Biomechanics
Prof. Varadhan SKM
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 47
3R Kinematics Forward and Inverse

Welcome to this video on biomechanics. We have been looking at kinematics of reaching and grasping.

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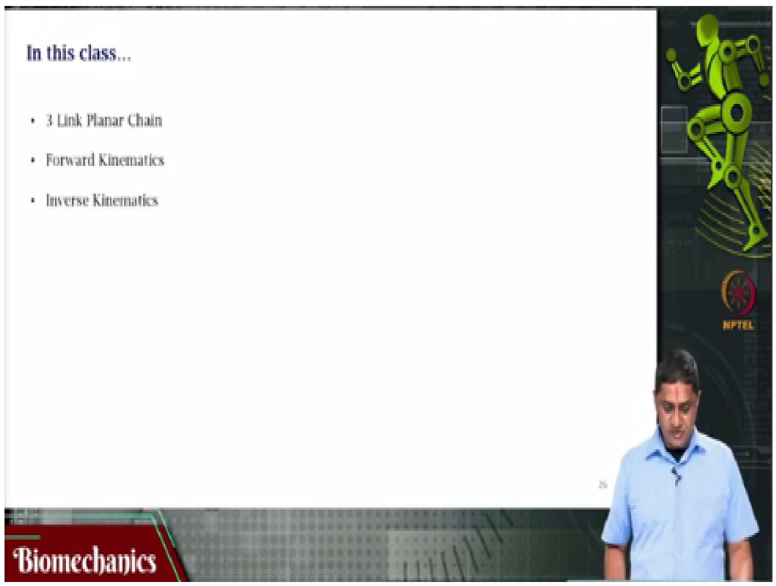

In this video we will look at a 3 link serial kinematic chain planar kinematic chain forward kinematics of 3 link kinematic chain and inverse kinematics of a 3 link kinematic chain.

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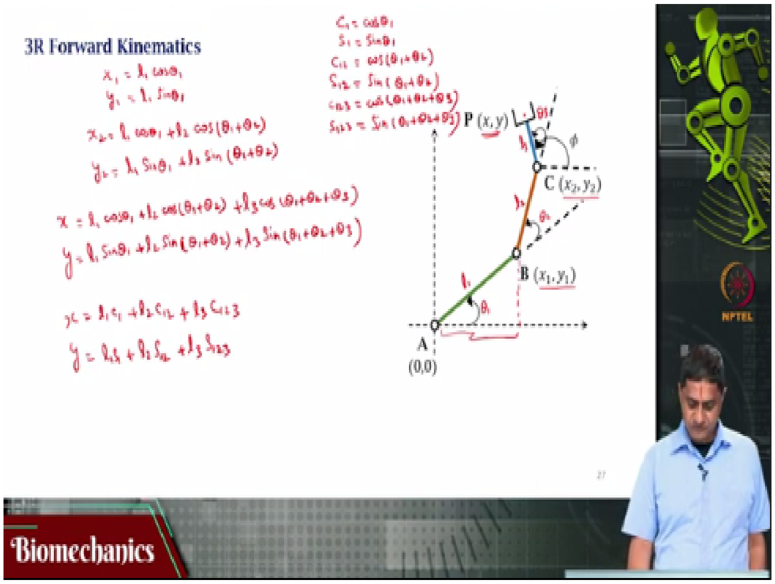

Consider this situation there are three links AB, BC and CP of link lengths l 1, l 2, l 3. Each of these is making the joint angle theta 1 theta 2 and theta 3 with respect to the previous link. In the case of link AB, it is making an angle of theta 1 with respect to the ground. So, this is known the question is, can you find the coordinate of the end point P can you find this x and y? Given that I know l 1, l 2, l 3 and theta 1 theta 2 theta 3.

How do you do this? Similar to how we did it for the 2R case we can proceed for the 3R case. Now before we proceed to find the coordinates of x and y and the coordinates of point P which are x and y we start by finding the coordinate of B which is x 1, y 1. What is this x 1 y 1? This is x coordinate is l 1 cos theta 1 is not it, x 1 is l 1 cos theta 1 and y 1 is l 1 sin theta 1. What about x 2? This is x 2 this is remember theta 2 is the angle at the joint rather the joint angle.

This is the angle that the link BC is making with AB. So, with respect to the ground the total angle will be theta 1 + theta 2. So, if I wanted to write out x 2 that is l 1 cos theta 1 + l 2 cos theta 1 + theta 2 and y 2 is l 1 sin theta 1 + l 2 sin theta 1 + theta 2. Now by extension I am interested in finding x and y, x is l 1 cos theta 1 + l 2 cos theta 1 + theta 2 + l 3 cos theta 1 + theta 2 + theta 3 and y is l 1 sin theta 1 + l 2 sin theta 1 + theta 2 + l 3 sin theta 1 + theta 2 + theta.

I mean this is the coordinate of the point P that is the x coordinate and the y coordinate. Of course, we kept a notation that c 1 is cos theta 1 s 1 is sin theta 1 c 12 is cos of theta 1 + theta 2 s

12 is sin of theta 1 + theta 2 c 123 is cos of theta 1 + theta 2 + theta 3 s 123 is sin of theta 1 + theta 2 + theta 3. By implementing by substituting for c 1 s 1 c 12 s 12 c 123 s 123. We get x as l 1 c 1 + l 2 c 12 + l 3 c 123 y is l 1 s 1 + l 2 s 12 + l 3 s 123.

This is the coordinates of the end point P using forward kinematics. This is essentially a relatively simple extension of what we did in the 2R case this is not very complicated.

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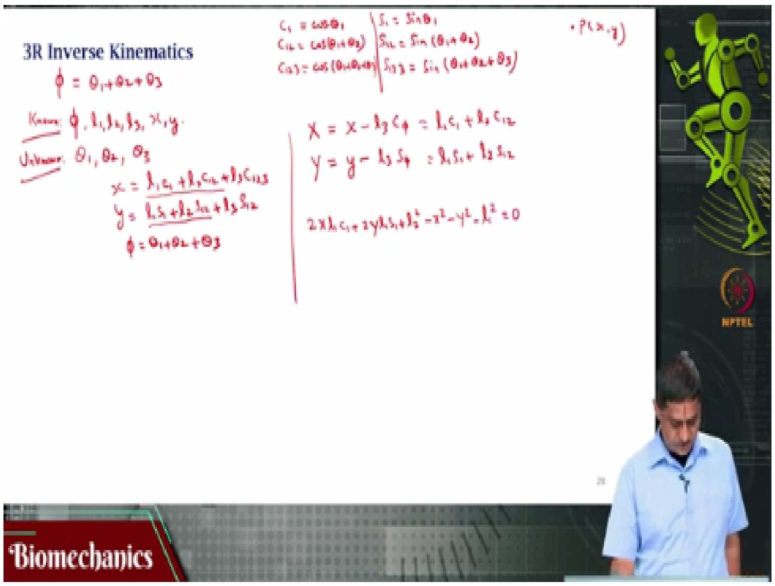

Now let us move on to inverse kinematics remember that the inverse kinematics even in the 2 link planar case was rather complicated. That is in the 2 link chain when the coordinates are given that is both x and y are given and link lengths are known. Finding joint angles is non trivial because there are two distinct configurations in which the same endpoint may be reached. We said this as elbow up elbow down configuration.

So, different configurations might lead to a given endpoint at least two different more than one will happen. That is in the simplest case of the 2 link chain. So, now we are looking at the 3 link case let us write down our notation for remembrance c 1 is cos theta 1 s 1 is sin theta 1 c 12 is cos theta 1 + theta 2 s 12 is sin theta 1 + theta 2 c 123 is cos theta 1 + theta 2 + theta 3 s 123 is sin theta 1 + theta 2 + theta 3.

One more thing that I am assuming here is I know the inclination of the last link with respect to the ground. The total inclination that the last link is making is something that I know, that is I know the value of theta 1 + theta 2 + theta 3. Let me call this with some variable phi this is assumed to be known. In the previous case I only assumed the coordinates of the end point P is what I know P of x, y is what I know.

But that will not be sufficient for me to solve a 3R inverse kinematics case because I only have two coordinates it will not be sufficient so I need an additional constraint. In this case I am assuming that I know theta 1 + theta 2 + theta 3 which I am going to call as phi. So, this is a big assumption we solve or we proceed with the solution with this assumption. So, for the inverse kinematics case what are the known's, phi is known remember.

Phi is known l 1, l 2, l 3 are known and x and y are also known what is not known theta 1 theta 2 I am writing plus theta 2 theta 3. Remember I know the sum theta 1 + theta 2 + theta 3 I do not know the individual values of theta 1 theta 2 theta 3 I know the sum theta 1 + theta 2 + theta 3. This is what I do I am interested in finding theta 1 theta 2 and theta 3, this is the problem at hand. Remember I have already written in forward kinematics the value of x is l 1 c 1 + l 2 c 12 + l 3 c 123 and y is l 1 s 1 + l 2 s 12 + l 3 s 123.

Something that I have written. In addition, I have I also know phi which is theta 1+ theta 2 + theta 3 this is what is node I have also written down the notations. Now what I am going to do is I am going to define two new variables I am going to call them capital X and capital Y. What is capital X? I am going to call this capital X as small x - l 3 c 5. Essentially this is this quantity is it not this is capital X.

Remember c phi it is c 123, is it not because phi is theta 1 + theta 2 + theta 3. So, c 123 is cos of theta 1 + theta 2 + theta 3 which is c phi and capital Y is small y - l 3 s phi. That means what is this what is capital X? Well, that is capital X and capital Y are those that are underlined which is l 1 c 1 + l 2 c 12 l 1 s 1 + l 2 s 12, that is it. Now this is what we solved in the two are inverse kinematics problem.

Remember this is what we solved in the previous video in one of the previous videos. So, this is the 2R inverse kinematics problem. Now I can proceed with performing the inverse kinematics similar to what I did in the previous case similar to what was done in the 2R case except the variables this time will be capital X and capital Y last time it was small x and small y. Other than that, there is practically no change. So, what I do is I am interested in eliminating say theta 2.

What I do is remember what we did I requested to please go back and review the previous videos. So, I am interested in eliminating theta 2 then I will get 2 x L 1 c 1 + 2 y l 1 s 1 + l 2 squared - x square - y squared - l 1 square = 0.

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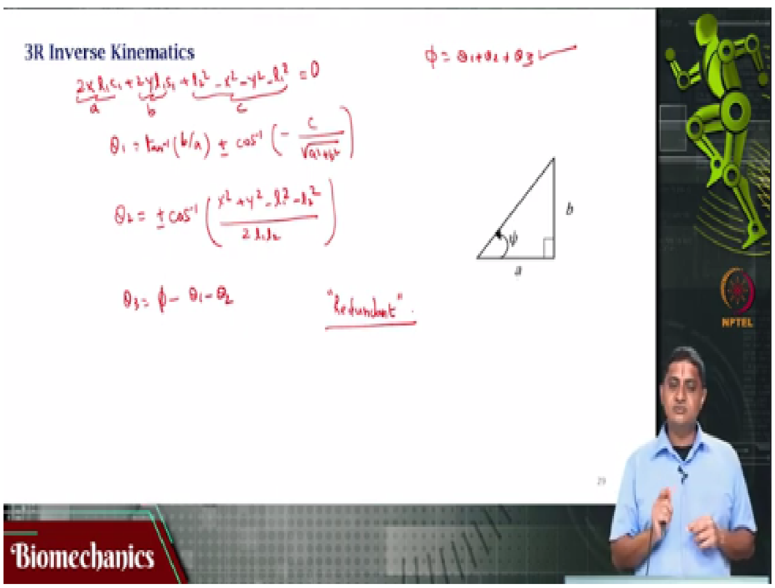

Now so if I eliminate theta 2, I will get 2 x l 1 c 1 + 2 y l 1 s 1 + l 2 squared - x squared - y squared - l 1 square is 0, therefore, now what I am going to do? I am going to call this as some quantity small a and that as small and this as some quantity small b and this whole thing as some quantity small c. Now there is this consider this right triangle with sides a b with this psi is the angle between them and the angle between a and b so it is a and b is 90 degrees.

Consider this right triangle as shown I can get using previous analysis or similar to previous analysis, I can write theta 1 as tan inverse of b by a which is psi that is psi plus as in plus or minus because there can be two solutions for this is it not plus or minus cos inverse of - c divided by square root of a square plus b square is it not. Perhaps we use just slightly different notation maybe we use capital A's and capital B's and capital C's does not matter.

But this is the general principle. Now similar to what we did in the previous video, I can find theta 2. How can I find theta 2? By squaring and adding the modified forward kinematics equations in terms of capital X and capital Y. If I do that, I will be able to find the value of theta 2 and that would be plus or minus cross inverse of capital X squared + capital Y squared - l 1 square - l 2 squared divided by 2 l 1 l 2.

And now that I know theta 1 and theta 2 finding theta 3 is easy because I know what is theta 1 + theta 2 + theta 3 which is phi. So, then theta 3 would be phi - theta 1 - theta 2. Now the question is can you do this without assuming phi = theta 1 + theta 2 + theta 3? Can you do this? The answer is no if you do not know the sum of theta 1 theta 2 and theta 3 or rather if you do not know the inclination that the final link makes with the ground.

If you do not have this detail, you will not be able to find theta 1 theta 2 and theta 3 remember this these kind of manipulators in which there are only two equations but there are three unknowns to solve this kind of manipulators are called redundant manipulators. Because a particular point in a plane can be reached with more than one configuration of the links. That is true even in the 2R case.

But in the 3R case it is super redundant because any if you take a particular point many different configurations of the 3 links can reach to that point.

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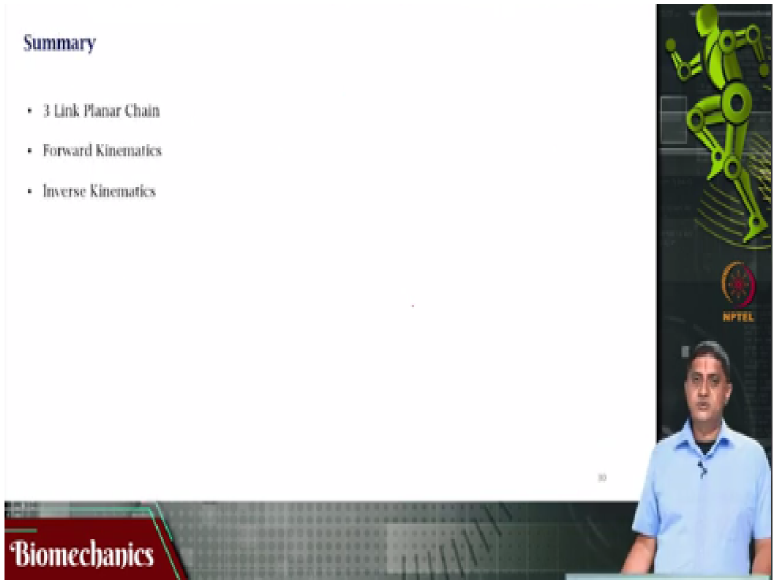

So, with this we come to the end of this video. So, in this video we saw a simple three link planar kinematic chain, forward kinematics of a simple three link kinematic chain and inverse kinematics of the three link kinematic chain. Importantly we assumed that we know the sum of the three joint angles theta 1, theta 2 and theta 3. So, that we can compute their individual values if you did not know the inclination that the last link makes with the ground.

You cannot compute the individual joint angles because the entire manipulator is redundant there will be many solutions to one particular just because there are two equations and there are three unknowns you cannot solve for this. Thank you very much for your attention.