Optical Spectroscopy and Microscopy

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Lecture – 16
Fundamentals of Optical Measurement and Instrumentation

So what we have seen so far is that we have developed a quantum mechanical framework for understanding the phenomenon of light matter interaction and then we obtained an expression for the rate of transitions happening in the matter upon shining light okay. Now in this process, we discovered that the rate of transition for a process in the process of light is in a way is exactly same when it goes from lower energy to a higher energy or a higher energy to lower energy.

In other words the order of the energy is immaterial, but rather what matters is which states actually the transition occurs in between and then thereby giving rise to the phenomena of stimulated emission, we talked about it, and then given this fact that the light can actually cause both absorption and emission equally likely, then Einstein argued through phenomenological rate equations and then equating it to the population distribution of Maxwell's at thermal equilibrium.

He argued that there has to be spontaneous emission and he showed that if you do account for that only, then you would be able to satisfy this Maxwell-Boltzmann distribution law. Thereby, we put in those principles, wrote down the rate equations for describing the population dynamics between these 2 states and then obtained an expression for the spontaneous emission rate and I told that we can actually experimentally measure these emission rates by measuring the fluorescence as a function of time in laboratory, but in doing so I told that the problem with this whole description.

The phenomenological description of Einstein is that he has to invoke in a sort of a quasi arbitrary manner that there has to be a spontaneous emission, it is kind of telling that it has to come without the framework and without there is no formal description of how this can actually come about. Now in order to see that it comes out naturally, I told that we need to treat the light by itself in a quantized manner, alright, as I was alluding to right in the beginning and this lecture is about that.

How do we go about treating light in a quantized manner and then use that description in light of our light matter interaction treatment that we have done with using time-dependent perturbation theory or TDPT and then show that spontaneous emission does not have to be arbitrarily put in and it comes out very naturally in this description, alright. So let us step in and see what this description is all about. The first thing that we realize is that we are going to have to describe light or for that matter even the chromophore of the atomic of the molecular system per se in terms of entities, right.

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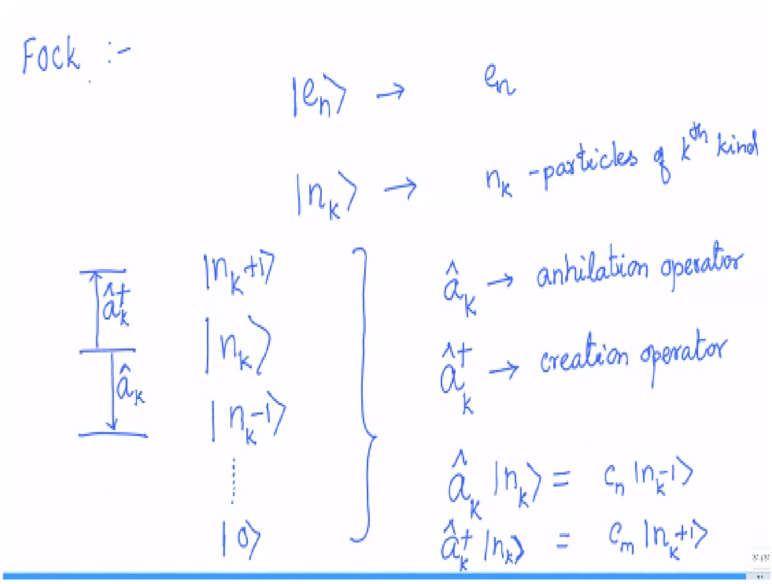

So in order to do this, the formal description was put forth by a Russian scientist called Fock. So what he said was that just the way you would describe an eigenket as a state vector corresponding to an atomic or an energy state in general with energy eigenvalue of en if we could describe states, these are called the number states where represented by let us say nk, now this corresponds to a state which holds nk particles, n particles of k identity meaning. So here what we are talking about n particles of kth kind that is what we are talking about in terms of nk, particles of kth kind.

So if you have to order these eigenkets are the Fock's states then they would be nk corresponding to nk particles, the nice thing about I mean since they are actually describing the number of particles, you would expect it should be possible to have nk-1 as well as nk+1 right. It is one particle more and one particle less, clearly you can go till a point where there are only 0, nk = 0 particles of kth kind, right.

So now in this description, what we are trying to order is really the particle numbers and we are kind of arranging or sorting these states in terms of their particle numbers and then a corresponding operator if we have to think of would be to be able to tell you how many number of particles are there in a given state. It turns out in order for us to write down an expression or write down an operator for that we need to understand two other operators and I am going to just state here an operator ak cap just to denote that it is an operator.

We will call it as annihilation operator and Hermitian adjoint of this operator we call it as ak dagger or we call this creation operator, right. Now, what they do? They have the properties that when ak operates on a state nk generates or moves the system from nk to nk-1, it annihilates a particle okay and these are all normalized basis kets, so they would spit out some constant, in here let us call it as cn, the constant is Cn, alright. Similarly we can write down an expression for ak dagger operating on nk which is given by Cm nk, sorry this is not k-1 here it is actually is k-1, nk+1.

It increases the particle count from nk to nk+1 okay, so where k is the k kinds of particles okay. Now so in the diagram that where we have ordered these states in terms of increasing particle number, then this would correspond to this operation, the operation of ak operating on nk would correspond to this process, a process where the system moves from nk to nk-1, this is again carried out by this the ak cap while on the other hand nk moving to nk+1 is done by start to be described by ak dagger right.

This is the property of the operators by themselves okay. So we start with defining the operators that have these properties. So now, it is convenient to think of that right because if you are talking in terms of energy eigenkets, then the measurement that we are interested in physical measurement is about knowing what the energy is. In this case, we are talking about Fock states or the number states, then what we are actually interested in is manipulating these numbers are extracting these numbers right.

Just the way you want to increase the energy or decrease the energy by any means of transitions, here the transition would be corresponding to creation operator or an annihilation operator making the system go from nk to nk+1 increase its number or nk to nk-1 to decrease its number, alright. So now once you define this, then we have an very interesting property again that I am going to state without necessarily proving you.

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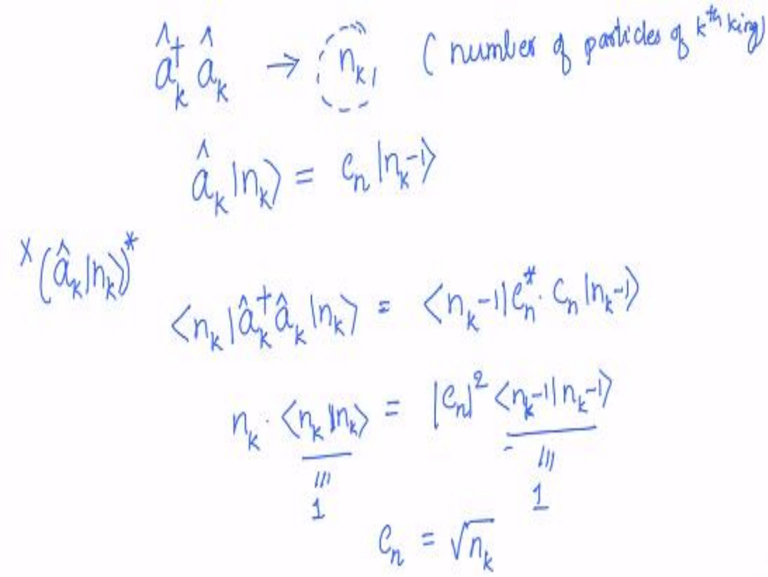

We know that ak cap takes the state from nk to one state below nk-1. Now if I were to think of a sequential operation right where I take the state one step down and then operate on with an a dagger, I mean first to annihilate and then bring it back, It turns out this sequence of operation corresponds to a number which is basically nk itself okay. Again, I am not going to prove this, your perhaps towards the end of the course as in an appendix video we can actually investigate this later, for our discussion what we need to know is we need to know this as a property of this operators ak dagger and ak.

So when ak dagger ak corresponds to a simple number nk period there is very simple operation. So clearly this is assuming that we are actually operating on an eigenket nk okay. Let us go back this. So I repeat, the dagger ak corresponds to nk so number of particles of kth king in a state n. So it corresponds to just this number okay. Now, it is interesting then we can actually look at what the values of the normalization constant Cn and Cm themselves are, right. So how do we get them?

So let us write down the annihilation operator first. So what this corresponds to is ak dag ak operating on state nk giving you Cn on nk-1. Now we could left multiply this both the sides by the complex conjugate of ak dagger nk itself and since the ak dagger is a Hermitian adjoint of ak, now if we were to left multiply with the complex conjugate of nk, right, the star represents a complex conjugate. So then, what we will have is the bra vector nk ak dagger ak nk equivalent to see we have taken the complex conjugate of this right and we know this is equivalent to Cn nk-1.

So what we could write is that nk-1 Cn the complex conjugate Cn nk-1. Now ak dagger so cap, cap here, now ak dagger ak is when operates on nk is going to give us the number nk itself. So this is equivalent to just a scalar, so we can pull it out as nk a simple number times nk equals Cn modulus square nk-1 nk-1. Remember these kets are the normalized basis kets of the Fock states. So this equates to 1 and so is this, thereby see we have an expression now for Cn itself as under root nk. Similarly, we can actually proceed forward to find out the value of Cm.

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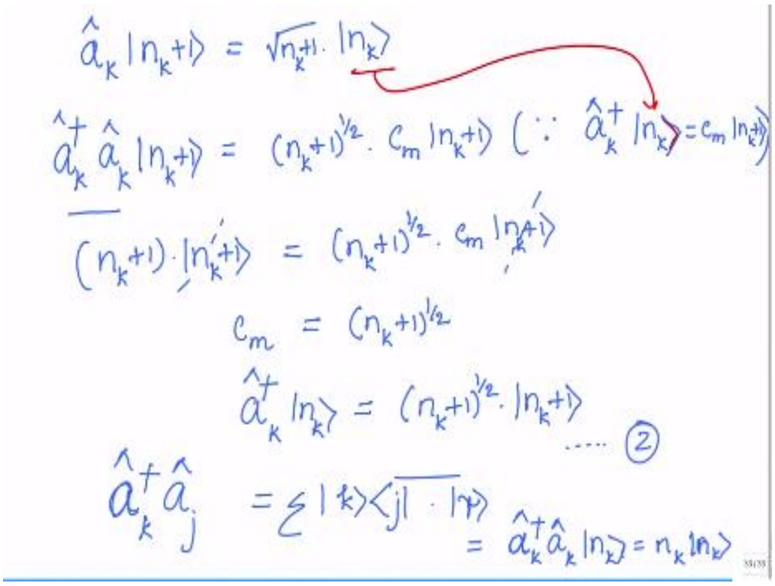

We will start out by writing down again the process of annihilation. The idea here is we want to proceed forward and find out an expression for the normalization constant Cm of the creation vector, not annihilation vector, creation vector, so but the trick that we are going to use is still we are going to use a small trick and we are still going to start with annihilation vector, it is annihilation operation itself ak operating on nk and we know that corresponds to under root nk times nk-1, sorry nk itself right. The expression we have written before.

Once again I repeat, so just the way we arrived at an expression for Cn the normalization constant for the annihilation operator, because of this now we would be able to write this whole process right, ak dagger nk as under root nk nk-1, alright. Now, I want to go ahead and obtain in a similar way the expression for Cm the normalization constant for the creation vector okay. So this Cm, so the procedure is pretty much the same except now here I am going to use a small little trick.

The trick is I am going to start up with the annihilation operator again except now I am going to start from a state nk+1 okay. Now that is going to give me from my previous expression nk+1 as my normalization constant nk. At this point, I am going to come and then operate with the creation operator ak dagger. Now we can write this as, I am going to just to avoid writing the roots and then write this half times Cm nk+1, right. So this is simply because operating on a dagger and both sides would correspond to me operating on ak dagger on nk from my previous line right, from this line.

So, this is coming here and now this there is nothing but Cm times nk+1 right. Now this result is from our definition of the a dagger operator itself, okay. So if you go 2 pages back, we have defined this, right here, ak dagger operating on a state nk alright. The ak dagger operating on state nk gives us Cm times nk+1 that is the definition of our creation operator, right. So, now what we can do is that we can make use of that and then write down the expression as the following. It is very highly convenient.

So now what we can say is that this ak dagger ak right is our number operator okay. So number operator spitting out the number corresponding to the state that it is actually, I mean it is not operator, it is going to give you a number that actually corresponds to the state nk+1 okay and that is from our definition. So that is going to be nk+1 times nk+1 giving you nk here okay. Now what we can do is that we could rearrange this whole term for Cm, this gets cancelled giving an expression for f cm as nk+1 to the power half or in other words we can write now safely a dagger as nk+1 to the power half times.

These are important relations, so I would like you to pay attention to this expressions, it is also good to number them and box them. So, I think we were looking at equation number 10, I remember previously, anyway, so let us call this as our equation 1 for today, so 1 and let us call this as equation number 2. So this follows directly from our definition of ak and ak dagger. In general, what you can actually think of this as the number operator, right, in general ak and aj as an operator that can be expanded in a basis of k and j okay.

These are called projection operators. So if you think of this expression operating on some state okay, some superposition state, what this does is that it is looking for the component of the component of the superposition states psi or chi along the jth eigenstate and then multiplies that by the k okay. So this as you will see when we were to look for a proof for the number operator becomes pretty simple when the k becomes equal to that of j assumes the form of a number operator, right.

Basically of nk giving you the nk itself and nk basis should not have changed because what all we have done is we operated upon the nk state, brought it down by one number and then increased it by another number. So, we should go back to the nk, but then because of this whole operation what we have is that we know now how many number of entities are present in this state nk okay. So, now that is very convenient and why are we actually looking at numbers and how is it related to the light matter interaction that we said that we are actually going to talk about that has to do with.

How we are going to use this process itself to explain both this entities, the light as well as the matter in terms of their numbers and then saying how the number of these entities in different states or in different modes change as their interaction proceeds. So the way we will do that is by using a small description of this entire process, the light as well as the matter in terms of the states, we will write down and then write an interaction Hamiltonian okay. So this is something that I am going to again state.

Once we do that, then we will see how it is quite natural in this description that we can fit in various different processes, I mean not just fit in, the various different processes emerges from this description and thanks to the simple description of this, the spontaneous emission comes out very naturally. We will do that in the next lecture. Thank you.