

**Optical Spectroscopy and Microscopy : Fundamentals of Optical Measurements
and Instrumentation**
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Lecture - 31

Hello and welcome to the Optical Spectroscopy and Microscopy course. So far what we have seen is are the different kinds of lasers, practical lasers and the laser outputs that are coming out from there and I said in the previous lecture that we will be talking about different kind of lasers, the pulse lasers to be specific.

And then before that, we were actually talking about some of the properties, special properties of the laser light itself. What we will do is to in order to better understand the generation of the pulse lasers and pulses from the lasers and how they work, the working principle of the pulse lasers, we would take a look into a little bit more properties of this light radiation, the special kind of radiation laser, laser light radiation.

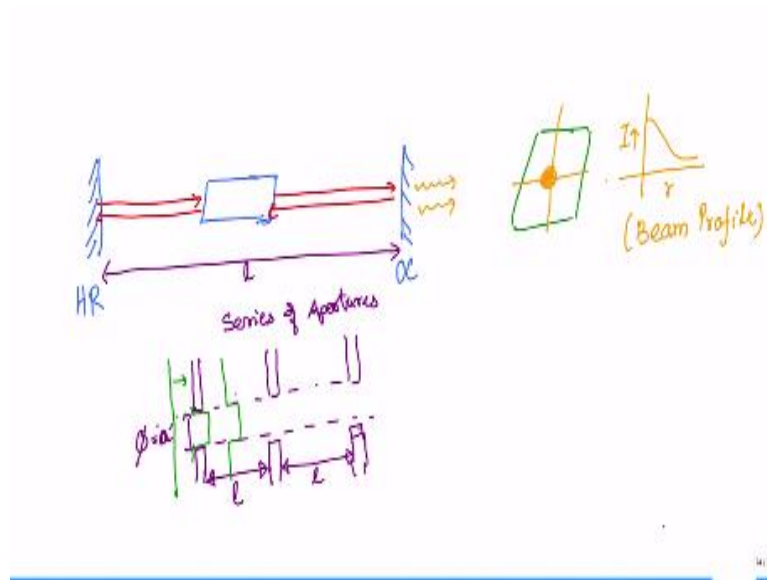
And particularly, we will be talking about something called as the modes. We would see what are these modes, how are they originating? What is the origin and then how they can help us understand how to generate laser pulses which can be very brief in time. So let us go into the course. So we saw in the previous lecture that the two defining properties of the lasers are the coherence and spectral brightness.

So now keeping that in mind also we saw some of the designs of the laser cavity and so on. Keeping that in mind now if I ask what happens to the laser light when it goes when it what happens to the stimulated emitted light when it goes into multiple round trips inside the laser cavity. And what kind of a bearing does it how in terms of the spatial distribution of the intensity.

Does the fact that it keeps going round and round and extracting many more similar to the emitted photons, right? That is the whole purpose of the laser, light amplification by stimulated emission. So in order to extract more and generate more of the stimulated emitted photons, we had to send them back into the cavity again and again.

So when we do that, is it going to affect or is it going to determine in some way the spatial distribution of these light photons when the light comes out of the laser? Clearly the answer is yes. If not, we would not be talking about this here. Now in order to understand how exactly does this round trips so to speak affect or could affect the spatial distribution let us take a look at the cavity itself, right.

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So we know that the laser cavity here is given by two mirrors. One of them is partial mirror and the other one is complete reflector. We call complete reflector as a high reflector and the partial meaning about 90%, 90 to 94% we call it as output coupler. And so the light the laser light comes out through the output coupler.

Now what we want to ask is, since we are sending this light back and forth in the cavity right, what happens to the intensity distribution across space? So I am going to actually draw a plane. It is equivalent to me putting in a small piece of paper across the laser beam, okay. So okay. So when we do that, what we you would see first thing is that you would see that it has a definite shape.

And if we were to plot this as a function of r I mean because it is a cross section we are going to we I mean we have two variables x and y or r and θ . And you would see it is symmetric, circularly symmetric mostly. And it has a typical distribution if you plot r as a function of intensity as a function of r it has a profile something like this. So we call this as the beam profile, okay.

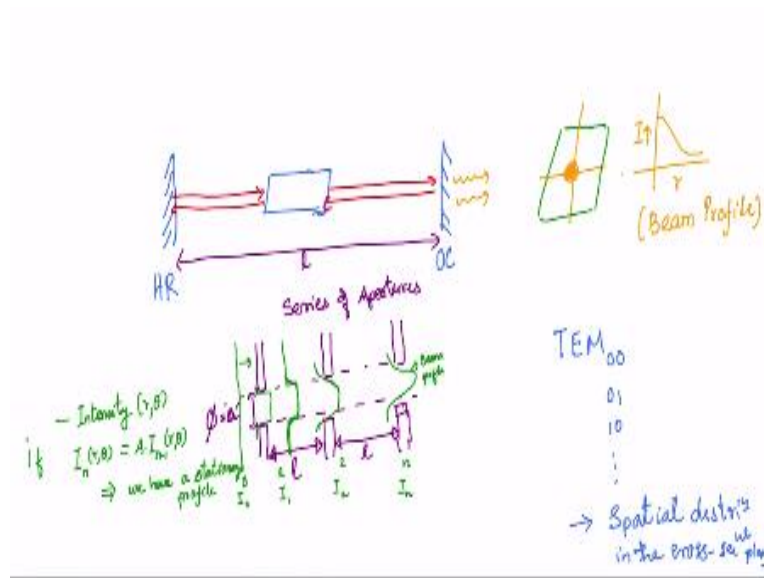
Now the question that we are asking is that can we tell anything about the beam profile knowing that the light is going to travel through this 2 meters repeatedly. First thing you notice is that because the light is traveling through back and forth inside this cavity what you end up having is that you end up having a behavior of light that is that can be approximated as a light that is going through multiple apertures kept at distance okay.

So you can think of multiple apertures and each of them kept at a distance corresponding to the length of the cavity, okay. So okay. So what is I mean where is this aperture coming from okay. This is basically these are series of apertures, okay. So each one of them is separated by this 1 the round trip distance or the cavity length you can think of and we can think of this apertures having some diameter equals a okay.

These are of the same size. This way of thinking arises from the fact that if you take a plane wave and then let us say the plane wave were to reflect from finite mirror, then we know that the plane wave does not have any defined it has an infinite extent right. However, when you subject the plane view to fall on a mirror that is of a finite extent and then take a reflection, now this mirror acts like a finite aperture.

And the reflected light or the reflected plane wave that is coming out from this coming out because of the mirror, is going to be diffraction limited because it is cut off at the cut off at the aperture edges. So it is it tends to it starts to diffract out. So if you if I were to represent this in terms of a plane wave that is extended over this big region, the light that is coming out from here would be just this, right.

All this light will be blocked. And what you will see is the emergence of this light, right. So when the light passes through, when it passes through here, it is going to be looking like that. It would not be so sharply defined. The edges would not be so sharp.
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Rather, what you will see is that the edges, if you take a closer look, it would not be so sharp, but rather as they progress forward, right. As they progress forward, you will start seeing because of the diffraction. Now if this beam even if it were to be exactly like this, when it makes one complete trip and then comes back and it is going to meet the same place, same mirror, same aperture, same limiting aperture.

So now what is going to happen is that gets clipped even further. So what you are going to have is a beam shaped something like this and eventually if it goes through enough number of trips, you would you would probably get a profile. This is in x direction or r direction so to speak where the beam profile now is stable, okay. Means what?

What we are representing in the green, this green line represents the intensity as a function of r, theta. Now I am going to tell that this beam profile has reached a stationarity or stable beam profile when if I were to represent that $I(r, \theta)$ okay and after n such passes is represented as the I of n r, theta. Now I am going to take a difference or if I can write the I of n r, theta as some amplitude modulated version of $n - 1$ r, theta.

Then what I am going to if this condition is satisfied, then what we imply is that we have a stationary profile. Remember the intensity, the total intensity could be different right because that is what is captured in this A here. But then the how it is distributed

across space that is represented by I of n r , θ . Now that we want it to be equal. That we do not want it to change.

But when you compare from here to here, I mean, let us call this as one, or let us say, this is 0 and 1 and so on and so forth. And this is my n . So if you were to compare between 0 and 1, clearly it is undergoing a change. It is, I mean particularly I mean, we should have called this as 0, so let us rename them.

So if we call this as 0 and this shape, the thing that comes out of from here, we call that as 1 and then the 1 that comes out emerges from here, let us call it as 2. So and then the guy that emerges from the n th aperture we call it as n , okay. This the intensity distribution here would be given by, I of 0. This would be I of 1 and so on and so forth.

What we see is that as we go from I of 0 to I of 1 and normalize to the total number of photons that are present inside in that beam profile, you will see that the beam profiles would be very different. I mean, they do not, they would not overlap. There will be differences. However, when you have when you see that the if it keep on passing it again and again and again, because of the diffraction, right.

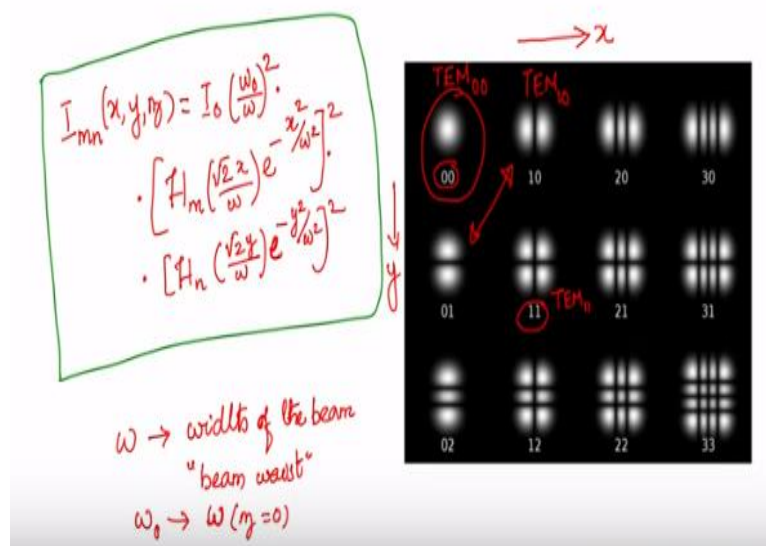
So it is a beam that is clipped off towards the edges and so it is actually spreads a little bit around the edges. So I am not going to be we are not going to be dealing with the modeling of this diffraction limited beam propagation and then getting the profiles. If you are interested, I would refer you to looking at Kirchhoff integrals, diffraction integrals and there are very nice way of capturing this diffraction limited spots on how they I mean that these diffraction features.

So but the point is that you would see that there will be spilling over of this light around these edges because they cannot maintain it that sharp edges over a long distances. And then when you send that beam to again into this aperture, over and over again it will reach stationary stage, okay. And then that stationary state we equate, we define it by in this following way. I of n of r t equals A of I of n minus r of t , okay.

So the intensity might change. Okay, that is A. But then the profile if normalized to that intensity and overlap them, they will overlap exactly the same way. Now this is the condition we like to impose and then go ahead and solve Maxwell's equation for propagation of light inside the cavity because it is actually the light that is traveling through the inside the cavity.

And we do that then the expression that we get okay, the expression we get is this, okay.

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The expression we get is as follows. I of mn we will worry about the mn in a bit little bit. I is given by x, y. These are the I mean you can I told you I have written it in terms of the Cartesian coordinates x, y and z is the direction of the propagation and x, y, z is given by I naught the photon content the total intensity at 0.00 origin. Omega naught by omega. Omega is basically the width of the beam, okay.

So we can write omega is basically is called the beam width or width of beam where the intensity has fallen down by 1 over e, okay. So that is how we define the beam width. It is also called as the beam based waist. And omega naught is the beam waist at z equal 0. So we write it as basically omega at z = 0. So now we put in I mean and then you have these two terms the H which is called the Hermite polynomial.

It is a function, mathematical function, the polynomial function and of order m and Hermite polynomial of order n okay. So order m in x axis and order n in Y axis to be

general. So the polynomials have different orders and then the 0th order, okay. So these different orders will have different distributions of intensity. And some of them I have actually taken it from the net and then have given it here for your reference.

What you can see here is that these are all these points here, right alright. And then that is the beam profile for the mode where $m = 0$ $n = 0$. It is a symmetric right in x and y there is the same Hermite polynomials. And you see that Hermite polynomial it is basically e to the power $-x^2$ by ω^2 . It is pretty similar to I mean, it is a Gaussian, right? If you put it to a 0th order, it is actually a Gaussian.

Now what you see is that since it is a symmetric in both x and y and then it has this I naught at the center, what you see is as you go radially out in all this theta direction you have a very nice smooth variation of the intensity dropping down to some point where it is where you can say that the intensity has fallen down by 1 over e^2 and then you say that, that is the intensity of the that is the beam width of the beam.

The second order or we can also call it as a second mode of the laser beam is given by you can propagate or you can go travel in both the x direction as well as in y . So and what you see here is that you have Hermite I mean I of this mode since it is a both m and n are equal to 0 , you also call this as TEM 00 mode, okay Transverse Electromagnetic mode 00.

Like that, you have the next mode where one of them is set to 1 and the other is 0 . So and then you have the corresponding mode. These two are corresponding nodes except and the different axis right. So 01 so that is why you see it being rotated. While both of them being set to 1 you have a laser, the distribution of the laser intensity pretty much like double I mean as shown here, okay.

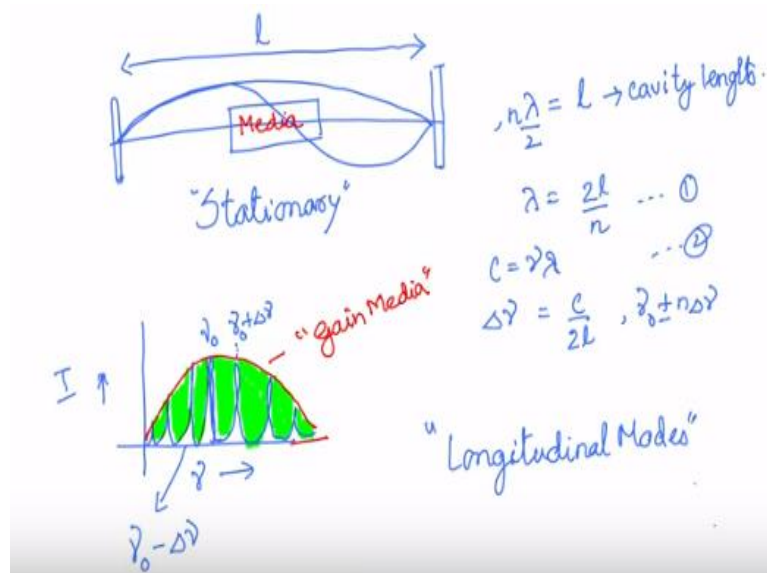
Double dumbbell if you like a crossed double dumbbell if you like and so on. We can actually go ahead and plot out the intensities, but usually the lasers that we get are tuned and then adjusted to have the mode to be TEM 00, okay. So there are quite a good reasons why you want to have this mode to be TEM 00.

Mainly because the center point, I mean the focus is pretty nicely I mean at the center, you have a very well I mean high intensity and then the focusing properties are very nicely different for it which is ideal to have. So you tune your lasers to be of this transverse electromagnetic mode 00. They are transverse because you are taking a transverse cross section, okay.

So now let us go back and what we have said is that okay this there are different modes and these are we call it as transverse modes okay 01, 10 so on and so forth. They talk to you about the spatial distribution in the distribution or in the cross sectional plane. Now what you want is that if you actually look into the laser cavity itself and then ask okay, now are there any other modes, okay.

This is one of the modes I mean this mode here represents different kinds of different solutions for the laser light generations keeping in mind one of the variance. In here what we are talking about the spatial distribution of the intensity across the direction of propagation.

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So you can actually if you think carefully there are certain other restrictions that the laser cavity imposes on the kind of laser light that can exist inside that laser cavity, okay. So this restriction comes from the fact that when you are actually making this laser light going reflected back and forth, what you are actually doing is that you are making them constructively I mean destructively interfere at the ends.

While constructively interfere every I mean at the places wherever you can, wherever it can. But the key point is that at the ends of the mirrors it has to be known so that you the laser, I mean the laser light can actually form what do you call it as stationary wave. Now this is not the only mode that for the for stationarity. It can actually, it can be a multiple of lambda by 2.

So you can actually have it like this. So all frequencies okay that or all lambdas. So all lambdas that correspond to this relationship, which is $n \lambda$ by 2 l should be equal to $n \lambda$ by 2 would be supported by this cavity and not any other wavelength right? So now this is serious restrictions, okay? So if you ask for what is the lambda here, so lambda is 2l by n sorry 2l by okay.

And this is the cavity length. It is useful to convert this in terms of the frequency of the light that we are using. We know that $C = \nu \lambda$. So what we are after is $\Delta \nu$ where the, this is the gap between two frequencies that can actually survive in this and if you actually work out you will see that using these two equations it is so you can very well easily see it is C by 2l.

Or to say that only given the length of the cavity only these frequencies okay I mean the frequency about the center, so there is a central frequency ν_0 and then the next frequency will be I mean after that either ν_0 plus or minus $n \Delta \nu$, okay. Only these frequencies will be able to oscillate inside this cavity. Any other frequency, you will not be able to form a stationary wave.

As a result what is going to happen is that the cavity will not be able to efficiently support the existence of such waves okay. So why is that? If you actually know this is about the cavity itself and then what can exist in a stationary mode. But if you introduce the laser medium here right, lasing medium. So the lasing medium has its spectra right, we know that.

So we can look into that spectra of lambda or nu for example. That is why we are writing nu. If you look at the spectra nu as a function and then the intensity this is basically the fluorescent spectra of the media right? So now that would show

something like this, okay, it is a continuous spectrum and there is no surprise here. So it is a pretty nice looking smooth curve.

So now this is, this media is also called as a gain media in the literature. This is primarily because it is by exciting this or taking this media into the to the excited state and then you are storing that energy and then you are extracting those molecules from the excited state, I mean you are stimulating those molecules in the excited state to come down to the ground state in an optical means.

Thereby you are extracting the energy that is stored in terms of an optical energy. So you have an optical gain okay. And that is why and that is mediated by this substance here. So we call it as gain media. And the gain media has a spectrum something like this. However, if in this spectrum, if you include that it is has to be inside this cavity.

Then what you are going to have is that you are going to have not all these frequencies all this intermittent frequencies, but you would have a comb structure coming in, okay with an uniform spacing of in both the direction okay. Both the direction around the central maximum μ_0 , you are going to this is our first mode. This would be given by $\mu_0 + \Delta\nu$. $\Delta\nu$ is $c/2l$.

And this would be, this would be given by $\mu_0 - \Delta\nu$. So thus suddenly if you take this medium and put inside the cavity, suddenly you see that the in-between frequencies right the in-between frequencies that are existing here, right those go missing because they cannot sustain they cannot exist inside this, sustain the oscillations. As a result it is these modes that survive in the cavity.

And what you see is a comb structure, comb kind of emission, when you plot it in terms of μ and you these modes the modes that are capable of existing inside the cavity because of this cavity length and the gain media limitation, we call them as longitudinal modes. And we will in the next class we will see the usefulness of this longitudinal modes, analysis of this longitudinal modes to what use they have in generating pulsed laser lights. Okay. Thank you.