Biomechanics
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Lecture - 50
3R Velocity

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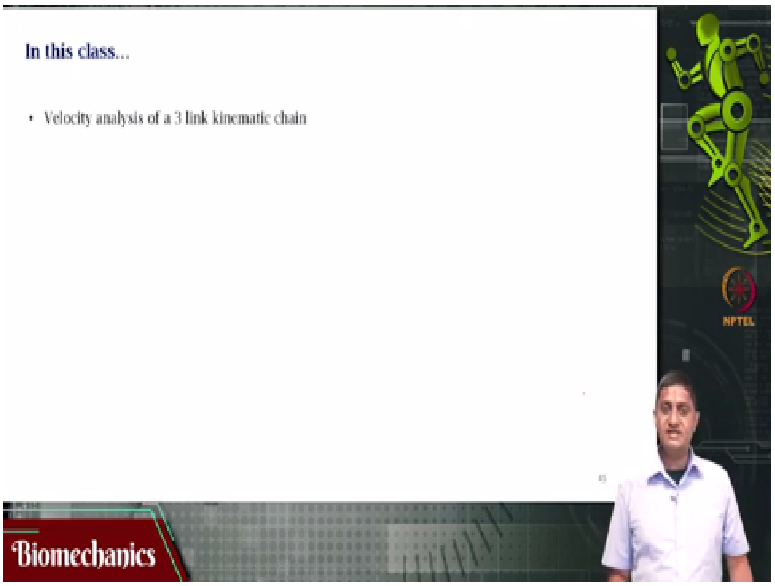

Vanakam, welcome to this video on biomechanics. In this video we will be looking at kinematics of three link kinematic chain specifically we will be performing a velocity analysis of a three link serial kinematic chain or the 3R manipulator.

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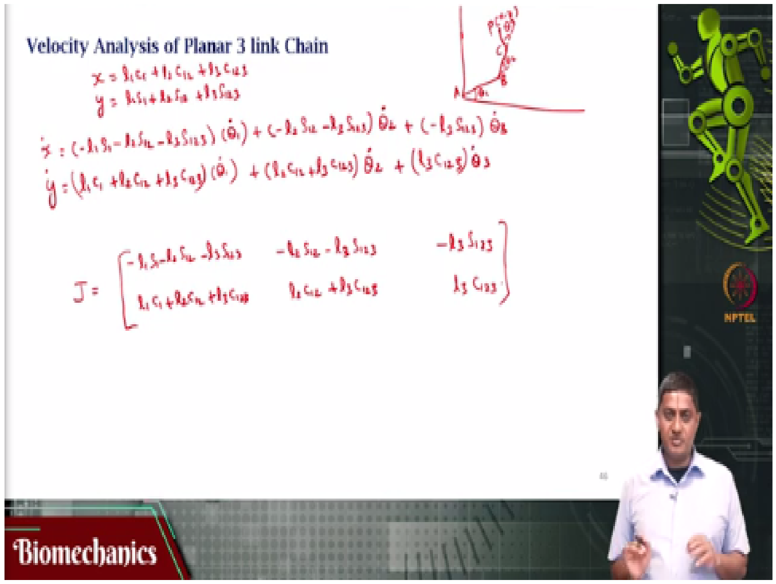

So, consider a three link kinematic chain theta 1, theta 2, theta 3 from forward kinematics I can find the endpoint coordinates of the point p as x is l 1 c 1 + l 2 c 12 + l 3 c 123 and the y coordinate is l 1 s 1 + l 2 s 12 + l 3 s 123. I differentiate this with respect to time x dot is - l 1 s 1 - l 2 s 12 - l 3 s 123 times theta 1 dot + - l 2 s 12 - l 3 s 123 times theta 2 dot + - l 3 s 123 times theta 3 dot.

Likewise, y dot would be l 1 c 1 + l 2 c 12 + l 3 c 123 times theta 1 dot + l 2 c 12 + l 3 c 123 times theta 2 dot + l 3 c 123 times theta 3 dot using chain rule. I request you to please spend a few minutes and check if this is correct it is correct but please check please do the math on your own please write out this entire differentiation and practice once. So, you are convinced that this is indeed correct this is correct but please check.

So, we did a similar equation in the previous video and we wrote out the Jacobian in that case. So, let us write out the Jacobin in this case. What is a Jacobin in this case? Will it be a square matrix? The answer is no because there will be three columns and two rows, that would be – l 1 s 1 - l 2 s 12 - l 3 s 123 - l 2 s 12 - l 3 s 123 – l 3 s 123. Here l 1 c 1 + l 2 c 12 + l 3 c 123 l 2 c 12 + l 3 c 123 here l 3 c 123. There are indeed three columns and two rows this is not a square matrix.

So, remember forward kinematics is a problem that is meant for solving at the school level, high school level, college level, undergraduate level. Real world problems almost always involved inverse kinematics solving for inverse kinematics.

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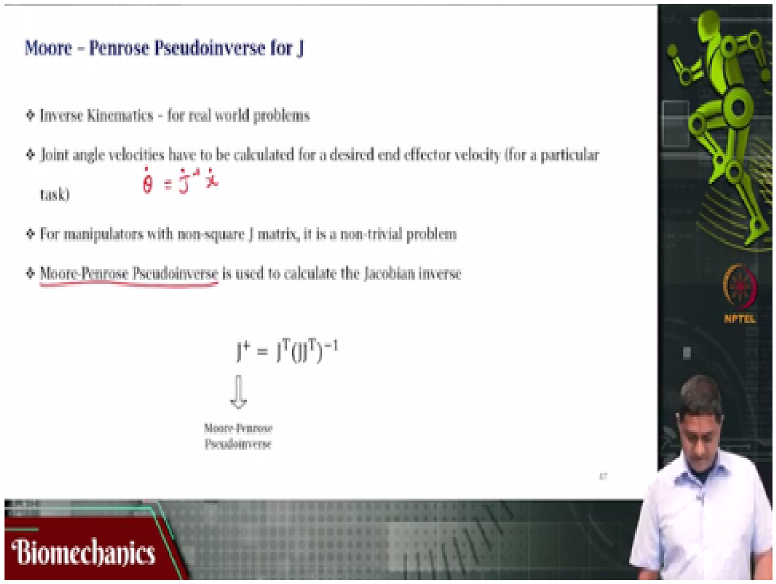

So, real world problems when they involve solving for inverse kinematics and I have an equation I have a Jacobian that does not have an inverse because this is not square the Jacobian matrix is not square. So, there is no J inverse J inverse is not defined for this case for the three link kinematic chain. Then but I am still interested in finding the particular joint angle velocities given that there is a known end effector velocity for a particular task for a particular configuration.

Let us say that there is a configuration I know the configuration and I am interested in finding joint angle velocities given that this is the end effector velocity given that this is the end effector can I find this in this case. It turns out that there is a way to find it but this is not a trivial problem this is not a simple matrix inverse problem like we saw in the two link serial kinematic chain problem.

So, we can find the inverse of three link kinematic change Jacobian but it is a non-trivial problem. That is in other words I am interested in solving for this the theta equation as the J inverse times x dot equation is it not all these are matrices. I am interested in finding this but I cannot find this, why? Because J inverse does not exist J inverse itself does not exist. Now what to do? That is the question that is the problem that I am faced with.

And this is only the three link chain there are many more chains like this now four links five links. So, there are redundant manipulators, manipulators with many you know links that are there all of which will not have a square matrix as a Jacobian. Now the question is can I find a general way in which I can use to find the inverse somehow? It turns out that there is such a solution that method is called as the Moore Penrose pseudoinverse method.

Which is essentially J + J transpose times JJ transpose the whole inverse remember this is not the Jacobian inverse, this is a pseudo inverse. When you say pseudo something that is not real. So, this is not comparable to the J inverse that you got in the two link case. The J inverse that you got in the two link case is an exact solution it exists at each configuration in the thrilling case what you are finding is a pseudo inverse remember this.

So, J inverse in this case is different from the J inverse that you got in the two links in serial kinematic chain case. This is some form of a let us say that this is an analytical trick, this is a trick to make it work where forcing it to work. This is the so-called Moore Penrose pseudoinverse case.

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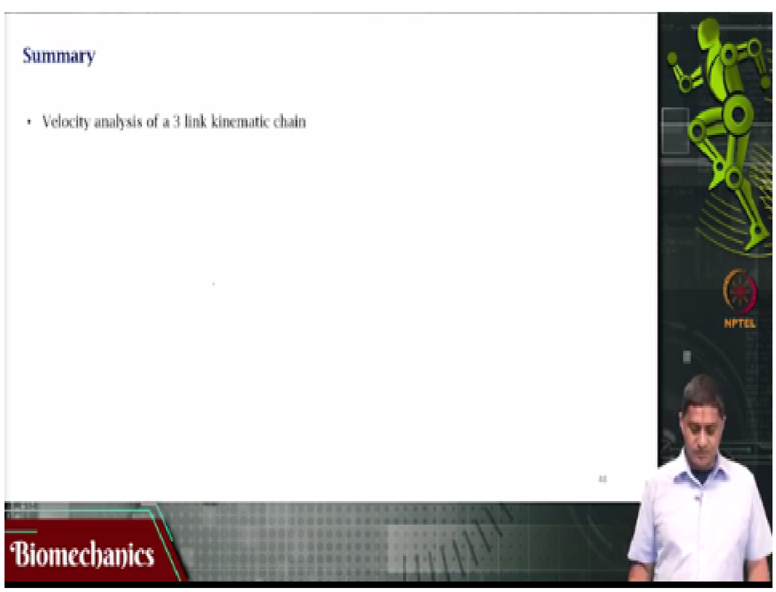

So, in this video they saw the velocity analysis of a three-link kinematic chain the forward kinematics. So, in other words we defined x dot and y dot in terms of theta 1 dot theta 2 dot and theta 3 dot. And we also saw how we can use a special method called as the Moore Penrose pseudoinverse technique to find the inverse of the Jacobian for the three link case for non-square matrices.

Remember that this is not a an exact method this is an analytical equivalent. So, this is a form of a mathematical trick. So, with this we come to the end of this video, thank you very much for your attention.