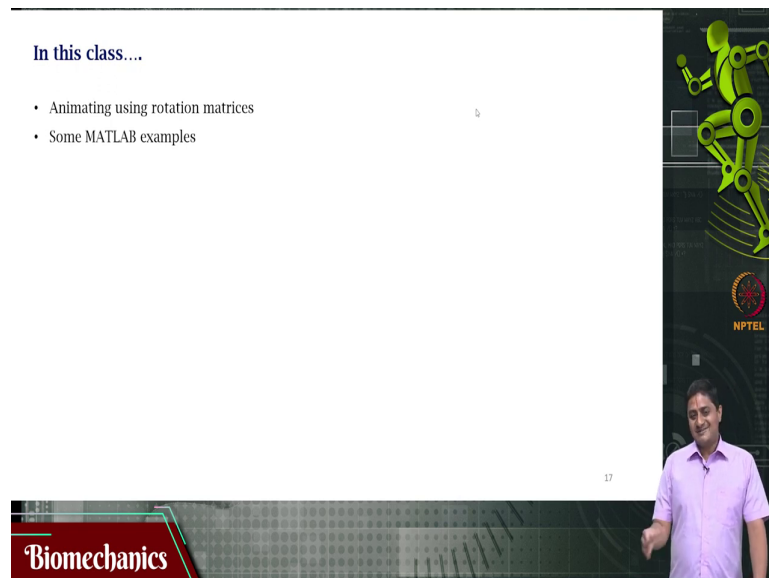


Biomechanics
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Lecture – 77
Animating Using Rotation Matrices- MATLAB Examples

Vanakam, welcome to this video on biomechanics we have been looking at some practical applications we were looking at how to measure segmental kinematics using various methods we had some discussion of rotation matrices in the previous couple of videos. We will continue our discussion on how to measure body segment kinematics and some methods of measurement.

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In this video we will be looking at how we can use rotation matrices to plot the rotation that is happening. We will also look at some real code and some pseudo code that shows some examples of how we can use Matlab to make these plots. Remember these are just examples and they are meant to illustrate the concept of the principle. And the real life or the real experimental measures will not be as simple as the ones that we show.

And are likely to be a little bit more complicated perhaps a lot more complicated. So, it is expected that a serious student of this discipline and someone who is more interested in using these principles in real life and in their experiments should go out and do more self learning over and over what is presented in this. So, this is meant more as a motivation for you to do some self learning.

There are plenty of tutorials and plenty of code and video tutorials available online that is going to help you reach there. This is going to take some investment of time because this does look a little scary and a little mathematical but at some point we will have to cross this. So, to achieve some deep amount of learning to achieve some good ends you need to invest the time and the effort. So, this is always the case this is the case in any discipline this is the case in biomechanics also.

Also remember that we are at the end of our course and we must expect to see some advanced principles being discussed. So, far this has been a course in which we have taken the basics and we have built a solid foundation. We will have to build some building on top of that. So, that is the AIM. So, hang in there and do this practice by yourself and see how it works and tinker with it and see what other changes you can make to this and how that helps you learn these principles better proceed with that note out of the way.


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Parameterization of rotations - Rotation matrix

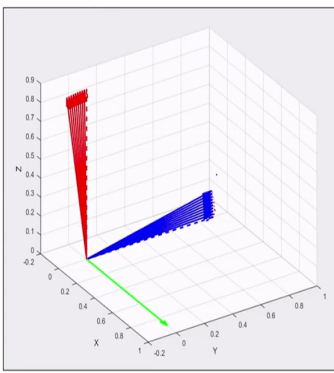
Animating using rotation matrices


for $\theta = 1:1:60$

$$Updated_{\theta}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$


\Rightarrow  Continuously updated

Quiver3 function





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First is how do we animate using rotation matrices. Let us say that you have some technique that is going to provide you rotation matrices on a time instant by time instant basis. Let us say that you are measuring 100 instances per second for example and this method that you are using whatever that method may be it might be an optical method it might be an electromagnetic tracking sensor or it might be Glove it might be some method it might be something.

That is giving you rotation matrices at the rate of 100 per second. So, it is giving rotation Matrix 100 rotation matrices every second. So, at one every 100th of a second it gives you a new rotation Matrix giving an idea about what is the kinematics of that particular segment that you are trying to measure. So, this is what is being given right. So, for example and you are interested in plotting this here we have taken a rotation about the x axis.

And I have also simulated that from one degree to 60 degrees the angle Theta is varying at the rate of 1 degree right it is getting updated let us say that this is happening. So, what would be the rotation Matrix when the rotation is happening about the x axis that would be this that would be that rotation Matrix that would be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Theta & \sin \Theta \\ 0 & -\sin \Theta & \cos \Theta \end{bmatrix}$ where Theta is changing from 1 degree to 60 degree at in steps of one degree at a time.

And let us say that this is getting continuously updated this is getting continuous update if you are measuring this, this will get continuously updated. But because here we do not have a measurement I am simulating this. So, how do you test whether a particular code is working in biomechanics as well as in other disciplines you always test it by using some using some simulated data you generate some simulated data and see if your algorithm is working and if it is able to plot.

If it is working and if it is able to plot then instead of the simulated data, if you just give measured data it will also plot that simple right. So, here what is being done is a rotation that is changing at the rate of one degree by the way this can be smaller or greater than one degree in actual measurement this might be much higher than one degree or much smaller than one degree remember actual measurement these are not actual measurements.

These are simulated data and you use a function quiver 3 to make this plot in Matlab this is specific to Matlab. There may be variance in other languages or packages of your interest. So, you might have to do some tinkering and you might have to use the help of Google or some other Forum to find out what's the appropriate function in the package of your choice right. Here we are taking the example as Matlab.

So, Quaver 3 is a function that is going to plot these arrows as if it is a quiver of arrows right. So, that is what it is plotting see what happens first it is plotting this one degree 2 degrees 3

degrees four to and so on and so forth until 60 degrees right now let us say that I am having this phone I'm having this phone that I am rotating right. Let us say that you know the axis are such that the what are the 3 possible access.

So, there is going to be an axis between the 2 of us between the screen and me orbit in the camera and me and there is an axis that goes from the left to right and are in my medial lateral direction. And there is going to be one axis that is going to go up and down like this and there is going to be one like this. And there is going to be one like there are going to be these 3 axis now in real life does rotation necessarily happen only about one axis.

Like this or like this or you know like this right it does not necessarily happen only about one of these axis a real life rotation is going to be like this something like that right this object is going to undergo a real life rotation that is happening in 3D. So, this example that you are having where you know one axis is about which this rotation is happening is a obviously a trivial example are not directly relevant to real life or experimental values.

Something to keep in mind is whatever the rotation that might happen. For example this object let us say for these axis that I defined let us say that is the rotation that is happening here in this case there is also a translation that is happening. But let us say there is a pure rotation that is happening. For example I am having this object in this orientation and it is undergoing a rotation like this pure rotation right.

It is undergoing a rotation like this. This is likely a 3D it is a rotation in 3D and what I can do is I can find a particular axis about which this rotation is happening right and that axis is likely to have components in all the 3 axis. So, that axis is going to have an x component a y component and a z component. So, that vector is going to have non-0 components in all the 3 because this is a rotation that is happening in 3D.

A question is how do you find this I will leave that as an exercise for you I will continue with this discussion for now. So, here is one way in which I can use rotation matrices to plot here is the pseudo code that we use for this.

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Parameterization of rotations - Rotation matrix

Animating using rotation matrices

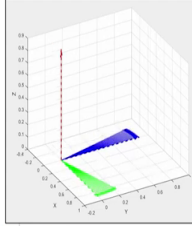

```

X=[1,0,0];
Y=[0,1,0];
Z=[0,0,1];
for thetad=1:1:60
    theta=deg2rad(thetad);
    R_x=[1,0,0;0,cos(theta),-sin(theta);0,sin(theta),cos(theta)];
    R_y=[cos(theta),0,sin(theta);0,1,0;-sin(theta),0,cos(theta)];
    R_z=[cos(theta),-sin(theta),0;sin(theta),cos(theta),0;0,0,1];
    Rotated_vector_X=R_x*[X',Y',Z'];
    quiver3(0,0,0,1,0,0,'g-', 'linewidth',2)
    quiver3(0,0,0,0,1,0,'b-', 'linewidth',2)
    quiver3(0,0,0,0,0,1,'r-', 'linewidth',2)
    hold on
    quiver3(0,0,0,Rotated_vector_X(1,1),Rotated_vector_X(2,1),Rotated_vector_X(3,1),'g', 'linewidth',0.5)
    quiver3(0,0,0,Rotated_vector_X(1,2),Rotated_vector_X(2,2),Rotated_vector_X(3,2),'b', 'linewidth',0.5)
    quiver3(0,0,0,Rotated_vector_X(1,3),Rotated_vector_X(2,3),Rotated_vector_X(3,3),'r', 'linewidth',0.5)
    pause(0.5)
end

```

R(1,1)=1 → x
R(2,2)=1 → y
R(3,3)=1 → z
R = R_x R_y R_z

Ry = for rotation about y axis
Rx = for rotation about x axis

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In this case what is being used is a rotation about see z axis how do we know this because reading this code. So, what is given is that initially x y z are given us this vectors 100 101 001 and where Theta is changing from 1 to 60 in steps of 1 degrees right I am converting this Theta to radian dig to Red of this and then I am defining the rotation matrices which are 100 0 cos Theta minus sine Theta 0 sin Theta cos Theta is the rotation about the x axis rotation Matrix for y axis is like likewise cos that are 0 sin Theta 0 1 0.

And minus sine Theta 0 cos Theta and the rotation of both the Z axis is cos Theta minus sine Theta 0 sine Theta cos Theta 0 and 0 0 1 how do you know about which axis the rotation is happening that is the axis that is going to have one either at 1, 1 or 2, 2 or 3, 3 if it is going to have one in at one, 1 in this Matrix the axis of rotation is X ok if the if the value of this Matrix at one, 1 is 1 then the axis of rotation is x.

If the value of the rotation Matrix if the if the element at 2, 2 is 1 then the rotation is happening about the y axis if in the rotation Matrix at 3, 3 you have one then the rotation is happening about the z axis. Of course I am assuming that the rotation is happening about one of these axis like I mentioned it is not necessary for rotation to happen about one of these principal axis it can happen about an axis that is having a projection non-0 projection into all these 3 axis.

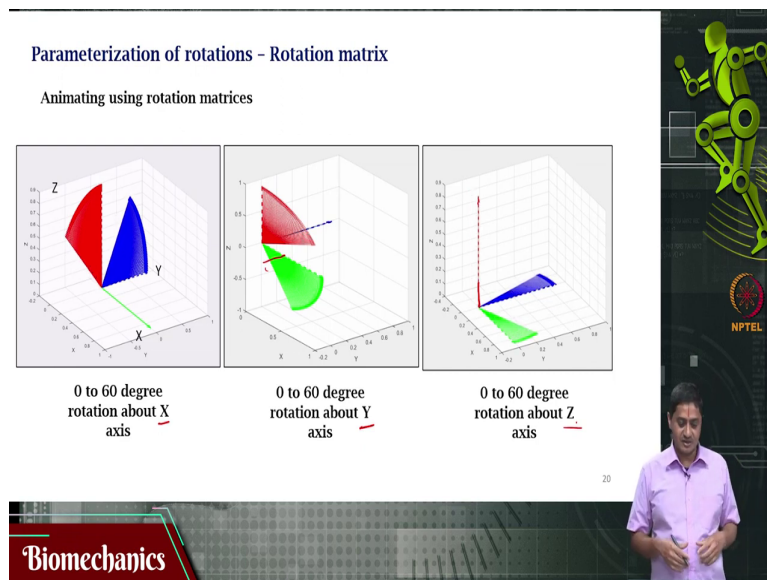
That is the more likely and the more realistic scenario here we are only simulating this. So, here what is happening is I am assuming that the rotation is happening about the z axis and then I am using quiver 3 to plot initially this the first arrow mark which is 0 0 0 1 0 0 and

likewise likewise for the other one 0 0 0 0 1 0 and 0 0 0 0 0 1 these. And then using hold on I am plotting the next Quaver.

Next arrow in the quiver and then I am you know and then I am continuing the for loop right and then rotating the vector at x 1, 1 1, 2 1, 3 likewise for 2, one 2, 2 2, 3 but what is this rotated Vector that rotated Vector is R is at times x dash y dash z dash is it not. So, here the rotation is happening about the z axis. Suppose I want to make this rotation about the y axis then I will change this line as R y times this then automatically the rotation will happen about the y axis for rotation about above y axis.

And R x for rotation about x axis ok and then I am going to the next value of theta because this is a for Loop that I am running remember this is simulation. So, I am running this for loop at values of theta jumping at the rate of 1 degree from 1 degree to 60 degrees. So, this is what I am doing. I can you know take this data from a measurement system and then input this and then perform the same analysis and so, this code will plot whatever is a rotation that is happening as by as measured by the system.

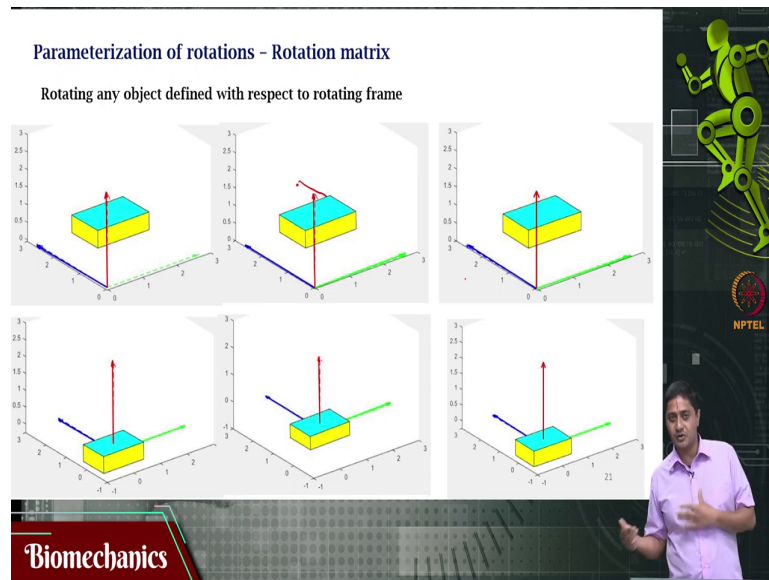
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Here again what is shown are 3 plots one that shows 0 to 60 degree rotation about the x axis. Another one that shows 0 to 60 degree rotation that is happening about the y axis here and another one that shows 0 to 60 degree rotation that is happening about the z axis. Essentially the same code has been utilized and just a small change has been made to plot this for us one more time this is only between 0 and 60 and is changing at the rate of one degrees and so on and so forth.

So, what is needed is for us to measure this and then plot appropriately again I am assuming that the rotation is happening purely about the x axis y axis or z axis this is not necessary and this is extremely unlikely to happen in real life or in the experimental situation. But here I am only interested in learning a principle or demonstrating a principle.

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So, suppose you have a rigid body or an object and I am interested in rotating this object with respect to a rotating frame. For example like this right this object is rotating in this case the object is rotating with the frame as the frame is rotating the object is rotating the object's orientation does not appear to change what is object then going through something for you to think. This object is rotating and it appears like its orientation is not changing.

So, that the frame itself is rotating the object is rotating along with the frame it is undergoing a pure rotation right it is undergoing a pure rotation here again there is this object that is undergoing a rotation about this axis about that blue axis whatever axis that is right about that axis. Likewise here the rotation is happening about the axis that is marked in red here again the orientation does not appear to change the object is rotating along with the frame.

So, the object it seems like the rotating frame is attached to the object in some sense R is having a fixed distance with the object that distance does not appear to change as the object is rotating the frame is rotating or as the frame is rotating the object is rotating. There is no change in the distance between the frame and the object something to keep in mind. Here are some more examples.

In this case what is happening is the object is placed at the origin of the axis the object is rotating and the frame is rotating ok in this case. Here also the same but the rotation is happening about a different frame different axis here also the same is happening the object is rotating with respect to a different frame. What is the difference between the top plots and the bottom plots.

In the top case the rotation is happening along with the frame and there is a fixed distance that is maintained with the origin of the frame in the bottom the object or the origin is placed at about the center of mass or some salient point on the body and it is rotating. So, a question is where do we place the origin that is not a trivial question and there is no one answer that is going to fit in all the conditions that depends mostly on the application right.

Specifically what is the particular application or the measuring system that you have in mind depending on that the frame might change where you place the frame might change. And a choice of a particular frame will or has the potential to give a higher accuracy a particular frame in specific cases might lead to a higher accuracy when compared with a different frame. So, you need to choose that particular frame that gives you the most accuracy in that particular measuring application.

For example if you are interested in you know finger moments I am interested in measuring flexion extension of the fingers right. I can expect to place a frame or an origin in the dorsal side of the pump somewhere distal to the wrist joint right something like that. But suppose I am only interested in measuring the orientation of the hand itself right that is a different problem that I am working with right.

So, the choice of the origin changes one thing that we have not looked at in these cases is the case of general plane motion. These are all pure rotations but in real life objects rarely undergo pure rotations there is always a small translation or a large translation that is associated with this in that case what would you do that is the question. And that it turns out is not a trivial problem right.

So, the motion in that case is a sum of or if you if it is not plain motion as I defined I just spoke about general brain motion it could be general motion. If it is General motion the

motion in that case is the sum of the translation motion and the rotation motion these 2 when summed up give you the general motion something to keep in mind. Here we have only looked at rotations.

Of course in many body parameters of Interest are in many movements of Interest mostly we are interested in rotations but that is not always the case there are cases in which general motion also happens. So, you are also interested in Translation and rotation. So, we need to keep this in mind as we are using these principles.

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Parameterization of rotations - Rotation matrix

Pseudo code - Rotating any object defined with respect to rotating frame

- Define the coordinates of the box (x, y, z coordinates of each point P1, P2, ...)
- Create each surface (say S1) using fill3 command-
 - For example `fill3([P1_x P2_x P3_x P4_x], [P1_y P2_y P3_y P4_y], [P1_z P2_z P3_z P4_z])` creates the surface S1

Rotation matrix for rotation about x (continuously updated)

Rotated points

$$\begin{bmatrix} P1'_x & P2'_x & P3'_x & \dots & Pn'_x \\ P1'_y & P2'_y & P3'_y & \dots & Pn'_y \\ P1'_z & P2'_z & P3'_z & \dots & Pn'_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} P1_x & P2_x & P3_x & \dots & Pn_x \\ P1_y & P2_y & P3_y & \dots & Pn_y \\ P1_z & P2_z & P3_z & \dots & Pn_z \end{bmatrix}$$

Original points

`fill3([P1'_x P2'_x P3'_x P4'_x], [P1'_y P2'_y P3'_y P4'_y], [P1'_z P2'_z P3'_z P4'_z])`

Suppose I am interested in rotating any object with respect to rotating frame. You can define the coordinates of the object it need not be a cuboid of the kind that we are defining say there are six surfaces in this cuboid of interest for me it need not be this but I am using some we need to start somewhere. So, you create each Surface by using the filter,nd for example here we are using the example as Matlab there might be other equivalents in the other packages .

So, you create this surface using the coordinates of the points of interest for example this is p 1 p 2 p 3 and p 4 each of this has an x coordinate y coordinate and z coordinate. So, you define p 1 x p 2 x p 3 x p 4 x p 1 by p 2 y p 3 y p 4 y p 1 z p 2 z p 3 z p 4 z. This creates that surface S 1. Now if we want to create the next surface S 2 I say fill with p 1 p 4 p 5 and p 6 and their corresponding x y and z components.

Likewise if I want to create the surface that is made by joining p 4 p 5 p 3 and p 7 I do the same likewise I create the surfaces that are not visible on the other side 3 more surfaces. And

so, I have all these six surfaces put together forming this body this rigid body that I am interested in. Now let us assume that a rotation is happening pure rotation is happening just about the x axis and that is getting continuously updated.

What is going to be the rotation Matrix this is going to be the rotation Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$ how do I know we discussed this in the previous class that is how I know that this is the rotation Matrix. And these are the original points these are the original points of interest. How many different points are there to define this object how many surfaces are there?

There are six surfaces how many different points are there? There are eight vertices for this cuboid there are only eight vertices and I need the coordinates of these eight vertices by the way it need not be a regular object like a cuboid that is not necessary. But you still need all the coordinates of all the points all the vertices of interest. So, these are the original points and then I continuously update data.

Here θ is getting updated as per the measures data say from a measuring system or I am using simulator data like I did in the previous slides right I simulated from one degree to 60 degree in steps of one degree. I can do that or I can measure I can place a measuring system such as an IMU or such as an electromagnetic tracker and I can get rotation Matrix that is getting updated at some rate say I am measuring one data point at every 100th of a second.

That is I am measuring 100 frames per second at equidistant times ok and I am filling all this then I will get the rotation at each of these points as a multiplication of the rotation Matrix times these vectors. So, this is essentially vector by Vector multiplication. This output Matrix is essentially a because there are 3 points now $P_1 x$, $P_1 y$ and $P_1 z$, 3 coordinate for each and I am going to multiply you know each column with each row and I am going to output right.

I am going to keep doing that and I do that I am going to get a set of rotated points which I can use to plot. So, that general rotation happening about many axis simultaneously are about one axis I that is having components in all the 3 principle axis that are shown will also be discussed but not in this in this video.

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Here is an example of measurement that we made long time ago using rotation matrices at that time we were just dabble with this IMU based system and look at the amount of cabling that is there plenty of cables and very large board that we used to use for measuring thumb kinematics. Remember you know measurement of kinematics of the thumb is a relatively complicated problem and we were able to achieve some amount of success with this as you can see in this video a reasonable amount of success.

And this is purely using rotation matrices we developed on this further and of course our system became smaller. And our method of measurement and display also improved substantially as you will see in future videos and you can also refer to our recent papers on this topic for more information. Here in this video what is shown are 3 segments moving relative to each other there is a segment coloured in cyan and one in magenta and one in yellow.

The corresponding points are defined corresponding coordinates are defined initially and then on each of these segments one IMU is placed and we are measuring this data using IMU and this IMU is outputting the rotation matrix. So, as the rotation matrix for each of this IMU is output we are computing the change in the orientation of that segment and we are showing that displaying that in real time that is what is happening.


Of course this is not as simple as what we saw in the previous slides because that involves only rotation about one axis at a time. Here you can see that rotation is happening about many axis although you know the rotations appear to be simple as performed by this student. You are seeing that the plots appear a lot more complicated than that.

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Summary....

- Animating using rotation matrices
- Some MATLAB examples



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Biomechanics

So, with this we come to the end of this video. In this video we looked at some examples of using rotation matrices for animations and we saw some Matlab pseudocode of how we can use Matlab to plot the data that comes out from rotation matrices. With this we come to the end of this lecture thank you very much for your attention.