Biomechanics
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Lecture-60
Voight Model

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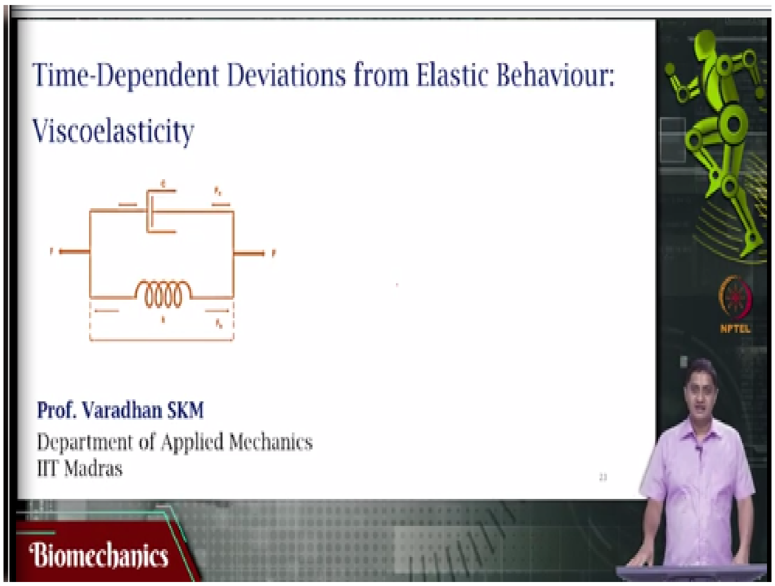

Vanakam, welcome to this video on biomechanics, we have been looking at viscoelasticity that is biological materials exhibiting both viscous and elastic properties. In the last video we looked at one model of viscoelasticity which is the Maxwell model of viscoelasticity. In this video we will continue our discussion on viscoelasticity which is our time dependent deviation in elastic behaviour, that is time or strain rate dependent changes in elastic behaviour.

In biological materials, our most biological materials are viscoelastic, so it makes sense for us to understand how viscoelasticity works.

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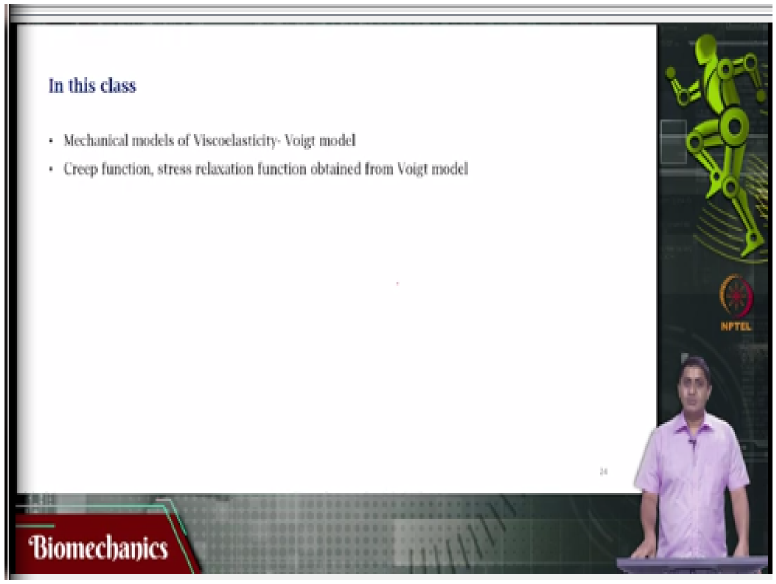

In this video we will be looking at another model of viscoelasticity, this model is called as Voigt model. And as we did with the Maxwell model, we will be looking at creep and stress relaxation functions for the Voigt model.

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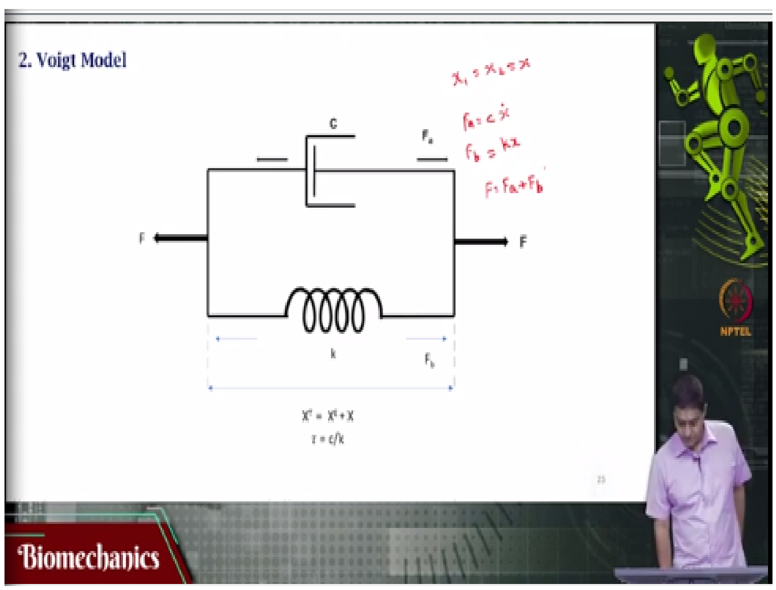

This is the Voigt model; this is a dashpot in parallel with the spring. In the Maxwell model what we had was a dashpot in series with the spring. Here you have a in parallel with the spring which means that there will be qualitative difference in the response of this model when compared with the Maxwell model. That makes sense but we do not know what exactly that is, in other words we have not yet done the mathematical formulation of this which we will do.

But just by examination of this model, I am looking at this model and I immediately know that the response is going to be qualitatively different, how? Because previously the force felt by the 2 elements in the Maxwell model was the same but the deformations were different. And we sums the definition to your total deformation and then we model the whole thing as a function of the deformation and then we converted it into forces and with appropriate initial conditions and so on, remember that is what we did.

But in the Voigt model the force that will be felt by the 2 elements will be the same but the deformations will not be the same, something to keep in mind. So, immediately it becomes obvious that the difference will be qualitative. So, I can immediately think that there will be some difference. So, this is the Voigt model in which the dashpot is in parallel with the spring. Remember, in the Maxwell model we had the dashboard and the spring in series with each other.

So, the results and the response to applied stress or strain or the applied force or deformation will be different between the Maxwell model and the Voigt model. Why because of the way in which I have formulated, because of the model itself. Because in the previous case in the Maxwell model these 2 are in series, now they are in parallel, so obviously there will be some behavioural difference between the 2.

The forces that when you pull this model on 2 sides the deformation on both sides will be the same but the force that is felt by each of these 2 elements will be different. What will be the force that will be felt at the dashpot? That is F a that is c x dot, I can say c x 1 dot but in this case x 1 which is the deformation at the element 1 and the deformation at the element 2 is some x. So, I can say that the force that is felt at the dashpot is c x dot.

And the force that is felt at the spring is some F b second element is some F b and that value is kx, these are the 2 forces that will be felt, x is the same. So, the total force that is felt by the whole system F is a function of the 2 forces F a n F b or I can say F 1 + F 2 or something like that.

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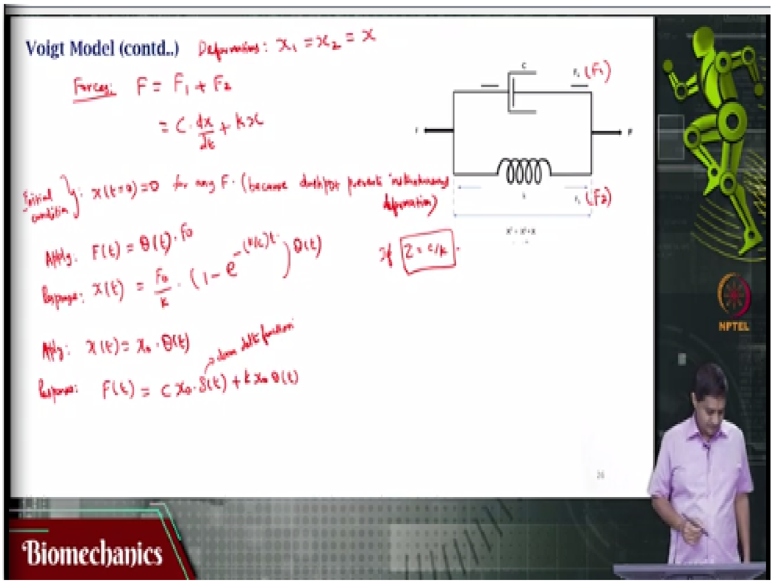

So, let us write this out. Defamations: x 1 felt by element one which is the dashpot = x 2 which is a deformation felt by the spring second element which is the spring the same as sum x**.** The forces F is the sum of the 2 individual forces F 1 + F 2, we will call this instead of F a I am going to call this F 1, instead of F b I am going to call this F 2. Because here I am using x 1, x 2, so it makes sense for me to use F 1, F 2.

What is F 1? F 1 is but dx 1 is dx that is C dx by dt or c x dot + F 2 is kx, I can say kx 2 but then x 2 is x, this is actually c x 1 dot but x 1 is x, so that is essentially c x dot or c dx by dt. F 2 is kx 2 but then x 2 is x, so that is kx, so this is c dx by dt + kx. So, what would be the initial condition for this model? The deformation x of t at time t = 0 is 0 that is the definition, that is the initial condition.

The deformation x of time t = 0 = 0, so deformation is 0. And this is true for any F. Why is this case because although the spring will immediately start to deform the dashpot as a whole prevents any immediate deformation, because dashpot prevents instantaneous deformation, this is the initial condition? Now if I apply a force, some force F of t is the theta of t as we discussed in the Maxwell model.

What would be x of t for this F of t, what would be the response? That would be that would be F naught by k times 1 - e power -k by c times t whole thing times theta of t. That is if c by k or if tau = c by k this would become 1 - e power - t by 2, this is the creep function. For an applied force which is the heavy side step function, if you have a step increase in force or stress the deformation or strain increases exponentially as in tau, where tau is c by k.

Because this is 1 - e power - t by tau, it is not e power - t by tau, e power - t by tau is a decreasing function this is 1 - e power - t by tau which is an increasing function. So, that happens and that is a long expected length because that is what you would expect from a viscoelastic material because this is the creep behaviour. So, as time passes the strain continues to increase for constant stress that is creep that is the behaviour that you would expect in a viscoelastic material.

And that depends on 2 things that is actually not a simple function of only one of these factors that depends on both c and k because tau is c by k. That depends on both the damping coefficient or the damping constant of the dashpot c and the spring constant k, both of this will play an role in this. Now suppose the application, suppose I am applying, instead of a force I am applying a deformation x of t.

Suppose I am applying some x of t which is actually this force that is applied is theta of t times F naught, some initial force times. Likewise x naught times theta of t if I am applying, that is the response and suppose I apply the response is F of t is c x naught times delta of t + kx naught times delta of t. So, how do we know all these things, how do I get this from some basic analysis of differential equations, now I request you to please revise this.

Because I know the initial conditions, I just have to substitute and redo some, so if I understand this, then I can quickly write this as F of t is c x naught times delta of t, what is delta of t? This is the direct delta function. So, what this means is that? Initially due to the dashpot there is going to be a sudden increase in the force as in the direct delta function but then that immediately settles down because as t increases, t is greater than 0, t is greater than T by 2.

For example, then the direct delta function does not exist anymore, the whole thing becomes 0. Then you only have the response due to the spring which is kx naught times theta of t which is a step change in the spring, only the spring response remains. So, initially there will be in certain increase in force and then that will decrease and then become the step function according to the applied deformation, this is the expected stress or force. Of course I request you to please check this mathematics by substitution whether these formulations are correct? They are correct but please do check for your own understanding.

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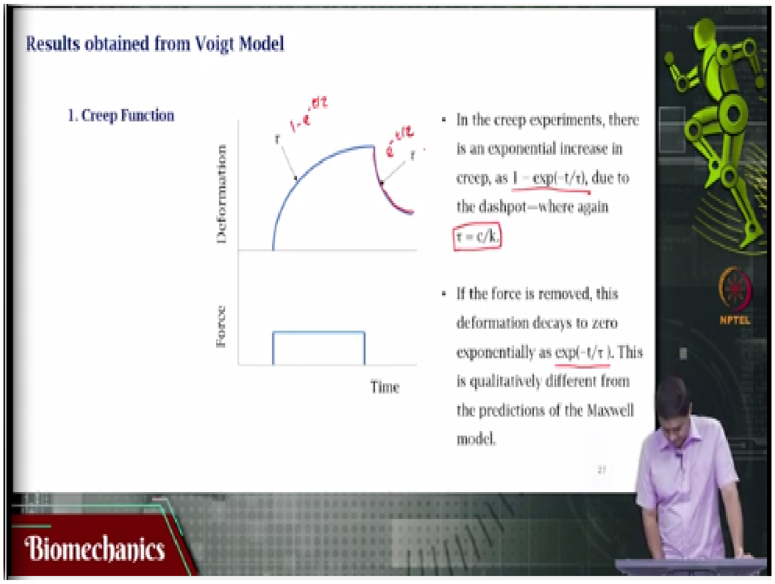

So, what are the predictions and what are the results from the Voigt model? In the creep experiments initially that is an exponential increase in creep as in 1 - e power - t by tau. Where do we know have this? That is this, 1 - e power - t by tau, why is this happening? Due to the dashpot I am having this increase are a response in which the deformation is increased as time passes, where tau is c by k.

Now when the force is removed what happens? This initial force is removed then this force decays as in e power - t by tau, this is also decaying but this is a decreasing function because this is 1 - e power - t by tau. By the way by now we must be in a position to identify which is e power - t by tau and which is 1 - e power - t by tau by just looking because this is exponentially decayed.

Compare this with the creep results of the Maxwell model and you will realize that there is a qualitative difference in creep between the Maxwell model and the Voigt model. There is a qualitative difference between the predictions of the Maxwell model and the Voigt model. On the one hand the creep function prediction by 1 seems to be more accurate or more acceptable or more realistic than the other.

So, it seems like both of these models are able to account for specific things but are they the full story is something that we will have to wait and watch. So, this is the creep result, so as I am increasing the force the response, the deformation increases as in 1 - e power - t by tau and if I remove the force it decays as in e power - t by tau.

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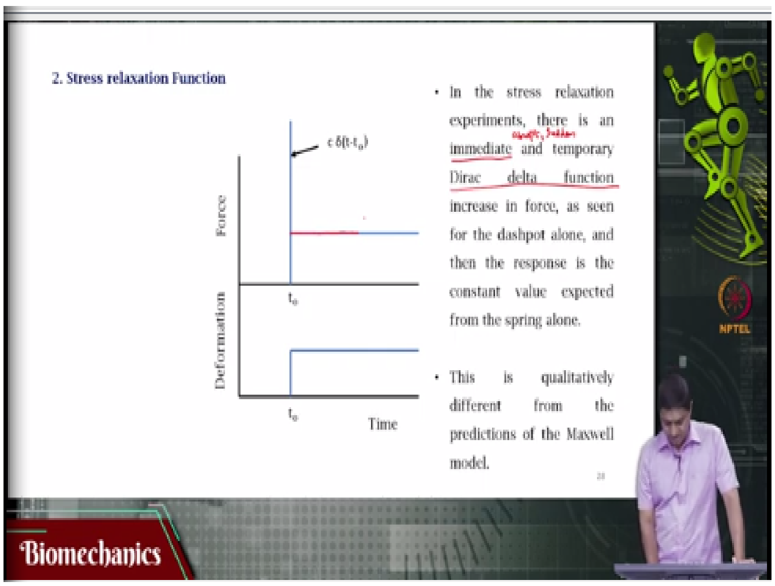

If I perform a stress relaxation experiment what happens? Stress relaxation experiment is if I apply a step change in deformation or strain what happens to the force or stress? That is the question. Well, if I apply a step change in strain there will be a direct delta function in force but that will last only a very brief period. Because direct delta function is going to last for a very small amount of time which is T divided by 2.

Where T itself is a very small amount of time, so it is a really small amount of time. So, at that time there is going to be a sudden increase in stress or force, a very large increase perhaps an immediate and sudden part temporary and immediate abrupt, sudden whatever, sudden large increase but temporary change due to the direct delta function. This is seen in the force, this is due to the dashpot alone, as seen in the dashpot alone.

And then as time passes very quickly the force settles down into the force that would be produced by the spring alone at this level. Once again this is also qualitatively different from the predictions of the Maxwell model, one challenge with this model is what is the physical meaning of this direct delta function delta of t? Physically is this realizable, this function is this realizable? And extremely large amount of force and an extremely small amount of time is physically not realizable.

So, this is a drawback or a challenge with the Voigt model, something to keep in mind that this model gives out a response that requires a very large amount of force to be produced in an extremely small amount of time which is not physically realizable. So, there are both disadvantages or some limitations to these 2 models Maxwell model and Voigt model.

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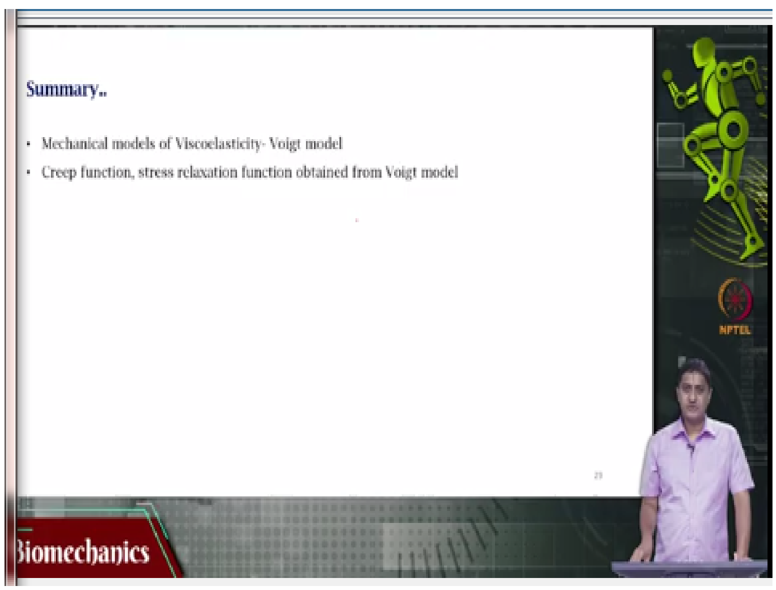

So, in this video we looked at another model of viscoelasticity the Voigt model and we discussed the creep function and the stress relaxation function using the Voigt model. With this we come to the end of this video, thank you very much for your attention.