Biomechanics
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Lecture - 67
Gliding Resistance, Tendon Wrapping and Friction Forces

Welcome to this video on biomechanics. We have been looking at biomechanics of soft tissues. Specifically in the previous videos we have been looking at the mechanical properties of tendons, we looked at stress strain relations we looked at the models of non-linear elasticity within the physiological are the toe region. We continue our discussion on tendons and tendon behaviour, in this video.

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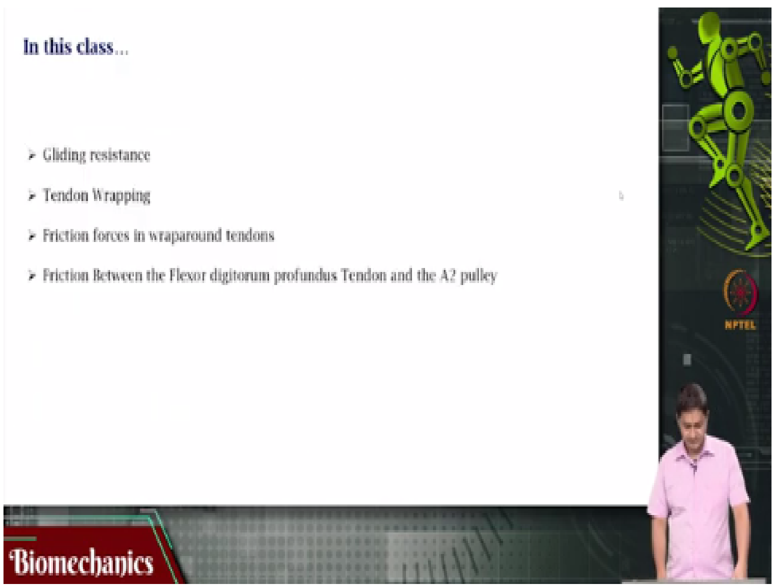

So, in this video we will be looking at gliding resistance, we will be looking at tendon wrapping and we will be looking at friction forces in tendons that wrap around the bone or forces in wrap around tendons. Those tendons that wrap around the bone we call as wraparound tendons. And using this methodology or using this approach or methodology an experimental measurement of the friction between flexor digitorum profundus tendon and the A2 pulley of that tendon.

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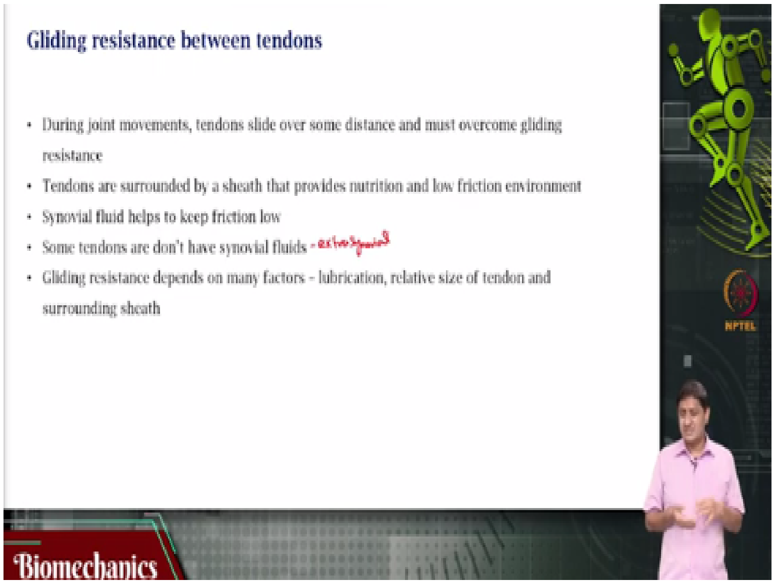

That has been measured. So, when there are movements that happen in a joint, tendons slide over some distances you know and they have a need to overcome this gliding resistance. Tendons are usually surrounded by a covering or a sheath some cover or some material that covers the tenderness sheath and the purpose of the sheet is to provide a low friction environment to provide nutrition.

One of the ways the body keeps the friction low is by the use of synovial fluids. Some tendons do not have synovial fluids and they are called extra synovial tendons. The tendons that have synovial fluids are called intra synovial tendons. So, the resistance so if tendons are going to slide and glide over this the amount of resistance the gliding resistance depends on many things. Of course, you would expect that this kind of friction will not have a contact area dependence but that is not actually true.

The relative size of the tendon and the surrounding sheath plays a role and lubrication the amount of lubrication the quality of lubrication plays a role. So, gliding resistance depends on lubrication relative size of the tendon and the surrounding sheath and such factors.

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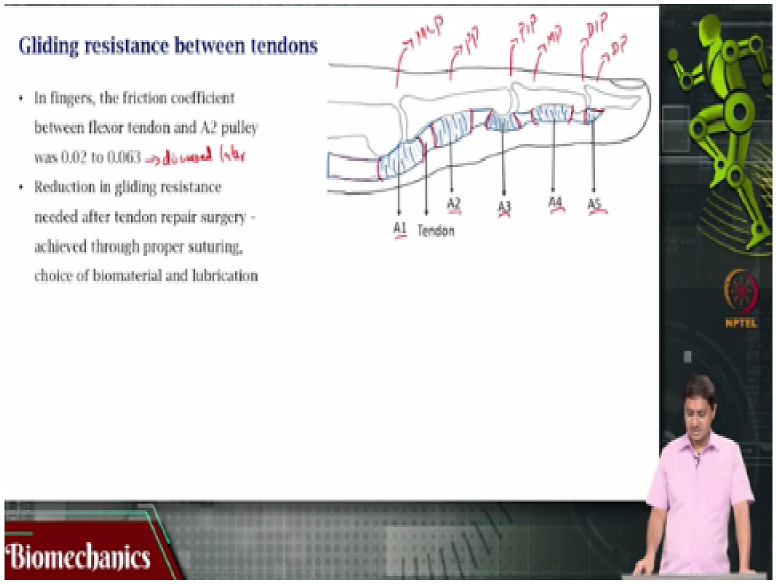

In fingers the friction coefficient between the fluxer tendon and the A2 pulley was measured to be between 0.02 and 0.063. This is the friction between the flexor tendon and the A2 pulley. We will discuss this experiment on the paper later. We will discuss this experimental approach and the corresponding paper a little later. There is a need to reduce gliding resistance after surgery that is performed for tendon repair.

So, after sports injuries or after any injury if the tendon is ruptured and the tendon repair surgery has been performed. There is a need to reduce the gliding resistance. This is achieved usually this is achieved through proper suturing and the appropriate choice of biomaterial and the quality and the quantity of lubricants lubrication that is available. What is shown here is the tendon this is the tendon form of the fingers.

And these are the various pulleys through which the tendon glides over through which the tendon traverses. These are called as A1 pulley A2 pulley A3 pulley A4 pulley and A5 pulley. Remember this is the whole finger this is the distal phalange this is the dip joint this is the middle phalange; this is the proximal phalange. This is the meta carpal phalangeal joint, am I correct. Can you check this?

You can check this with your understanding of anatomy that we discussed long time ago. The beginning of the course when we; discuss the finger anatomy hand and finger anatomy. We have discussed this in much greater detail by just looking at the picture I have described what is the distance phalangeal, what is the dip joint, what is the middle phalange, what is a pip joint and what is the proximal phalange, what is the MCP joint.

Remember what is DP is distal phalange. DIP is distal interphalangeal joint, PAP is proximal inter phalange joint, MCP is meta carpo phalangeal joint. The proximal phalange is called PP, and the middle phalange is called MP just check this once. So, these are the five pulleys that are of interest for us these are called as A1 pulley A2 pulley A3 pulley A4 pulley and A5 pulley. The coefficient of friction between the tendon.

And that A2 pulley is the one that is that was measured using the experiment that we will discuss later. This is that pulley that we are talking about.

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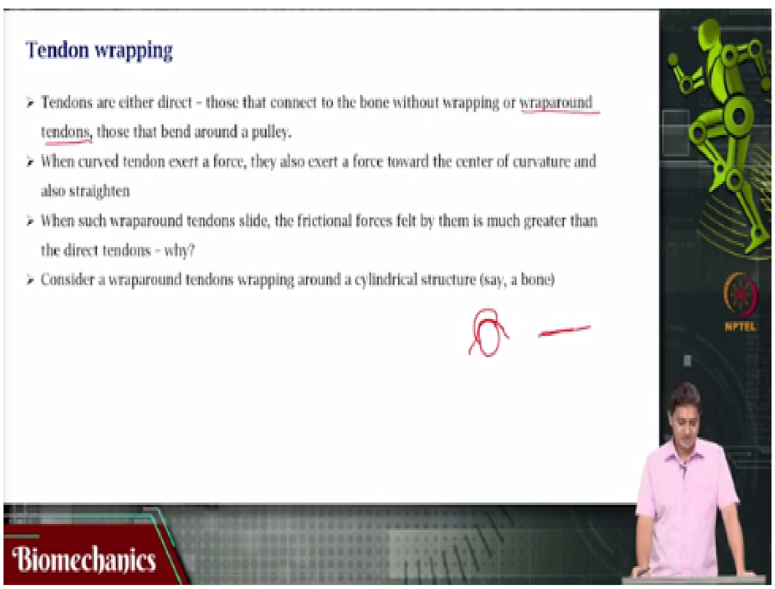

Now tendons either directly connects to the bone are they wrap around the bone and then connect. So, those tendons that connects to the bone directly without wrapping around it or the direct tendons. Sometimes tendons wrap around a bone a wrap around a pulley and then connect. This wrapping around a pulley are called as are those that wrap around the bone or wrap around the pulley are called as the wrap around tendons.

Those that do not do that are called as the direct tendons. When curved tendons when they exert a force usually what they do is that they also produce a force towards the centre of curvature. So, they are curved and so when they produce a force, they tend to straighten a little bit because they also produce a force towards the centre and they also straighten a little bit. When these kind of wrap around tendons slide the friction felt by them is much greater than the friction felt by the direct tendons.

Why is this, the case? So, let us say there is a bone a cylindrical structure and a tendon wraps around corner stays in contact and then goes out like this and goes out like this. The friction felt by this tendon is much greater than the friction that is felt by a straight direct tendon. Why is this the case? We will have to check why this is the case because one likely reason is that the contact area increases.

But you would say that the contact area does not affect simple principles like a simple friction mechanisms such as coulomb friction. But actually, this is not true in many cases contact area tends to have a very important role in deciding the amount of friction and the amount of resistance. In terms of friction may be coefficient of friction might not change but the amount of force that is felt will be affected by the contact area.

Well, if that is the case would that happen in a flat surface also is another question. Mostly this also happens in a flat surface but the amount of friction the amount of frictional force that will be felt in a wraparound or in or in materials that wrap around a cylinder for example in this case is much greater why is this the case we will have to see and that is what we will be discussing in most of the remaining part of this class.

Now let us consider wrap around tendons that wrap around a cylindrical structure. By the way remember we discussed this; the case of the pulley and what is the purpose of this kind of pulleys and wrapper wrapping around. We discussed this for example, one example that immediately comes to my mind is the case of the patella. Remember the tendon from the quadriceps rats around the patella. Why is this? Because there is a need to increase the momentum.

So, for mechanical purposes there is this pulley kind of mechanism that is present then this. So, let us consider a cylindrical structure such as that one and a tendon that wraps around it and connects here and then another so one is the proximal end and the other is the distal end. And we are interested in studying forces that are felt are exerted by this tendon, that is of interest for us.

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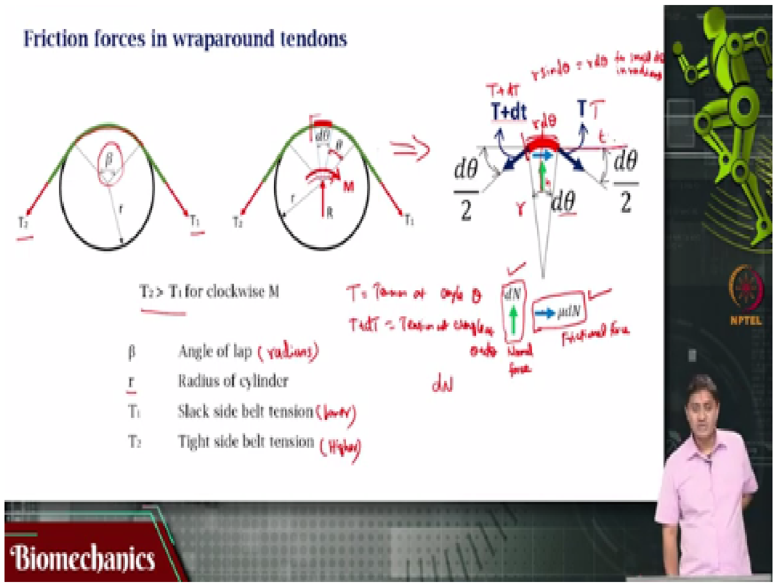

Now that situation can be modelled similar to a cylinder that on which a belt has been wrapped around or wound around. Let us say that the rope has contact with this cylinder for an angle beta in radians and this rope or this belt has two tensions T 1 and T 2. If there is an external rotation that is applied as shown in this picture an external rotation capital M that is applied, T 2 can be assumed to be greater than T 1 for the sake of discussion.

Let us assume T 2 is greater than T 1 in this small r is the radius of the cylinder T 1 is the tension that is felt on the slack side or this is the lower tension T 2 is a tension that is felt on the tight side or the higher tension. Now let us consider a differential area a really, really small area somewhere in the middle of this contacted areas somewhere I am considering a very small area that is formed by this sector with angle d theta, the d theta is really small and infinite small quantity.

So, we zoom in to this element and what we see is drawn here so, there is this extremely small sector that one that is due to the angle d theta the radius is r. Now what would be the length of this element? That would be r d theta. Well technically a that is actually let us say it properly this is going to form the opposite side of the right triangle. So, that would be r sin d theta but that would be r d theta for small d theta in radians.

So, because of this reason this is actually r d theta, how did I say this. This is why you have to go back and check the particular videos where we learned how to resolve vectors. When we say this is actually r sin d theta but sin d theta = d theta for small d theta is actually a you know from high school mathematics. Sin d theta is d theta for small d theta and radians. A question is what are the forces on this differential element? What are all the forces on this differential element?

Well on this side of the differential element the tension if the tension is some T as shown here this is T tension, on the left side it will be some T + sum dt. Why would that be the case? Why the tension on the left side will be higher? Because T 2 is greater than T 1, so as I am moving to the left the tension will be a tad bit more on the left side, not necessarily a lot more that will actually be an infinite small increase an extremely small increase in tension.

Why would it be extremely small? Because the element size is also extremely small. It is an infinite small element because of that reason the increase in tension would also be very small now our interest is to study assume that an equilibrium has been established and study the nature of this static equilibrium in this element that is of interest for us. This is our goal. Our goal is to assume that static equilibrium has been established and we are interested in studying this static equilibrium in this small element.

So, we have capital T on the right and we have capital T + dt on the left. As I mentioned tension at the angle let us say remember that angle has been marked to be theta. So, tension T is the tension at a angle theta T + dt is the tension at angle theta + d theta. That is the tension at that point after d theta. What would be the normal force? The normal force theta + d theta, what would be the normal force?

The normal force is the force that is felt here that is marked in green here and its value will be sum dN to be a really small normal force dN is it not, because it is an infinite small element so it is going to produce a very small normal force d capital N, dN a differential force, an inferential extremely small force. If the normal force is dN what is the frictional force? Well for coefficient of friction mu the fractional force would be mu dN that is drawn and shown in blue here.

That is the friction force which is mu dN, dN is the normal force mu dN is the frictional force. Now let us call this direction as the t direction and this direction as the n direction for historical purposes this is called as a this, we are using an coordinate system called the normal tangent coordinate of the Nt coordinate system. It does not really matter for this, that is the n direction and this is the t direction.

I am going to write out the equations of static equilibrium in the t direction and in the n direction. You can do it for x and y also the results will not be different.

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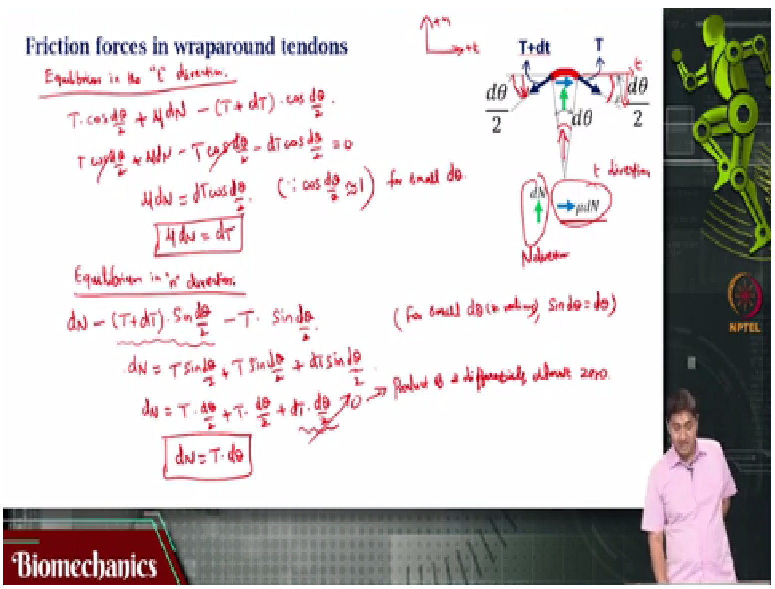

So, equilibrium in the t direction is in the tangential direction, what would that give us? That would give us this is the force that we want but that but t is acting in this direction but we are one we want the force in that direction is it not, where that angle, what would that angle be that is actually an interesting question but you can find out, you can try to use your knowledge of geometry and verify whether this angle is actually d theta by 2.

For this angle to be d theta this angle would be actually d theta by 2 based on our high school geometry I know this and this angle would also be d theta by 2. Again, do not ask how do I know check this is correct. These two angles on the sides will be d theta by 2. I am interested in the force in that direction that is the t direction that would give me actually T times cos d theta by 2, is it not because that is the adjacent side to that angle is it not.

So, that would be a cos d theta by 2. What other forces are there in the t direction? The positive t direction, this is the positive t direction. The positive t direction I also have the frictional force which is mu times dN. In the negative t direction, I have - T + d capital T times cos d theta by 2. I can expand this T + dT cos d theta by 2 as T cos d theta by 2 + mu dN - T cos d theta by 2 - dT cos d theta by 2 = 0.

Rather these two will get cancelled T cos d theta by 2 and - T of d theta by 2 will get cancelled. So, I will have mu dN - or mu dN = dT cos d theta by 2 but for small theta cos d theta by 2 is actually one, for small theta because d theta already is an infinite small quantity. Here we are talking about d theta by 2 that is almost like cos 0 is it not, d theta by 2 is very close to 0 because of this reason cos 0 is 1 because cos 0 is 1 this will be mu dN = dT.

Since cos d theta by 2 is approximately 1, for small d theta is it not, d theta is infinite because of this reason mu dN = dT. Now let us discuss equilibrium in the n direction, that is the n direction is it not. Let us write out all the things that are there in the n direction, what is the force in the positive n direction? That is the reaction force that is dN, dN is in n direction mu dN is in the t direction equilibrium in the n direction gives dN - T + dT times sin d theta by 2.

Because of this force not this force are the component of T + dT in the negative n direction, this is the + n direction and this is the + t direction. In the negative n direction, there will be a component of the belt force, that component what would that be, that would be sin d theta by 2 times T + dT is it not. That is that component is it not, that is sin d theta by 2 times because it is the opposite side.

Those who are still confused about; how I am doing this go back to our first few videos that we discuss how to resolve vectors. So, that would be sin d theta by 2 times T + dT but we are not done yet that is this t force also that comes into the picture and that would be - T times sin d theta by 2. Taking the T dT sin d theta by 2 terms to the right side I am having dN = T sin d theta by 2 + dT sin d theta by 2 + dT sin d theta by 2.

Now for small d theta in radians sin d theta = d theta from high school trigonometry I know that for small d theta in radians sin d theta is d theta. So, I am going to use that here dN is T times sin d theta by 2 let me write this again here T times sin d theta by 2 would actually be d theta by 2 + T times sin d theta by 2 would be actually a d theta by 2 + this would be almost 0, why because it is the product of two differentials that would actually, let me write this anyway.

That would be dT times d theta by 2. Now that is almost 0, why is that because it is the product of two infinitesimal quantities that will be even smaller than those two quantities dT times d theta by 2 is a very small and extremely small quantity. So, I am neglecting that the product of second order differentials and just neglecting that from the analysis. The sum of T d theta by 2 and T d theta by 2 will give me T d theta is it not. So, that would be dN = T times d theta.

Why am I doing this? Because this is the product of two differentials almost zero. So, dN gives me T times d theta, this is what I get from the n direction mu dN gives me dT this is what I get from the t direction.

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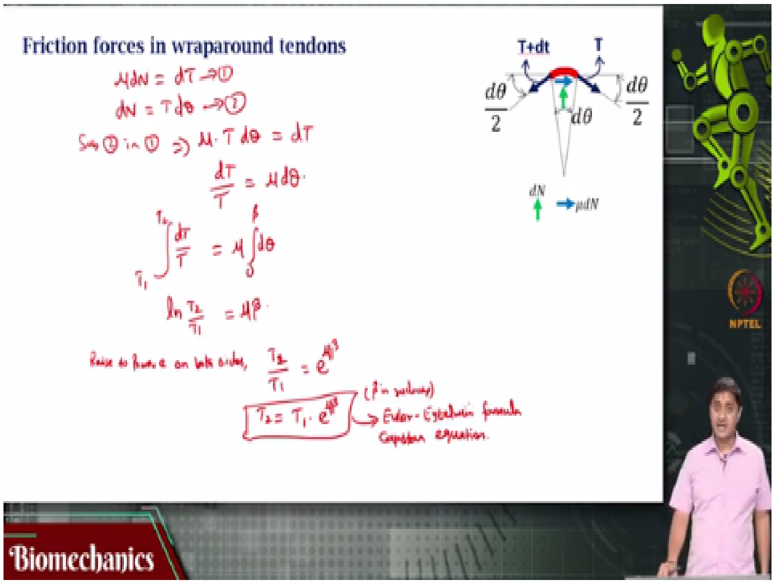

Let me write this in the next slide and continue the discussion, mu dN = dT and dN = T times d theta this is what we have. Now substitute for dN in the first equation, I am going to call this first equations 1 and 2 sub 2 in 1, what do I get? Mu times T d theta = dT are dT by T = mu times d theta. Integrate this between the limits of tension and the angles. What are the two tensions? T 1 and T 2. What are the angles? 0 and beta.

Remember when we first formulated the contact area is known to be beta is it not that is the contact area T 1 and T 2 are the two tensions, I want to integrate this. Integral dT by T between the tensions T 1 and T 2 = mu times integral 0 to beta d theta, this will give me lon T 2 by T 1 = mu beta raised to the power e on both sides. This will actually give me T 2 by T 1 is e power mu beta or rather T 2 is T 1 times e power mu beta.

This is a relationship between T 2 and T 1 that I am interested in understanding. This is called remember beta is in radians this relationship is called as the Euler Eytelwein formula or the Capstan equation. This is the Euler Eytelwein formula or the Capstan equation.

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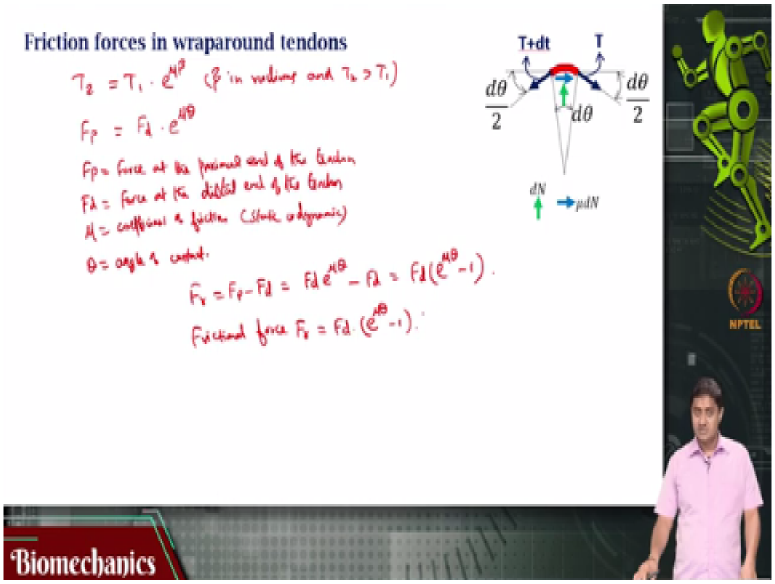

The Euler Eytelwein formula is T 2 T 1 e power mu beta where beta in radians and T 2 is greater than T 1 this is the Euler Eytelwein formula. In biomechanics this is usually written as the force on the proximal side of the tendon is the force on the distal part of the tendon times e power mu theta, where theta is the angle of contact, mu is the coefficient of friction F d is the force at the distal end of the tendon.

And the F p is the force of the proximal let me write this out F p force at the proximal end of the tendon F d is the force at the distal end of the tender. Mu is the coefficient of friction it can be static or dynamic coefficient of friction static or dynamic. Theta is the angle of contact, what would be the frictional force F r? F r is the difference between F p and F d, is it not. That would be the difference between F p and F d that would be F d times e power mu theta – 1, is it not.

Or let me rewrite that that would be F p - F d which is substitute the value for F p which is F d e power mu theta - F d that would give me F d times e power mu theta – 1. So, the frictional force F r is F d times e power mu theta – 1. So, the frictional force is a function of the coefficient of friction and the angle of contact, that is what we learn from this mini derivation. Now a question is how is this directly relevant in biomechanics? How is this directly relevant biomechanism?

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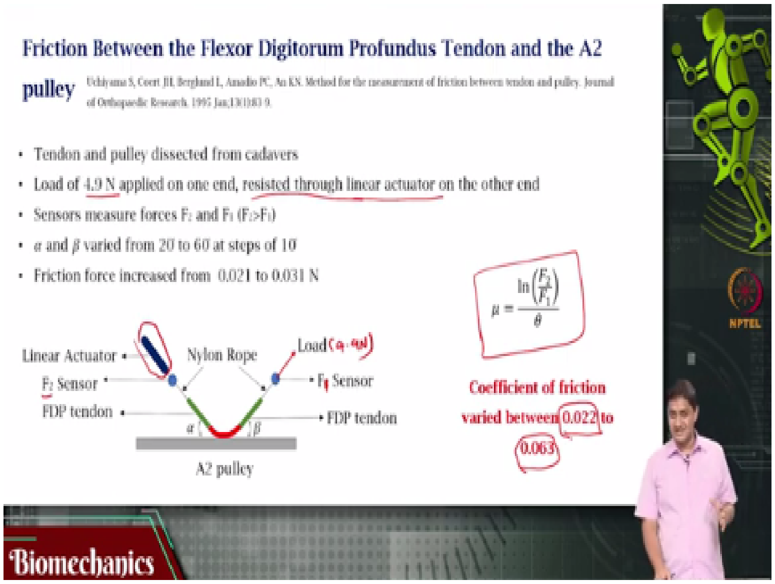

Let us take a simple example an experiment that was performed in the 1990s the early 1990s. That they used a similar principle or essentially the same principle to find the friction between the flexor digitorum profundus tendon and the A2 pulley. They took the pulley and the tendon from cadavers essentially harvested from cadavers and they applied a known load of 4.9 Newtons is applied at a 4.9 Newtons is applied at one end of the tendon.

On the opposite end a linear actuator is resisting this here this is the linear action this is resisting this force and there are two force sensors in series with this arrangement if F 1 sensor and F 2 sensor, that is F 1 sensor and this is F 2 sensor and they are making two different angles alpha and beta with the A2 pulley the tendon two ends of the tendon are making two angles alpha and beta respectively with the A2 pulley.

And what they found was that under they changed alpha and beta, they change alpha and beta so as to change the contact angle. And they found that the frictional force changed from 0.0 to 1 to 0.031 and they used this formula to compute the coefficient of friction between the A2 pulley and the tendon and they found that the coefficient of friction between the A2 pulley and the flexor digitorum profundus tendon is between 0.022 and 0.063.

These are really small numbers remember that these are well lubricated systems. So, it is not abnormal or it is not entirely unheard of when these kind of small coefficients of friction are presented in the biological system. This is one of the ways in which we could use the previous derivation or previous principles. In our understanding of how tendon wrap around causes changes in friction and behaviour.

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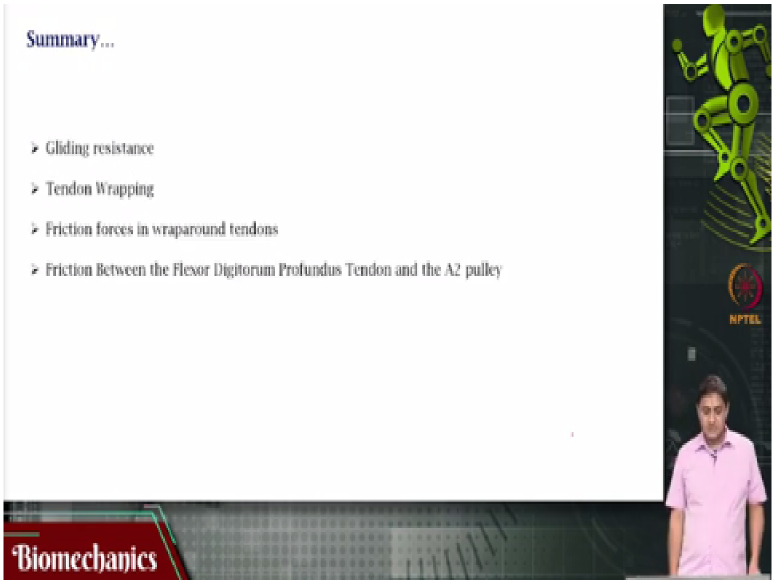

So, in this video we looked at gliding resistance, tendon wrapping around bone and how that changes quantities in friction. And friction forces in wrap around tendons using a small derivation we found the friction forces in wrap around tendons and friction between flexor digitorum and profundus tendon and the A2 pulley experimental measurement of our experimental estimation of this in using cadaver tendons. With this we come to the end of this video. Thank you very much for your attention.