Biomechanics
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Lecture - 48
D-H Parameters

Welcome to this video on biomechanics. We have been looking at the kinematics of grasping and reaching. In the previous videos we looked at two link serial kinematic chains and three link serial kinematic chains.

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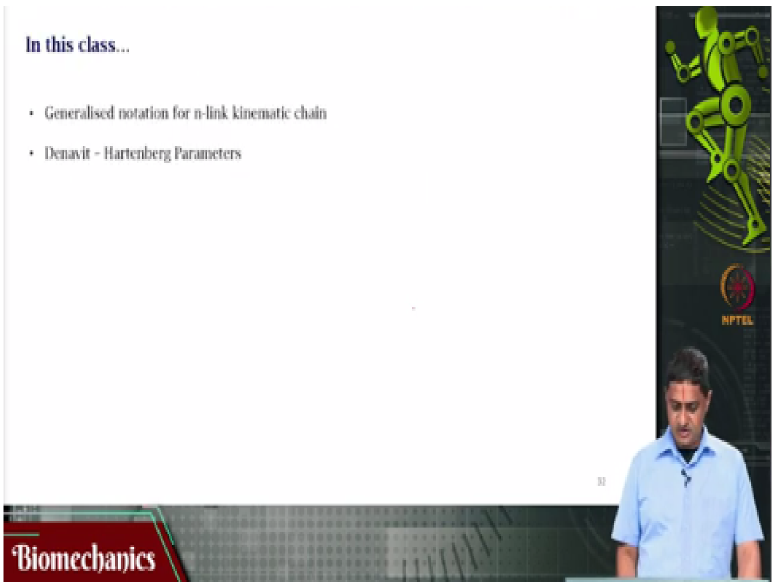

In this video, we will be looking at a more generalized notation for n link serial kinematic chain. What would be a method that you would use to compute the n point coordinates when there are n links with different link lengths? The popularly used method in the field called as DH parameters Denavit Hartenberg parameters, D H parameter.

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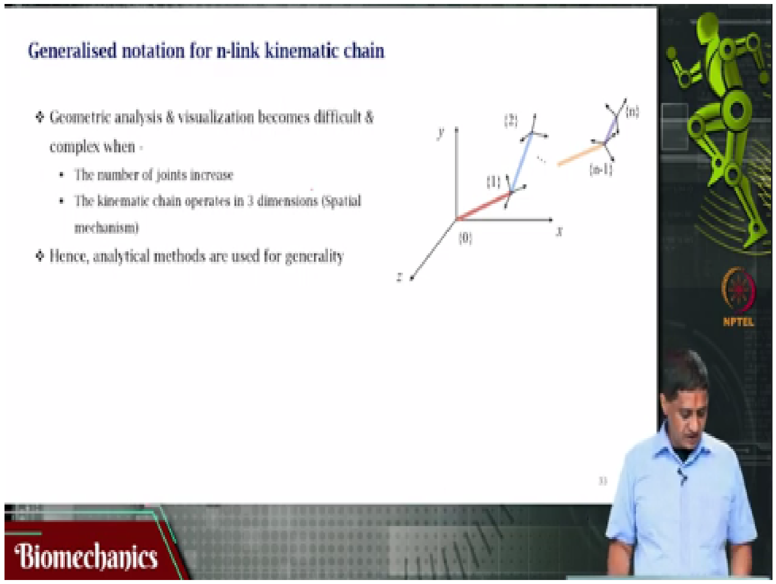

As it is in the 2 link chain and the 3 link chain we saw that the forward kinematics itself becomes complicated after some point. The inverse kinematics is a little too complicated unless you have some assumptions you really cannot even solve for the inverse kinematics either there must be some constraint or there must be some. This constraint can be a geometrical constraint. This constraint can be a physical geometrical constraint that a particular link cannot go beyond a particular point.

Or this constraint can be such that moment in a particular joint is somehow linked to movement in a different joint. That some constraints you need to have to be able to uniquely identify or uniquely solve for the inverse kinematics equation. And this complexity keeps on increasing as the number of links proceed. We saw that in the previous case because tooling kinematic chain we solved that was not too bad it was relatively easy.

The moment we jump to three link suddenly the complexity is jumped and as the number of links keep on increasing the complexity keeps on increasing. So, also so far, we have assumed only planar kinematic change. But then real-world robots and real-world humans can make movements that are not in a plane. Right now, the moment that I am making is in space is in 3D. Technically you can solve for this technically you can still use trigonometry and first principles to solve for this.

But it will be a little too tedious a little too difficult to solve. So, we are in a situation where we need to use some analytical methods some generalized analytical method to solve for this both forward kinematics and inverse kinematics. So, this was first suggested by Denavit and Hartenberg 1955. So, Denavit and Hartenberg used some standard notations, some specific notations that are always followed using their method.

These are every joint is represented by a coordinate frame. There is a coordinate frame that is placed at every joint every single joint and this is such that and this coordinate frame is placed such that the Z-axis is always the joint axis. The Z axis of the joint is always a joint axis this is how it is placed by definition. For example, if the giant is a revolute joint then Z-axis will be along the direction of the axis about which the joint rotates right or if it is a prismatic joint then the Z axis will be along the axis about which the joint translates.

So, by using these methods by using this simple approach a manipulator a multi-link manipulator becomes a whole bunch of setup coordinate frames, only coordinate frames are there in this. The links alone how do you define? The links are defined by the relative position and orientation of the airframes of the coordinate frames. So, the shape of the link is eliminator, what kind of link this is that is eliminator.

This approach is called the DH parameters or the Denavit Hartenberg convention and this consists of four parameters in total two joint parameters and two link parameters.

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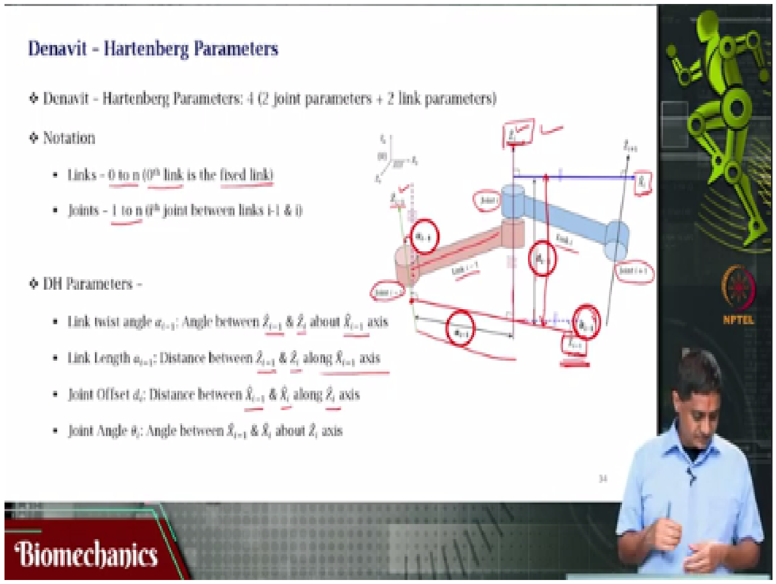

Two joint parameters and two link parameters are used in DH convention and what is the notation? Links are always from 0 to n and the zeroth link is the fixed link or the ground, joints are always from 1 to n. So, the ith joint is between links i - 1 and i. So, remember that link starts from 0 whereas joint starts from 1. So, that means that the first joint is between the zeroth link and the first link, something to keep in mind.

Consider these two links i - 1 and i and there are three joints that are involved these are joint i - 1, joint i and joint i + 1. These are connecting the two links link i - 1 and link i the Z axis is always along the joint. We mentioned this previously that the Z axis is always along the joint axis. The ground is also shown in the figure as shown you know the fixed link zeroth link is also shown. Now the question is how to find the axis and how to find the Y axis, Z axis is relatively easy to define because that is the joint axis.

How do you find the X axis that is the question. So, it turns out that you define the X axis as the common normal between the Z - 1 axis and the Z axis. The X i - 1 axis is defined as the common normal between the Z i - 1 axis and the Z i axis. So, this is the Z i - 1 unit vector and this is the Z i unit vector. Now the question is what is X i – 1? That has to be because this is a plane that has to be the unit vector.

Because this is space that is that vector that makes an angle of 90 degrees with both the Z i - 1 axis and the Z i axis that particular common and there will only be one of them. So, that particular common normal is the X i - 1 axis this is it. Now the point of intersection of the X i - 1 axis with the Z i - 1 axis the point of intersection of the Z i - 1 axis and the X i - 1 axis is the origin of the i - 1 coordinate frame.

Now how will you find the Y i - 1 axis that is the origin but you still have to find the Y axis? You use the right hand thumb rule because you know the Z axis you know the X axis find the Y axis, use the right hand thumb rule to find the Y i - 1 axis. Likewise repeat this process for Z i and you know remember for the case of Z i you will have to do it for X i axis that is the X i axis is the normal between Z i and Z i + 1 between these two and so on and so forth.

Repeat this process then find then find Y i axis then find Z i + 1 then find you know X i + 1 then find Y i + 1 so on and so forth. This repetitive iteratively you will have to keep doing it. This is the framework that it looks a little complicated but with some practice and with some understanding we will be able to catch with this. So, this is just the description. Now what are the four DH parameters defined?

One is alpha i - 1 this is the angle between Z i - 1 axis and Z i axis about the X i - 1 axis this is that angle that is alpha i - 1. This is called as a link twist angle. Then link length a i - 1 this is the distance between Z i - 1 and Z i axis along the X i - 1 axis. Remember that is not this link length rather this distance. Something to keep in mind that the link length is the length along the X i - 1 axis, distance between the two axis Z i - 1 and Z i along the X i - 1 axis that is called link length.

And then there are two joint parameters. Joint offset d i which is the distance between X i - 1 and X i along the Z axis. So, this is a Z i axis, this is the Z i axis the distance between the X i - 1 axis and the X i axis along the Z i axis that is this distance. This is called as the joint offset d i then joint angle theta a. This is the angle between X i - 1 axis this is and X i axis that about the Z i axis about that axis that is theta i - 1.

So, once you define the joint parameters and the link parameters from all the joints and for all the links in the manipulator then you can construct what is called as a Denavit Hartenberg table or the DH table that contains n rows corresponding to n links under four columns. What are the four columns? Linked twist angle, link length, joint offset and joint tackle these four. Then you can use this DH table to find the forward kinematics equation and then solve for them.

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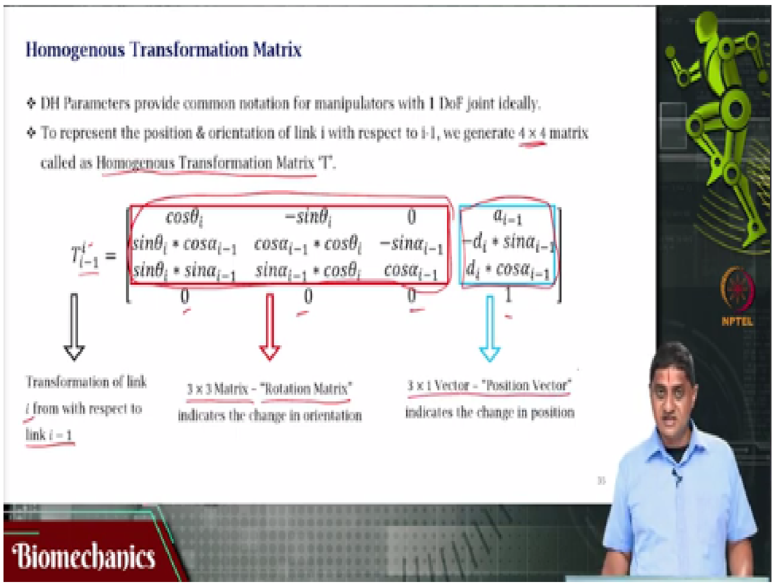

To do this we construct what is called as a homogeneous transformation matrix T that is a 4 by 4 matrix. This is found by multiplying four individual matrices which are essentially two rotation matrices corresponding to alpha i - 1 and theta i two joint rotations and two translations corresponding to you know a i - 1 and d i. There are only four parameters two are angular parameters and two are distance parameters so that this is done.

Why are we doing this? Because the frame i - 1 is transformed to frame i. There is a transformation that is happening for the frame i - 1 to i and so on and so forth. In future we will be discussing a bit more detail in about rotation matrices. We will be discussing more details about rotation matrices in future videos. For now, let us just say that this 4 by 4 homogeneous transformation matrix is obtained by using two angular parameters and two distance parameters.

Now what would this transformation matrix look like? This is how it will look like. This is the transformation between i - 1 and i. How do I know this? Because that is the notation it says T subscript i - 1 superscript i is the transformation between the i - 1 frame and the ith frame. In this I have this 3 by 3 method square matrix, this 3 by 3 square matrix is the one that is giving an idea about the overall rotation that is happening the total rotation not individual rotation.

So, total rotation that is happening because there are two rotations that are happening one involving theta i, the other involving alpha i - 1. There are two rotations that are happening. This 3 by 3 matrix gives me a sense of the total rotation that is happening. This column gives me a sense of you know the translation that is happening and I have added the last row 0 0 0 1 just to make the whole transformation matrix square.

Otherwise, this is a you know there are only three rows and four columns. To make it square we have added the fourth row so once again this transformation matrix T is transformation of link i with respect to the link i - 1, i - 1 to i. And this 3 by 3 matrix is the rotation matrix we will see more details in future indicates the total orientation change, the total change in orientation. This is the position vector 3 by 1 vector which indicates the change in distances or the positions.

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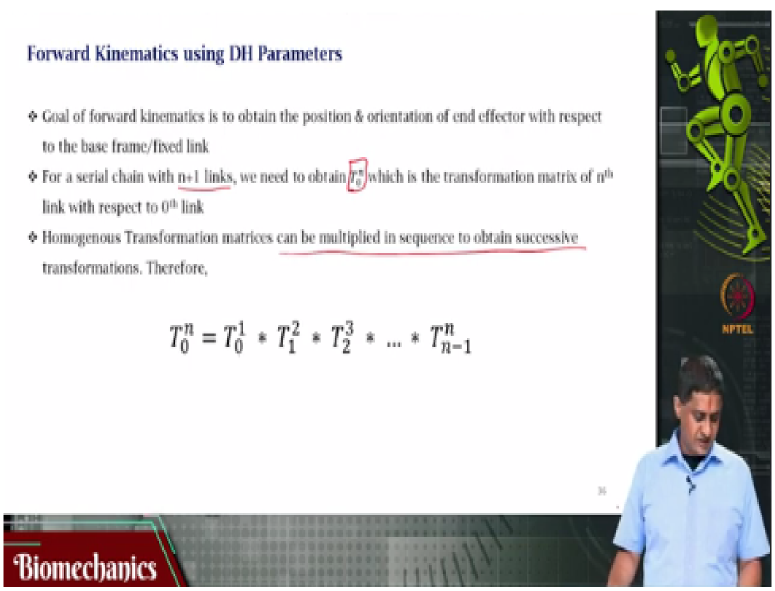

Now forward kinematics essentially is used to find position and orientation of the end effector. I am not interested in knowing the individual link parameters in a multi-link chain. I am interested knowing the endpoint coordinates, position and orientation of the end effector with respect to the base and with respect to the ground with respect to the fixed link. If there is a serial chain with the n + 1 links I am interested in finding this t, 0 to n which is the transformation of the nth link with respect to the zeroth link.

That would mean that there are many matrix multiplications that I will have to perform. Homogeneous transformation matrices like the one that we saw in the previous slide they will have to be multiplied in sequence to obtain the transformations. So, therefore you will have this T 0 n is T 0 1 + T 1 2 so T 0 1 times T 1 2 times T 2 3 times etcetera T n - 1 to n. Remember the multiplication is done in this direction.

So, these transformation matrices are multiplied on the left hand side. So, to obtain the last T matrix to the 0th that is the last T matrix on the nth frame to the T matrix on the zeroth frame. If I perform it in the other direction for example it will not give me the correct result. Why? Because matrix multiplication is not commutative, I will not get the same result. So, we must be very cautious about how we apply this transformation matrices.

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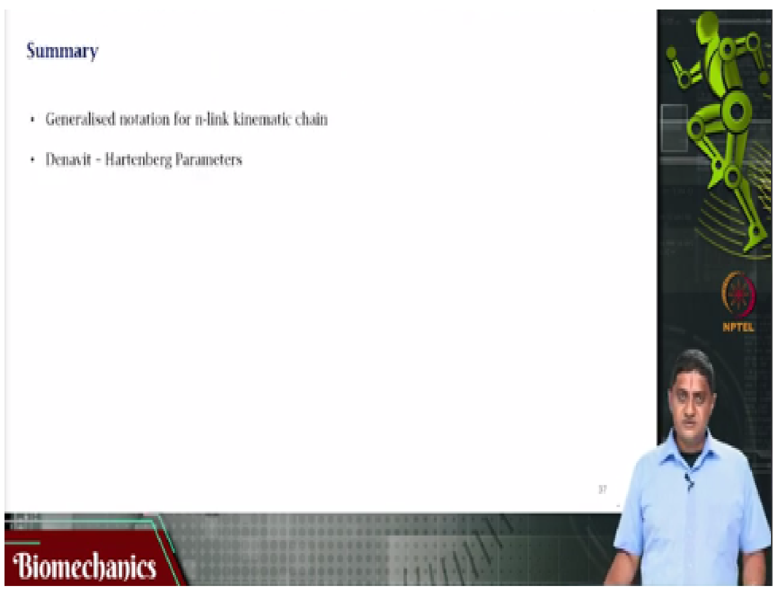

So, in this video we saw a generalized notation for studying an n link serial kinematic chain and Denavit Hartenberg convention or the Denavit Hartenberg parameters for defining and understanding the n point coordinates of the in the forward case in the forward kinematics case for describing a transformation matrix. Thank you very much for your attention.