Biomechanics
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Lecture-58
Viscoelastic Modeling

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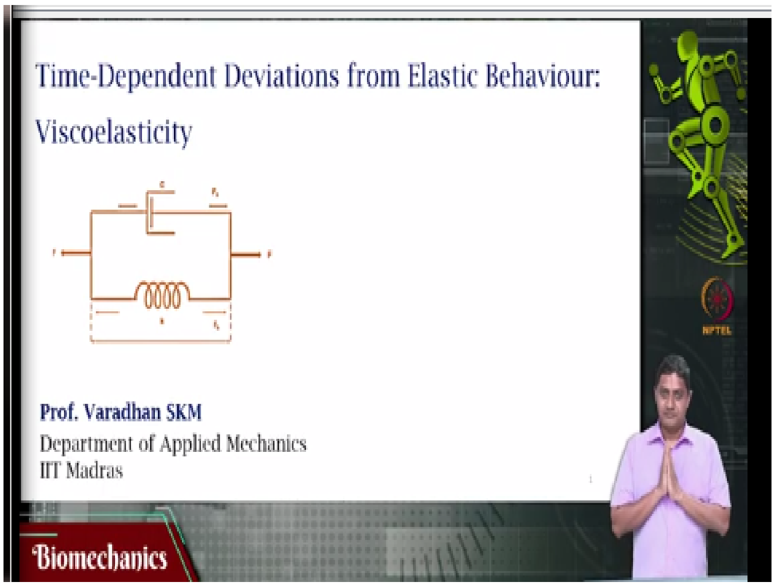

Vanakam, welcome to this video on biomechanics, we have been looking at mechanics of biological materials in the previous weeks we have looked at bone for example and the behaviour of bone as a biological material. And we said that we have assumed for most part bone is a rigid body and perform some analysis. But bone undergoes changes dependent on time, it is elastic behaviour changes depending on time, this is called as viscoelasticity.

Time dependent deviation, now changes in the material behaviour due to time this is called viscoelasticity or materials that exhibit both viscous and elastic behaviour.

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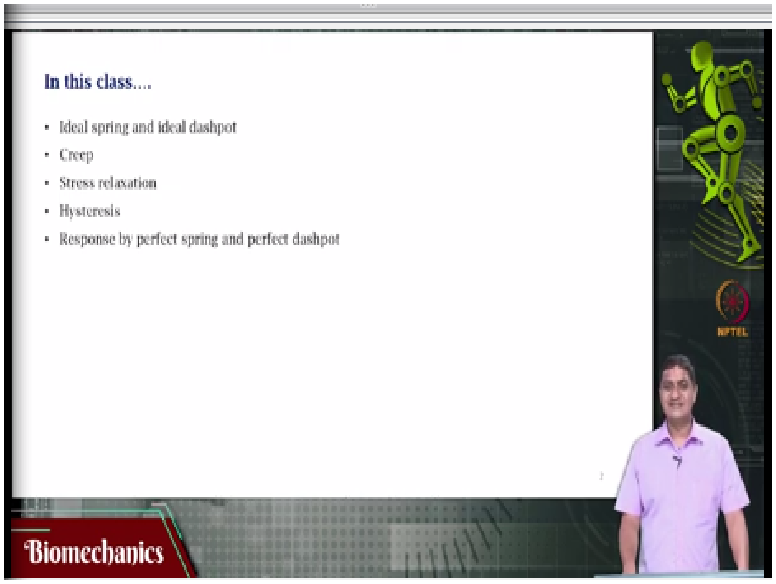

So, in this class or in this video we will be looking at the characteristics of an ideal spring and an ideal dashpot. What is creep, what is stress relaxation, what is hysteresis? These 3 are manifestations of viscoelasticity and response of a perfect spring and a perfect dashpot.

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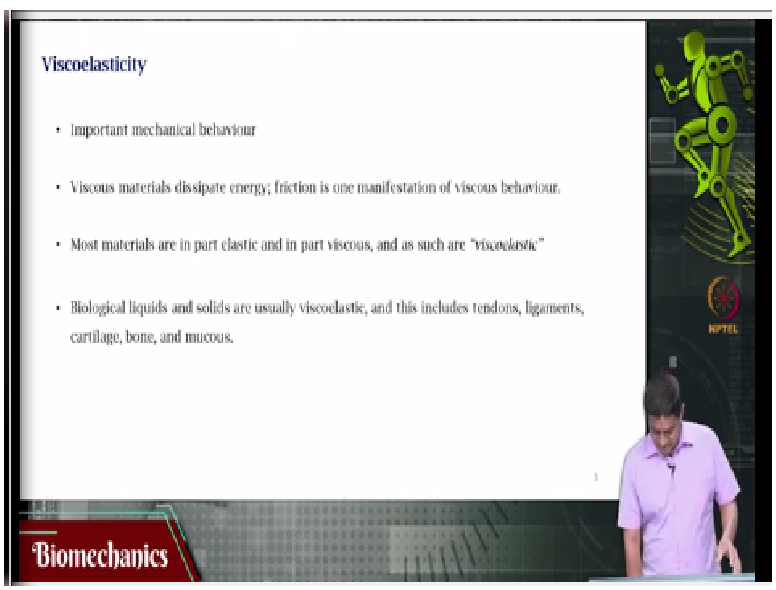

Viscoelasticity is a critical mechanical behaviour, it turns out that most viscous materials dissipate or expend energy. For example, friction is one of the manifestations of viscous behaviour, many materials show both viscous and elastic characteristics. Although most engineering materials we consider as elastic materials or viscous materials, we model them as either elastic or viscous.

So, from an engineering viewpoint or many engineering materials tend to show either elastic or viscous behaviour. But biological materials such as bones or soft tissues such as tendons, ligaments, cartilage are relatively harder tissue such as bone usually or almost always show viscoelastic behaviour. That is they show both viscous characteristics and elastic characteristics which makes it slightly difficult, slightly challenging to model.

So, that means that modeling of these materials if you use only an elastic model for the bone for example that is not going to capture the entire behaviour, there are going to be cases where this is not going to work, there are going to be situations in which this model is not going to be completely true where this model is going to fail you. It is useful to know the regions or the ranges of forces or ranges of environments in which this might not work. But what is more useful is to model the bio materials as viscoelastic materials?

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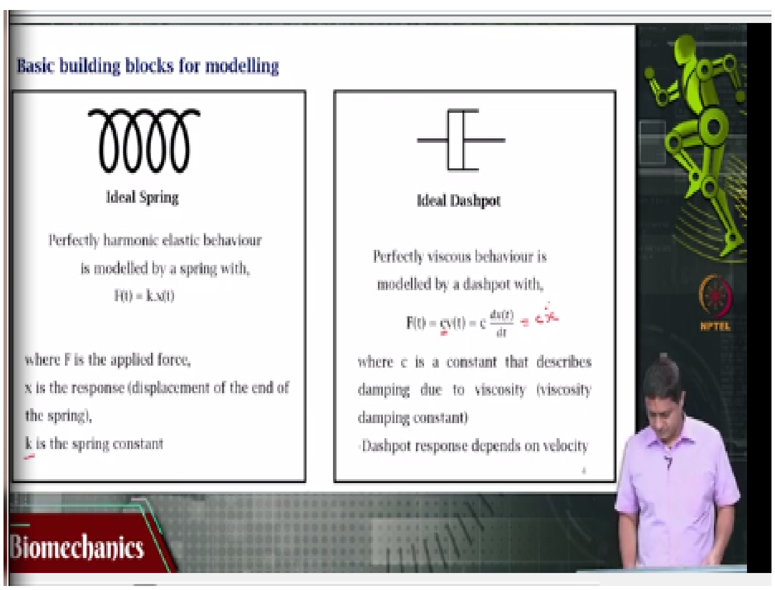

So, for our models we use an ideal spring, an ideal dashpot of course when you say ideal that means that most practical materials including engineering materials are not ideal, we understand that but you have to start your modeling somewhere. So, an ideal spring shows perfectly harmonic elastic behaviour once modeled by this equation F is kx or F as a function of time F of t is k times x of t, where k is the spring constant, x is the harmonic responses or the displacement at the end of the spring.

So, there is displacements that you are measuring which is x and k is the constant of proportionality, usually the spring constant. The ideal dashpot is exhibiting this perfect viscous behaviour that is it shows resistance to velocity. So, the ideal dashpot shows perfect viscous behaviour that is F is proportional to velocity that is F is cv, where c is the constant of proportionality and v is the velocity. What is v in terms of displacement?

That is x dot or dx by dt. So, if I am writing the force as a function of time F of t then F of t is c times the velocity v of t or rather c times dx of t by dt, where c is a constant which is the constant of proportionality. That is essentially damping due to viscosity or it is a viscous damping constant. And dx by dt are x dot is the velocity, we know this, this is another notation for velocity, dx by dt is x dot.

So, what we notice here is that dashpot behaviour depends on the velocity of with which you move that means the more the velocity the more will be the force that will be filled but not displacement. Whereas spring it is a displacement or the deformation that plays a role, for a dashpot it is not the displacement but rather the velocity that plays a crucial role, these are of course ideal models.

So, most materials including some of the engineering materials will not show this ideal behaviour. So, if you take even an ideal spring it might not show the ideal behaviour, so there are some regions within which it will show this ideal behaviour after which it may not show this ideal behaviour, so something to keep in mind. But here we are interested in looking at some building blocks, some ideal materials that we take are an ideal building blocks that we take and then we use these to further build our models.

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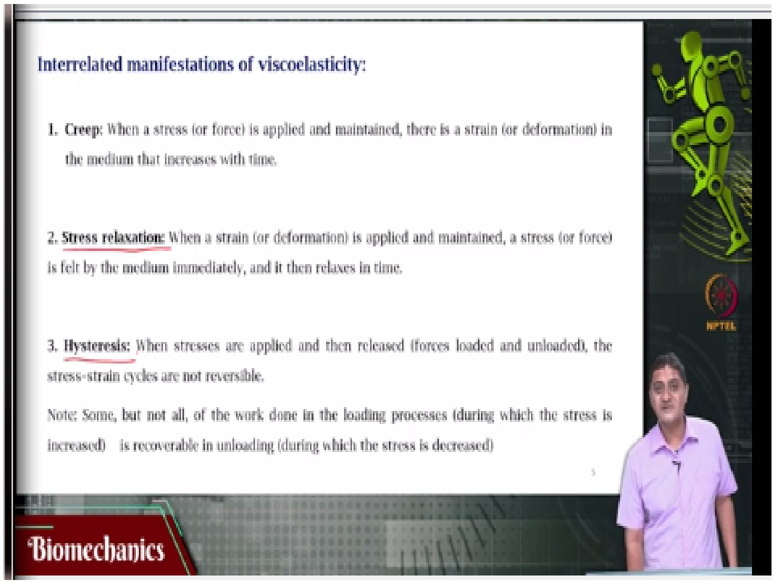

Now what are some manifestations of viscoelasticity? What would happen if a material is viscoelastic? So, when you apply a stress or a force and you maintain that stress, strain continues even after the stress is maintained at a constant level. That is the strain continues to increase with increase of time, so as time passes the strain continues to increase but the stress itself is not increased, so the force itself is not increased.

The force is applied and then maintained at a high level not increased but the deformation continues to increase, this is called as creep. Then when you apply a deformation or a strain and you maintain this deformation then a stress is developed immediately and this stress is very high, the stress that is developed is immediate and at a high level but as time passes the stress keeps going down.

In the case of stress application the stress is applied and the deformation field keeps on going up as time passes, with passage of time the strain keeps on increasing that is creep. But when you apply a strain the stress development is immediate and with passage of time the stress keeps going down or it relaxes, that is called relaxation of stress. And then you have an interrelated quantity or an interrelated parameter which is hysteresis.

When you apply stress and then you release the strain behaviour or the loading and unloading behaviour in the stress strain cycle are not exactly reversible. That means the path that you take to increase the strain and to decrease the strain are different, so there is path dependency in the stress strain curve. This will lead to a situation in which there is an area in the stress strain curve that is enclosed by the path taken.

So, because of this reason the energy that is expended in developing the strain is not completely recovered in the viscoelastic material. So, some of the energy is dissipated, some of the energy is gone, it is something that you cannot recover. So, some but not all of the work done in the loading process is recoverable during unloading. So, that means as you increase the stress the strain increases as you decrease the stress the strain decreases but in a different path, so some of the energy is lost this is called as hysteresis.

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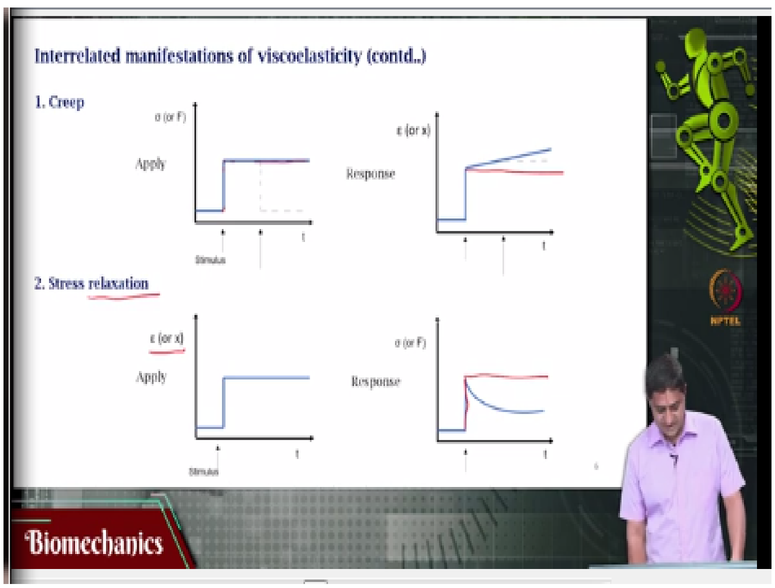

So, to explain when you apply a stress immediately and you maintain at a constant level the strain development is immediate but then as in an ideal spring for example you would expect this kind of a behaviour. But that is not what is happening, it goes to a high level and with passage of time it continues to increase, so this is called creep behaviour. So, there is a difference with passage of time, there is a difference in behaviour with time this is why viscoelasticity is called as time dependent deviation in material behaviour.

So, with passage of time the strain felt keeps increasing this is called creep. Now when you apply a strain or a deformation the development of force or stresses almost immediate but as time passes there is a relaxation. Why is this happening? You would expect that the stress developed is this and then it is maintained at this high level.

But there is time dependent deviation in the material behaviour in which the stress keeps decreasing as a function of time and then maybe it settles down at a different lower level but there is a time dependent drop in this behaviour. This is called stress relaxation or in other words the stress is relaxed or the stress is reduced, stress relaxation.

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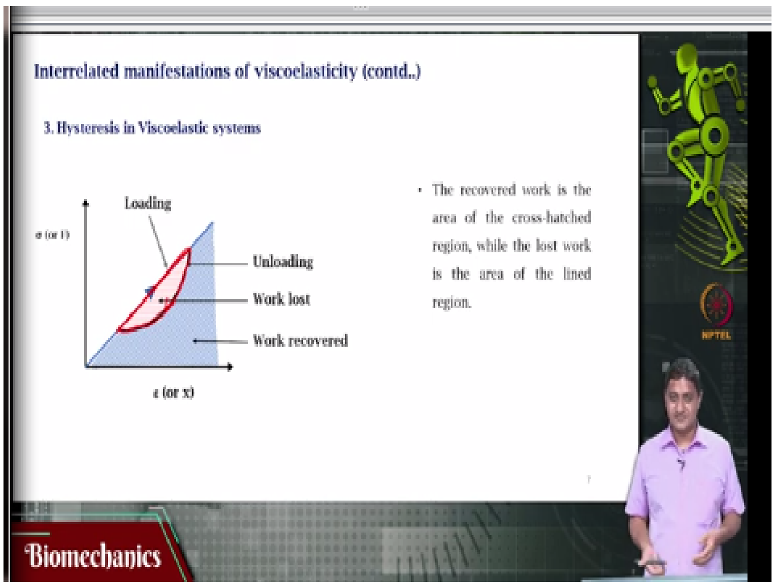

And then you have path dependency, one is time dependency the other is path dependency. So, I am loading in this path then I am increasing the strain in this path and then I am decreasing it. As I am decreasing it I would expect that the material return back like this in an elastic material. If it is an elastic material as I am increasing the strain the stress increases and then as I am decreasing it will come back in the same path.

Actually I have not even run the same path but along the same line. But in a viscoelastic material what happens is that as I am unloading that is the path that is taken by the unloading curve. That means only some of the energy that is expanded in loading is recoverable, that is the blue lines are the ones that are recoverable and the red lines, the area that is shaded in red that work or that energy is lost, this is called as hysteresis, so there is path dependency.

So, the way you go and the way you return will lead to a situation in which there is a loss of energy, this is something that is not expected in a perfectly elastic material. So, this is a manifestation of viscoelasticity.

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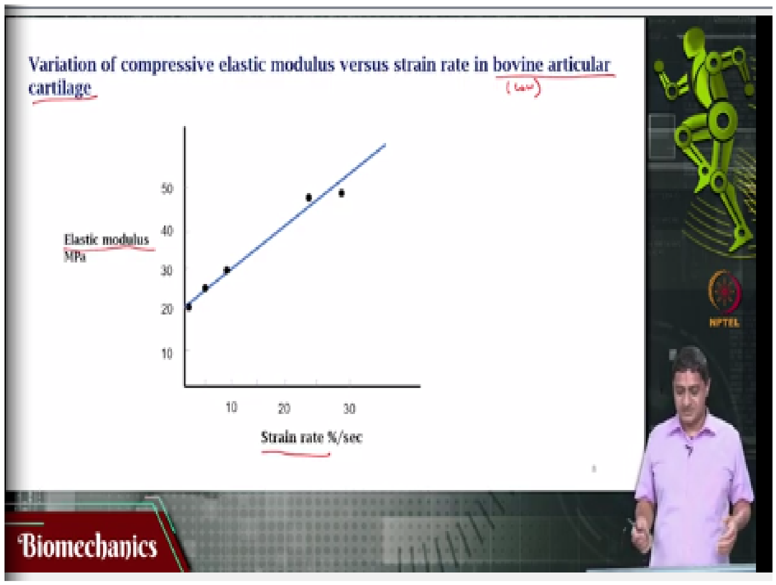

Here what is shown is a plot between strain rate and elastic modulus. Remember, elastic modulus is a relationship between stress and strain but it turns out in some material in this case the material used is bovine articular cartilage, so biological material, articular cartilage of cow. So, in this material with the change in the rate of application of strain, as I change the rate of application of strain the modulus or the elastic modulus changes.

It is not the stress that is changing, depending on the rate at which I am applying the strain the modulus itself changes. For a perfectly elastic material modulus is a constant, this is something that we have seen in previous classes. The modulus is a constant for an elastic material but in a viscoelastic material depending on the rate at which I am loading and unloading, depending on the rate at which I am applying this strain there is change in the elastic modulus.

This obviously is not an elastic material; this is a manifestation of viscoelastic behaviour. So, in other words this is time dependent or rate dependent change in elastic behaviour. So, the same amounts of strain when you apply are 2 different rates the response will be different. The applied strain is the same but the rate at which you apply whether you apply it fast or whether you apply it slow depending under the modulus will be different.

That means it is not the strain that is having a major effect or the strain might have some effect but it is not only the strain that is having an effect but the rate at which you apply the strain also has an effect, something to keep in mind. Which is why the y axis is not stress, the y axis is elastic modulus, so the modulus itself changes. That means this behaves as different elastic materials at different strain rates.

This is not a single material or it is not a single elastic material or in other words this material exhibits rate dependency or dependency of applied strain rate that is it is an viscoelastic material or this is one of the manifestations of viscoelasticity.

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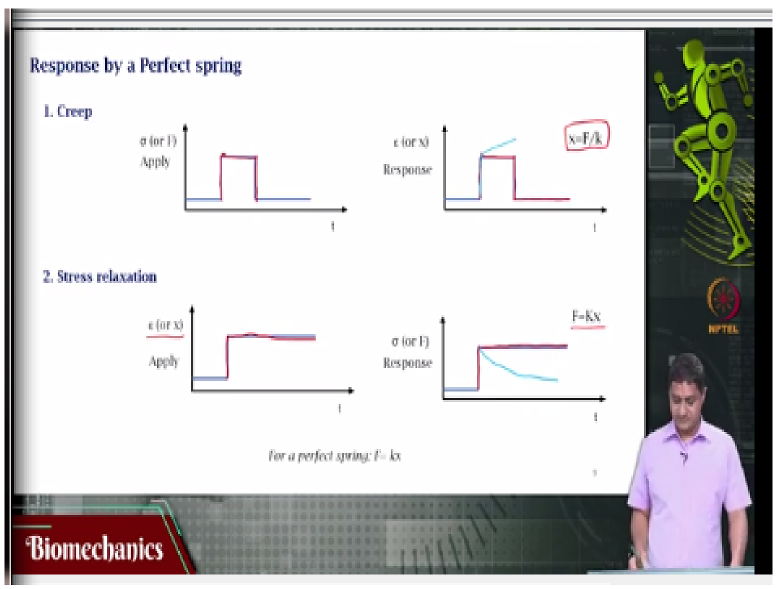

How would a perfect spring respond? Well, a perfect spring if I apply a stress I would expect a corresponding change in strain and then it goes up and it stays up and then it goes down, as it goes down the strain also goes down. What about stress relaxation? Suppose I apply a strain or a deformation, as the strain increases the stress also increases the force felt also increases. And then as the strain is at a constant the force felt also remains at this constant, why is that?

Because this force is Kx and we know that this deformation is F by k because the force applied and the spring constant does not change, the force applied changes according to this characteristic strain will follow the stress. Likewise the force applied is or the force felt is a function of the spring constant times the displacement and I am applying a displacement and the force field will be F = k as you would expect, this is the ideal spring characteristic.

This is a perfect spring or an ideal spring; of course we saw what happens during the application of stress sustained in a viscoelastic material. What would happen? You would expect that in an elastic material you would expect this to be the characteristic but in a viscoelastic material what you see is that the strain will go up and then as a function of time the strain continues to go up although the stress itself remains a constant.

This is called creep, this is creep in a viscoelastic material. And in a viscoelastic material the stress relaxation would be like this, the stress increases and with passage of time it relaxes to a new constant low level.

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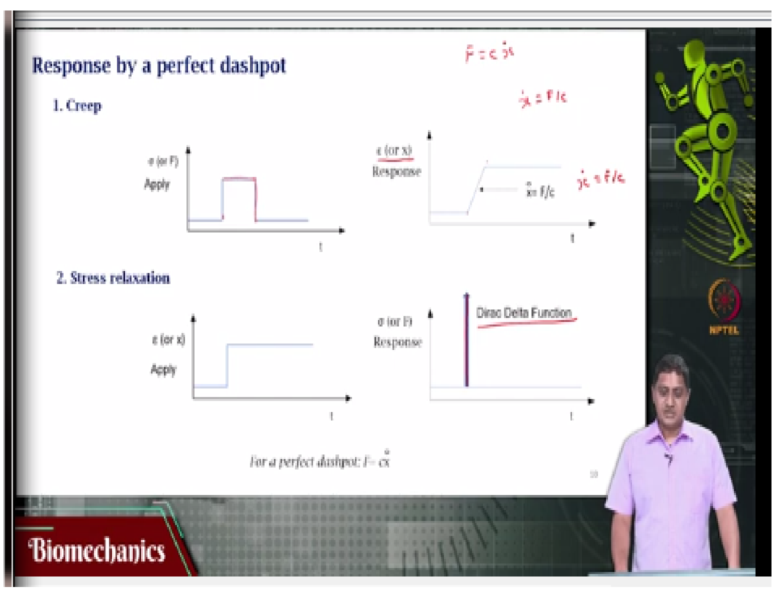

What about a perfect dashpot? In a perfect dashpot what you would expect is? You are applying a stress and then you are maintaining it at a constant level. Remember, in a perfect dashpot F = cx dot, that is the equation relating velocity and force. So, now I am applying a force or a stress and I am maintaining at a constant level and then I am removing. Then the strain will be such that the velocity will be constant like this.

So, you are applying, as you are applying x dot is F by c, but what is plotted here is X. The response is X of the strain that is plotted, since x dot rewriting this x dot is F by c here. Since it is not x that is constant but rather x dot this is what you will find. And if I apply a strain that is x what would be the force that will be felt? Because that will be an integral rate that has to happen immediately or in time 0 which is going to be this direct delta function.

In an extremely amount of time an extremely large amount of force is developed which can be characterized or modeled by using this direct delta function. Of course this is for a perfect dashpot, we will have to see what happens in the non-ideal dashpot.

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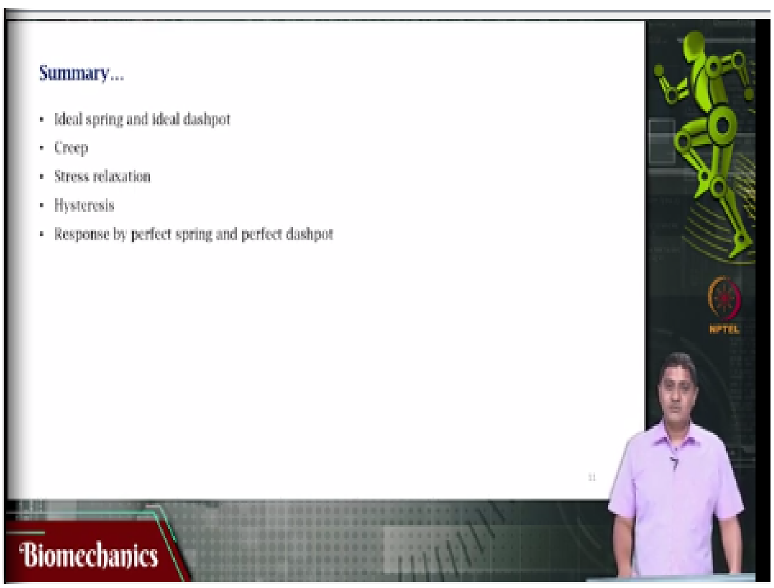

So, in this video we saw what is an ideal spring, what is an ideal dashpot, we saw what is creep and what is stress relaxation and what is hysteresis? And we saw the response by a perfect spring and a perfect dashpot. With this we come to the end of this video, thank you very much for your attention.