Chemical Reaction Engineering II
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Lecture - 31
Measurement of residence time distribution

Friends, in the last lecture, we looked at what is residence time distribution and what are non ideal reactors. And we initiated discussion on; what are the experimental methods to measure the non ideal situation inside a reactor. So, let us look at little bit more into it. So, the method to do that is basically to inject a tracer.

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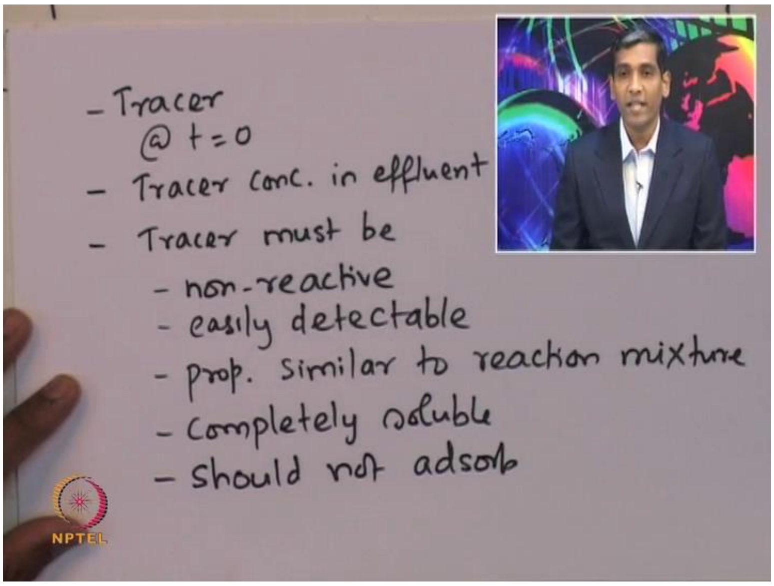

We need to inject a tracer into the reactor. And let us say at time t equal to 0 a certain concentration of tracer is actually inserted in injected into the reactor. And then, the effluent concentration the tracer concentration in the effluent is measured. So, now if we measure the tracer concentration in the effluent, then this gives an idea about the non ideal behavior of the reactor.

Now in order for this to conduct such an experiment and to measure the tracer concentration, the tracer has to possess a certain properties. And the tracer must be the tracer must be non reactive, otherwise the tracer is not going to reflect the true non ideal behavior, because in some amount of tracer which is actually injected into the reactor, is now going to be consumed in some reaction. And it should be easily detectable. The experimental methods that are available the measurement techniques that are available, should be sensitive enough to detect the tracer concentration, even small amounts of tracer concentration in the effluent stream.

Then, the properties of the effluent must be similar to that of the reaction mixture. So, the properties must be similar to that of the reaction mixture, otherwise it reflect the different behavior. So, in order to capture the non ideal behavior of the reactor, the properties must be similar, for example, viscosity etcetera. And then, it has to be completely soluble in the reaction mixture. And it should not adsorb onto adsorb on to the reactor walls; otherwise some of the tracer is actually consumed. So, it does not reflect the non ideal behavior completely. So, now the question is how do we measure the residence time distribution. So, there are 2 injection methods.

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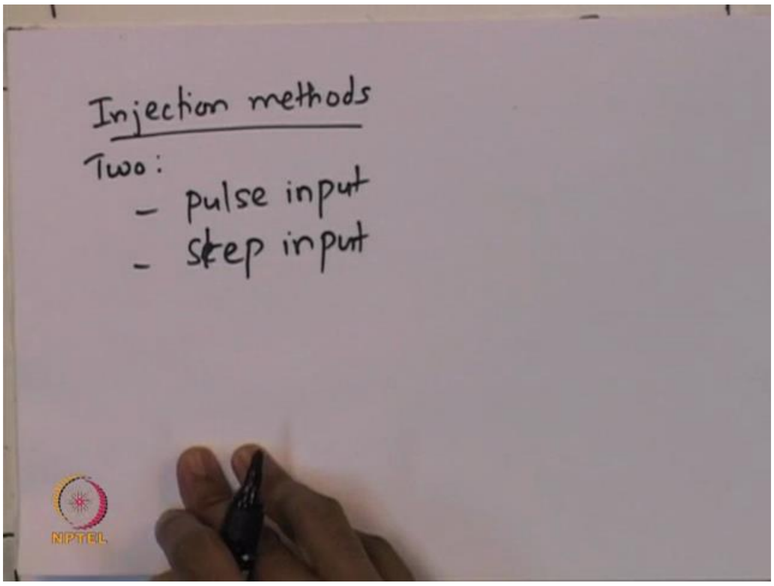

There are 2 methods which are commonly used, which is in order to detect the residence time distribution, which is a reflection of the non ideal behavior of the reactor. And the 2 methods are: the pulse input method and the step input method. These are the 2 methods that are used experimentally to measure the residence time distribution. So, let us look at little bit more deeply into the pulse input method and later we look into the step input method, as to what these 2 different methods are and what are the purpose of these 2 methods.

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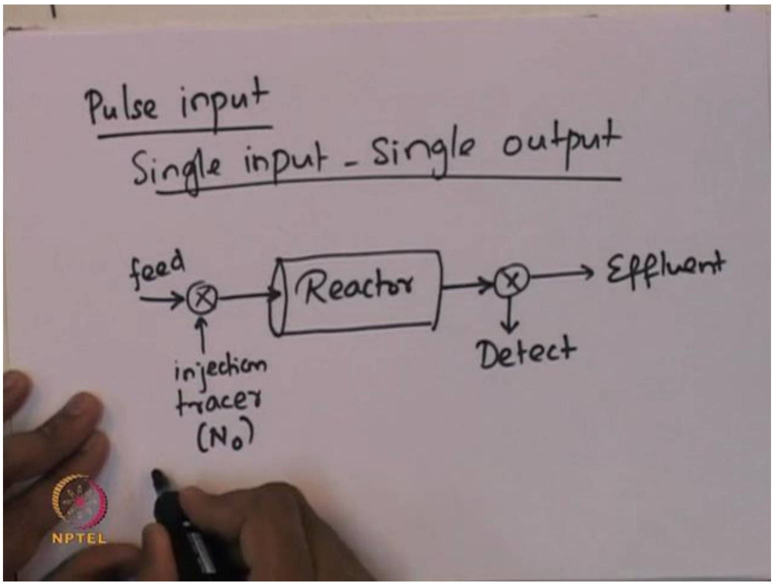

So, let us look at the pulse input method. So, now suppose we consider a single input single output system, that is, suppose if there is a reactor. And then there is a single input stream to the reactor and the single output stream to from the reactor. And suppose, if there is a feed stream which is present here and let us say that we put a, an inject we inject the tracer along with the feed, which actually goes in to the reactor. And may be the total amount or total concentration of the tracer is N naught and it has to be a pulse tracer. So, we will describe in a short while what is meant by a pulse tracer injection.

Then, suppose there is an effluent stream and then we can actually detect the concentration of the tracer in the effluent. So, this is a reactor and it has a single input and a single output system.

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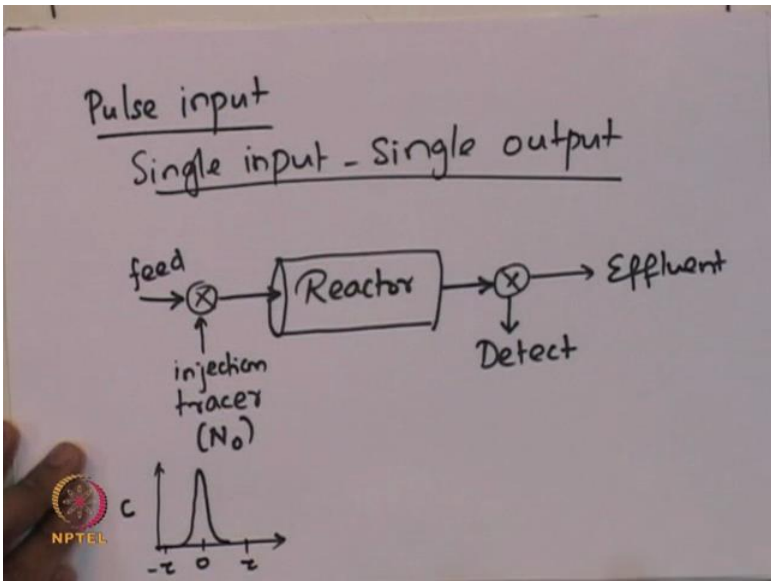

So now, the tracer that is injected has to be a pulse at a certain time. So, let us say at time t equal to 0, we inject a pulse of tracer, that is, a certain concentration of the tracer, in as short time as possible it is actually injected into the reactor. So, that is what is called as a pulse input. So, suppose if this is the concentration of the tracer and this is minus tau sometime before and sometime later. So, there is sometime before the injection point, there is no tracer in the reactor and sometime later also there is no tracer in the feed. So, you just give a short pulse and then and then leave it.

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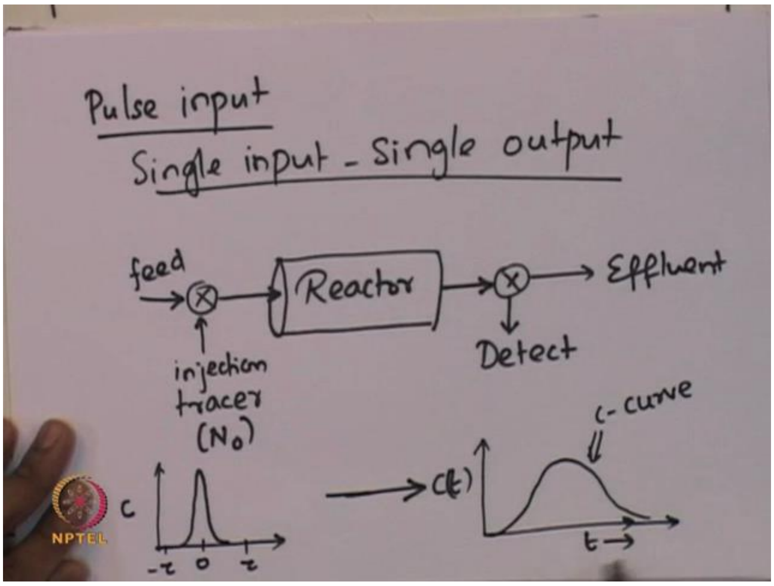

Now, once we do this, if we monitor the concentration of the tracer in the effluent stream of the reactor then, the kind of concentration profile that 1 will expect is basically it looks like this. So, this is time and this is the concentration of the tracer as a function of time. So, this kind of a curve is what is called as a c curve in residence time literature. So, the concentration of the tracer as a function of time is what is called as the c curve. So, now suppose if we know suppose if, such is a system question is how do we find this concentration curve? So, how do we find this?

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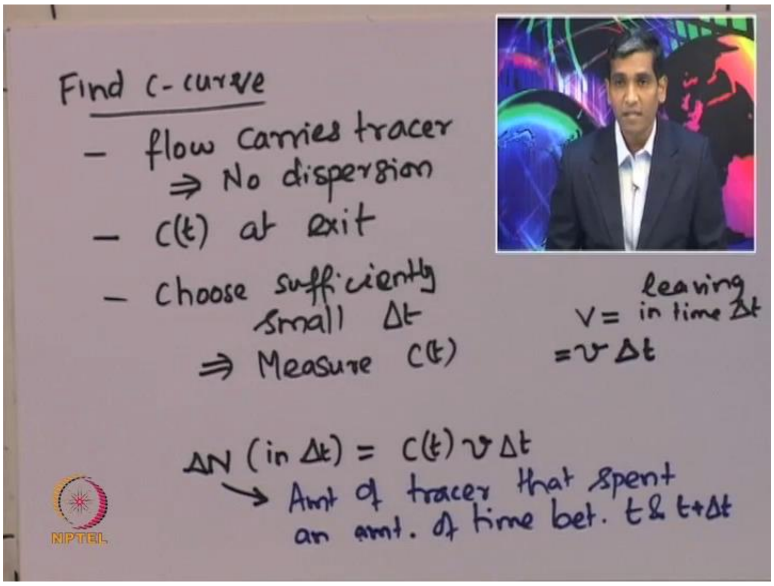

We need to find the c curve. And how do we find the c curve. So, we have to make some assumptions. So, if we assume that only flow fluid flow through the reactor this is the only p only 1 which is actually carrying the tracer. And if we assume which means that, there is no dispersion between the point of injection and the entrance of the reactor. And later in 1 of the lectures, we will actually account for the presence of dispersion in the reactor.

Suppose if we assume that there is no dispersion. And then we measure the concentration of this tracer at the exit, that is, the effluent stream. Now, the trick to measure the concentration or to construct a c curve, is basically to choose a very small time step delta t and the delta t should be chosen such that, the concentration of the tracer in the effluent stream within this delta t does not change significantly. And then, measure the concentration of the tracer at every delta t which is very small.

So, measure the concentration. So, choose sufficiently small delta t and then measure the concentration C of t. Now, once we measure this concentration and suppose if v is the volumetric flow rate of the effluent stream and the volume of fluid that actually is leaving the reactor in the time delta t volume in time delta t. So, leaving in time delta t for volume of the fluid leaving in time delta t should be equal to the volumetric flow rate at the effluent stream multiplied by the corresponding delta t.

So, therefore, the amount of tracer that is actually leaving the effluent stream in time delta t that is given by the concentration of the tracer in the effluent stream at that point, multiplied by the volumetric flow rate v multiplied by delta t. So, that gives the total amount of tracer that actually leaves in this time delta t. Now what is this mean? So, this is the amount of material; this is basically the amount of material, amount of tracer that spent an amount of time between t and t plus delta t.

So, what this means is that, this is the amount of tracer which has actually reside spent the amount of time between t and t plus delta t. So, that is the total amount of tracer in that times, that has actually spent that much time inside the reactor. So, now if N naught which is the total injected as we observed in the schematic before.

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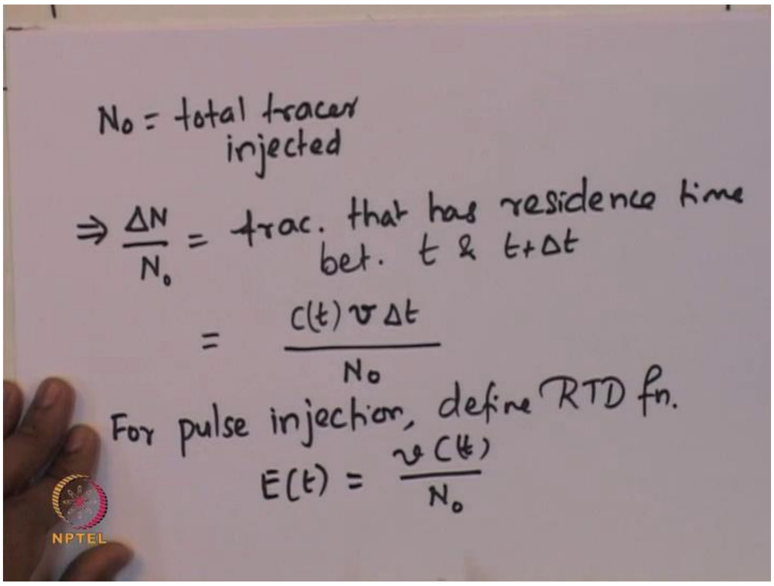

If N naught is the total injected total tracer. So, that is the total amount of tracer which is actually injected in to the vessel. Then, delta N by N naught is nothing but the fraction that has residence time. So remember, residence time is the time spent by the material inside the reactor, whose residence time is somewhere between t and t plus delta t. So, the time that certain amount of material has spent inside the reactor, if that is between t and t plus delta t and that will be the fraction of that will be the material which is going to come out of the vessel, in that time delta t.

And therefore, the fraction that has residence time between t and t plus delta t is simply given by delta N divided by N naught. And that is equal to c which is the concentration of the tracer in the effluent multiplied by the volumetric flow rate of the effluent stream into delta t, which is this whole numerator is nothing, but delta N divided by N naught.

Now suppose, for pulse injection, if we define an RTD function E of t which is basically given by v into C of t divided by N naught. So, that is the definition. So, that is the concentration of the tracer in the effluent stream multiplied by the corresponding volumetric flow rate divided by N naught. So, that is the amount of that, is the if we define that as the RTD function, then we can write the fraction that has the residence time between t and delta t as delta N by N naught.

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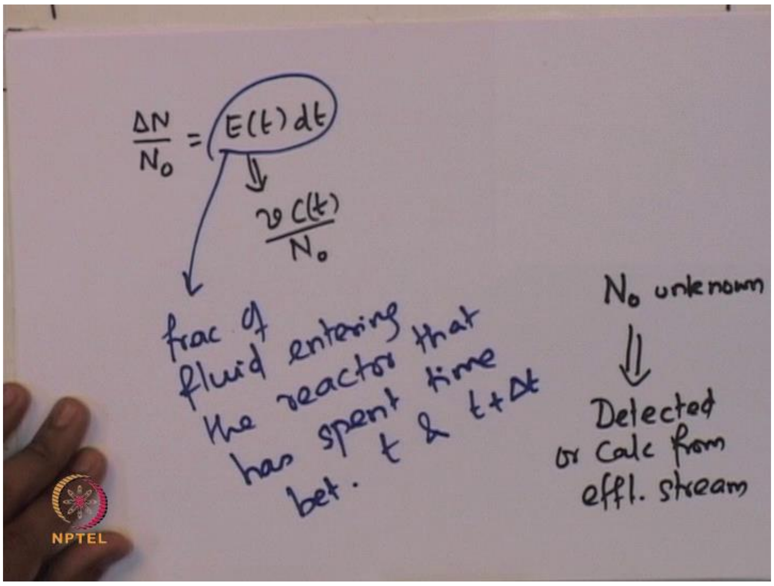

That is equal to E of t into dt. So, what is this E of t represent? E of t is the RTD function which is v into C t divided by N naught. What it represents is; it basically describes how much time different fluids elements have actually spent in the reactor. So, this quantity E of t which is the residence time distribution function, it quantifies or describes how much time different fluid elements, have actually spent or have resided inside the reactor before they leave the reactor.

So, that is an important property of the reactor, which is actually captured in this residence time distribution function. So, what can we do with this? So, if E of t is basically describes the amount of material that is actually residing in the reactor, then E of dt is basically, so this quantity; it represents the fraction of fluid entering the reactor entering the reactor that has spent time between t and t plus delta t. So, it is very important to understand this function E of t times dt, that is the fraction of the fluid that is basically delta N by N naught, which is basically the fraction of fluid total fluid that enters the reactor and has spent time between t and t plus delta t.

So, suppose if the total amount of tracer that is actually fed into the vessel a N naught, if that is not known. Suppose if it is unknown, which may be the case at many situations. If that is unknown, then the amount of N naught can actually detected from the effluent stream. It can be detected or calculated from the effluent stream concentration of the tracer in the effluent stream. So, how do we do this?

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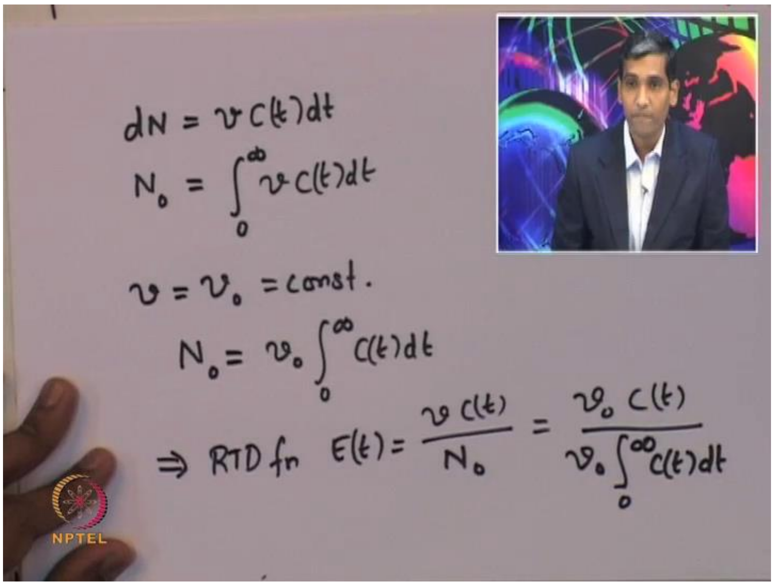

So, dN is essentially given by the volumetric flow rate v multiplied by C of t into dt. So, therefore, N naught which is the total amount of tracer that is actually fed into the reactor, remember it is a pulse tracer; that means, there is a sudden injection of the tracer at a certain time and very short time. And therefore, the total amount of a tracer that actually leaves the reactor from 0 time to infinite time, should be equal to the total amount of tracer that has actually been fed into the reactor as a pulse.

So, therefore, the N naught will be 0 to infinity multiplied by v into C of t into dt. So, that is the total amount of tracer that has actually been fed into the reactor. Suppose if we assume, that the effluent stream volumetric flow rate is constant, then N naught which is the total amount of tracer that is been fed inside is given by v naught into integral 0 to infinity C of t into d t.

So, therefore, the residence time distribution function E of t; that is given by v into C t divided by N naught and that is given by v naught. Because the effluent stream volumetric flow rate is maintained constant, we assume that it is constant divided by v naught into integral 0 to infinity C of t dt.

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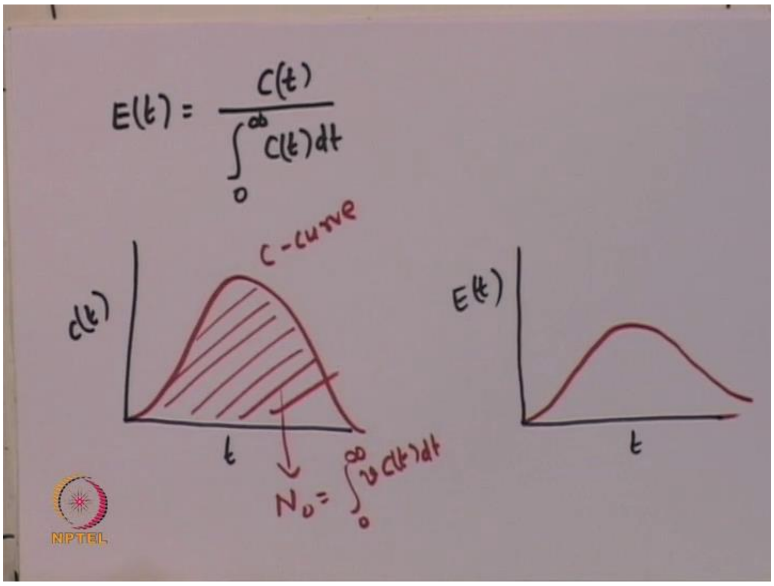

So, from here we can easily see that the residence time distribution function E t is essentially given by the concentration of the tracer at any time, divided by the total amount of tracer, that is actually leaving the reactor or the total amount of tracer that is actually entering the reactor. So, if I plot this function. So, suppose the concentration versus tine graph. So, the effluent concentration as a function of time, if it looks like; suppose if it is a bell shaped curve, so that is the concentration of the tracer in the effluent stream as a function of time. Then the area under this curve; so this area under this curve is essentially N naught which is the integral 0 to infinity v into C of t dt. So, weighted area under this curve essentially gives the total amount of tracer that has actually been fed into the reactor.

So, now if I plot the corresponding residence time distribution function; so E of t. Then, the curve essentially looks like this and this is, so if we know what is the C t versus or the c curve. So, this is the c curve. So, if the c curve is known then we can actually find out the e curve. But in order to find out the E curve from c curve, we need to be able to integrate the expression or find the area under the c curve. Only if you are able to find the area under the c curve, we will be able to construct the residence time distribution function. So, now from this we can actually find out what is the material which is that is leaving the reactor with a specified residence time, specified range of residence time.

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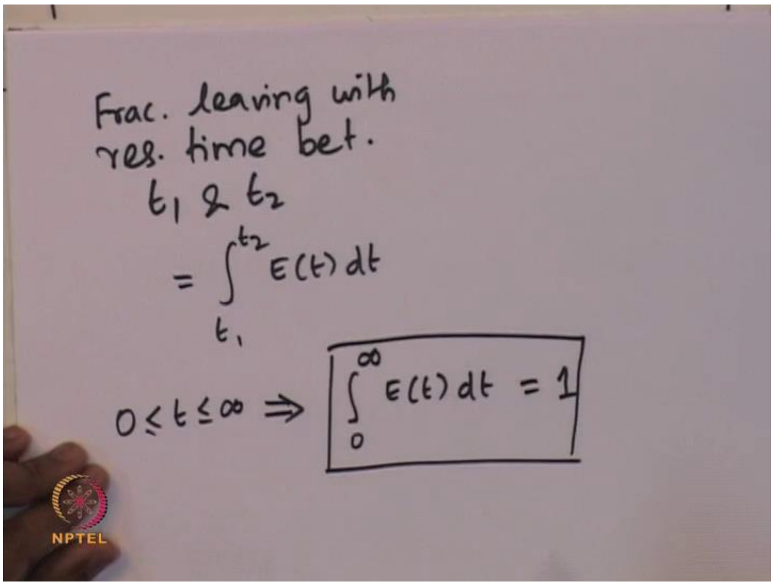

So, suppose if I want to know; what is the fraction of material leaving with residence time between t 1 and t 2. So, I want to know what is the fraction of the material which is been fed into the reactor and that has spent the residence standard that has spent sufficient times, but the time that they spent is between t 1 and t 2, certain time t 1 and t 2. So, that can that fraction is essentially given by integral t 1 to t 2 E of t dt.

So, if we know the residence time distribution, then there are lot of properties or lot of information about the reactor can actually be extracted from the RTD function. Now suppose, if I want to know; what is the fraction that leaves the reactor, with residence time from time t equal to 0 all the way to infinity; which means, what is the amount of material which leaves the reactor with the residence time in the whole spectrum of time.

So, that will simply be given by for 0 less than t less than or equal to infinity, that fraction will simply be 0 to infinity E of t dt and that is equal to 1. Which means; the area under the RTD function curve is 1, which is true, because it is a fraction and so the area under the curve should be equal to 1. Now, the question is how good is this pulse input method? Can it be used in all situations in order to find out the residence time distribution function? And that is not and that is really not the case. There are certain limitations.

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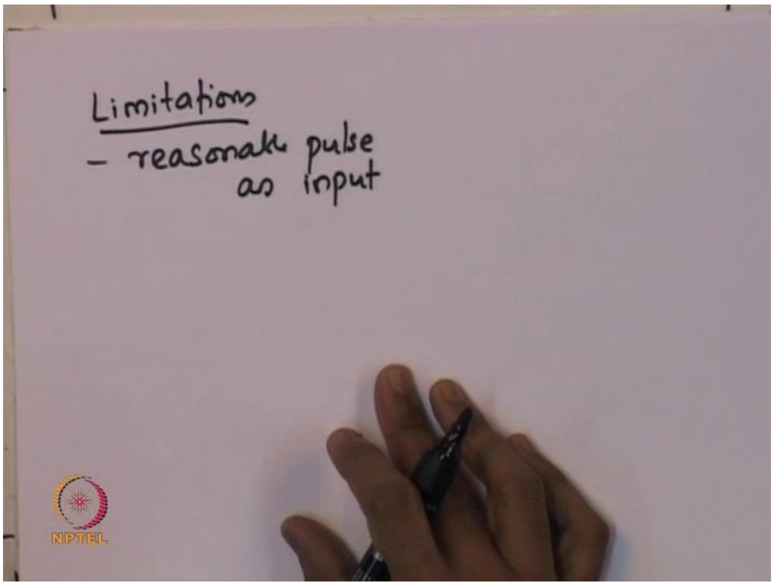

So, the limitations are; so it is important to obtain a reasonable pulse as the input. So, that is often not very easy to do. So, it is very difficult to get reasonable pulse as input. Why is that, because the injection has to be done at a very short period and a sufficient quantity of the tracer has to be injected in this very short period. So, that is not a very easy task and it is not very easy to perform such an experiment, where the tracer is actually injected into the stream, in a very short time and sufficient quantity.

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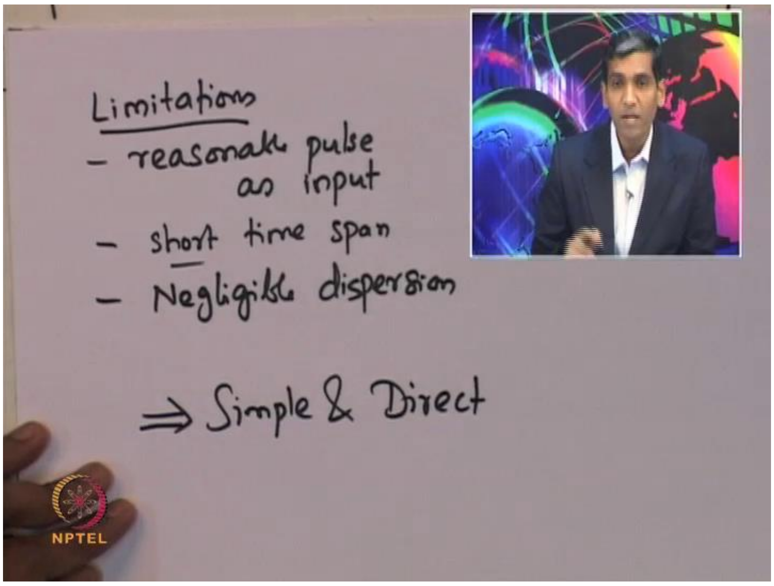

So, injection has to b in a very short time, it has to be in a very short time span. And more importantly; the reactor must have negligible dispersion between the point of injection and the entrance of the reactor, over which very limited control can actually be established on the reactor, 1 cannot control the extent of dispersion. So, if the reactor does has reasonable dispersion, then this pulse input method simply does not provide a clean way to find the residence time distribution.

However, pulse input is basically, it is a very simple and direct method, provided these 3 conditions are actually satisfied. Now another important kevyet, associated with the pulse input method is that.

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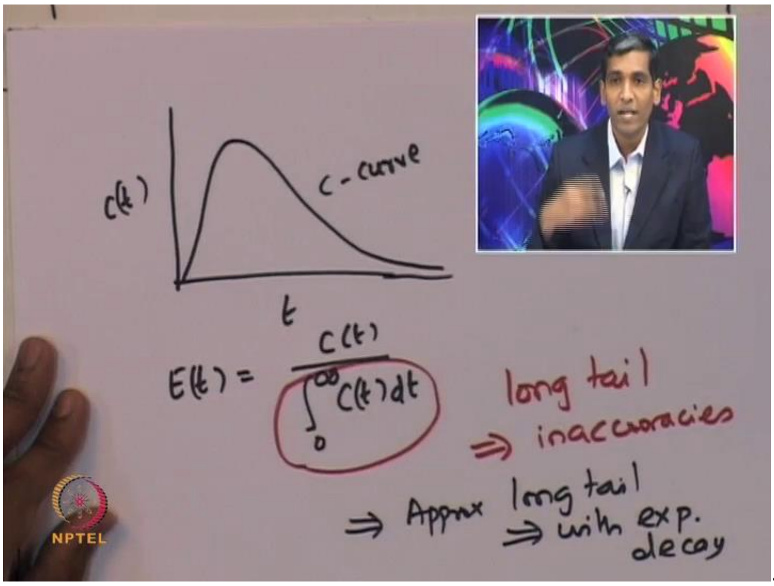

If I look at the concentration curve, in certain situations there can be a very long tail which is present in the c curve. So, this is the c curve. So, now, the concentration curve has to be integrated. Remember that Eof t is basically given by C t divided by integral 0 to infinity C t dt provided, that the effluent volumetric flow rate is constant there is no dispersion etcetera.

So, in order to perform such an integration, 1 needs to do an numerical integration of this of this curve; this c curve. And if the tail is very long, then it is not easy to integrate this expression numerically and it can actually cause inaccuracies in the integral. So, therefore, the long tails can actually leads to inaccuracies in estimating this integral. And therefore, 1 specific reconciliation is; 1 specific method in order to get over circum when it is problem is, basically to use approximate ling tails.

If long tail is observed, approximate long tail with exponential decay. So, assume that the long tail is exponentially decaying. And so now, there is it is an analytical function and it can be used to find out the integral C of t dt to a reasonable accuracy. So, therefore, if there is long tail, then it can lead to all kinds of numerical issues, in order to estimate the RTD. So, let us quickly summarize; what are the steps; that is involved in actually finding the E curve.

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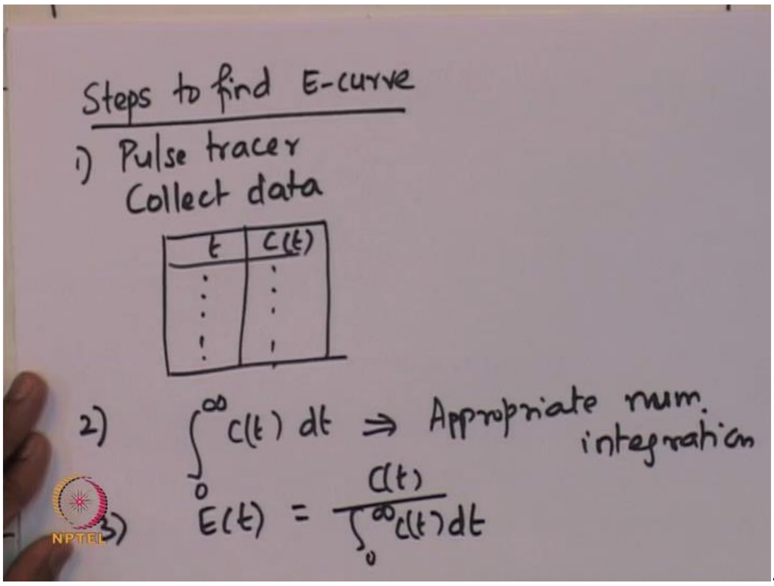

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So, steps to find E curve. The first step is basically to inject a tracer. So, inject a pulse tracer and then collect data on the way to. And then by collecting data, 1 would actually get a table of time versus C of t. So, at different times, we can 1 can actually find out what is the concentration of the tracer in the effluent. Now once this data is available, then 1 needs to find out this integral C of t dt by using appropriate numerical integration scheme. By using appropriate numerical integration, 1 can actually find out what is this integral between 0 to infinity C t dt, which is basically the area under the c curve. And the last step is to find out the E of t which is C t divided by integral 0 to infinity c t times d t. So, that is the area under the curve.

So, these are the 3 steps that has to be followed, in order to find the E curve. So, now, if you know the E curve, we know the RTD function. And we have estimated the E curve, using the concentration of the tracer at the output. Now the input that was given in order to estimate such an a c curve, is basically a pulse input. Now the question is; is there a connection between the input concentration profile and the output concentration profile? So, the answer is; yes there is.

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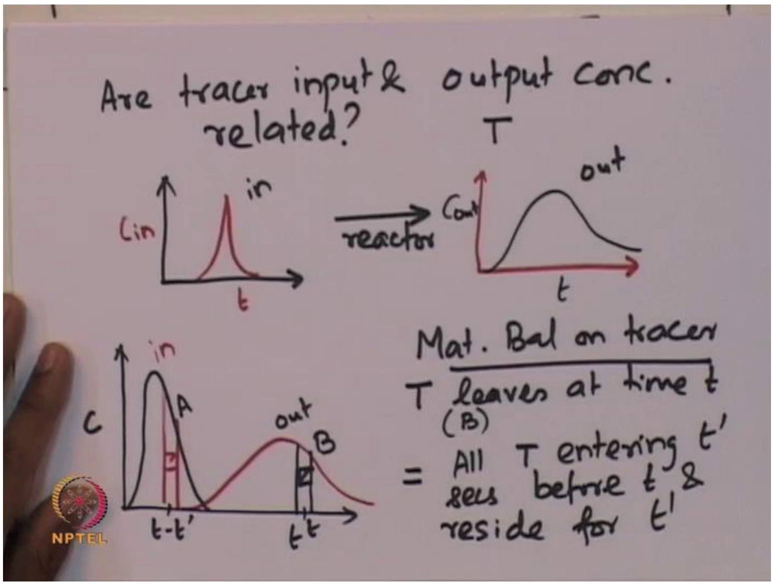

So, are tracer concentration concentrations related are they related? So, suppose if I call T is my tracer, suppose T is the tracer that is actually fed into the reactor. And now, if the input to the reactor is basically given by this function; suppose this is the input concentration to the reactor; that is the input concentration profile. And after going through the reactor, the species the tracer is actually measured in the effluent stream. And let us say if the concentration of the tracer in the effluent is given by this curve here. So, that is C out. So, that is the in and that is the concentration of the tracer in the outlet stream or the effluent stream.

So, now suppose, if I place them in the same graph. So, the objective is to find out what is the connection between this concentration profile and this concentration profile. So, suppose, if I place them in the same graph and that is my input stream. And suppose the corresponding output stream is this. Now, I can write a material balance, to find out the find out the relationship between the concentration of the species, that is actually coming out of the reactor at a certain time and how is that connected to the concentration of the tracer at the inlet.

So, suppose I take a specific rectangle, specific element at a certain time t. And let us assume that, the fluid element which is present in this small box. And similarly, if I take a small element here, which is essentially at t minus. So, it is exactly the fluid stream that is entering the reactor at t prime time smaller than the time t, at which I am monitoring the concentration of the species in the outlet stream. So, now, I can a material balance on the tracer, essentially T which is the tracer which is leaves at time t; that should be equal to which is basically the 1 in the rectangle B.

Suppose if I call this as a rectangle B and this is rectangle A. And the amount of tracer that leaves at time t, should be equal to all tracer T entering t prime seconds before t and they stay exactly for t prime seconds in the reactor and reside for t prime. So, basically the amount of tracer that leaves at this time should be equal to all tracer that has actually entered t prime time before this time t and they have resided for that span of time t prime.

So, that is the material balance, which connects the amount of tracer that actually leaves the reactor and the amount of tracer that actually enters the reactor at a certain time t. So, now this can actually be rewritten as…

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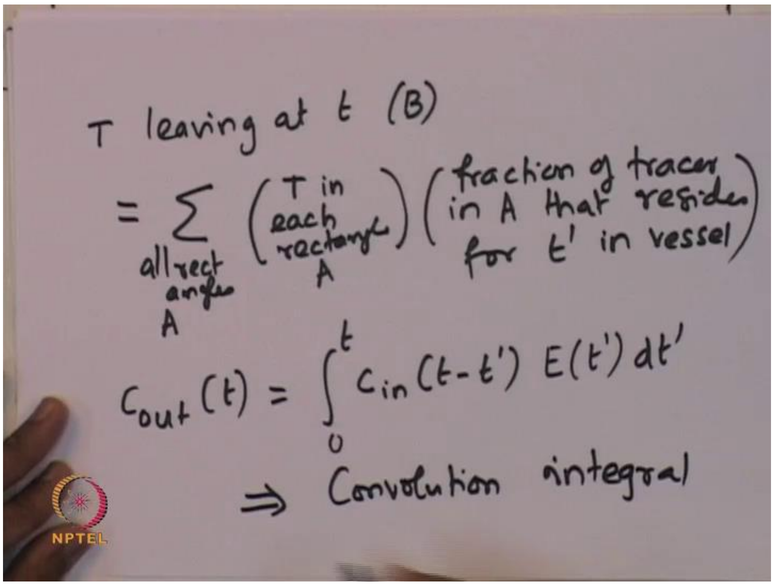

So, that amount of trace that is leaving at t, that is basically rectangle b that should be equal to sum over all rectangles, all rectangles A that is, all rectangles in the feed stream multiply sum over all rectangles in feed stream, that is sum of the amount of tracer in each rectangle multiplied by the fraction of tracer in the rectangle A that stays for that resides for t prime.

So, the amount of tracer that actually enters the reactor at a time t prime before the time t, which is basically the time at which the certain tracers species is actually leaving the reactor and I am attempting to connect that to the amount of material that is actually come into the reactor. So, that should be equal to sum over all rectangles; that is the amount of tracer that is present in each of the rectangle multiplied by the fraction of tracer in that particular rectangle that resides in t resides for t prime seconds in the vessel.

So, now, if I plug in the corresponding quantities; so if I write this expression in terms of the integral, it will be the concentration of the species in the outlet stream, that should be equal to 0 to t, that is the total amount of time that I am monitoring multiplied by the concentration of the species in that rectangle. So, this corresponds to the concentration of the tracer in each rectangle, which is actually injected t exactly t prime time before the time at which the concentration, of this particular species is being monitored, multiplied by the fraction of the species that actually spends t prime time inside the vessel.

So, that is the relationship between the concentration of the tracer, that is actually fed into the vessel and the concentration of the tracer that actually leaves the vessel at any time t. So, this kind of an integral is what is called as the convolution integral. So, this is relationship between the input and the output concentration is actually very useful to find out various aspects of the reactor using the RTD function. So, next let us look at the input case.

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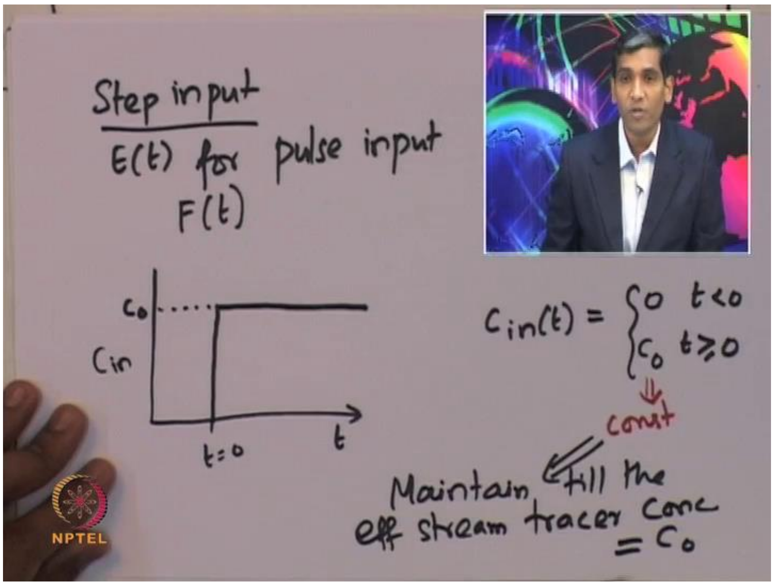

So, let us look at the step input case. In the case of pulse input, we actually found out the E curve. In a similar fashion, here we are going to define a curve called F curve, which is actually called the cumulative distribution function. So, now, in a step input case what is done is; suppose you take a single input single output vessel and then at some time t equal to 0. So, suddenly you provide a certain concentration of the tracer as a feed. You inject a certain concentration of the tracer into the feed. And then the concentration of the tracer is actually maintained.

So, basically it is a step function. And let us say that, this concentration is sum C naught. So, this is the inlet concentration of the tracer into the feed stream and as a function of time. So, now the; so C in mathematical representation of that will be is c n as a function of time will be 0 if it is; if t is less than 0 and that should be equal to some constant C naught if t is greater than or equal to 0.

So, we assume that, we are able to provide a constant concentration of the tracer into the reactor. We assume that we are able to feed a constant concentration of the tracer into the feed stream. And till what time do we have to maintain it constant. So, we have to maintain it constant till the effluent stream tracer concentration is equal to C naught. So, we keep, we maintain the concentration of the tracer in the feed at some constant value C naught, until the effluent stream the tracer concentration in the effluent stream is exactly equal to or almost equal to the concentration of the feed, concentration of the tracer which is actually maintained in the feed stream.

So, now once this is done, then we can actually measure the amount of while, doing this we can actually measure the concentration of the tracer in the effluent stream. So, soon after the step input is given.

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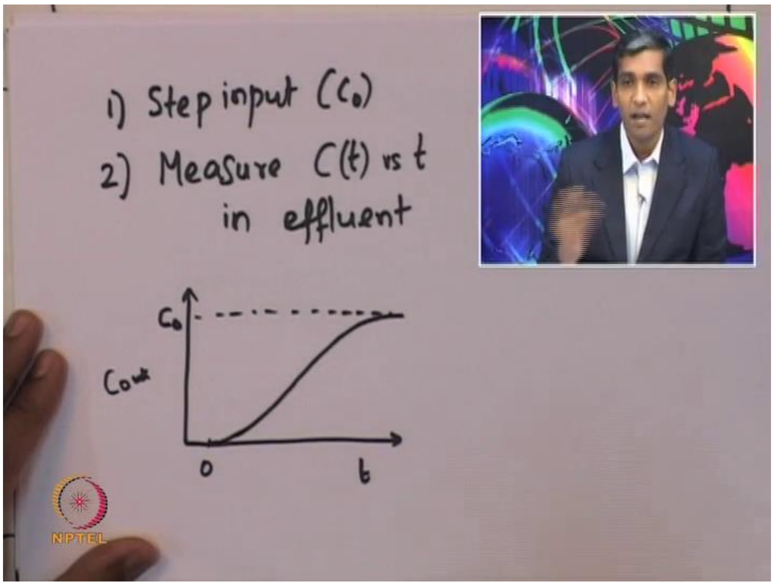

So, the first step is to give a step input, let us say at some concentration C naught. And the second step is to measure the concentration of the tracer, as a function of time in the effluent. So, that is the second step in the step input experimental method. And then if we look at the schematic of the outlet concentration of the tracer, it will essentially look something like this, where as a function of time. The concentration of the tracer is now going to slowly increase from 0. The time when the input is actually provided, time when the step input is provided, there is no tracer that is present in the effluent stream.

So, therefore, at time t equal to 0, the it will start at 0 concentration. And then, the tracer is now going to slowly appear, move through the reactor and then it is now going to appear in the effluent stream. So, once the concentration is measured, the concentration slowly increases, till it approximately reaches the concentration the constant concentration, with which the tracer is actually being maintained at the at the input of the feed stream to the reactor.

So, till it reaches that concentration, 1 has to measure the concentration of the tracer in the effluent stream. Once it reaches that concentration, then the step input can actually be stopped and the experiment is done. So, now once we measure this concentration versus time data what can we do with this.

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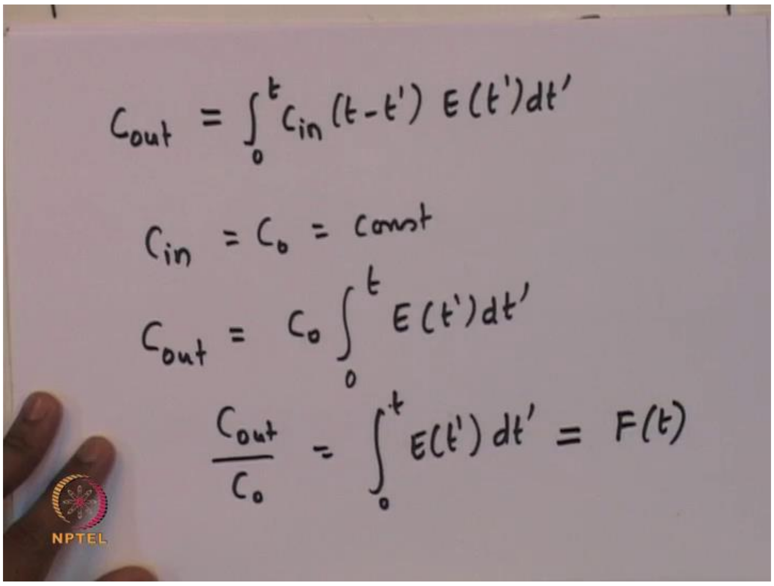

So, we know from the convolution integral, from the convolution formula that the concentration of the tracer in the output, is actually related to the input concentration as 0 to t C in into t minus t prime multiplied by E of t prime into dt prime, where this is the input concentration; concentration of the stream in the concentration of the tracer in the feed stream and E of t is the fraction of the material that actually spends exactly t prime amount of time inside the reactor, that is the residence time of that fraction of element is basically t prime.

So, from here, because C in the in, because it is a step input and the concentration of the tracer in the feed is actually maintained constant at C naught. So, this is basically equal to C naught and that is a constant. And therefore, the C out will be C naught into integral 0 to t E of t prime into dt prime or C out by C naught, that is equal to integral 0 to t E of t prime into dt prime. So, this integral this integral over the over the E curve or the RTD function is; what is called as the cumulative distribution function F of t.

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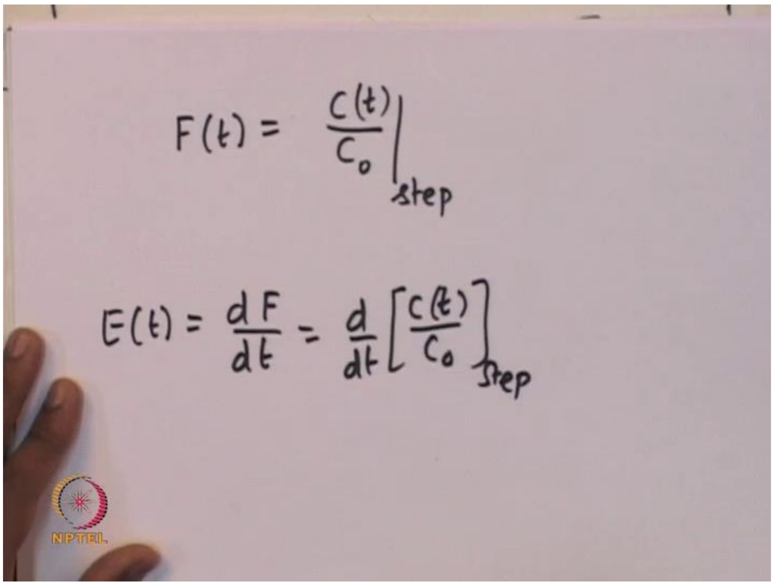

So, F of t is essentially the concentration of the tracer in the output divided by the total amount divided by the concentration of the tracer which is actually maintained in the feed stream. And this is for the step input function step input method. So, therefore, E of t can easily be shown that, that is equal to d first differential of F with respect to time or that is equal to d by dt of C of t divided by C naught.

So, F basically says what is the fraction of that fluid which is actually leaving the reactor, with a certain time at a with a certain residence time. So, what are the advantages of the step input method?

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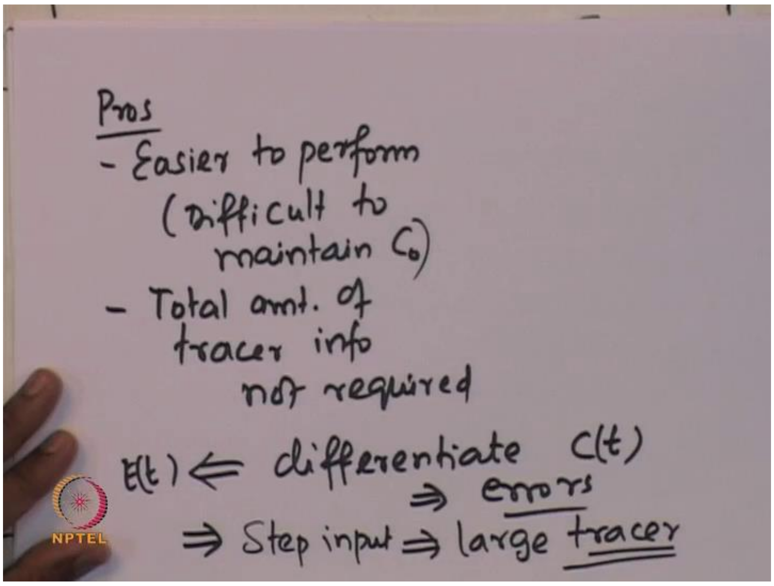

The advantages are that; so it is easier to perform this experiment, because it can actually the experiment involves maintaining a certain concentration of the tracer at a certain level. 1 may not be able to maintain exactly at that level, but it is much better than giving a pulse input, because the pulse input certain fixed quantity of the tracer has to be feden in a very short time period.

So, therefore, it is easier to perform the experiment, although it may be difficult to maintain a certain fixed concentration C naught. And that is because of certain experimental limitations. And also an important advantage with this method is that, the total amount of tracer that information is not required. So, remember that, in a pulse input case in order to find the total amount of tracer, 1 needs to actually integrate the concentration or the C curve and if there is a long tail then, there are inaccuracies that may creep in into this in into this integral, that is the total amount of tracer. So, such kind of a integration is not required for the step input method.

However, in order to find the E curve, 1 needs to actually differentiate the data. So, differentiate the data and that may actually lead to some errors. So, some errors may creep in, while actually differentiating this the data the differentiating the concentration of the tracer as a function of time in the effluent stream. Now another major drawback of this particular method is that,, because these because 1 has to maintain a step input, the step input will require a large quantity of tracers. And if the tracer is actually an expensive material, it is not a very economical method to actually conduct an experiment to measure the RTD function.

So, what are; are there other techniques or other tracer techniques to measure the RTD function? The answer is of course, yes.

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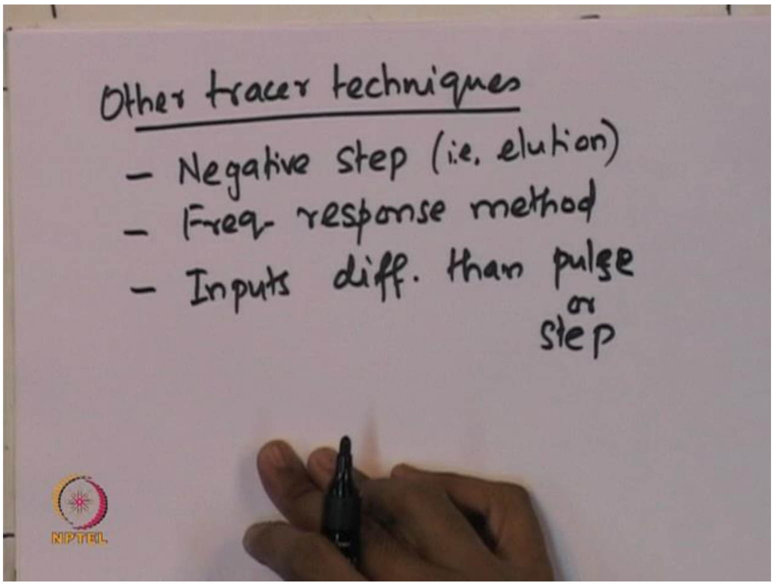

So, other techniques are; they are other tracer techniques, but these are not very commonly used, because it is difficult to perform these experiments. So, 1 is the negative step tracer, where the where the concentration of some tracer is now suddenly, it is actually decreased. And it is not very easy to perform such an experiment; basically there is an elution or dilution of the tracer in the feed stream.

Then, another method which is used is called the frequency response method. And third method is; basically 1 can use inputs which are than pulse or step method. So, there can be other types of inputs that can actually be provided. And these are not very common methods. And so we it will not be discussed in this course. So next, after finding the RTD, 1 needs to know what are the characteristics of this RTD function, because we are interested in finding the non ideal behavior of the reactor. And we said that the RTD function, somehow it captures some information about the non ideal behavior of the reactor.

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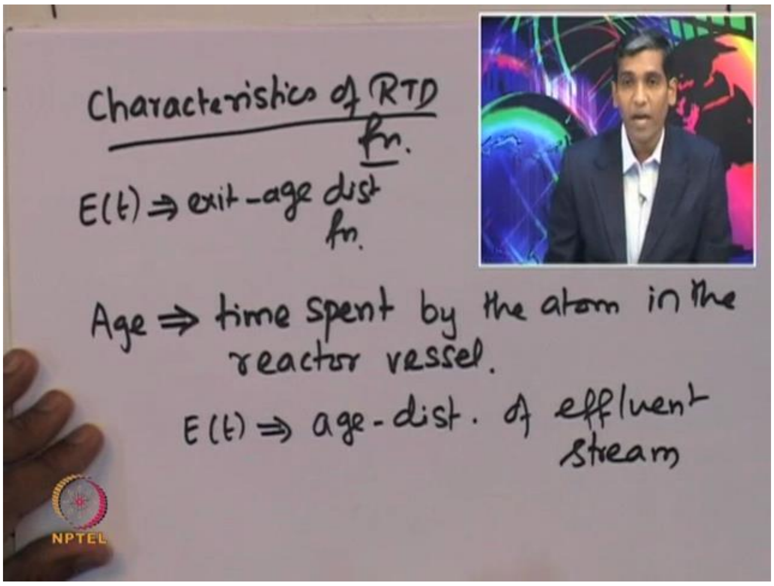

So, we can now estimate what are the characteristics. So, the characteristics of RTD function perhaps, may provide some clue about the characteristics of the non-linear behavior of the reactor itself. So, suppose if E t which is the RTD function which is also called as the exit age distribution function. And let us define what is meant by age of let us say an atom or a molecule of a material.

So, the age is given its defined as the time spent that is, the time that is spent by the atom or molecule in the reactor vessel. So, that is what is defined as the age of that particular atom or molecule of the material which is actually being considered. So, therefore, clearly E of t is essentially the age distribution of the effluent stream. So, age distribution as measured by the concentration of the species in the effluent stream, that is what is the RTD function E of t. So, now there are some definitions. So, we can now define what is this F t and E t.

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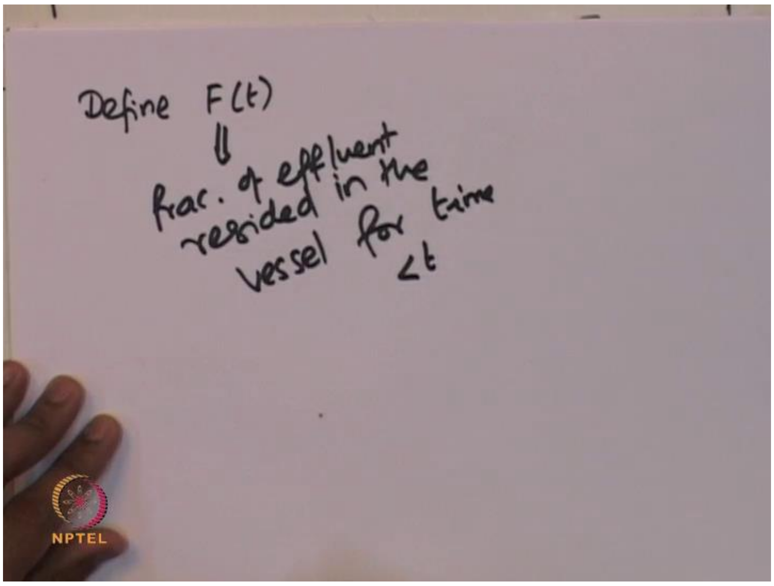

So, we need to define in order to characterize the RTD function in order to decipher some of the properties, we need to define some quantities. And the first 1 is F t. So, F t is essentially the fraction of the effluent, which is actually resided for a time which is smaller than time t. So, F of t is basically fraction of effluent, resided in the vessel for time less than t. So, now, how is that defined?

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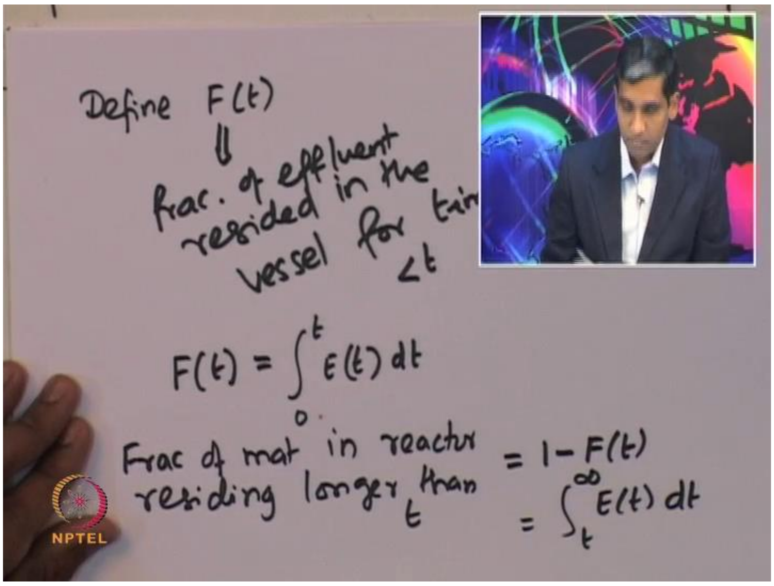

So, F of t as we have seen in the step input case. So, it is actually defined as integral between 0 to t E of t dt. So, that is the fraction of the effluent that has actually resided inside the vessel for a time which is less than t. Now 1 can also define; what is the fraction of the effluent, which is actually; what is the fraction of the material, which is actually residing inside the reactor for a time which is longer than t.

So, 1 can define; fraction of material in reactor, residing longer than time t. That is; what is that fraction of material which is actually staying inside the reactor, for a time which is actually greater than t. And that is essentially given by 1 minus F of t and that is equal to integral t to infinity E of t dt. So, sum of these 2 is; obviously, equal to 1.

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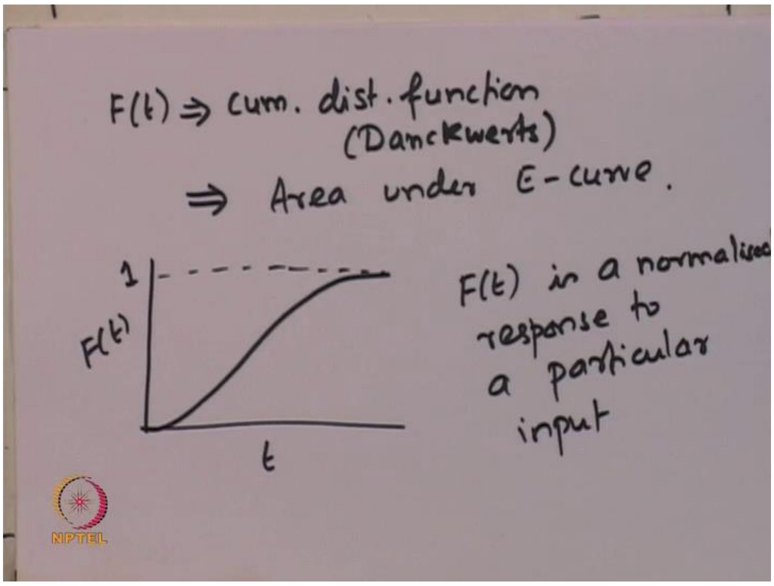

So, F of t as mentioned earlier is nothing, but a cumulative distribution function. And such a definition was actually given by a Professor. P. V. Danckwerts. Danckwerts was the 1 who came up with this definition or this name for F of t. And this is basically area under the E curve. So, as you can read from the integral. It is the area under E curve. And that is given by; if you sketch the F t versus time. So, that will be. So, the maximum possible value that F curve can take is exactly equal to 1. So, F t is essentially normalized is a normalized, it is a normalized response to a particular input. So, F t is a normalized response to a particular input; that is actually fed into the reactor.

So, let us summarize; what we have actually learnt in today's lecture. So, we have looked at; what are the properties of the tracer which is required for it to actually function as a tracer to estimate the residence time distribution function. And then the 2 injection methods which are; the pulse input and the step input method to actually decipher the residence time distribution function. And what are the different curves like the concentration curve and the E curve and the F curve, which basically characterizes the residence time distribution in the pulse and the step input method. Then we looked at what is this convolution integral, which connects the concentration of the tracer to the input of the reactor and the concentration of the tracer as measured in the effluent stream.

Thank you.