Chemical Reaction Engineering - II
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Module - 11
Lecture - 52
Measurement of RTD II

So, let us quickly summarise what are the steps that is involved in actually finding the E-curve.

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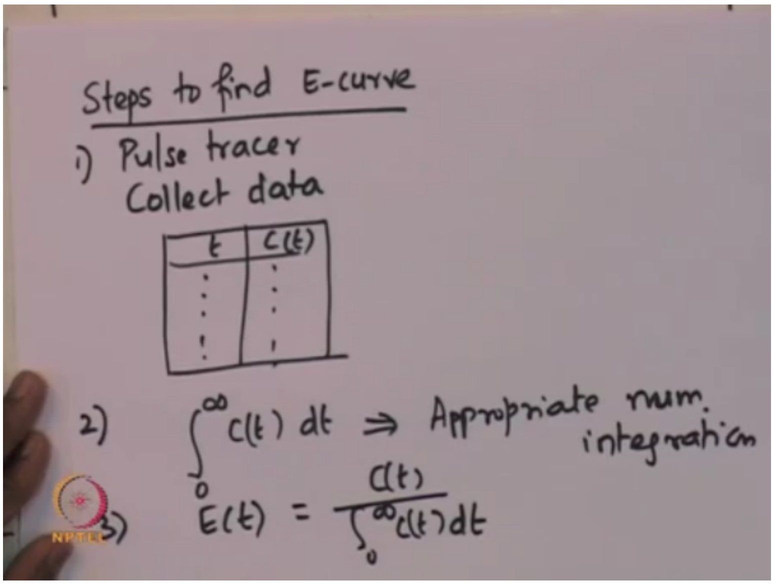

So, steps to find E-curve. The first step is basically to injector a tracer. So, inject a pulse tracer. And then, collect data. The way to, and then, by collecting data one would actually get a table of time versus C of t. So, at different times we can, one can actually find out what is the concentration of the tracer in the effluent. Now, once these, this data is available, then one needs to find out this integral C of t d t by using approximate, appropriate numerical integration scheme.

By using appropriate numerical integration, one can actually find out what is this integral between 0 to infinity C t d t which is basically the area under the C-curve. And the last step is to find out the E of t which is C t divided by integral 0 to infinity C t times d t. So, that is the area under the curve. So, these are the 3 steps that is that has to be followed in order to find the E-curve.

So now, if we know the E-curve, we know the RTD function and we have estimated the E-curve using the concentration of the tracer at the output. Now, the input that was given in order to estimate such a C-curve is basically a pulse input. Now, the question is, is there a connection between the input concentration profile and the output concentration profile. So, the answer is, yes, there is.

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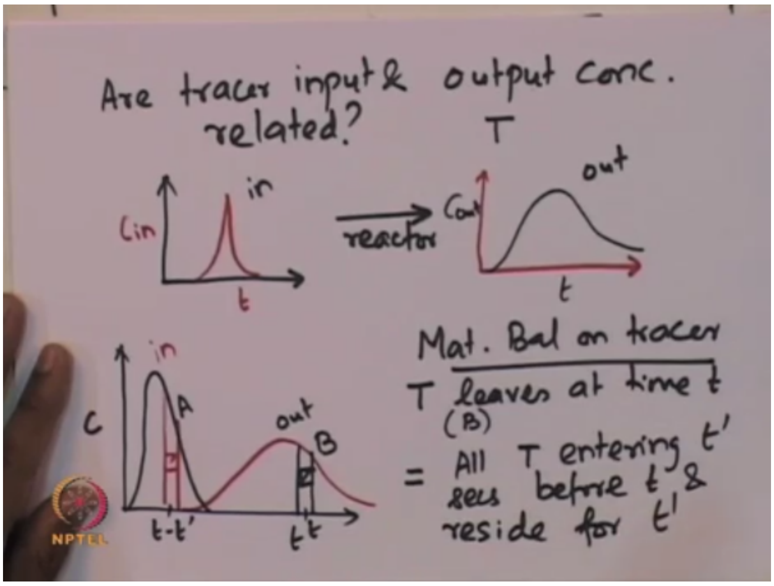

So, are tracer concentrations related? Are they related? So, suppose if I call T is my tracer. Suppose T is the tracer that is actually fed into the reactor. And now, if the input to the reactor is basically given by this function. Suppose this is the input concentration to the reactor. That is the input concentration profile. And after going through the reactor, the species, the tracer is actually measured in the effluence stream.

And let us say if the concentration of the tracer in the effluent is given by this curve here. That is C out. So, that is the in and that is the concentration of the tracer in the outlet stream or the effluent stream. So now, suppose if I place them in the same graph, so, the objective is to find out what is the connection between this concentration profile and this concentration profile. So, suppose if I place them in the same graph and that is my input stream.

And suppose the corresponding output stream is this. Now, I can write a material balance to find out the relationship between the concentration of the species that is actually coming out of the reactor at a certain time. And how is that connected to the concentration of the tracer at the inlet. So, suppose I take a specific rectangle, specific element at a certain time t. And let us assume that the fluid element which is present in this small box.

And similarly, if I take a small element here, which is essentially at t –, so, it is exactly the fluid stream that is entering the reactor at t prime time smaller than the time t at which I am monitoring the concentration of the species in the outlet stream. So, now I can write a material balance on the tracer. Essentially, T which is the tracer which is leaves at time t, that should be =, in which is basically the one in the rectangle B.

Suppose, if I call this as rectangle B and this is rectangle A. And the amount of tracer that leaves at time t should be = all tracer T entering t prime seconds before t. And they stay exactly for t prime seconds in the reactor. And reside for t prime. So basically, the amount of tracer that leaves at this time should be = all tracer that has actually entered t prime time before this time t. And they have resided for that span of time t prime.

So, that is the material balance which connects the amount of tracer that actually leaves the reactor and the amount of tracer that actually enters the reactor at a certain time t. So now, this can actually be rewritten as.

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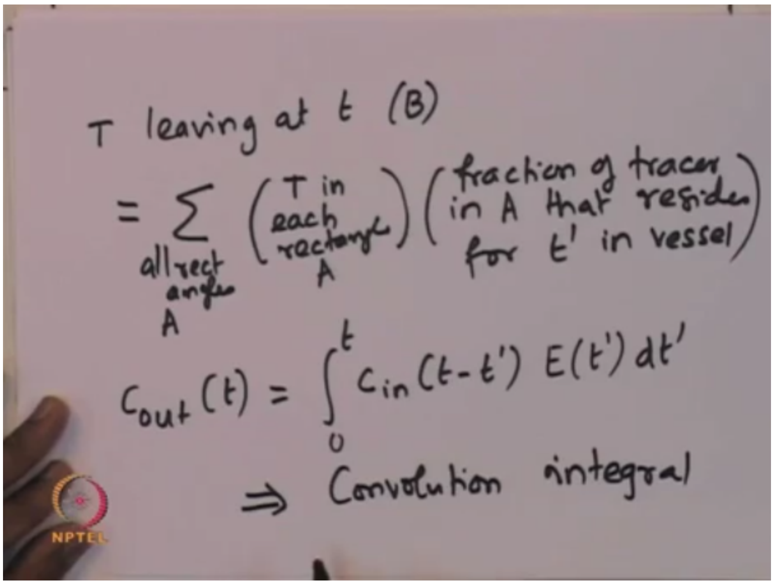

So, the amount of tracer that is leaving at t, that is basically rectangle B. That should be = sum over all rectangles. All rectangles A, that is, all rectangles in the feed stream, multiply, sum over all rectangles in feed stream. That is, sum of the amount of tracer in each rectangle multiplied by the fraction of tracer in the rectangle A that stays for, that resides for t prime. So, the amount of tracer that actually enters the reactor at a time t prime before the time t, which is basically the time at which the certain tracer species is actually leaving the reactor.

And I am attempting to connect that to the amount of material that has actually come into the reactor. And so, that should be = sum over all rectangles. That is, the amount of tracer that is present is each of the rectangle multiplied by the fraction of tracer in that particular rectangle that resides in t, resides for the t prime seconds in the vessel. So, if I now plug in the corresponding quantities.

So, if I write this expression in terms of the integral. It will be the concentration of the species in the outlet stream. That should be = 0 to t. That is the total amount of time that I am monitoring. Multiplied by the concentration of the species in that rectangle. So, this corresponds to the concentration of the tracer in each rectangle, which is actually injected t, exactly t prime time before the time at which the concentration of this particular species is being monitored.

Multiplied by the fraction of the species that actually spends t prime time inside the vessel. So, that is the relationship between the concentration of the tracer that is actually fed into the vessel and the concentration of the tracer that actually leaves the vessel at any time t. So, this kind of an integral is what is called as the convolution integral. So, this relationship between the input and the output concentration is actually very useful to find out various aspects of the reactor using the RTD function. So, next, let us look at the input case.

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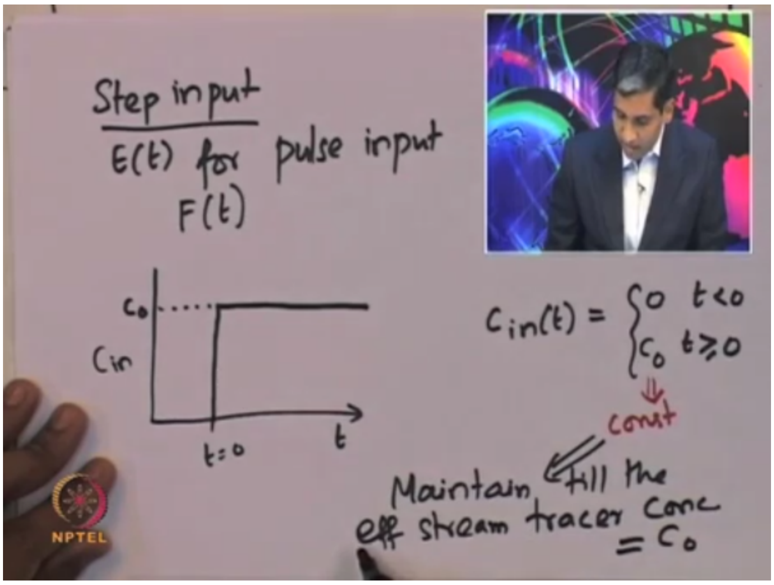

So, let us look at the step input case. In the case of pulse input, we actually found out the E-curve. In a similar fashion, here we are going to define a curve called F-curve, which is actually called the cumulative distribution function. So now, in a step input case, what is done is, suppose you take a single input single output vessel. And then, at some time t = 0. So suddenly, you provide a certain concentration of the tracer as a feed.

You inject a certain concentration of the tracer into the feed. And then, the concentration of the tracer is actually maintained. So, basically it is a step function. And let us say that this concentration is some C nought. So, this is the inlet concentration of the tracer into the feed stream and as a function of time. So now, the, so C in, mathematical representation of that will be C in as a function of time will be 0 if it is, if t is < 0.

And that should be = some constant C nought if t is greater than or = 0. So, we assume that this is, assume that we are able to provide a constant concentration of the tracer into the reactor. Assume that we are able to feed a constant concentration of the tracer into the feed stream. And, till what time do we have to maintain it constant? So, we have to maintain it constant till the effluent stream tracer concentration is = C nought.

So, we keep, we maintain the concentration of the tracer in the feed at some constant value t nought until the effluent stream, the tracer concentration, the effluent stream, is exactly = or almost = the concentration of the feed, concentration of the tracer which is actually maintained in the feed stream. So now, once this is done, then we can actually measure the amount of, while doing this, we can actually measure the concentration of the tracer in the effluent stream. So, soon after the step input is given;

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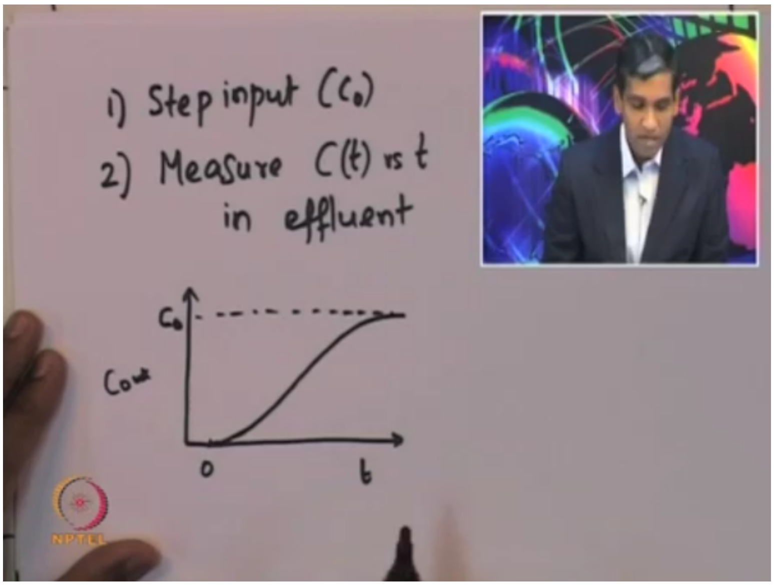

So, the first step is to give a step input, let us say at some concentration C nought. And then, the second step used to measure the concentration of the tracer as a function of time in the effluent. So, that is the second step in the step input experimental method. And then, if we look at the schematic of the outlet concentration of the tracer, it will essentially look something like this. Where, as a function of time, the concentration of the tracer is now going to slowly increase from 0.

The time when the input is actually provided, time when the step input is provided, there is no tracer that is present in effluent stream. So, therefore at time t = 0, the, it will start at 0 concentration. And then, the tracer is now going to slowly appear, move through the reactor. And then, it is now going to appear in the effluent stream. And so, once the concentration is measured the concentration slowly increases till it approximately reaches the concentration, the constant concentration of the, the constant concentration with which the tracer is actually being maintained at the input or the feed stream to the reactor.

So, till it reaches that concentration, one has to measure the concentration of the tracer in the effluent stream. Once it reaches that concentration, then the step input can actually be stopped and the experiment is done. So now, once we measure this concentration versus time data, what can we do with this? So, we know from the convolution integral, from the convolution formula that the;

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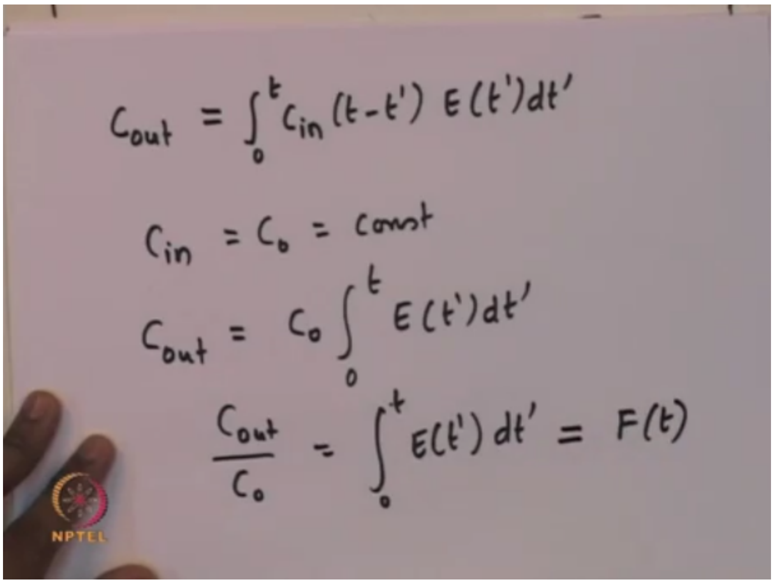

The concentration of the tracer in the output is actually related to the input concentration as 0 to t. C in into t – t prime multiplied by E of t prime into d t prime, where this is the input concentration, concentration of the stream in the, concentration of the tracer in the feed stream. And E of t is the fraction of the material that actually spends exactly t prime amount of time inside the reactor. That is the residence time of that fraction of element is basically t prime.

So, from here, because C in the, because it is a step input, and the concentration of the tracer in the feed is actually maintained constant at C nought, so, this is basically = C nought and that is a constant. And therefore, the C out will be C nought into integral 0 to t E of t prime into d t prime. Or C out by C nought, that is = integral 0 to t, E of t prime into d t prime. So, this integral over the E-curve or the RTD function is what is called as the cumulative distribution function F of t.

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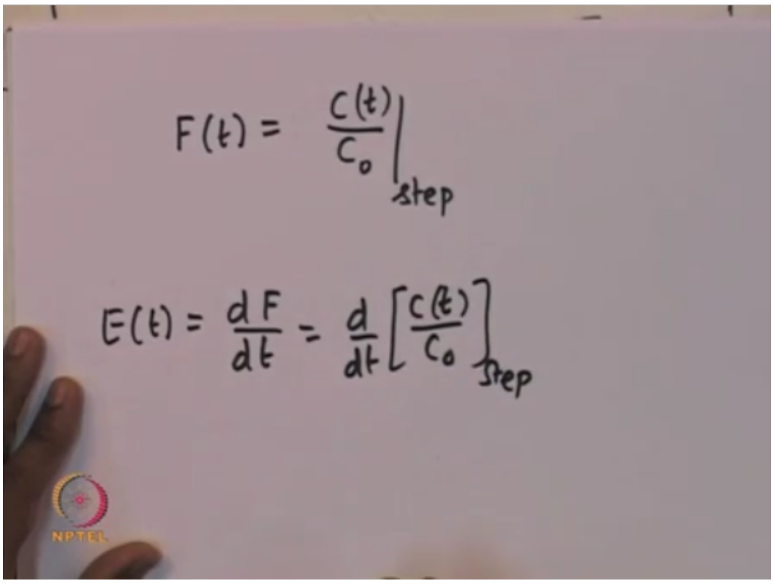

So, F of t is essentially the concentration of the tracer in the output divided by the total amount, divided by the concentration of the tracer which is actually maintained in the feed stream. And this is for the, step input function, step input method. So therefore, E of t can easily be shown that, that is = d, first differential of F with respect to time. Or that is = d by d t of C of t divided by C nought. So, F basically says what is the fraction of that fluid which is actually leaving the reactor with a certain time at a with a certain residence time. So, what are the advantages of the step input method? The advantages are that:

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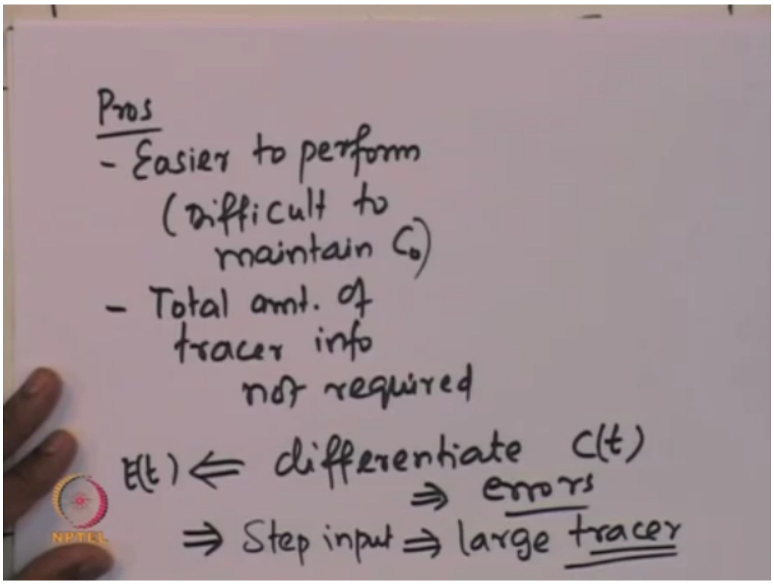

So, it is easier to perform this experiment because it can actually, this experiment involves maintaining a certain concentration of the tracer at a certain level. One may not be able to maintain exactly at that level, but it is much better than giving a pulse input because the pulse input, a certain fixed quantity of the tracer has to be fed in, in a very short time period. So, therefore the it is easier to perform the experiment although it may be difficult to maintain a certain fixed concentration C nought.

And that is because of the experimental limitations. And, also an important advantage with this method is that the total amount of tracer, that information is not required. So, remember that, in a pulse input case, in order to find the total amount of tracer, one needs to actually integrate the concentration or the C-curve. And if there is a long tail, then there are inaccuracies may creep in into this integral. That is, the total amount of tracer.

So, such kind of integration is not required for the step input method. However, the, in order to find the E-curve, one needs to actually differentiate the data. So, differentiate the data and that may actually lead to some errors. So, some errors may creep in while actually differentiating this, the data, the differentiating the concentration of the tracer as a function of time in the effluent stream.

Now, another major drawback of this particular method is that the, because these because one has to maintain a step input, the step input will require a large quantity of tracers. And if the tracer is actually an expensive material, it is not a very economical method to actually conduct an experiment to measure the RTD function. So, what are, are there other techniques, or other tracer techniques to measure the RTD function? The answer is of course, yes.

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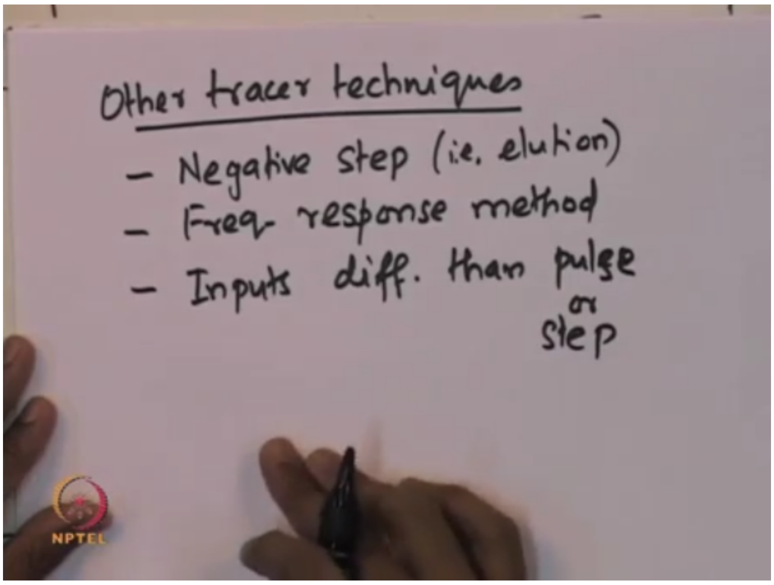

So, other techniques are, there are other tracer techniques, but these are not very commonly used because it is a, it is difficult to perform these experiments. So, one is the negative step tracer where the concentration of some tracer is now suddenly, it is actually decreased. And it is not very easy to perform such an experiment. Basically, there is an illusion or dilution of the tracer in the feed stream.

And then, another method which is used is called the frequency response method, it is called the frequency response method. And in a third method is basically, one can use inputs which are, than pulse or step method. So, there can be other types of inputs that can actually be provided. And, these are not very common methods. And so, we will, it will not be discussed in this course.

So next, after finding the RTD, one needs to know what are the characteristics of this RTD function. Because we are interested in finding the non-ideal behaviour of the reactor and we said that the RTD function, somehow it captures some information about the non-ideal behaviour of the reactor. So, we can now estimate what are the characteristics.

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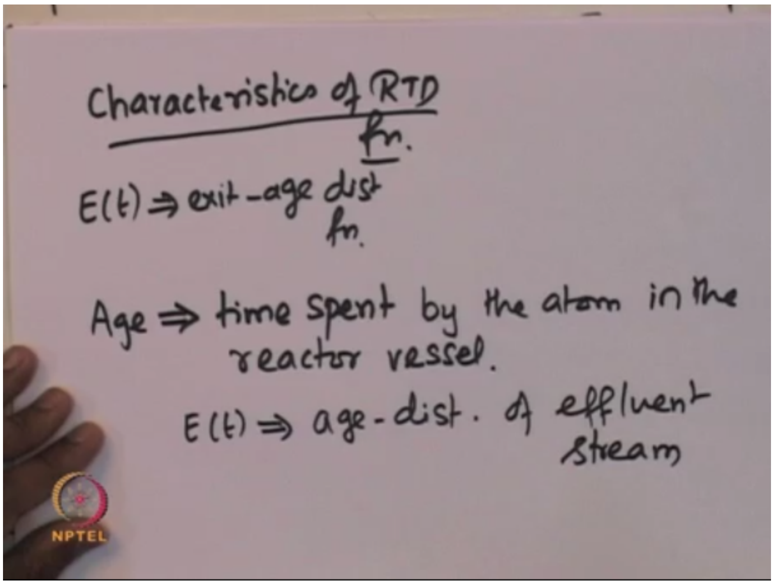

So, the characteristics of RTD function perhaps may provide some clue about the characteristics of the non-linear behaviour of the reactor itself. So, suppose if E t which is the RTD function, which is also called as the exit age distribution function. It is also called as the exit age distribution function. And let us define what is meant by age of let us say an atom or a molecule of a material. So, the age is given, is defined as the time spent.

That is the time that is spent by the atom or molecule in the reactor vessel. So, that is what is defined as the age of that particular atom or molecule of the material which is actually being considered. So, therefore clearly, E of t is essentially the age distribution of the effluent stream. So, age distribution as measured by the concentration of the species in the effluent stream. That is what is the RTD function E of t. So now, there are some definitions. So, we can now define what is this F t and E t.

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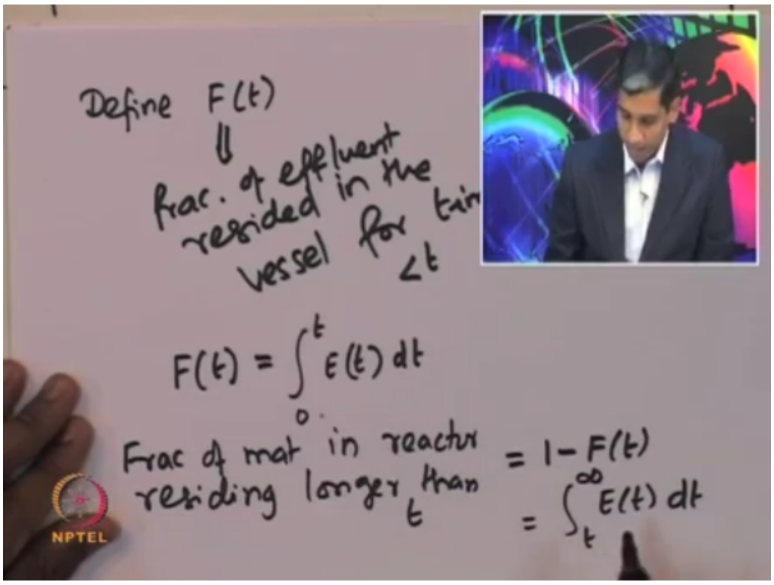

So, we need to define, in order to characterise the RTD function, in order to decipher some of the properties, we need to define some quantities. And the first one is F t. So, F t is essentially the fraction of the effluent which is actually resided for a time which is smaller than time t. So, F of t is basically fraction of effluent resided in the vessel for time < t. So now, how is that defined.

So, F of t as we have seen in the step input case is actually defined as integral between 0 to t E of t d t. So, that is the fraction of the effluent that has actually resided inside the vessel for a time which is < t. Now, one can also define what is the fraction of the effluent which is actually, what is the fraction of the material which is actually residing inside the reactor for a time which is longer than t.

So, one can define fraction of material in reactor residing longer than time t. That is, what is that fraction of material which is actually staying inside the reactor for a time which is actually greater than t. And that is essentially given by 1 – F of t. And that is = integral t to infinity E of t d t. So, sum of these 2 is obviously = 1.

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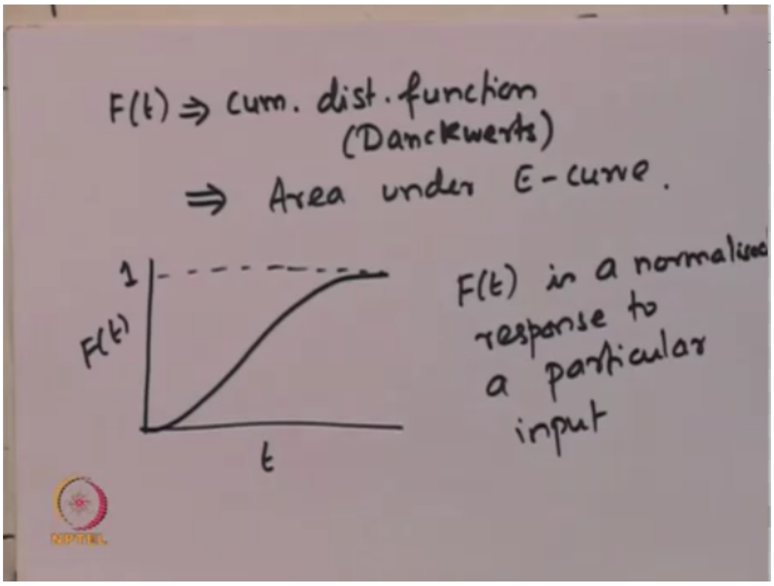

So, F of t as mentioned earlier is nothing but a cumulative distribution function. And such a definition is actually given by professor P. V. Danckwerts. Danckwerts was the one who came up with this definition or this name for F of t. And this is basically area under the E-curve. So, as we can read from the integral, it is the area under E-curve. And that is given by, so if we sketch the F t versus time. So, that will be; so, the maximum poodle value that F-curve can take is exactly = 1.

So, F t is essentially normalised, it is a normalised response to a particular input. So, F t is a normalised response to a particular input that is actually fed into the reactor. So, let us summarise what we have actually learnt in today's lecture. So, we have looked at what are the properties of the tracer which is required for it to actually function as a tracer to estimate the residence time distribution function.

And then, the 2 injection methods which is the pulse input and the step input method to actually decipher the residence time distribution function. And what are the different curves, like the concentration curve, and the E-curve and the F-curve which basically characterises the residence time distribution in the pulse and the step input method. And then, we looked at what is this convolution integral which connects the concentration of the tracer to the input of the reactor and the concentration of the tracer as measured in the effluent stream. Thank you.