Biomechanics
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Lecture – 78
Composite Rotation Matrix and Relative Orientations

Vanakam, welcome to this video on biomechanics we have been looking at practical applications and how we can measure body segmental kinematics and how to represent them how to plot them and how to graphically represent them using software. We looked at some Matlab examples of this in the previous class.

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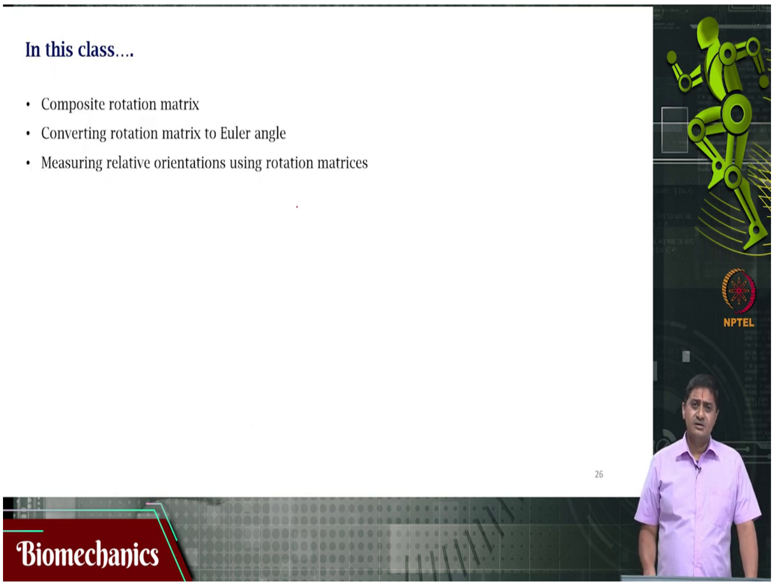

In this video we will look at the composite rotation Matrix that is a rotation Matrix that has an angle different angles about the 3 principle axis. So, in the previous videos we looked at rotations that happen only about one of the principal axes at a time. In this video we will be looking at what happens when a composite rotation happens that is a rotation that involves notation about all the 3 principle axis.

How to convert rotation matrices to Euler angles and how to measure relative orientations using rotation matrices, how to use rotation Matrix to find orientation difference this is what we will be looking at.

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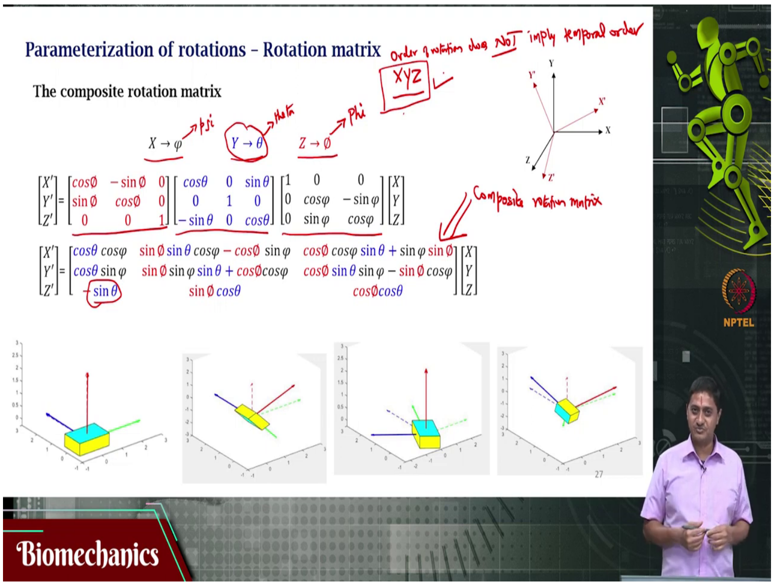

Now consider this case where I am having an angle change or a rotation of Psi an angle of Psi happening about the x axis an angle of theta happening about the y axis and an angle of Phi happening about the z axis. For clarity that is Psi that is theta and that is Phi just writing this for clarity and let us say that the rotation is happening in the order x y z. What does this mean? Does it mean that first the x rotation is happening and then the way rotation is happening and then the z rotation is happening is that what it means or what does it mean.

Actually it x y z order of rotation does not necessarily imply a temporal order it does not mean that the x rotation is first happening in time followed in time by the y rotation and followed in time by the z rotation this is not what it means. What this means is the considered order for representation is x y z. This is the order of representation that is used this does not necessarily imply or mean and almost never the case that this is the temporal order in which the rotation is happening.

The rotation will happen in whatever order it happens that and it may not even happen about any one access at a time. Simultaneously it might be happening about many access all of these 3 principle axis. So, this does not imply a temporal order. Order of rotation does not imply a or does not imply temporal order right. Now but you still for representation sake you need to identify what is the considered order of rotation that is x y z that means the x rotation Matrix followed by the y rotation Matrix followed by the z rotation Matrix is from right to left.

Remember matrix multiplication how does it happen from right to the left that is how it happens remember from childhood from high school this is how we perform matrix multiplication. So, if the order of rotation is x y z then the z rotation Matrix will come first followed by the y rotation Matrix followed by the x rotation Matrix this will imply the actual considered order of rotation is x y z that is what it means.

And after some multiplication after matrix multiplication this is what I will get as the; this is what I will get as the composite rotation Matrix looks like a very scary thing right. There are many things and it is not clear how to identify the rotations about age by looking at this because you are not just going to have this you are going to have numbers. When you are measuring simultaneously about all these 3 axes using a measurement system such as an IMU you are going to have some numbers that are going to be presented.

How do you know what is the angle that is made about each of the principal axis. So, that you can plot right how do you know that that is what we will see in the rest of the video. So, here rotation is happening about this green axis right and in this case rotation is happening about that blue axis and here rotation is happening about that red axis whatever that axis is x y z axis. And here that is the rotation that is happening where that is this rotation happening about which axis.

That is not clear right let us play it one more time. In the first case the rotation is happening about the green axis is it not. The second case it is one more time it is the second case it is happening about the blue axis and in the third case it is happening about the set about the red axis right. Now what about this one this is the composite rotation that we are speaking about this. Combines rotation happening about all the 3 axis in this case maybe it looks like all the 3 axes are undergoing the same amount of rotation but that need not be the case in real life.

It can undergo different and almost always undergoes different amounts of rotation about the 3 principle axis which is why you are having Psi theta and Phi as the 3 angles about the 3 axis. So, you are going to have a rotation that looks like this right. The question is how do you compute the rotation that is happening about each of the principal axis for your understanding and for your representation sake.

Here it is simulated data and so, I am changing all the 3 acts simultaneously and the code is just plotting this out but it is difficult for me to visualize when this happens unless I understand the rotation that is happening about each of the axis it is somewhat difficult for us to visualize in this case this is simulated data. So, it looks nearly perfect and very symmetrical and very easy for me to understand but real life that need not happen right.

So, because if this rotation Matrix is the one that is being output by an IMU then it is not clear for me how to visualize or it is not very simple for me to issue this. How do we then look at well it. Seems like there are ways in which I can you know find those specific angles for example I can start out with this I am only having sine theta. So, if I am getting that element value then I can find the value of theta as sine inverse right.

That will give me the angle about the y axis for example right if this is the composite rotation Matrix. Remember this is true only for this order of rotation remember this. This is K this composite rotation Matrix works only for this order of rotation not for any other order of rotation. So, this is an example you cannot memorize this for example remember this.

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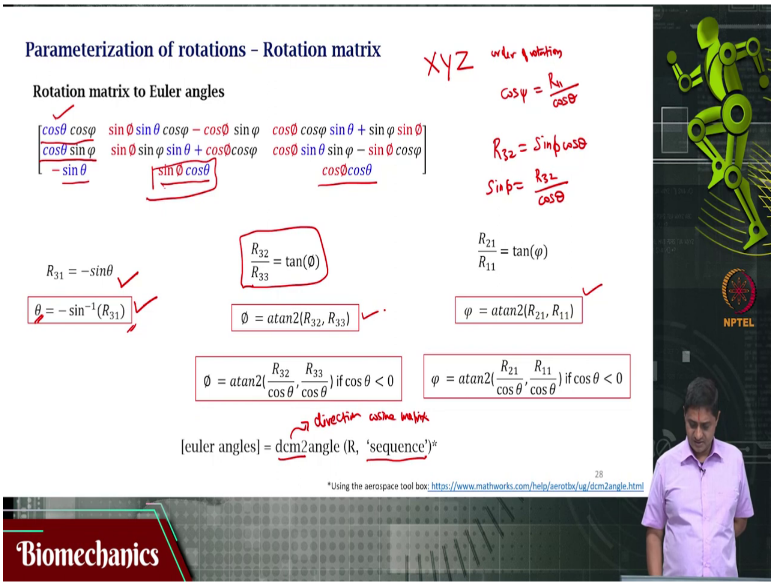

So, the element 3 1 is minus sine theta. So, I can find theta as minus sine inverse of this rotation Matrix element 3 1 right straight away straight forward I am getting theta. Now are there other ways of finding phi and Psi well you know if I look at these two 3 2 and 3 3 there is a common element cost data between them. If I divide one by the other coset is going to get cancelled and I will have tan phi as R 32 by R 33. Once again remember this works only for this order of rotation.

Because for different orders of rotation you will have different composite rotation matrices I will speak about that in just a minute for this order of rotation I am having this composite Matrix and from that I can find Phi as tan inverse of R 32 by R 33. Likewise if I take these two if I divide 1 by the other I am going to get tan psi and I can find Psi as tan inverse of R 21 and R 1 R 21 divided by R 11.

I can do something else if I first find theta well if I first find theta there are other ways of doing this there is no single way to do this for example if I have found data right using this method then I can take this value that is R 32 R 32 is sine Phi cos theta and since I know theta I already know theta how do you know I just formed it in the previous step from that I can find sine Phi R 32 by cos theta where theta is already known from this step from the first step likewise for likewise here also I can find cos Psi as call I can find cos cos Psi as r 1 1 divided by cos theta.

So, Psi is cos inverse of R 11 by cos theta theta is already known for me there are many ways of doing this there is no single way this works for the most general case. So, this is a simple case. So, I can right if I take these elements and you know I can straight away take R 31 R 32 R 33 and R 21 and R 11 and I immediately get this right. So, using this I can find the angles the Euler angles from the rotation matrices.

This is using a manual method or an algorithm. If you are having the aerospace toolbox in Matlab by the way this is not a default toolbox that comes with Matlab. If you have this toolbox then converting from the rotation Matrix to either angles is DCM to angle of the rotation Matrix you still need the sequence because otherwise the system will not understand you still need to say whether it is x y z or whatever else right.

So, you will still need that sequence as an input to this toolbox to this function DCM to angle to compute your Euler angles. What is this DCM? DCM is this Matrix of direction cosine direction cosine Matrix right direction cosine Matrix to angle that is another way in which you can represent rotation Matrix right.

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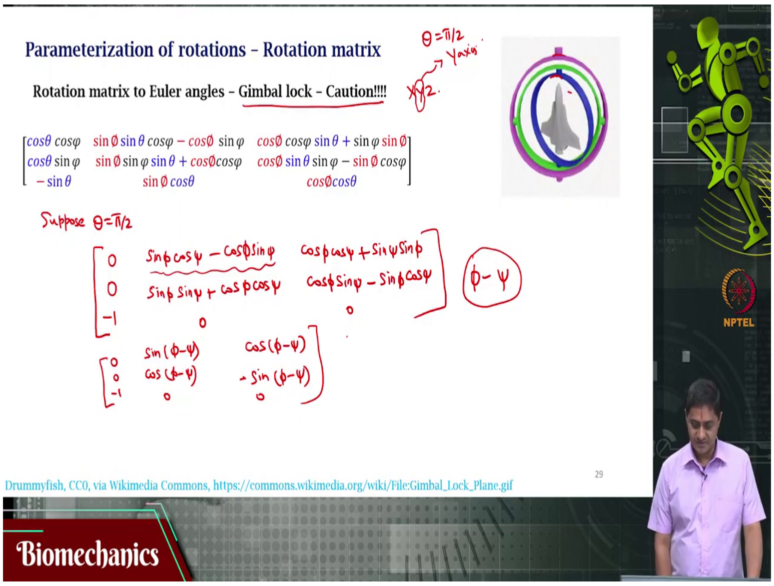

There is an issue that comes when you try to convert from rotation Matrix to other angles that is Gimbal lock this is a problem that we will discuss in much greater detail in a future video. For example what happens here is that initially this blue frame right is at a different angle then it goes and locks itself to or aligns itself in line with the magenta frame now right. In this case what happens is that the measurement will no longer have the same number of degrees of freedom as expected.

It is not like there is a physical lock that is happening but measurement will no longer be uniquely represented there are some measurement issues that come into this picture we must discuss. We will discuss this in much greater detail in future. So, when you do this you need to be a little cautious about the conversion from rotation Matrix to Euler angles because sometimes. So, this is an issue that comes only when you are dealing with Euler angles right something to keep in mind.

So, when you are converting and when this Gimbal lock happens there is some uncertainty or some ambiguity about the way you represent data you need to be sure about how you are representing this data. Otherwise because you are only using software to plot we will all use software to plot like Matlab. Matlab will plot whatever is given to it because it is not having a brain of its own whatever data is given.

Only as humans as experimenters we know that some sets of data make sense and some angles are impossible to happen but a system does not know these things. Whatever is given to it, it will plot something to keep in mind. Now let us just briefly discuss this Gimbal lock problem for the case when theta goes to 90 degrees suppose theta equal to 90 degrees or Pi by 2 radians right. In the xyz sequence remember the sequence is x y z.

Then what happens to this Matrix substitute everywhere theta equal to 90 degrees and let us rewrite. This Matrix is going to take a little bit of time but let us do that cos theta well cos 90 is zero and again the second element cos 90 is 0 minus sin theta is minus one sine 90 is 1 but it is minus sin theta. So, that will be minus one. And the next one is having sine Phi sine theta cos Psi right.

So, that will become because sin theta is 1 that will become sin Phi cos Psi minus cos Phi sine Psi minus cos Phi sine Psi already I am getting some hints about what is happening because this looks like a trigonometric identity but let me proceed the next one is cos Phi cos Psi sine theta sine theta is one. So, I am going to write this as cos Phi cos cos Psi + sine Psi sine Phi. The next element is sine Phi sine Psi sine theta because sin theta is one this will become sine Phi sine Psi + cos Phi cos Psi this is also looking like a trigonometric identity.

The next one is cause 3 sine theta sine Psi because sine theta is 1 that will become cos Phi sine theta is one sine Psi minus sine Phi cos Psi. The next one is sine Phi cos theta cos thetas zero the next one is cos Phi cos theta but for theta 90 degrees cos it is 0. This is the rotation Matrix that I get when theta goes to 90 degrees as seen in this the blue measure here is going to 90 degrees when it goes to 90 degrees this is what I will get.

Let me apply the trigonometric identities that I know here what would that be? That would be 0 0 -1 this one what is this? This is sin Phi cos Psi - cos phase sine Psi that is sine of sine of Phi – Psi. The next one is cos cos phi cos psi + sine Phi sine Psi right that is cos of Phi minus Psi the next one again is sine Phi sin Psi + cos Phi cos Psi that is again cos of Phi minus Psi. And the next one is cos Phi sine Psi minus sin Phi cosine that is actually minus of that is actually minus sine of Phi minus Psi.

Then the next two elements are already zero because cos theta is zero. What this tells me is that now the description of this rotation is dependent only on two angles what are these two angles are actually only one degree of freedom. This one degree of Freedom it turns out is this angle difference between Phi and psi right. The difference between the angles these two angles is the only degree of Freedom that appears to matter here right.

Now from this what I am understanding is that it appears like there is a reduction in the number of degrees of freedom earlier I had a 3 degrees of freedom. Now it appears like only this is all that matters for describing this rotation and remember this happens only when theta equal to Pi by 2 what is theta that is rotation about the second axis second principle axis in this sequence the y axis in this case.

Now we will have a longer discussion about this in a future video but this is something that we want to keep in mind that suppose theta equal to Pi by 2 or suppose the second angle goes to Pi by 2 then the whole system or the whole representation breaks down and we are going to have quite a bit of complexity that is going to appear in this or quite a bit of ambiguity that appears here this is called Gimbal lock.

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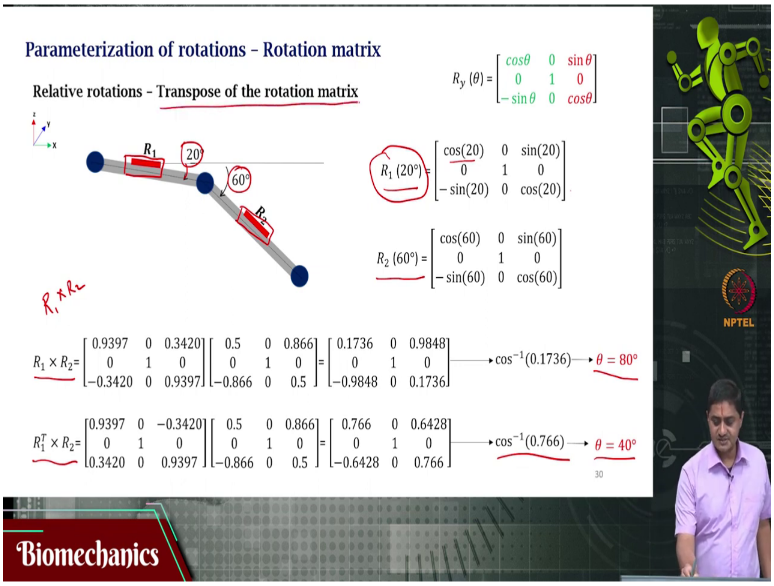

Now I am interested in measuring relative rotations this is what I am interested because what I am working on is a finger kinematics orientations of finger with respect to each other I am interested in relative rotations. Now let us say that I am getting one rotation Matrix with respect to some inertial frame of reference that can be that can be one more IMU that is placed at an inertial location a location that is not moving.

Assumed to not move by the way there are always moments. But I am assuming to that this point will have relatively much less movement than the moments that I am interested in measuring. And then I am measuring R 2 which is also measuring the rotation Matrix with respect to that earlier the previous the only other inertial reference frame I am getting these two. Let us say the first one is measuring an angle of 20 degrees the second one is measuring an angle of 60 degrees right.

Then I will have two rotations and represented as cos 20 0 sine 20 0 1 0 and minus sin 20 0 cos 20 as the first rotation Matrix and the 60 rotation Matrix will look like this right now if I multiply r 1 and r two If I multiply the two rotation matrices R 1 and R 2 I will get you will have to do the actual computations for 20 degrees and 60 degrees and input the actual cosines and signs and do the actual multiplication you will get the value of theta the net rotation as 80 degrees.

But suppose I am multiplying the transpose of this rotation Matrix this rotation Matrix R 1 and transposing and then multiplying it with R 2 then I am going to get theta as 40 degrees. So, the total rotation that is happening I am getting as the product of the net rotation that I get from the product of R 1 and R 2 but if I'm interested in relative rotations between R 2 and R1 if I am interested in relative rotations I multiply R 1 transpose with R2.

And whatever is a rotation Matrix I get from that I try to get the angle and that angle I will get us the difference between these two. So, if I am interested in finding relative rotations I will have to use transpose of the first Matrix rotation Matrix.

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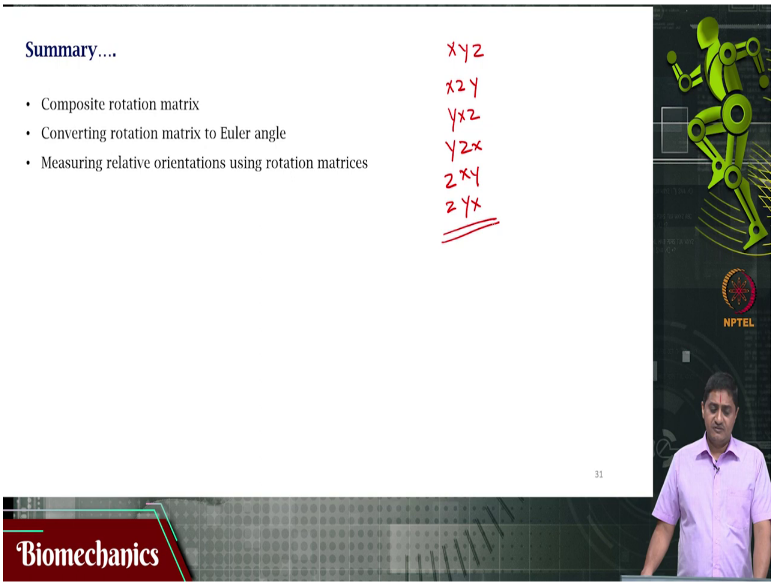

Something to keep in mind is the order of the Euler angles the sequence matters. If I have 3 principal access what are the various ways in which the order can go let us write those out I can either have x y z or a can have x z y or I can have y first y x z or I can have y z x or I can have z x y or I can have z y x. If the principal axis is not repeated right then these are the various ways orders in which I can find the Euler angles.

These are the various Euler angles possible if the orders are not repeated if the axes are not repeated I am going to have these six possibilities. I can have other cases where the one of the principal axis repeats then it is going to be a complicated problem to deal with which we will not discuss as part of this video. So, we will stop here in this video we looked at composite rotation Matrix and how to convert rotation Matrix to Euler angles.

And how we can use rotation Matrix to find relative orientations and we also just begun our discussion or just touched upon one example of the case of Gimbal locks with this we come to the end of this video. Thank you very much for your attention.