan beta. If you had done that the simplification would not have happened, something to keep in mind. Constantly, I was tempted let us do this do not try to do this.

So, we have now arrived at the solution. What was asked from us was finding F M and F J. And if I had marked this as tan beta, you can still solve this but it would have been a lot more ugly or a lot more difficult. You can still solve this. This classic look a sin of beta – theta is coming from the trigonometric identity, cos theta sin beta – cos beta sin theta is sin of beta – theta. So, this gives the expression for F M and F J in terms of W theta and beta.

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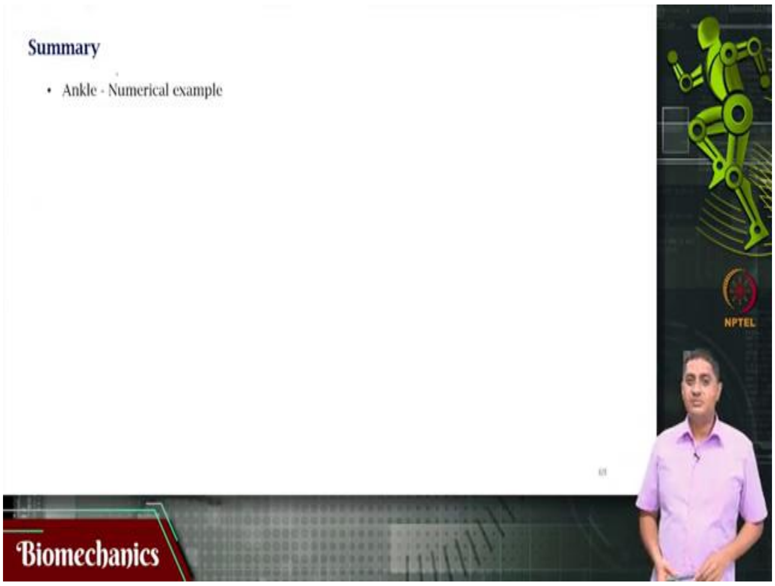

So, with this we come to the end of this video. In this video we saw a simple problem involving statics at the ankle joint. Thank you very much for your attention.