Biomechanics
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Lecture - 45
Forward Kinematics and Workspace

Welcome to this video on biomechanics. We have been looking at kinematic chains, we gave the example of the upper limb as a serial kinematic chain. Remember the lower limb can also be considered to be a serial kinematic chain. In fact, we discussed that when the lower limb is constrained by the floor which is when you are walking. When you are walking the lower limb is constrained by the floor.

So, that is a form of closure, it is a closed kinematic chain. There is one constraint at the hip and there is another constraint on the floor. So, any movement of this kinematic chain which is the thigh shank chain or the upper leg lower leg chain can happen only between these two constraints. In other words, but the hip and the floor constitute the constraints within which the leg can move likewise reaching.

While we are discussing reaching of course we restrict our attention to plane are reaching within the plane what are the various constraints that are there something to keep thinking about.

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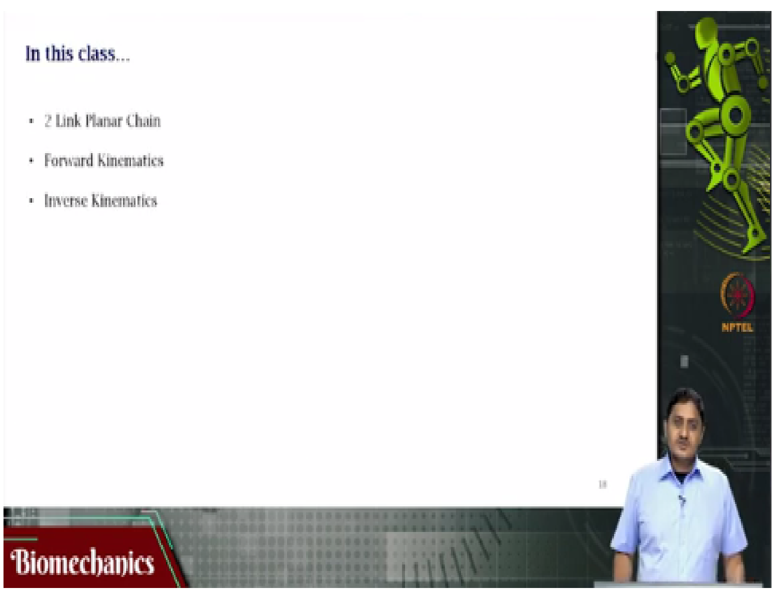

So, in this video we discuss the kinematics of a two link planar chain it is forward kinematics and it is inverse kinematics. We start our analysis with first principles and we proceed to develop the method for this type of analysis.

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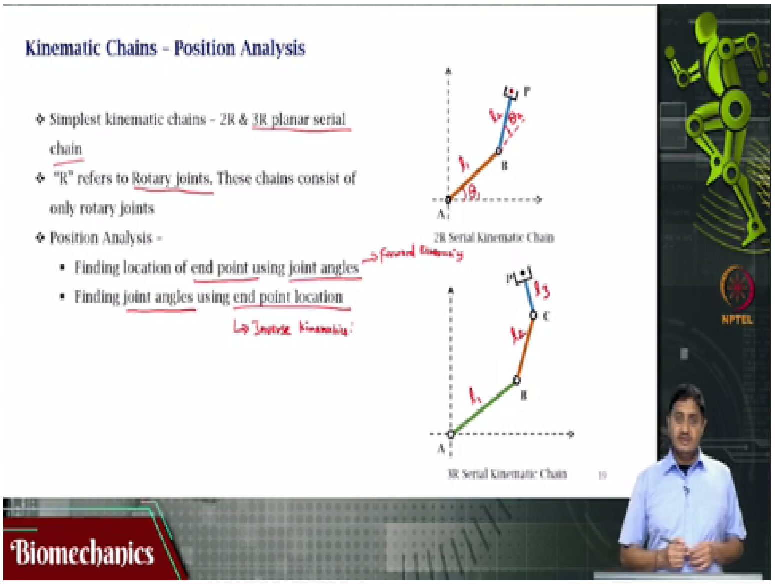

So, consider the simplest kinematic chain, a 2 link planar serial chain. We will also be focusing on 3R other three link planar serial chain which we will do in a future video. For now, we restrict our attention to a 2R planar serial chain that is of this type. So, there is one link AB I am going to call this link length as sum l 1 and another link B P I am going to call that link length as l 2. So, two are serial kinematic chain in the case of the 3R change that is an AB with length 1 that is a B C length l 2 and there is a C P with link length l 3.

Of course, R here refers to rotary joints these chains consist of only rotary joints for the purpose of this discussion you assume that these chains have only rotary joints. So, now a question is suppose I know the angles that these links make with the horizontal or with the previous link or with the ground as the case may be can you find out the end point coordinates. This is the big broad question before us.

In other words, suppose I know the joint angle at A and the joint angle at B suppose I know that the joint angle at A is some theta 1 and the joint angle at B is sum theta 2 with respect to the previous link. Remember this very critical to remember. This theta 2 is with respect to the previous link AB not with respect to the ground. The question is given that the linked links are l 1 and l 2 can you use principles of trigonometry and geometry to arrive at the coordinates of the point P this is the broad question.

Can you find the location of the end point using joint angles and link length? Link lengths are always assumed to be known for example this is called as forward kinematics. Let us say I know the link lengths and I know the endpoint coordinate that piece is something that I know I also know the link lengths l 1 and l 2. Can you tell me the joint angles theta 1 and theta 2 is the other question. This is for the simple case of the tooling chain.

When the number of links increase this becomes complicated as we will see in future videos. Now if I need to find the joint angles using end point this is called as inverse kinematics. I know the end point and I want to know the joint angles. It turns out that one is you know more challenging than the other we need to perform the analysis and find out which one and why that is the case.

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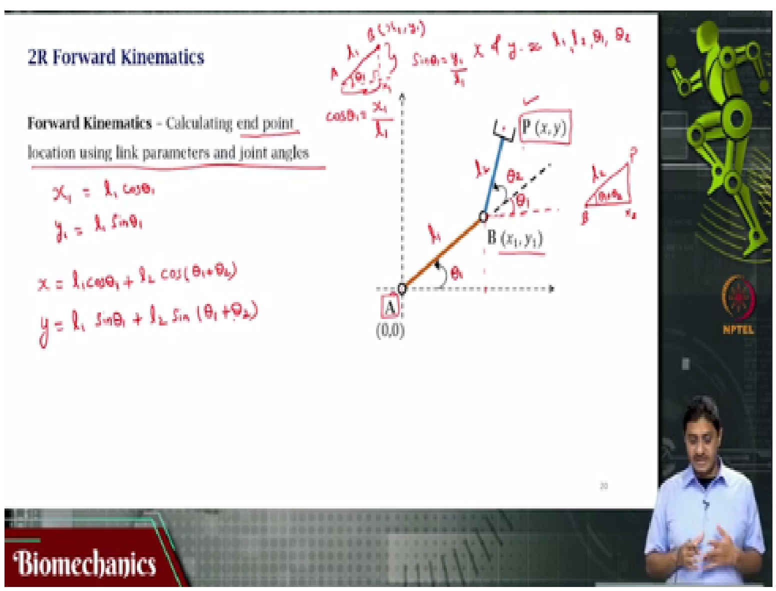

So, let us start our analysis by looking at this two link planar kinematic chain. The question is find the end point coordinates using link lengths and joint angles. I know this theta 1 and I know this joint angle as theta 2 this is given to you and the link lengths are given to you as l 1 and l 2.

Can you find out the coordinate of the point P? In other words, can you express x and y as a function of in terms of l 1, l 2, theta 1, theta 2 some functions of theta 1 theta 2 for example.

Can you do that; that is the question, how do we proceed with this analysis. Before we do, let us perform a simpler analysis of this case. Now what I am going to do? I am going to take the same link length l 1 and the same theta 1. Now question is can I find out the coordinate B in this. The answer is yes, I can find out the coordinate B using theta 1 and l 1, how? What is the coordinate of B?

It has two coordinates and then x coordinate and y coordinate and that is the x coordinate is it not, that is the x coordinate that is the adjacent side of the right triangle AB x for example. The adjacent side of this right triangle is the x coordinate of the point B. So, I could then express this as let us call this as I am calling these two points as x 1 and y 1. This B is going to have coordinates x 1, y 1.

The question is what is x 1? x 1 is the adjacent side of the right triangle AB x well I know theta 1. So, cos theta 1 is adjacent side by hypotenuse, adjacent side is A x and since I know theta 1 I can actually write out x 1 as l 1 cos theta 1. How am I getting this? Because cos theta 1 I am writing in here cos theta 1 is x 1 I have told this is the coordinate is the adjacent side x 1 divided by hypotenuse AB. What is the length of the hypotenuse AB that is l 1.

So, in this I know l 1 and I know theta 1. So, the only unknown is x 1 so x 1 is l 1 times cos theta 1. It is Y coordinate is this is it not, that is the opposite side or the side opposite to theta 1. Now I can write out sine theta 1 as y 1 divided by l 1, it is not the opposite side divided by the hypotenuse. Hypotenuse in this case is l 1, the opposite side is the coordinate y 1 it is a y coordinate of this point B.

So, that means I do not know y 1 I can write out y 1 as l 1 sin theta 1 is it not I am taking l 1 to the other side that means that y 1 is l 1 sin theta 1. We have not yet solved the problem this is just the beginning. Now but this is not what I am interested in, I am interested in what is a big question I am interested in finding the x and y coordinate of the point P not the point B. Point B anybody can find out a high school student a 10 standard student can immediately solve this and say hey what is the big deal.

I am not interested in finding just B that helps me finding the coordinates of the point B helps me but it is not sufficient. I need to find out the coordinate of the point P. Remember the link BP is at an angle of theta 1 + theta 2 to the horizontal, how do we know this? Because it is having an angle of theta 2 with respect to the previous link and the previous link is having an angle of theta 1 with respect to the ground.

So, the net inclination of this link with respect to the ground is actually theta 1 + theta 2. Now I can write out well actually if I am interested, I can perform a similar analysis I will leave this as an exercise to you. So, in the right triangle B x 2 P, in this right triangle I am interested in finding the location of the point P as a function of the link length l 2 and the inclinations theta 1 and theta 2. Remember how I got theta 1 and theta 2, theta 1 + theta 2.

How did I get this? Because theta 2 is the inclination of the link BP with respect to the link AB, but because the link AB is already at an inclination of theta 1 with respect to the ground the net inclination of the link BP is theta 1 + theta 2 with respect to the ground that is how I got this as theta 1 + theta 2. Perform an analysis similar to what I did with the previous triangle. In the previous case I had an x I could call this as capital X 1 for the purpose of generality.

So, I am having this X 1 and then I am having an X 2. Now could you find out the location of point P as a function of theta 1 + theta 2 and l 2 in the B frame of reference with respect to the coordinate B then I can move this to the coordinate A. So, that is the idea, you could try this as an exercise, you could try this. But I will write out the final answer of this point x, y as a function of the coordinate system located at A can, I write this out.

The answer is yes, I can write this as x is l 1 cos theta 1 which is that plus l 2 cos of theta 1 + theta 2 similar to the previous analysis and then I can write out y as l 1 sin theta 1 + l 2 sin of theta 1 + theta 2. Remember it is not l 2 cos theta 2 and it is not l 2 sin theta 2. It is l 2 cos of theta 1 + theta 2 and l 2 sin of theta 1 + theta 2, something to keep in mind. This is the required coordinate of the end point P which is a combination of the two link links and the two thetas that are involved with respect to the ground.

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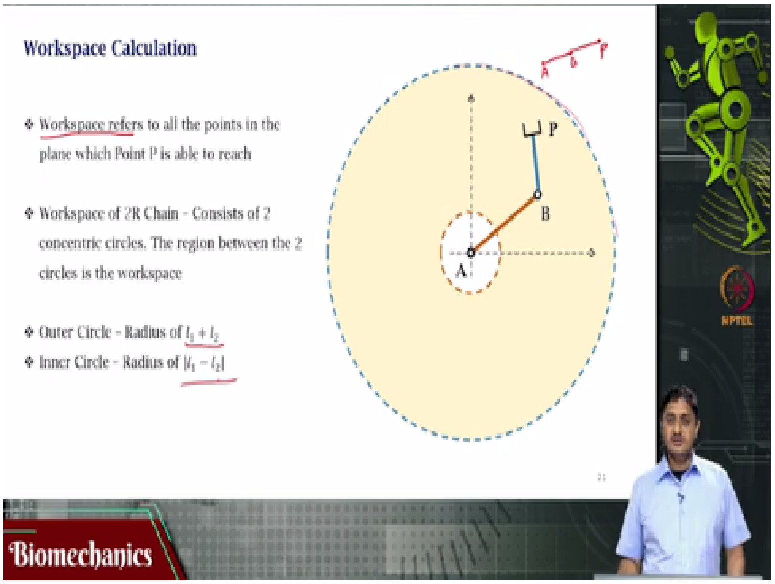

We are interested in computing workspace. What is this workspace? It refers to the set of all points in the plane within to which this point P will be able to reach. For example, in two link chain in a 2R chain it consists of two concentric circles the region between the two circles is the workspace. So, I have one concentric circle which is one of these two concentric circles which is the outermost circle.

When will this be true when AB and BP are parallel to each other or in other words the two links are such that are the or the entire chain is elongated completely that is theta such that theta between AB and BP is 180 degrees or it is just like this ABP. When that happens, I will inscribe this circle this big huge circle that is this circle, I will be producing this circle and when P is such that when it is close to that point.

When it is rotated around such that the angle is rather not 180 degrees but rather zero degrees you will have another circle produce this smaller circle. Note that in this white smaller circle within this white smaller circle the end effector cannot enter. Why? Because the link length is such that it actually l 2 in this case is assumed to be smaller than l 1. Because of this reason I actually cannot go beyond this circumference of this smaller circle.

So, the set of all points or the area that is shaded in yellow in this case is a set of all points to which this point P or this end effector P can move to this set of all points to which this end effector P can move to is called as the workspace of this kinematic chain. Remember this is a function of the link links l 1 and l 2. The outer circle has a radius of l 1 + l 2, the inner circle has a radius of modulus of l 1 - l 2.

Of course, if l 2 is greater than l 1 this inner circle will essentially vanish but for the purpose of discussion, we must have this more you know; non obvious case. So, with this we come to the end of this video. In the next video we will be looking at inverse kinematics. Thank you very much for your attention.