Biomechanics
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Lecture-59
Maxwell Model

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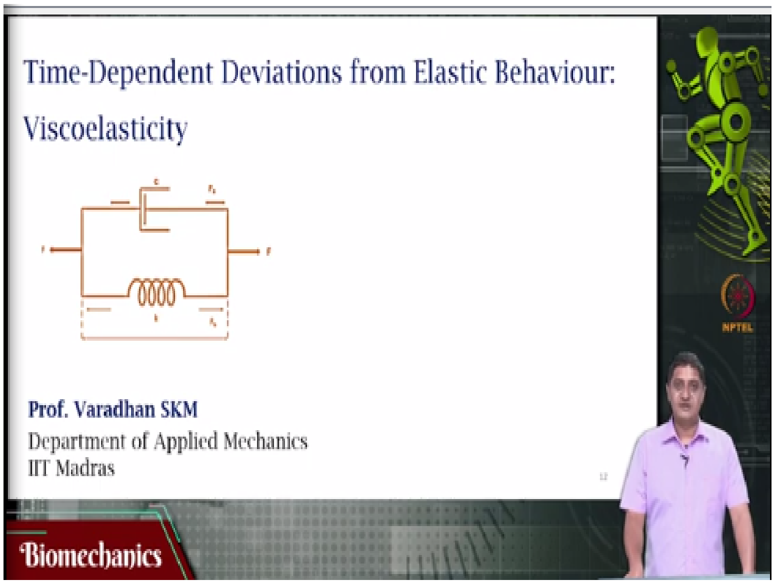

Vanakam, welcome to this video on biomechanics. We started looking at viscoelasticity in the previous video and we will continue our discussion of viscoelasticity in this video. What is viscoelasticity? We mentioned this, materials that exhibit or show or manifest both viscous and elastic behaviour are called viscoelastic materials. Most biological materials or almost all the biological materials of interest to us exhibit viscoelastic behaviour.

That means that there will be a change in behaviour as a function of time and as a function of applied strain and there will be strain rate effects, there will be time dependent deviation in elastic behaviour that is depending on time there is a change in behaviour.

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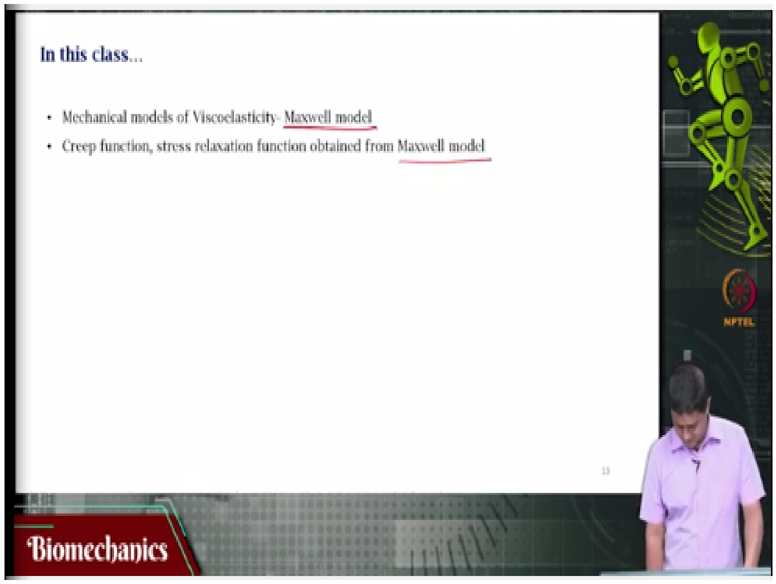

So, in the previous class we looked at ideal spring, ideal dashpot, we looked at what is a creep function and what is a stress relaxation function and how does creep and stress relaxation function look like for an ideal spring and an ideal dashpot? And we also predicted, we also guessed that in a viscoelastic material how it may look like? But we do not have a model for that. And we also said there is another manifestation of viscoelasticity which we called as hysteresis.

In this class we will be looking at mechanical models of viscoelasticity, one model we will start with in this class that is called as Maxwell model. And we will plot creep function and stress relaxation function for Maxwell model.

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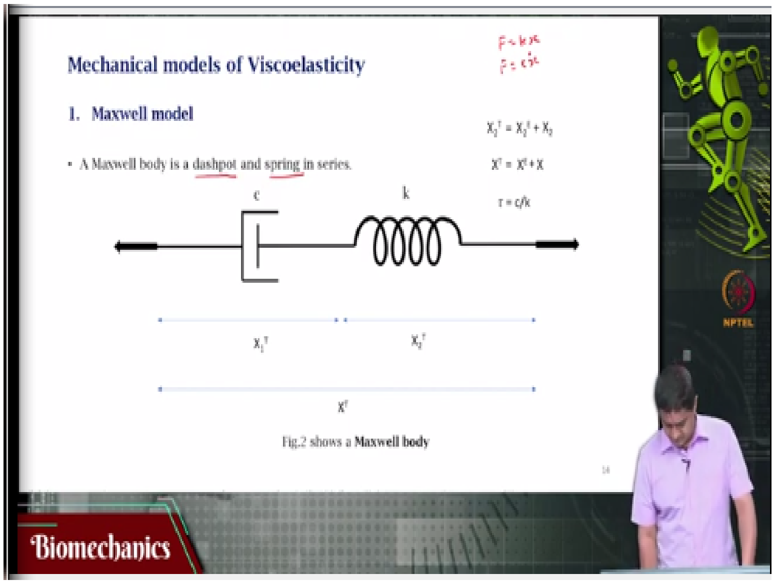

What is Maxwell model? A Maxwell body or a Maxwell model of a material is a spring and then dashpot in series with each other, in series that is a dashpot and a spring. Now before getting into the analysis and derivation what this means is that? If I apply a force at the 2 ends of this body the force will be felt equally by both the dashpot and the spring but the deformations will not be the same.

The deformations in the spring, for example is dependent on the force applied but the deformation in the dashpot is not directly related to the force. Because it is dependent on velocity in a dashpot, remember for a spring F = kx and for a dashpot F = cx dot. And their properties, the property of spring and dashpot are defined by their corresponding constants, for a spring it is k, for a dashpot it is c, it is a damping constant c for the dashpot.

But you are applying only the same force and the same force is felt by both these but the deformation is not expected to be the same and it will not be the same, x is different for spring and dashpot. And it is x dot that is dependent or that is going to change based on force, something to keep in mind as we proceed with the analysis.

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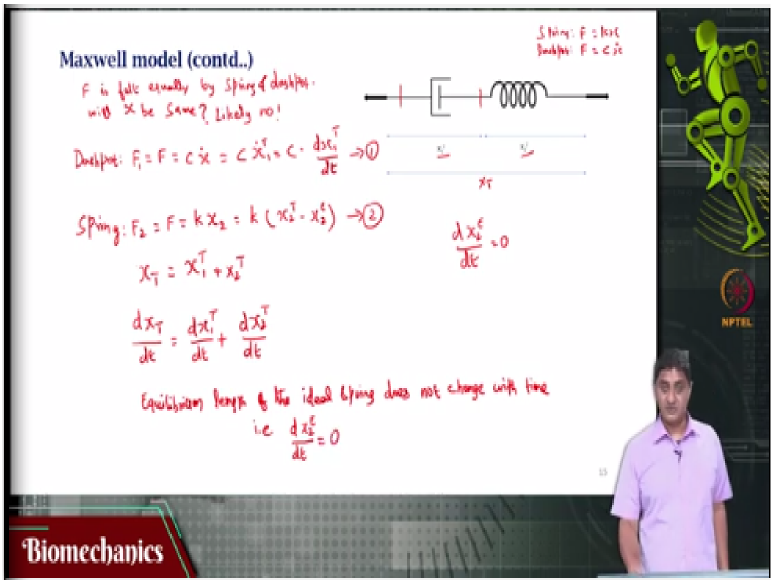

Let us write it down, so we remember. F is felt equally by both, by spring and dashpot, will X be same? That is the question, remember this is having both the spring and the dashpot in series, X is not expected to be the same. Likely no or almost always no, likely no, why is that? Because for a spring F is kx and for dashpot F is cx dot, so you are not expecting x to be the same, I can write this down.

For the dashpot F is I am going to call the force applied are felt at the dashpot as F 1 which is F actually but for convenience I am going to call this F 1 because the deformations I am going to call as x 1 and x 2. The total deformation X T, the total deformation is a function of the defamation felt at the dashpot and the deformation felt at the spring. But first let us deal with the forces; F is c x dot but what x is this?

This is this x are x 1 because it is not the entire x, it is not the total deformation, it is the deformation felt at the dashpot alone. So, this is C X dot, that is correct but it is not C X total dot but rather C X 1 dot or expanding C dx 1 by dt, another notation. Now here we are saying X 1 T, here also I will say X 1 T. What about spring? For the spring I am going to call that force as some F 2 but then F 2 is also F because the force is felt equally by both. F 2 is F and that is k x but in this case that is X 2 k X 2 T.

And what is that? Remember for a spring that deformation is change from its equilibrium or resting length, let me write that down. Let me write this as kX 2 and what that is k times the current deformation minus the equilibrium position of the resting length. I am going to call this as equation 1, this is equation 2. Now what is the total length? Well, the total length is, the total length X T is the length at the dashpot which is X 1 T + the length at the spring which is X 2 T.

Now I can differentiate this, you will soon realize why I am differentiating it, so hang on. So, dX T by dt is dX 1 T by dt + dX 2 T by dt, so I am differentiating throughout with respect to time. A question is does equilibrium length change with time? In other words X 2 E does it change with time? Well, this is an ideal spring, it is equilibrium length is something is a constant for that spring, it is resting length, it is equilibrium length is something is a constant for that ideal spring.

So, dX 2 E by dt is 0 in other words let me write this down for clarity I am writing this down equilibrium length of the ideal spring or the spring in general does not change with time, that is dX 2 E by dt is 0, that is how I am getting it.

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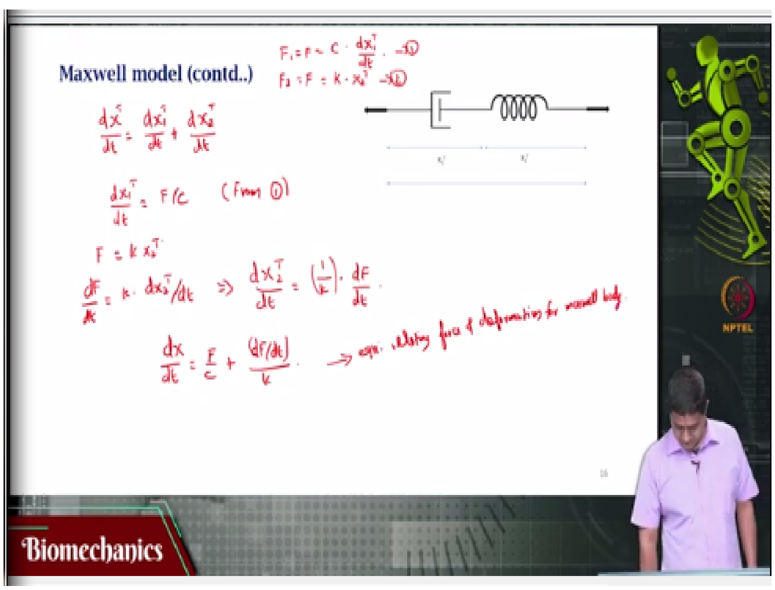

So, dx by dt is dx 1 by dt + dx 2 T by dt. Now let us also for clarity let me rewrite equation 1 and equation 2 in this slide. What is equation 1? Equation 1 is F 1 is C dx 1 T by dt and the force felt by the spring F 2 is which is F that is K X 2 T this is equation 1 and this is equation 2. Now I can write dX 1 T by dt as F by C from equation 1, I can write dx 1 T by dt as F by C. And what is dx

2 by dt? That is dF 2 by dt times k, because if I have to differentiate equation 2, what is equation 2?

That is F 2 is F, F is k X 2 T. Now if I differentiate on both sides I get dF by dt is k times dx 2 T by dt, this is what I am getting. Let me rewrite this as dX 2 T by dt is 1 by k times dF by dt so this is 1 by k times dF by dt. Now I have expressions for dX 1 by dt and dX 2 by dt, now I can substitute them. So, I get dx by dt as F by C + dF by dt the whole thing divided by k, 1 by k times dF by dt is dF by dt divided by k. Now what is this?

This is the equation relating force and deformation for a Maxwell body. Because there is both the effect of the spring and the dashpot in this case, this is the equation that relates both force and deformation for a Maxwell body.

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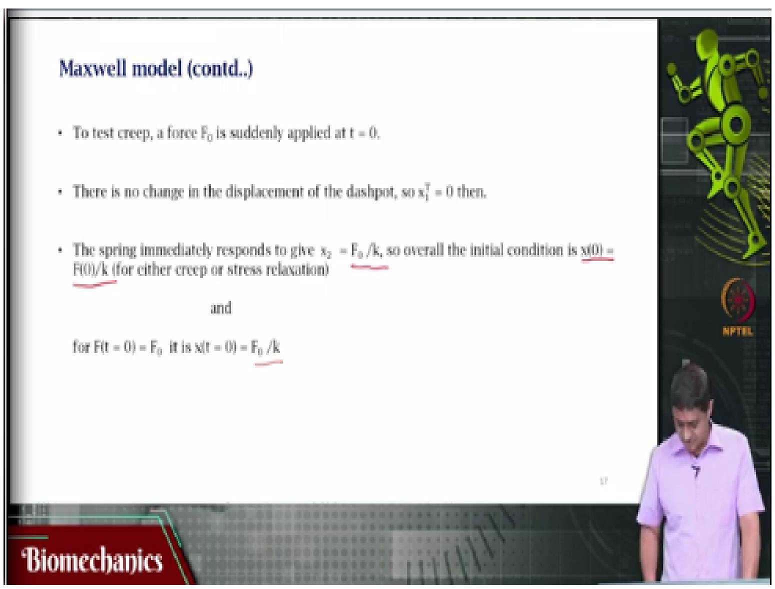

Now if I want to test creep, what would I do? I am applying a force of a known force F naught at time t = 0 and I am checking what happens in the spring and the dashpot? Well, the first thing is that because the application of force is sudden and abrupt, at time t = 0 there will be no deformation in the dashpot. In the dashpot there will be no deformation because it is that force is c x dot but the spring will immediately respond and that deformation that will be filled is F naught by k.

So, at time t = 0 the force that is felt is related to the displacement as x of 0 is F of 0 by k, this is initial condition for both creep and stress relaxation, this is the initial condition. So, that is F of t = 0, the x is F naught by k, why is this? Because the dashpot cannot immediately respond, at time t = 0 there is no displacement in the dashpot. That does not mean that there is no displacement ever in the dashpot, at time t = 0 there is no immediate response.

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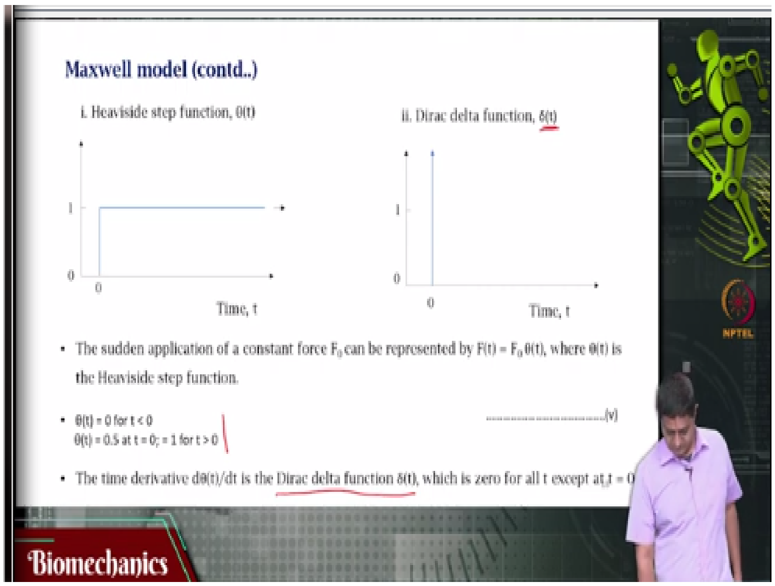

Now how do you model a sudden application of a force? You model this as a step function. So, you use this mathematical function Heaviside step function. So, the application of a constant force is F of t is F naught times theta of t, where theta of t is the Heaviside step function given by this which is 0 for time t less than 0 and at time t = 0, at time t = 0 it is 0.5 and time t greater than 0 it suddenly 1. Just at time t = 0 it is 0.5, this is the Heaviside step function.

What is the time derivative of this Heaviside step function? That is d theta by dt, that is the direct delta function delta of t, what is this? This is at time t = 0 this is 1 or very large value and everywhere else it is 0, well, this is actually not a physically realizable function but we are discussing theory here. So, theoretically at exactly time t = 0 d theta by dt is very high and at all other values d theta by dt is 0, this is the direct delta function delta of t.

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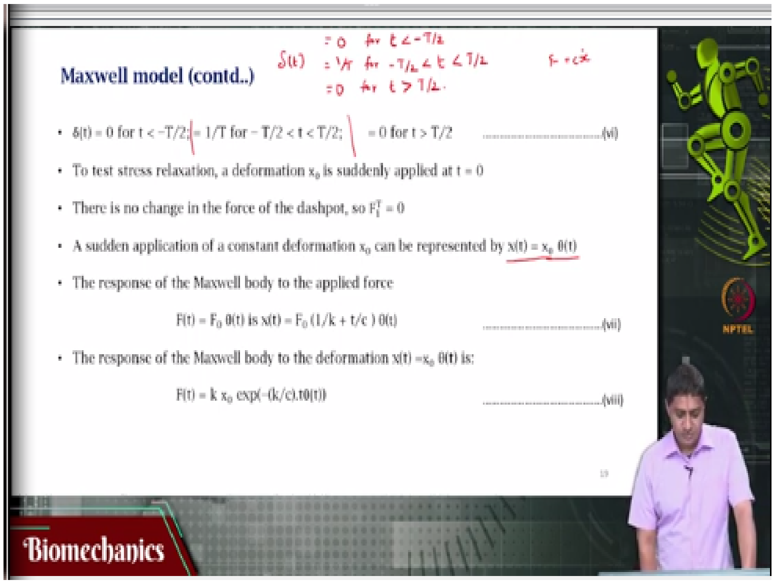

What is the value or how to model this direct delta function? Well, if I take a really small time window T when the time is less than half of T or in other words if delta of t minus this is equal to 0 for when t is less than - T by 2 and is 1 by T for a t between - T by 2 and + T by 2. And is again 0 for t greater than T by 2 that is what is written, I am just rewriting it for clarity. Now if I want to test stress relaxation I apply a sudden deformation x naught at time t = 0.

When I suddenly apply a deformation at time t = 0 there is no immediate change in the force that is felt, so F 1 of t is 0. This sudden application of the constant deformation can be represented as x naught times the Heaviside step function theta of t. Now what will be the response of the Maxwell body to the applied force F? there will be some force F naught times theta of t, suppose I am applying a force F of t which is F naught times the Heaviside step function theta of t.

It will be x is F naught times 1 by k + t by c, times theta of t. Now suppose I apply a deformation X of t is X naught theta of t is k times X naught times e power - k by c times t the whole thing multiplied by theta of t.

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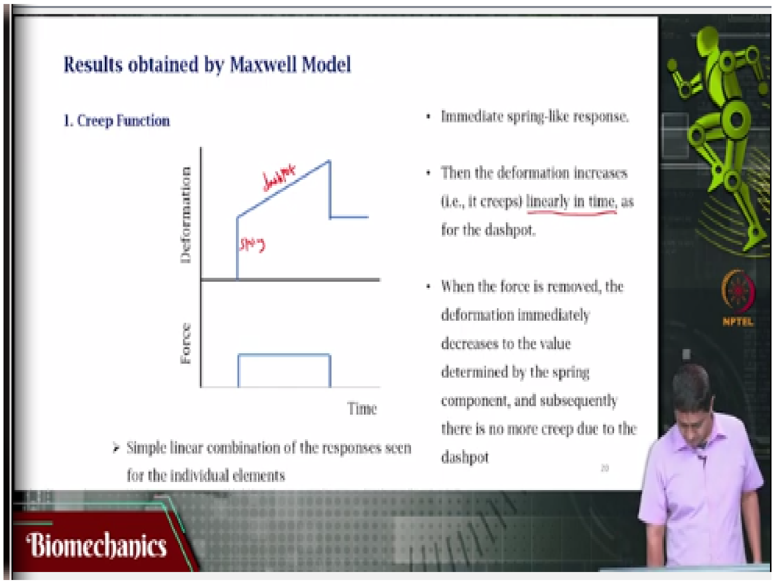

Now let me try to discuss this. So, what would be the creep function? Well, I am applying a force and I am measuring the deformation, there is an immediate spring like response. That is you know their deformation increases and then the deformation continues to increase in time, why is that? Because initially the spring is recruited and then slowly the effect of the dashpot comes into the picture, so slowly there is an increase in deformation with passage of time.

That is it creeps linearly in time because of the dashpot, this is due to the spring and this is due to the dashpot and then I am removing the stress or the force that is applied. And then there is immediate response to the spring value and then it remains a constant. When the force is removed deformation immediately decreases to the spring component and then there is no more creep, the creep disappears due to the dashpot.

So, this is you know a simple linear combination of individual responses of the 2 elements. So, there are 2 elements spring and dashpot what is happening is a simple linear response. So, the creep response is a combination of or a linear combination of the 2 individual responses of the individual components.

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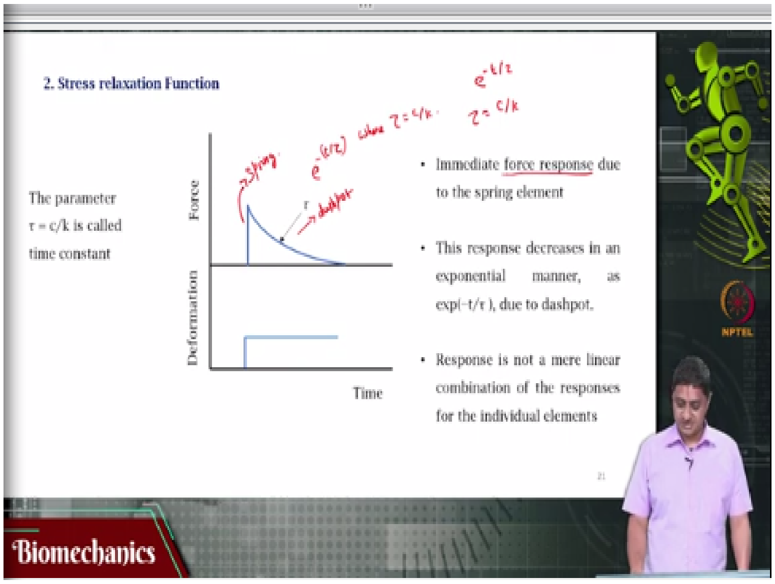

But suppose I apply deformation, what happens is there is an immediate response due to the spring or there is an immediate development of force due to the spring, this is immediate force response due to the presence of the spring. But then this response decreases with time in an exponential manner as in e power -t by tau where tau is c by k. This is due to dashpot and this is due to the spring.

In this case you cannot say that this response is a mere linear combination of the 2 individual responses. No, actually e power -t by tau is not linear by definition, an exponential function is not linear and also that tau itself is a function of both the spring constant and the damping constant, tau is c by k it is dependent on both the spring and the dashpot. So, it is not merely a linear combination of the spring and dashpot individual responses.

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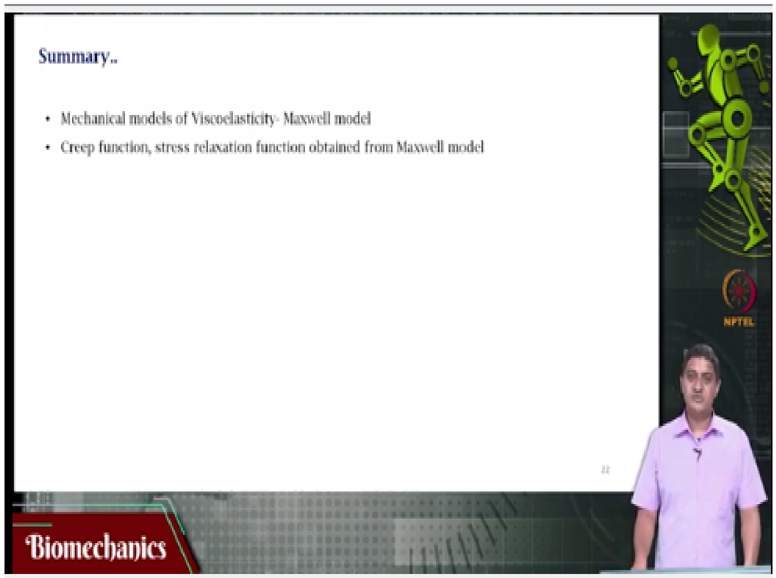

So, what we have seen? We saw one model of viscoelasticity which is the Maxwell model and we saw how the Maxwell model is modeling or is predicting the response to stress or strain. And we saw how creep function is a mere linear combination of individual responses of the 2 elements but the stress relaxation function is not a mere linear combination but rather an exponential function that decays as in tau which is a function of both the spring constant and the damping constant c. With this we come to the end of this video, thank you very much for your attention.