

**Biomechanics**  
**Prof. Varadhan SKM**  
**Department of Applied Mechanics**  
**Indian Institute of Technology-Madras**

**Lecture - 54**  
**Wolff's Law and Hookean Behavior**

Vanakkam. Welcome to this video on biomechanics. We have been looking at the properties of biological materials or the mechanics of materials of biological materials. Specifically, in the previous videos, we were looking at bone as a biomaterial or bone as a biological material and its properties or its mechanical properties as a biological material, and how it compares with some engineering materials. We will continue that discussion in this class.

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**In this class...**

- Wolff's law of bone remodeling
- Passive and active components
- Hookean and non-Hookean behavior

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So in this video, we will start our discussion with what is called as Wolff's law. So when you say a law, you are looking at very exact relationship. So when you say a law, you are expecting a mathematical or an exact relationship. But this is more like a semi empirical law. It is not a law in the Newton's law sense of the word law, okay? It is slightly different. Wolff's law of bone remodeling.

And important consequences, the crucial consequences of Wolff's law of bone remodeling, what it means for us in real life. And passive and active components of the response of these biological materials. And Hookean behavior and deviation from Hookean behavior of biological materials.

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**Wolff's law** of bone remodeling. *Muscle response is Active + Positive*

Incorporates three concepts inherited from 19th century anatomists: *Inorganic - Hydroxyapatite  
Organic - Collagen*

- ❑ Bone is deposited and resorbed to achieve an optimum balance between strength and weight.
- ❑ Trabeculae in cancellous bone tend to line up with the directions of principal stresses that they experience.
- ❑ Both phenomena occur through self-regulating mechanisms that respond to mechanical forces acting upon bone tissues.

Wolff's contribution to the "law" consisted of the proposal that trabecular bone tended to be formed during growth and development in orientations that corresponded to principal mechanical stresses that acted on the bone, and that hypothetical mathematical laws could explain this process.

So what is Wolff's law? This combines concepts or ideas that were already known or already studied or described in the 19th century. First is, bone is deposited and reserved to achieve an optimum balance between strength and weight. We will discuss this and the details of what this means in just a little bit. So let me first read what this is. So bone is deposited and reserved.

So bone continuously adjusts to the mechanical environment in which it is present. So we can restate it like this and we will describe the details in just a little bit. The trabeculae in the cancellous bone tend to line up or tend to align itself with the direction of the principal stresses or the most important stresses that it undergoes or it experiences. So again, pointing to the environment in which it is working.

And both of these phenomena occur through self-regulating mechanisms that respond to mechanical forces acting upon bone tissues. So that means that that is a constant adjustment. Of course, the time cycles of these adjustments will vary. But we are not looking at yearly adjustments, we are looking at, you know, timely response of the biological system to the stresses and strains, to the forces and elongations that it goes through.

So and this timely response, we are looking at the timelines of days to weeks as a response, so we are or at best months. So we are not looking at a response that happens after one year. So remember, this is starkly different from the engineering

materials that we would see. Imagine you have a building that is built for, that is designed to take up a particular load.

Or you have a beam that is designed to you know, to take some maximum load. And I am loading it just slightly more or a tad bit more or almost at the, at its maximum or slightly less than its maximum limit. Would this beam adjust its characteristic so it can take a little bit more load tomorrow or next week or next month? That is not going to happen?

Because this beam is designed by humans and the beam itself has no life or no mind of its own or no active or constantly responding mechanical properties. It is a passive material that is designed with a specification in mind. And you are not supposed to load it almost at the, at its maximum load. You are supposed to load it at much lower levels, right?

But the bone in contrast can take such loads and then what it will do oh, it seems like we have to go. It is not like the bone is having a mind of its own. But it is more like oh, this bone is getting loaded almost at its maximum, better to strengthen it, because the biological system, the human body, all biological systems bones in other animals also realize that there is this constant adjustment.

So remember, form follows function. The structure of the bone will follow the function. So if you are using it more, you are going to have more strength developing. So this deposit of bone, when we are discussing bone is deposited what do you mean by that? What do you mean by when you say that the bone is deposited? What you mean is that the organic and the inorganic material gets deposited into the bone, right?

So when you say bone, you are talking about compact bone or the cortical bone, and you are talking about the cancellous bone or the trabecular bone. So when you say deposited, you are talking about deposit of both of these, also deposit of both inorganic. What is inorganic material in bone, we know this. This is hydroxyapatite. An organic material composed of calcium and phosphorus.

And organic material or the organic matrix, which is mostly collagen. Not just the materials, the directions in which this will happen, right? Trabeculae and the cancellous bone tend to line up with the directions of the principal stresses. So the direction of the collagen fiber will also vary depending on the stresses that it takes. We would think that the bone is a passive material.

It is a passive material for most purposes, unlike the muscle, right. So when we discuss the muscle we said its resistance and its response, mechanical response, is a function of two things, right? We said this. Muscle response is active plus passive. So there is a rubber band type of response, like the tendon response, ligament response, rubber band response, which is the passive response.

Then there is the response that happens due to the neural innervation that it happens. This is the active response. The bone does not have such an active response. But here we are saying that the bone adjusts. The bone does adjust, but it is not immediate like the muscle response. So in that sense it is not, even in the muscle it takes a few milliseconds. But we are talking about, so it all comes down to this timescale that you are looking at.

In timescales of a few seconds, few milliseconds, few hundreds of milliseconds to a few seconds, muscles immediately develop an active resistance for example. These are of course called as reflexes, voluntary responses and so on. So there is a resistance that the muscle develops, the person wants to resist a force that is being applied externally. That is an active response.

So what do you mean by active? Somewhere in the central nervous system, a decision is made to resist that external force and that decision is implemented within seconds. Now in the case of the bone also if the load that it is taking is very close to the maximum load that it can take, the bone starts responding, but this response timescale is not in the order of milliseconds and seconds, like in the case of bone.

So from one point of view, you can even say that the bone is somewhat of an active material, but with a much larger timescale. So it takes weeks for the bone to deposit more of this inorganic hydroxyapatite and organic collagen and align the collagen in

such a way that it will maximize the strength so that the next attempt of the same exercise or the same load lifting this bone can take a higher load.

From that point of view, you might even say that the bone is an active material. It is a controversial point, you can take it for what it is worth. But the point is that the bone has a tendency to remodel itself. Now that is the key point that you need to take from here. That the bone has the ability and the tendency to remodel itself, which is why this Wolff's law is called as Wolff's law of bone remodeling.

So depending on the loads that the bones take, they may have a higher strength or a lower strength. Simply put, in the field of exercise and kinesiology, where I come from, this is simply called as the use it or lose it principle. If you do not use the bones, it is expensive to maintain bones at a very high level of strength, because you know, you need to have this higher amount of hydroxyapatite collagen, and all of them aligned and well optimized.

So if you keep on adding this hydroxyapatite and collagen, the weight keeps on increasing, right? So all the inertial properties will, you know will change according to the weight increase. So it is not an optimum solution or an efficient solution to have a very high, you know, highly strong or a very strong bone, if you are not going to use it.

So it makes sense for the system to keep enough strength in the bone that you can do the work that you are doing, you are usually doing, but not too much that it costs a lot in terms of weight, but not too less, but not too weak, that it breaks down even for doing activities of daily living. So that is an optimization that is happening.

And this optimization, again, you know it is important to crucial, absolutely crucial to look at the time scale in which this is happening. The timescales that you are looking at is weeks and months. It is not seconds and minutes. But it is also not years, which is why it is important to keep your bones at an optimum level of strength. So what it means practically for us, for all humans, in particular, for those who are getting older.

If you are older, as you get older, there are many problems that come, we have discussed this previously. Bones have a tendency to lose calcium, especially in women, leading to more pores right, we discussed this, more pores, more holes. We discussed this when we discussed the structure of bones and the types of cells in the bones. That is the so called disease of pores in the bone. What is it called?

If you are not able to remember, can you please check the previous video. It is called osteoporosis, is it not? So it makes sense for you to for all of us to take enough calcium and keep the bones at a relatively high level of strength, maybe it is not efficient, but that will help you, you know, keep the bone strong.

And in case of accidental misuse or in case of some accidents, there is a higher probability that a person with a stronger bone will escape with smaller fractures or smaller complications, when compared with a person who is having osteoporosis. Because a person having osteoporosis or a person having a very weak bone, if an accident, accidents can happen to anybody, right?

If an accident happens to a person who is having weak bones, then the bones will practically collapse right, will be broken into many pieces and is very difficult to set them, especially when you are older. Especially when you have a heavy body weight. You are having multiple problems here. So which is why it is important, it is crucial to keep the bone strength at a high level, especially as you are getting older.

That way you are reducing the probability that you are going to suffer because of bone fractures in case there is an accident. You cannot say well, I would not have an accident. There is no such guarantee. So it makes sense for us to keep the bone health at a high level. Why? Because you can escape with lower, it is like an insurance that you have against accident.

I mean, I am not talking about the personal accident policy that you have. I am talking about our own personal accident policy which is exercise. So weightlifting has a tendency to keep depositing more and more hydroxyapatite and more and more collagen of more and more bone material and in optimal fashions, so that people can lift heavier weights in future.

So that way, they can take greater loads without collapsing or without breaking. So which is why weightlifting or strength training not only strengthens the muscles, not only makes you healthier in general, but also improves bone health. Of course, optimum nutrition is another point. So you need a lot of calcium and vitamin D, for this to happen. So there are many dimensions.

So I am not talking about the nutrition part here. So if you are taking enough calcium and vitamin D, and if you are, you know, doing strength training, the possibility that your bones will be strong is higher. So this is, you know, due to this remodeling principle. Let us say your body weight is high, and you are having a sedentary lifestyle. Then what happens is that your bones are not used to heavy stresses, right?

Because you are not really lifting weights, you are not doing strength training. Other than your own body weight, the weights that you lift is quite minimal. Because of this reason, and because you do not do a lot of walking, even the weightlifting of your own body weight you are doing just between your parking lot and your office and your parking lot and your home, then it is pretty minimal exercise, physical activity that you give yourself.

And because it is more expensive to maintain tissues like bone tissues, the body will optimize in favor of lower bone weight and lower bone strength, as opposed to maintaining a relatively high weight, because maintaining a high bone weight is an expensive process. So and you are wondering, really is this how it works? Because these are not like concrete. So that is where the difference comes.

So this is not concrete. This is not a, you know, a steel tube. Bones are biological materials that are constantly undergoing biological change as a function of the forces and stresses that it takes. So something to keep in mind. So just why you have this Wolff's law of bone remodeling. Also consider that if a person is bedridden for example, right? That means that the person is not moving at all for months or weeks.

So in that time, the bones tend to lose strength, which is why it is important for them to get back into the exercise regimen relatively gradually. As they develop more and

more strength, they will have to keep, you know, they will have to keep on gradually increasing the difficulty level of their exercise. So they cannot straight away get back to their previous exercise regimen if they were exercising previously, right?

So something to keep in mind because the bones tend to lose strength if you are bedridden for a long time. Another example is the case of space travel. If someone is going to International Space Station, spends about a few months there and coming back, because there is no gravity, right? The bones tend to lose strength. They do perform exercises within the space station.

But still, it can never match the dynamics that a person undergoes on planet Earth, right? So there are, again goes on to show the importance of how dynamic this tissue is, the bone tissue is, right? All this points to one simple thing. The takeaway message is that keep exercising, keep your bone health at a relatively high level so that, you know you may be in a position to avoid major accidents or major fractures if you ever get into an accident.

That is the insurance that you have. So what exactly is Wolff's contribution to this law? So when you say law, what is this law? This is not a law in the Newton's law sense of the word law. It is not an exact relationship. It is more of a semi empirical law that is composed of multiple concepts.

So Wolff's contribution to this law is essentially the proposal or this idea that trabecular bone tended to be formed during growth and development in orientations that correspond to the stresses. So how they are forming the orientations right, are dependent on the mechanical stresses that it takes. And that hypothetical from mathematical law may be in a position to explain this process.

So you may be able to explain how much is the orientation or how aligned the trabecular bone is to a given loading condition. You can, you may be able to explain this with the help of simple correlations is the expectation. So that is the law part of the Wolff's law.

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## Mechanical perspective

*Muscle force = Active force + passive force*

- From a mechanical perspective, the biological components can be either passive or active.
- Passive components, such as bones and tendons, respond to outside forces, in ways that are either independent or dependent of time.
- Active elements, muscles, generate forces.
- Muscles are indeed active elements, but they also have some properties of passive components, and when they are modeled, the model must include both their active and passive properties.

Now remember I already mentioned this, biological materials can be either passive or active. Technically bone is a passive material, but then again I just mentioned that bone is also an active material just with a longer timescale. So it all comes down to this how do you classify is a function of many things. So it may be considered active from a longer timescale.

So but at short timescales of hours and minutes, bone is practically a passive material. Tendon is also a passive material, but remember tendons can also become stronger. Tendon is also an active material in the longer timescale, in the larger timescale, the bigger timescale. Tendon can also be strengthened, muscles can be strengthened, bones can be strengthened, tendons can be strengthened.

Essentially, your body is a dynamic body. It is not something that you got and this is what you got. It is something that changes according to the needs, right? At short timescales less than a day right, hours and minutes, bones and tendons are essentially passive components and they respond to outside forces in ways that are either dependent on time or independent of time. But muscles have both active and passive responses.

We mentioned this. So muscle force is active force plus passive force. Passive force is coming from material properties like the rubber band like properties. In that sense, it is similar to a tendon, right? But active force is not coming from the material

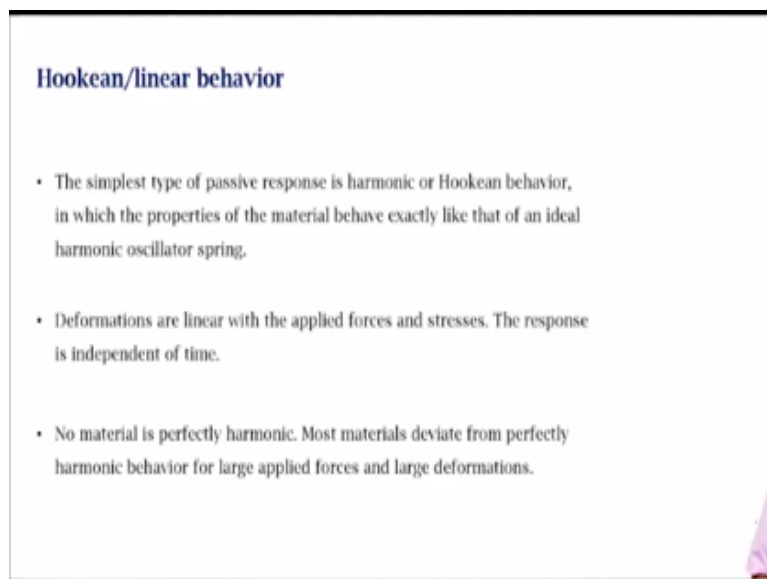
properties it is coming from neural command that is causing an active contraction, what we saw in excitation contraction coupling and sliding filamentary.

So a neural command is causing due to a conformational change in the actin myosin system due to a conformational change that the myosin head goes through to attach to that and then pull causes a deformation, causes of force. This is active force development, right? This is not purely due to materials, this is due to something else. Energy is expended to achieve this right, we know this.

So this is why muscles are considered active, right? These are active elements, muscles, they generate forces, right? These are indeed active elements, but they also have some passive component. So you are going to have a passive component of force and an active component of force. At any point in time, if there is a total muscle force, it is essentially a sum of these two. So when we are modeling the force, we must include both the passive component and the active component.

Now, at a given force level, if you want to estimate how much of this is coming from the active force and passive force, there are you need some advanced simulation and advanced analysis techniques, it is possible but it is not necessarily simple or easy, okay? It is possible to do this, but that does not mean that it is easy to do it, but it is possible.

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**Hookean/linear behavior**

- The simplest type of passive response is harmonic or Hookean behavior, in which the properties of the material behave exactly like that of an ideal harmonic oscillator spring.
- Deformations are linear with the applied forces and stresses. The response is independent of time.
- No material is perfectly harmonic. Most materials deviate from perfectly harmonic behavior for large applied forces and large deformations.

So if I am looking just at a passive response right, the simplest passive response is the Hookean behavior or harmonic behavior in which the properties of the material behave like the ideal spring or the ideal harmonic oscillator spring. In this case deformations are linear with the applied forces and stresses and the response itself is not dependent on time, it is actually independent of time.

But technically speaking, there is no such material that is perfectly harmonic. So whenever we assume, whenever we discuss these things, we discussed an ideal spring. That means that a practical spring is not ideal. So something to keep in mind. No material is actually perfectly harmonic.

And most of these materials, they almost always have some deviations from ideal behavior right, ideal harmonic behavior, in particular for large forces or large deformations, which is why the study of large deformations is a relatively advanced concept and infinitesimal deformations or really small deformations is a relatively preliminary or elementary concept with which you start your analysis, right?

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### Non-Hookean behaviour

- A material can deviate from a Hookean behaviour with the deformation depending nonlinearly on force or stress, and yet this deformation can still be reversible (elastic behaviour).
- This means that the material returns to its initial state when the stress is removed both in the linear and nonlinear parts of this elastic regime or region.
- For larger stresses, the material is no longer elastic because it undergoes plastic deformation, which is irreversible.
- This means that the material never returns to the same size or shape when the stress is removed.
- For even larger stresses, there is fracture. One glaring example is the fracture of bones.



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So whenever there is a deviation of a material behavior from the Hookean behavior, so deformation depends nonlinearly on the applied stress or applied force, but it might still be reversing. That means it might still be elastic or it might still exhibit elastic behavior. Just because it is nonlinear, does not mean that it is not elastic. A spring can be a nonlinear spring, okay? It is not  $F$  is equal to  $kx$ .

There is some nonlinear component, but it will still return back to its original state you know. It has elastic property, just that it is a nonlinear spring, okay. This means the material returns to its initial state, when the applied force is removed. And both the linear and nonlinear components of the elastic regime can be modeled, right?

But if you keep on applying the force or if you keep on increasing the stress, the material is no longer elastic, because after some point, it reaches its point of maximum elastic yield or the point at which it can exhibit the last elastic behavior after which it undergoes a deformation, where it will not come back to its original state. That is called as plastic deformation or non-reversible, it is not reversible.

Again that means that it is not going to return back to its original you know configuration, when the stress is removed. If you keep applying the force or stress further, then what happens? The material breaks down or there is fracture and example is the fracture of bones.

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#### Summary...

- Wolff's law *A bone remodeling.*
- Passive and active components *A force.*
- Hookean and non-Hookean behavior

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So in this video, we saw Wolff's law of bone remodeling that is passive and active components of force and there is Hookean and non-Hookean behavior of materials. Crucial is a point to remember a takeaway message is that we need to exercise to keep our bones healthy, especially in older adults, especially in women so that the bone health is retained at an optimum level so that they do not break their bones for small faults or for even in small accidents, something to keep in mind.

So it is important to lift weights and train using weights or do strength training exercises. Not just running or walking, strength training exercises or lifting weights, helps to keep the bone healthy. Thank you very much for your attention.