Biomechanics
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Lecture-62
Viscoelasticity in Bones

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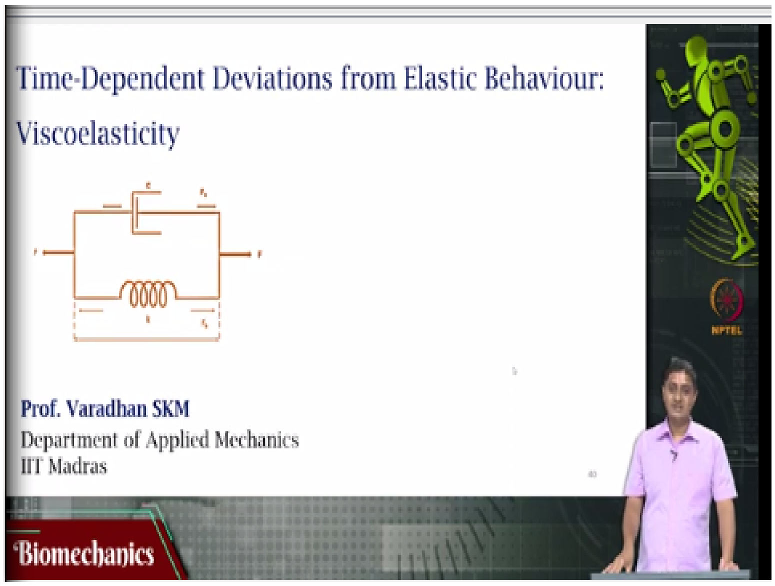

Vanakam, welcome to this video on biomechanics. In the last few videos we have looked at time dependent deviations from elastic behaviour that is viscoelasticity in biological materials. Specifically we looked at 3 models of viscoelasticity which is the Maxwell model, the Voigt model and the standard linear model or the Kelvin model.

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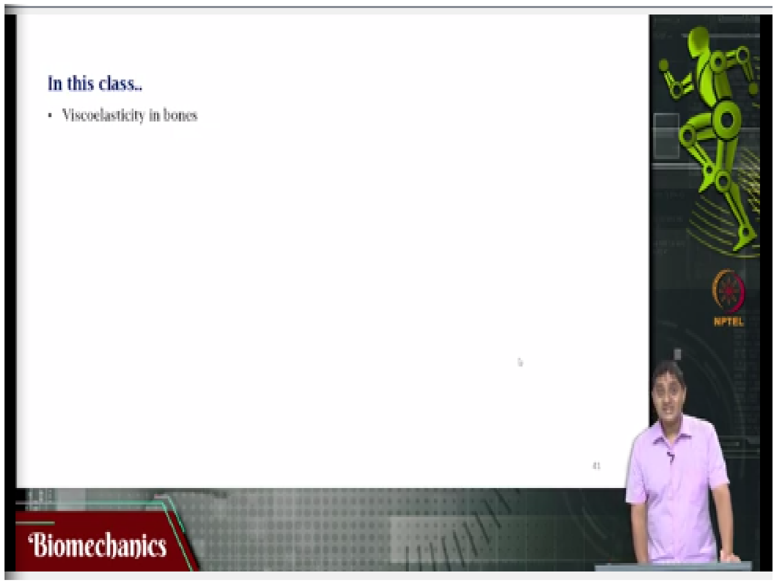

In this video we will be looking at viscoelasticity in one particular biological material that is bone or we will be looking at the viscoelastic modeling of bones.

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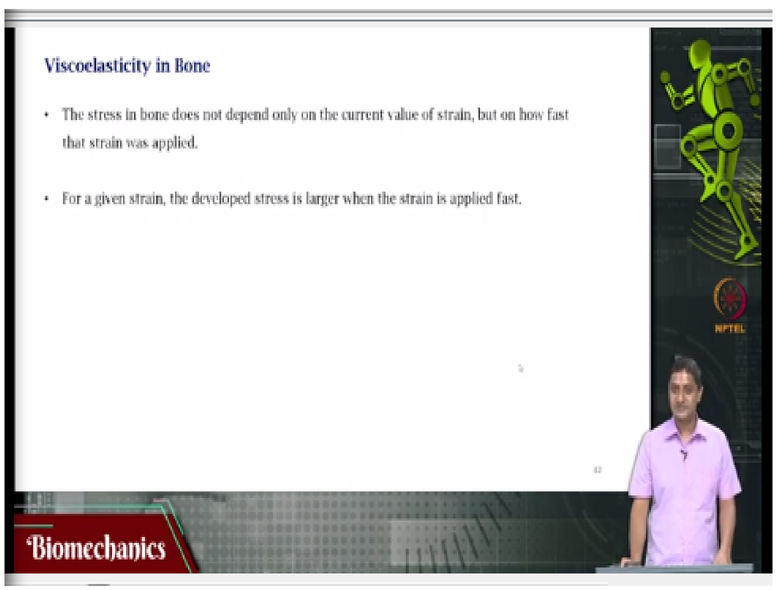

So, the stress developed in a bone, it is not just dependent on the value of the applied strain but rather also dependent on how fast or slow the strain is applied. So, it is not just the absolute value of the strain that matters, of course the absolute value of the strain matters but that is not the only factor, how fast or slow you apply that strain also matters. So, for a given strain the applied stress is larger when that given strain is applied very fast.

But then what do you mean when you say fast and slow? What is fast application of strain and what is slow application of strain? That is not clear, we will discuss that. But if you apply strain fast then the stress is larger, so that means what? That means that the stress and strain are not purely linearly related, it is not like if I apply a larger strain the stress will be larger. Yes, that will happen even in an elastic material but not just that.

If I apply a given strain faster I will develop a larger stress means it is not merely the strain that matters but also the strain rate that matters or the time rate of change of strain matters. The time rate of change of deformation or dx by dt matters. So, that means that there are other things that come into the picture and these are manifestations of viscoelasticity which we can try and model using our previous understanding of viscoelasticity.

For example, we can start with the Kelvin model and try to predict what would happen in bone, let us make that attempt. Also the strained response depends on how fast or slow the stress is applied. So, when you apply a stress what happens to the strained response? So, suppose for a given load you apply that load very fast then the deformation is low, the same load if it is applied over a period of time then the deformation is high. So, that means the rate of force loading matters that is dF by dt also matters.

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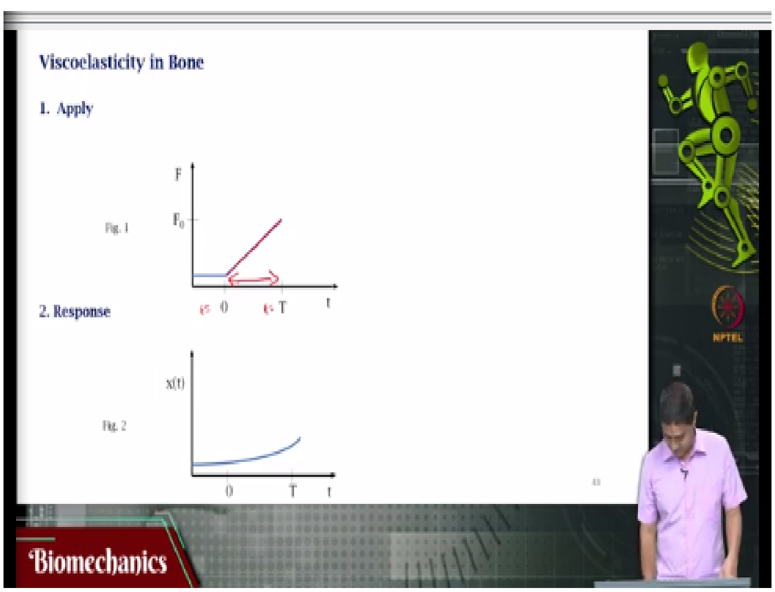

Let us say that we are applying a load F naught that varies linearly in time between time 0 and time t = T, this is t = 0 and this is t = T and there is a linear variation in force. I am measuring experimentally what is happening in the bone, what is the experimental response of the bone in deformation and I am finding something like this. This does not look linear and it is not linear, now let us try to understand why?

This is the experimental observation or can we try and explain this experimental observation using our model or is our model good enough to explain the actual experimental observation? Because if not, if our model is not able to explain the experimental observation maybe either the experiment has not been conducted properly which it is not in this case the experiment has been conducted properly or the model needs to be updated. So, we can check whether the predictions match the experimental observations.

This is bread, butter and water for all experimentalists, this is something that we do all the time, this is something that we do every day, we have a theory, we make a prediction and then we do an experiment and check if that prediction matches with our theory. If not either the experiment has not been properly conducted or the theory has to be updated, daily life for an experimental researcher, let us proceed. So, a linear change in force does not lead to a linear change in deformation is what we observe, let us see what this means.

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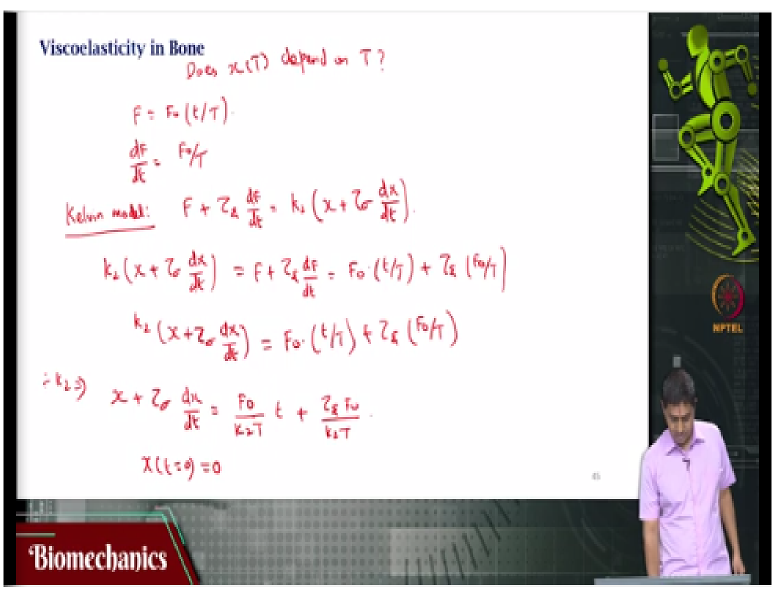

In particular our interest is to obtain a relationship between F and x that also incorporates T. The question is does x of T depend on T not t of course it depends on t but does it depend on T because if it does that means the lower the window, let us go back to that slide and check. The lower this time window the response will be different the higher the time window the response will be different. Is T part of your response equation? That is the question.

Of course t will always be part of the solution, the question is, is T part of the solution? That is the question. Let us say the force is F naught by times t by T that means dF by dt is F naught by t, what is our Kelvin model? What is our standard linear model? That is F + tau epsilon dF by dt = k 2 times x + tau sigma dx by dt, now this is the Kelvin model or the standard linear model. Now substitute for F and df by dt in this Kelvin model, that is F naught times t divided by T, so we rewrite this as k 2 times x + tau sigma dx by dt = F + tau epsilon dF by dt, what is F?

F is F naught times t divided by T + tau epsilon times. What is dF by dt? That is F naught divided by T or let me rewrite this, that is k 2 times x + tau sigma times dx by dt = F naught times t divided by T + tau epsilon, then F naught divided by T. Or I can divide by k 2 throughout what will I get? I will get x + tau sigma dx by dt is F naught by k 2 T times t + tau epsilon F naught divided by a k 2 times T. What is the initial condition? Initial condition is X at time t = 0 is 0 or the deformation at time t = 0 is 0.

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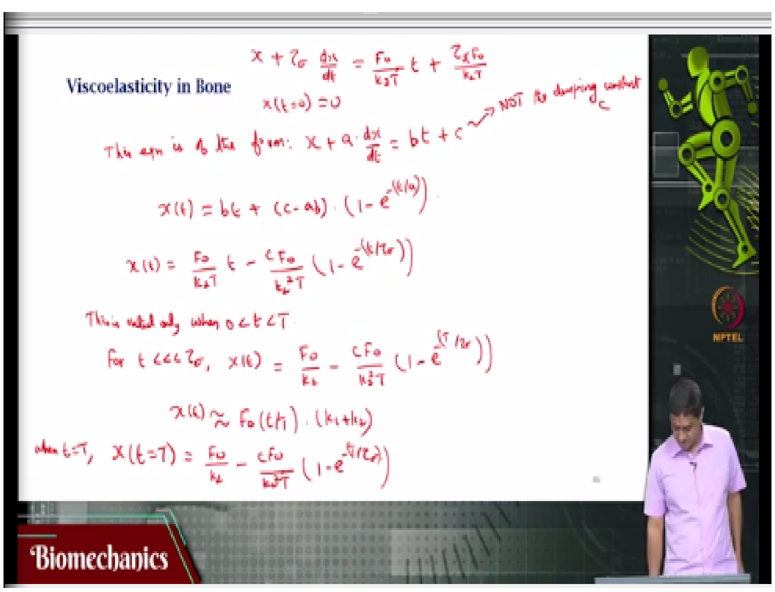

Let us write this in the next slide, that is x + tau sigma dx by dt is F naught by k 2 T times t + tau epsilon F naught divided by k 2 T with x at time t = 0 is 0. What is the form of this equation? This equation is of the form, what is the form? X + a times dx by dt = bt + some constant c. Note that this is not the damping constant c, this is a different constant, this is not the damping constant of the dash pot, this is different.

What is the solution to this differential equation? With x at time t = 0 is 0, this has the solution x of t is bt + c - ab times 1 - e power -t by a, this is the solution. How do you know this? From my knowledge of differential equations, you know this from your knowledge of differential equations. Now check this form of the equation by substituting into the original equation and you can actually try and find the values of the constants a, b and c.

You can check that, that I will leave that as an exercise, do that and also check whether this values of a, b and c are something that you can obtain from this. So, what would be the final solution? The final solution would be x of t is. So, this would be the final solution for deformation. So, this is valid only between 0 and T, whenever time is between 0 and T this solution is valid.

Now when t is much less than the time constant tau sigma what happens? For t that is much less than tau sigma what would be x of t? x of t would be F naught by k 2 - cF naught k 2 square T times 1 - e power - T by tau sigma. So, when t is much less than tau sigma this would be the expression for deformation that I would get. This x of t is approximately, not exactly. This x of t is approximately F naught times t by T times k 1 + k 2.

Now the question is what would be the deformation at the end of the force ramp? That is what would be the definition at x of t = T? When t = T the deformation x of T is or rather let us rewrite this and deformation x of t = T = F naught by k 2 times - cF naught by k 2 square T times 1 - e power - T by tau sigma. So, this is the expression for the definition when t = T, this is what I would get as the definition. So, at the end of the force this will be the definition that I will get.

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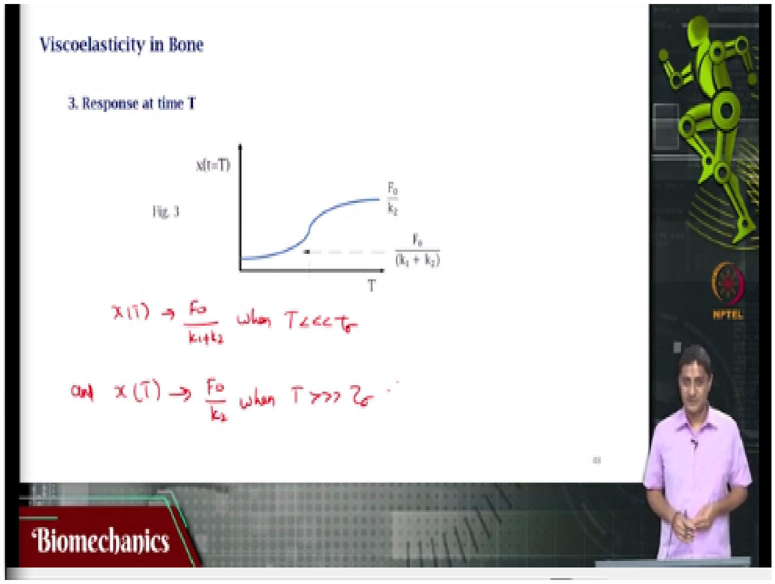

This is plotted in this solution; in this figure this solution is plotted. So, what does this mean? This means when you say that you are applying the force fast or slow what it means is that compared to the corresponding time constant how fast or slow is the force applied. Compared to tau sigma, in this case tau sigma is the time constant, compared to tau sigma how much fast or slow is your time of application of the force, if it is comparable or if it is much smaller the response is different.

So, in these limits x of T is F naught divided by k 1 + k 2 when T is much, much smaller than tau sigma and x of T is F naught divided by K 2 when T is much, much greater than tau sigma. And this model does agree with the experimental observations. What is experimental observation? That deformation is lesser with faster loading and deformation is more with slower loading, this is what we observe in experiments and this is what we get with this model.

So, our model is able to capture with relatively good accuracy the experimental observation that means that this is a reasonable model. Of course, so that means again one more time what is fast and slow? Fast and slow really mean how much comparable are your application times with respect to the corresponding time constant, that is what is meant by fast and slow. It is not a subjective measure, I think this is fast, I think this is slow, it is not our opinion, it is compared with that materials tau sigma, that particular specimens tau.

Compared with that time constant how fast or slow is this, that is the question? Of course we have not really done the stress relaxation experiment. I request you to try that, I will leave that as an exercise, I request you to try that as an exercise.

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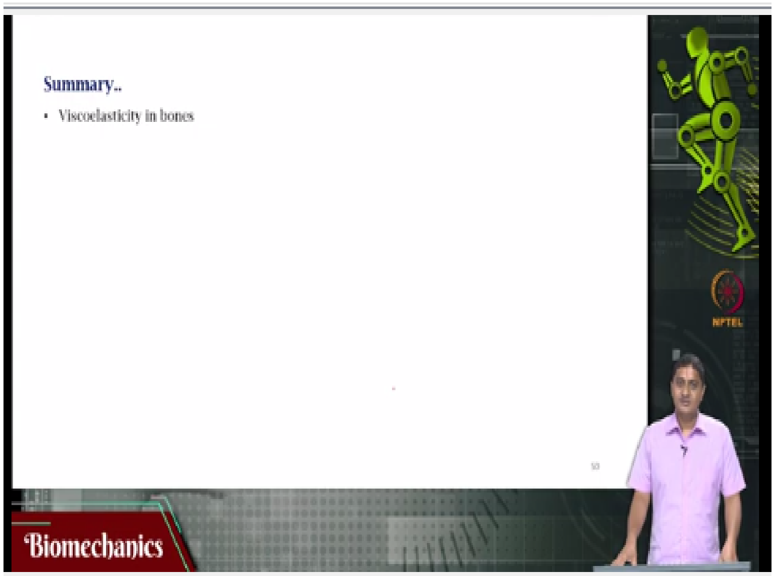

So, with this we come to the end of this video, thank you very much for your attention.