Natural Gas Engineering
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Module No # 02
Lecture No # 10
Wellbore Performance Relationship (WPR)

Hello everyone today's lecture is on wellbore performance relationship in this lecture we will understand the mathematical expression for the wellbore flow profile.

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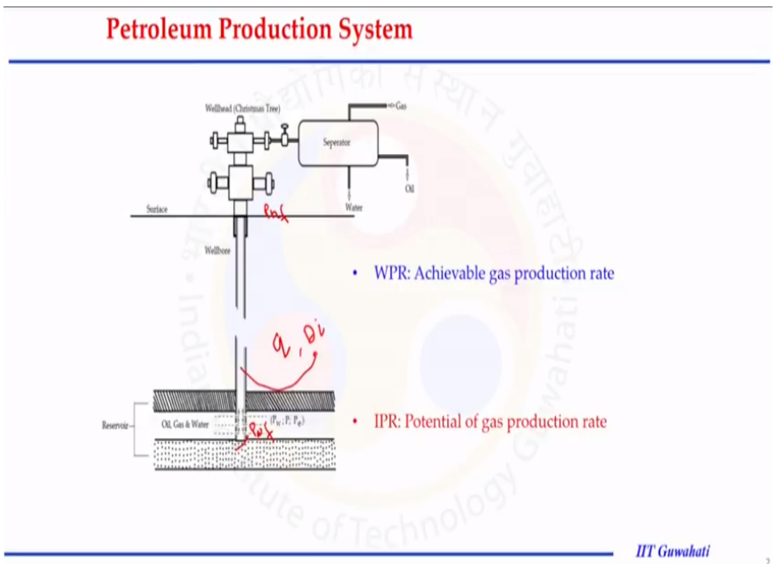

Let us revisit our production system that is where we have established the relationship for the IPR the bottom part where we understand how the flow through reservoir is happening and the gas is reaching to wellbore condition and the bottom of the wellbore where the pressure Pwf this is very much required because it tells us the potential of gas reservoir how much gas is available underneath or in the reservoir domain how much maximum production can be achieved.

In today's discussion we will extend our knowledge for wellbore section that is from Pwf to surface that we say Phf. So Pwf is bottom hole pressure at wellbore and Phf is pressure at well head. Because of the pressure difference again the flow will happen and the relationship between this pressure gradient or pressure draw down with flow rate Q rate with which the flow can be achieved from this well bore and the dimension of the well bore like the diameter internal diameter this wellbore is a part of today's discussion.

This is very important because WPR allow us to calculate the achievable gas production rate we will see before we are going to understand the mathematical relationship how the pressure Pwf and Phf are related to production rate Q and the other parameters of the fluid and the geometry of the wellbore.

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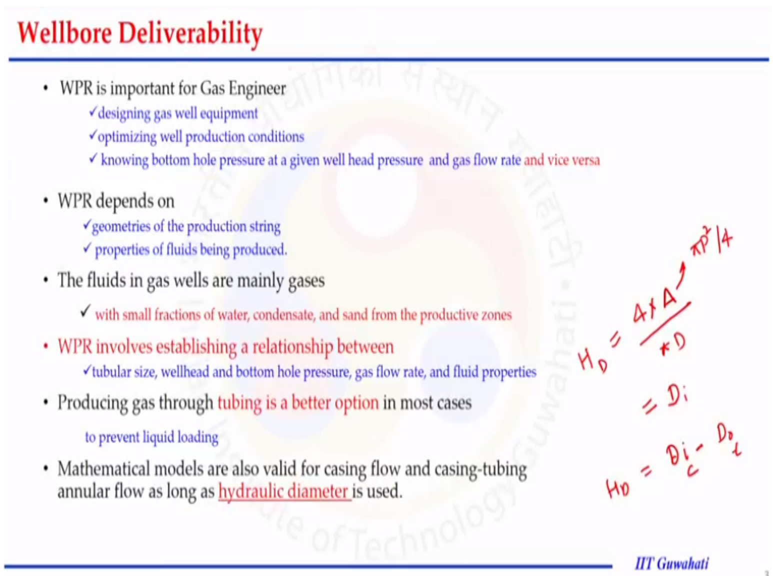

Let us understand some of the basic part of WPR that is comes under wellbore deliverability it is important for gas engineer because it allow us designing gas well equipment if well understanding of the WPR relationship is achieved the designing of gas well equipment will be much easier not only the deigning part it is also help us in optimizing.

The well production condition and what condition the well should be producing or should be the head Phf what should be bottom Pwf pressure and the diameter of the tube should be chosen and what rate the production should be accomplished that is matching the surface facilities the requirements in terms of capacity of the gas and the pressure the surface facilities can handle effectively.

Knowing bottom hole pressure at a given well head pressure and flow rate so once we develop the WPR relationship you will see at the end of this class in terms of example we will discuss when we could establish this relationship we can calculate the bottom hole pressure that is very much record in IPR relationship that can be calculated with WPR once we know what is the well head pressure and what rate the well is producing or vice versa.

If two are known third one can be calculated WPR of course depends on the geometry of the producing string. So the production is happening throw the tube, through the casing or through the annular depend on the what size of that tubing is how long the production string is the WPR depend on that as well as the properties of fluid being produced.

Most of the cases in natural gas production it is gas but similarly most of the time gas along is not being produced gas is always associate with some amount of it might be in a very small amount some amount of liquid that could be the oil, water some amount of some small solid particles those are also being produced along with the gas through this production string. That production string could be just tubing, casing or the annulus.

The fluid in gas well are mainly gases but as I mentioned the condensate well will produce oil and gas even the gas well which is gas well under the reservoir conditions when it is producing the pressure draw down is happening and as we understand from our phase diagram class the phase may get change or the fluid may enter in the envelope of two phase reason where some liquid is also getting produced.

And when this is happening the liquid that is coming out of the gas phase under the local condition of temperature and pressure that will get accumulated in some reason in the wellbore and will create a problem because this phenomena called the liquid loadings and comes under special issues where certain action should be taken to take out the valuable liquid from the wellbore reason.

Sometimes the production is not ceasing in the wellbore WPR involves relationship between tubular size well head bottom hole pressures gas flow rate and of course of the properties of fluid. Our fluid is not ideal fluid it is a mixture of hydro carbon gases and non-hydro carbon gases and because of that the conditions if condition are changing like the local temperature and pressure the fluid properties especially viscosity, compressibility factor, density those are the function of temperature and pressure will also changed.

We will see in today's class not only the properties when the properties are changing the equation those represent particular phenomena becomes complex and solving those equation need certain approximation then only we can achieve the desired result. The desired result is how my flow rate through this wellbore is related to this big pressure draw down that is spread over several thousand feet depth from Pwf is from the bottom of the wellbore to the head of the well and that relationship is called as WPR.

As mentioned production can be achieved from the tubing from the annulus or from the casing part but experience source producing through tubing is the better option because it prevent liquid loading the superficial velocity of the gas and liquid. If liquid is present in the system the relative superficial velocity decide the liquid accumulation is happening or not or what is the liquid loading possibility in the system and experience shows if it is happening through the tubing the chances are less.

Mathematical model are also well it for casing flow at casing tubing annular flow as mentioned only difference will come in mathematical equation which is derived for the tubing flow. If the diameter of the tubing in express is replaced by the hydraulic diameter by the definition hydraulic diameter is four times cross sectional area divided by wetted parameter. So if we go by definition of hydraulic diameter we can write this HD that is 4 times cross sectional area.

If the tube is slender the cross sectional area the fluid is flowing is a pie D square by 4 and the betted parameter means the ahhh parameter of this tube is pie D. So if we do that for a slender tube it comes out as the diameter or the internal diameter of the tube. So in case of annular flow the HD will be like D internal diameter of casing – outer diameter of tubing. So just replacing the diameter in the expression by the hydraulic diameter we can use the same mathematical expression for flow through annulus.

So let us go ahead with the understanding of this basic WPR why it is important and how to do it we can set up the mathematical equation by considering the pie could be either the horizontal vertical or inclined.

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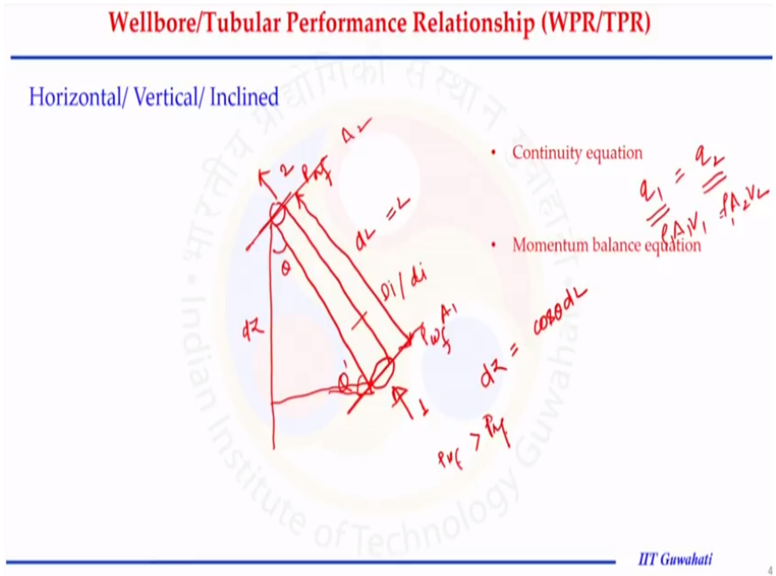

So let us see this is our tubing through which the flow is happening and we are assuming the flow is happening from this direction to this direction from one to two or I can say this is the place where the pressure is Pwf bottom part of the well and this could be Phf. This tube could be horizontal vertical and inclined but in case of the wellbore production mostly it is vertical we can adjust that thing with the help of angle theta either we can use this angle or this angel depend on the angle we are choosing with the sin form or Cos form will come into expression.

We can say the length the tubing is that is the spread over several thousand feet we can see the length is del L or just L. And the vertical height is del check so we can establish the relationship and that relationship says we are having del z = Cos theta. So now flow is happening from 0.1 to because of the pressure difference and the pressure difference is Pwf is higher than the Phf and this pressure energy will push the fluid to travel from bottom part to top part of the wellbore.

The diameter of this tube is internal diameter we can say Di depend on the unit system it is mentioned either it is mentioned capital Di or small di. In general in US field unit system if it is mentioned as small di means the unit associate with this diameter is inches and if it is capital Di is field or otherwise it is specified the symbol along with it unit. So for this system when the flow is happening vertically from point 1 to point 2 because of the pressure difference we can set up the continuity equation or the momentum balance equation.

Continuity equation simply says there is no accumulation of the mass is happening whatever is entering at this point let us say cross sectional area A1 and leaving at this cross sectional area 2 is same there is no accumulation in the system. And when it is happening we can say q1 = q2 so the flow rate or volumetric flow rate or mass flow rate we are assuming here the density is constant at one point and 2 point that may not be correct what we are saying for the assumption the flow rate is same and A1 V1 = A2V2.

Otherwise it should be rho A1V1 = rho A2V2 but from the continuity we could understand there is no accumulation happening in the system and we assume the density is not changing much the cross sectional area of the tube is also constant or almost constant throughout the tubing part then the velocity will also be constant and that gives us the flexibility to assume the contribution of the kinetic energy in the momentum balance equation is negligible.

So when we talk about the momentum balance equation momentum balance means the momentum getting in and out of the system the difference between in and out the balance is by the forces acting on the system before these forces could be kinetic energy could be gravity could be friction forces could be shaft work done on the system or other type of the forces assumption we can make only gas is flowing.

If we assume other than gas, liquid or solid compound or water is also being produced the expression will be more complex more difficult. So let us start with the single gas phase flow system and the flow through the tubing is govern by the first law of thermo dynamics that is the conservation of energy principle and that by setting up the mechanic balance equation to a system we can get the relationship between the parameters those are responsible for this conservation of energy for this flow that is happening to the tubing.

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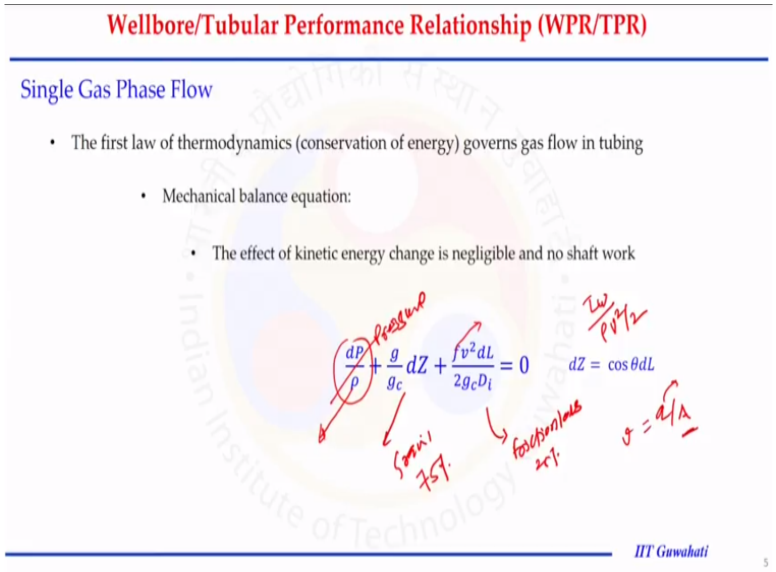

If we do so we can see considering there is no change in the cross sectional area the kinetic energy changes in the system can be ignored similarly there is not external device installed across the system so no shaft work is done on the system it means shaft work is also negligible so the pressure energy in the system this is pressure gradient is balanced by the gravity and the fractional losses.

Depend on the pipe roughness the frictional losses will happen in the system and the gravity will play a major role in fact when the tube is vertical and the flow is happening because of the pressure gradient as we see in our fluid mechanics class also we can set a very simple mathematical equation depend on what is happening in the system we can establish the relationship very easily in the terms of dependent and independent parameter.

Similar we are doing here but it is little bit complex and the complexity is because our system is not ideal system certain properties are not constant they are the function of local condition so let us see in the mechanical balance equation what we are having the pressure energy gravity energy and friction losses or come up in the form of mechanical energy balance equation and the summation of all these is equal to 0.

So the pressure energy that is responsible for this flow is balanced by the gravity and friction forces depend on the pressure gradient we are having the flow will happen in fact in a vertical flow system the gravity contributes around 75% and the frictional loss is account around 25%. So the gravity is a dominating mechanism of dominating force or dominating part of the energy change that is actually governed the flow throughout the vertical tubing.

We will discuss this in the next slide how the parameters appearing in this expression can be replaced to make this equation solvable. Because you see here the pressure term the gravity is appearing the natural gas is real gas gravity may not be constant. Most of the cases as been considered natural gas is compressible fluid of course density is not constant in that case and the f which is a frictional force factor called is the friction factor F that comes out by the definition of stress by rho V square by 2.

We can get if we know the tau, (()) (17:21) or we can convert this expression into other form of Reynolds number and can use some charts from our fluid mechanics class we are understand a Moody chart is available that is relate how the friction factor or the moody friction factor should be calculated when we know the flow through a pipe is happening and under what condition that flow is happening it is happening under the laminar condition or in a turbulent condition or in between like in a transient condition .

We will see that in the next slide important is this f that is also function of Reynolds number, Reynolds number is a function of q the volumetric flow rate that we will also see in the next slide and the q is the function of temperature and pressure. So this F is also going to create a problem it is not easy to put some value and take it out similarly the velocity V = q / A and this q again depend on the local condition temperature and pressure.

So we will see how we can replace the parameters those are making this equation difficult to solve how we can replace those parameters in some other term or in some familiar term those we know or we can understand how to calculate from the understanding of fluid properties.

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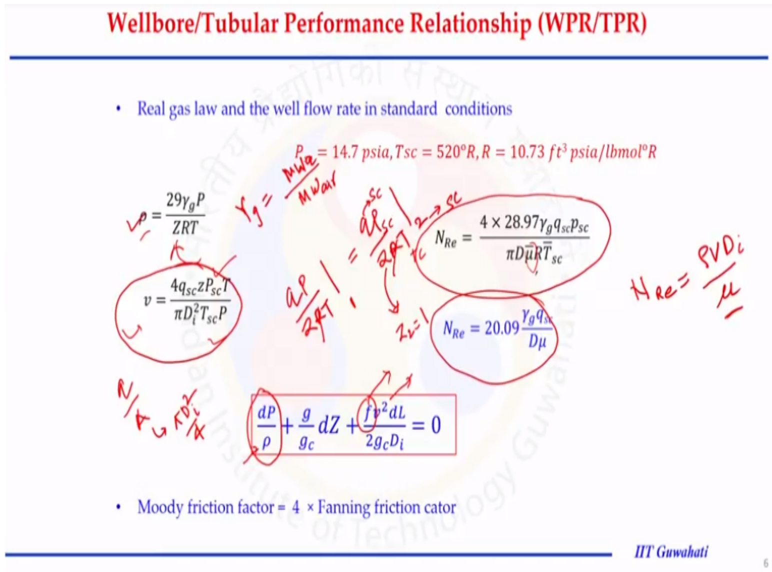

So we say the density rho = 29 gamma g P / ZRT this we understand from our properties of natural gas class how to establish this relationship here gamma g = molecular weight or apparent molecular weight of the fluid or the gas divided by the molecular weight of air.

So indirectly the gamma g is accounting the type of fluid or the gas is being produced from a gas well and in other terms what are the composition of that gas. How much percent of particular dominating hydro carbon, non-hydro carbon compounds can be represented in the form of gamma g accounts for the compositional variation from particular well we may be producing just a constant composition until unless some extreme conditions are happening like pressure is declining some liquid accumulation is happening or the phase accumulation is happening.

That is separate if the composition of the gas or known we can calculate the molecular weight know the molecular weight we can calculate gamma g putting gamma g here we can calculate rho but again see here rho also depend on Z is compressibility factor and that compressibility factor is also dependent term that depends on temperature that also we had seen in our properties of natural gas is class how to use some empirical correlation or big chart to calculate the compressibility.

We can use (()) (20:40) method to calculate the compressibility that gives us little bit easiness in creating a excel sheet where putting the data we can get the compressibility. But compressibility factor is required similarly here for the velocity as mentioned last slide we can write q /A A is cross sectional area pie D square by 4 or we can write internal diameter of tube q is flow rate at a particular temperature and pressure.

So we can use the relationship that says at a particular condition qP / zRT at 1 condition for a real gas = qP / zRT by second condition and the second condition we can assume as a standard condition under that standard condition we know what is the pressure value what is the temperature value we can cancel out this R and the Q value that will be denoted by now Qsc because this Q is estimated assuming the estimation of flow rate or the Q value is done at standard conditions.

The z value even the gas is real gas but we considered the z value at standard temperature and pressure condition as once the z2 will become 1. By doing this we know how to put those value and get the expression for velocity putting area putting this relationship of real gas we got this expression here Psc and Tsc are standard pressure and temperature we know in a field unit system the P is 14.7 psia and Tsc is 520 degree ranking R value at the field unit system is 10.73.

If we put the numerical value we can get the value of velocity in some numerical term also. Now Reynolds number the parameter that characterize the type of flow is happening or required to calculate the friction factor. So the Reynolds number rho VD/ Mu. Now if we put rho value velocity value in that expression and this Di internal diameter of the pipe and the viscosity of the fluid that is being flow from the tube. If we put everything we will see the expression here getting like this.

Here the question comes at what condition the viscosity should be calculated because again the viscosity is a function of temperature and pressure. So the viscosity of the fluid that is flowing through a tube can be calculated under certain average temperature and pressure condition and that is was the bar on the Mu is appearing in the expression it means the Mu is calculated at average condition.

The average could be chosen like the well head temperature and pressure and the bottom hole temperature pressure and the average or some other mean the average could be applied to calculate the viscosity of the fluid. Otherwise it is very difficult because viscosity will also change at point to point. When we put this standard condition and pressure condition and get the expression for the Reynolds number it appears very simple pressure like the rho divided by Mu it says under the US field unit system.

The Reynolds number = 20.09 gamma Z Qsc is Q estimated at standard condition divided by D diameter of the pipe and Mu the viscosity of the gas that is flowing through this tube. So now let us see our balance equation that we had in this last slide that says the pressure term here now we know we can replace this rho we see here the F we see the next slide but know how to replace this V we know that thing and the F could result in a numerical value again numerical value depends on the Reynolds number and the roughness of the pipe.

If roughness of the pipe is known the Reynolds number at which the fluid is flowing through a tube or a pipeline is known we can calculate using the moody chat we can out this values in this expression and can derive the mathematical expression that is much familiar compared to this one we will do in the next here important part is the F that is appearing here is it a moody friction factor or a finding friction factor.

There is a relationship between Moody and fanning, Moody says four times of the finding friction factor whenever it is not mentioned it is a moody friction until unless it is specified it is a finding friction factor in the expression or in the chart which we are going to use the relationship of moody and friction factor should be adjusted to at the accurate understanding or the value of the friction factor that is going to use in this mathematical expression.

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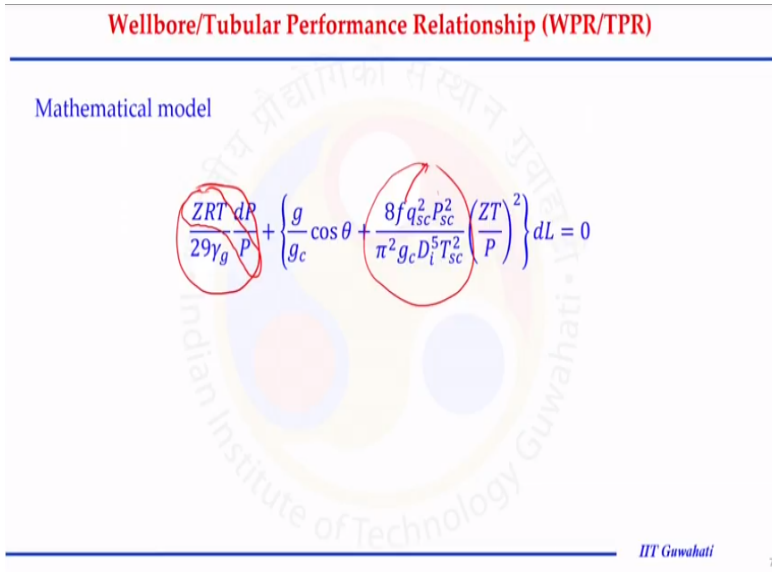

So let us see how to see this expression when input the value other than this F that we will see how to calculate it the equation will appear like this so this is still the pressure term we just replace the density and here we had replace the velocity. So the first term the pressure term not depend on PZRT similarly second term is also depend on ZT/P and if we can establish the relationship this equation can be solved otherwise this equation cannot be solved in the current form we have to mix certain assumption.

Assumption in terms of how to calculate the compressibility factor how to make the assumption may compressibility factor can be integrated when we are going to integrate this equation similarly for the pressure and temperature we will see later on let us understand first how to estimate the friction factor.

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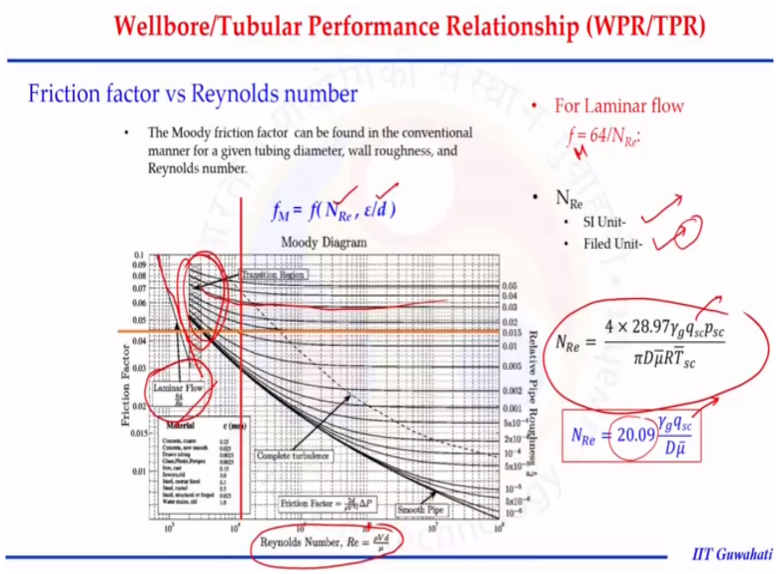

So this chart shows about the friction factor that says Reynolds number and this chart is for the Moody's friction factor I already mentioned the finding friction factor of the different they are having the relationship of Moody's friction factor = 4 times of the friction factor here you will see the Moody's friction factor similarly fanning will also be a function of Reynolds number and f salient /d is called active roughness of the Pie.

This is the value f salient this could be provided by the vendor which is supplying the pipeline or it is not given there are several literature. Those allow us to calculate the effective roughness of the pipe. Important is the Reynolds number if the Reynolds number is known and the roughness of the pipe is none we can read this chart and can calculate the friction factor value depend on in which reason the flow is happening if it is happening in the laminar reason the expression is simply 64 / Reynolds number.

Here it is moody friction factor you will see other chart when it is finding friction factor this will be having 16 / NRe. But in the Moody friction factor it is 64 / NRe so let us talk about the moody friction factor only now so when the flow is in the laminar reason it is having a relationship like the Reynolds number the friction factor is declining and it is not a function of roughness of pipe. The rough ness of the pipe plays a major role when the flow is in turbulent reason or transient reason when the gas is flowing with a significant high velocity.

And that is the case in the case of the natural gas because of significant pressure difference the gas is flowing under the turbulence condition and in that case f silent by d needs to be known before we can read this chart because you can see here there are several lines those are inclined and then becoming flat. Flat means of certain Reynolds number the value is not depending on the Reynolds number it is just depend on the roughness of the pipe.

In the transient region becomes little lesser and miss the effect of roughness of the pipe just concentrate in this reason only and we can calculate the effectiveness factor if we know the value of Reynolds number and we know Reynolds number depends on the q or gamma g can be estimated if we know the composition of the gas but the flow rate depends on the temperature and pressure.

And if the temperature and pressure are changing flow rate will change and when flow rate is changing Reynolds number will changing the f value which use using the chart and any other correlation will also get changed. So important is putting some iteration method to estimate the value of Reynolds number that is satisfying the condition of the flow rate. The flow rate at which the fluid is being produced another thing is NRe in the SI unit system or in the field unit system.

If it is represented in field unit system the rho VD by Mu kind of system will be there we can adjust the other terms when it is at US field unit system because of the unit chosen to represent different parameter those are appearing in the expression those are empirical coefficient may appear in that similar thing is appearing in there because of the adjustment of the parameter the diameter some time it is in inch sometime it is fit we are not converting using Di small that means the diameter in inch the numerical coefficient that is appearing will get change.

So we should be careful when we are dealing with a difference unit system the expression may end up with some numerical coefficient because of the unit chosen to express one of the parameter that is appearing in the expression. So let us go ahead with this understanding we want to know the Reynolds number if Reynolds is none we can calculate the friction factor.

Of course the effectiveness ahh roughness of the pipe or the effective roughness of the pipe is also required.

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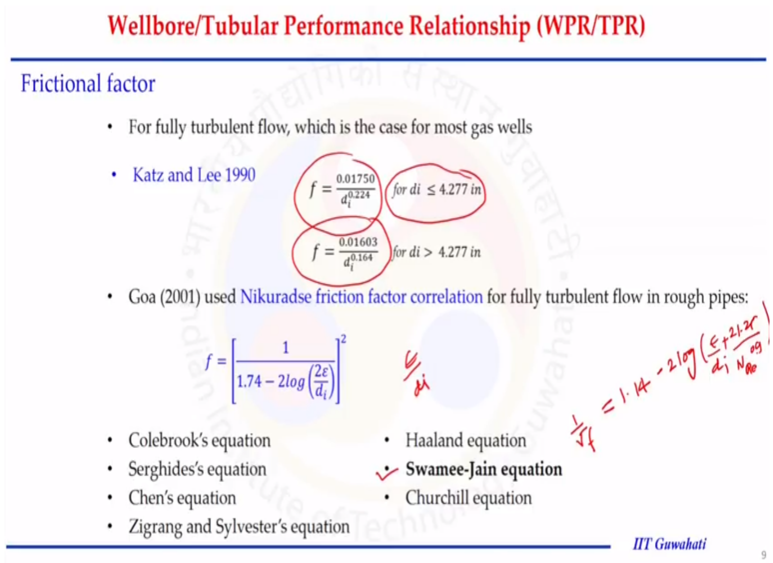

This is not only the way we can calculate the pipe roughness or this is not only the way we can calculate the friction factor there are several correlation depends on this chart or on other understanding reported in the literature that is how to calculate the value of F. As already mentioned the gas mostly flowing in this wellbore is mostly follow the turbulence flow and the fully turbulence flow several expression given in the terms of diameter of the pipe in terms of the effective roughness of the pipe or in terms of Reynolds number or the combination of all three.

So here Katz and Lee they had given the expression for fully turbulence flow condition and the expression is divided based on the diameter that is they said if the diameter of the tube which is used to produce the gas is lesser than 4.277 less than or equal. The expression can be F = 0.01750 divided by diameter to the power 0.224 and if the diameter is greater than 4.277 the expression will be different and that says that numerator part as well as in the denominator the power of diameter will also get change.

So depend on the diameter Katz and Lee had given this expression Goa use the Nikuradse friction factor correlation for fully developed turbulence flow in rough pipe and that they had consider this is not depending on along the diameter of the pipe but on the effective roughness but this is described by f silent by di and the f salient value is something that is specific to pipe that is used to produce the flow.

So either it can be calculated or should we supply by the vendor but this relationship simply says the f can be calculated just by f / di relationship we do not need the Reynolds number but there are several correlation are reported in literature and there are develop based on the understanding of the flow through the pipeline how pipeline roughness and diameter and the flow regime that is laminar transient and turbulence occurring in the system the correlation can be used.

All are good enough in certain condition especially the Swamee and Jain equation is considered better than the Colebrook equation and Chen's equation or some other all are good but depend on which one is chosen what is the formation is a level and what accuracy the results are required as mentioned Katz and Lee they mentioned depends on diameter only Nikurdse relationship depends on f salient by di only and similarly if we write about the Swamee and Jain equation that says how the friction factor is related to the other parameter like 1/ root f = 1.14 – 2 log f salient by d, di of course it is internal diameter + 21.5 / Reynolds number to the power 0.9.

So swamee and Jain consider Reynolds number also in the estimation of friction factor and that is knowing the expression for friction factor and correlation propose in the literature we can calculate the friction factor value.

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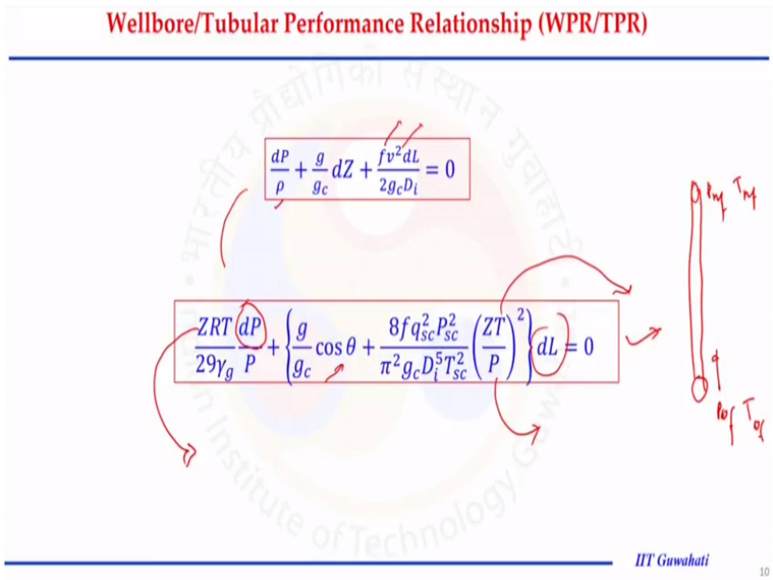

And once we know the friction factor value now in this friction expression we know how to replace rho how to replace f how to replace velocity and when we did arrangement of this equation replacing this terms we could get this expression. This expression simply says here this is pressure and this is length so the entire tube length in a incline condition because the cause the that is appearing here in the incline position the expression says how pressure drop is related to length for the entire length of the wellbore.

If we can solve this equation we can get the relationship but see here the expression is still having the complexity it cannot be solved so easily because the Z is appearing here the compressibility similarly the temperature similarly the pressure we know the flow through the well bore is having because of the pressure for example we are having this vertical, tubing and the flow is happening of course the Pwf is higher than Phf and at intermediate position the pressure should be different or between Pwf and Phf.

Similar the temperature is also different here that is Twf and Thf temperature is also different they cannot be taken constant and when temperature and pressure cannot be assume constant in a actual case then how compressibility can be taken constant. So after adjusting the parameter in the mechanical balance equation we could the establish the relationship for a single gas flow system.

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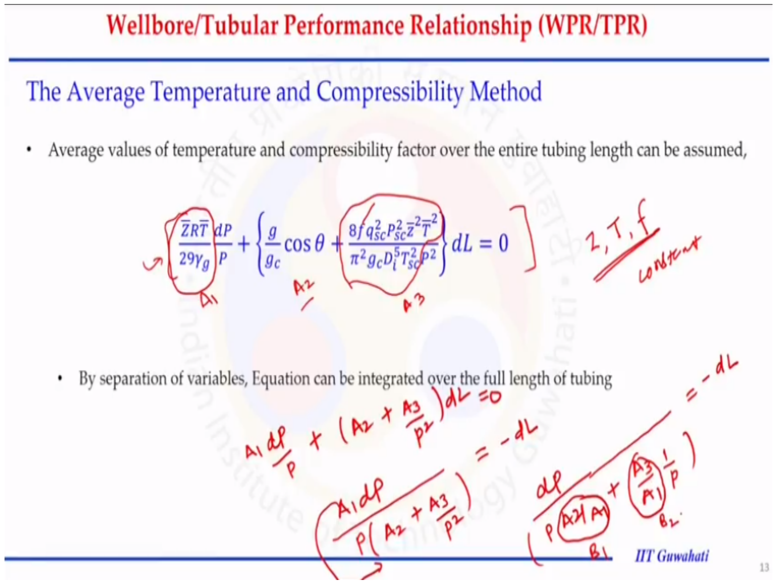

This is an ordinary differential equation which is governing only through the single gas flow through the tubing it is equally applicable for the casing and annulus as already discussed we just need to replace the diameter by the hydraulic diameter. This equation is still very complex because our fluid is real fluid it is complex in the nature it is properties changes when we are going to deal with fluid at different temperature and different pressure conditions.

The fluid is compressible that is the density and viscosity very along this tubing line because the temperature and pressure also changing some assumption can be made like temperature is changing linearly with respect to the lenses we are going down from the surface the length is increasing temperature is also increasing that we can approximate it is happening linearly but compressibility that is the function of temperature and pressure need to be calculated locally.

And that is put the condition on solving this equation and this equation cannot be solved as an ordinary differential equation this is the difference between the ordinary differential equation we establish when we are solving for the ideal gas but in this case we are getting a complex system. So certain approximation are required to solve this equation there are several approximation have been reported in the literature to solve this equation but two of them are widely use in the gas industry one of them is the average temperate and compressibility factor method and the second is Cullender and smith method.

We will go through one by one about both the method so the next method the average temperature and compressibility method that is assume the calculation of temperature those are appearing in that equation previous equation we had seen temperature was in inside pressure was inside and the compressibility was inside. So after assuming like any mean we can calculate the value of compressibility, temperature and f constant value we can assume the entire length temperature we can just average out by choosing well head temperature and the bottom hole temperature.

The average value similarly can be done or compressibility also and for a local segment in the tubing length if there are other ways available we can calculate the average value of compressibility and temperature. And when we can do that the thing we can take those term out of this derivative term and this equation will take this same. And in this equation if we also assume F is constant then you see we can take this is entire term as constant.

Because as we know the constant value of compressibility R value is numerical value temperature value will be numerical value and if we know the composition of our gas we know the gamma g. So we can write this as a constant and this is A1 similarly this is also constant value A2 if we know the inclination if it is horizontal particular angle we know this is going to be a constant and this is A2 and here also other than P square we can say everything after putting this stand up pressure. temperature, pie value, compressibility factor value, average temperature value.

This is going to be A3 if I write this equation in a simplified form here in this equation we had assume all these are having constant value. So if I arrange this equation this will take safe line DE/E + A2 + A3/ P square by dL = 0. Now we can apply the separation of variable concept to solve this equation. Let us further simplify this equation and if I do so what I am going to get is A1 dp/ P A2 + A3 / P square = - dL.

Before we are going to intricate let us further simplify this we can take this A1 out here and we will get this equation in this form PA2 / A1 + A3 / A1 1 / P square. And on the right hand side we are having – dL so we can lump the parameter again and A2 / A1 we can write as a B1 and A3 / A1 is B2.

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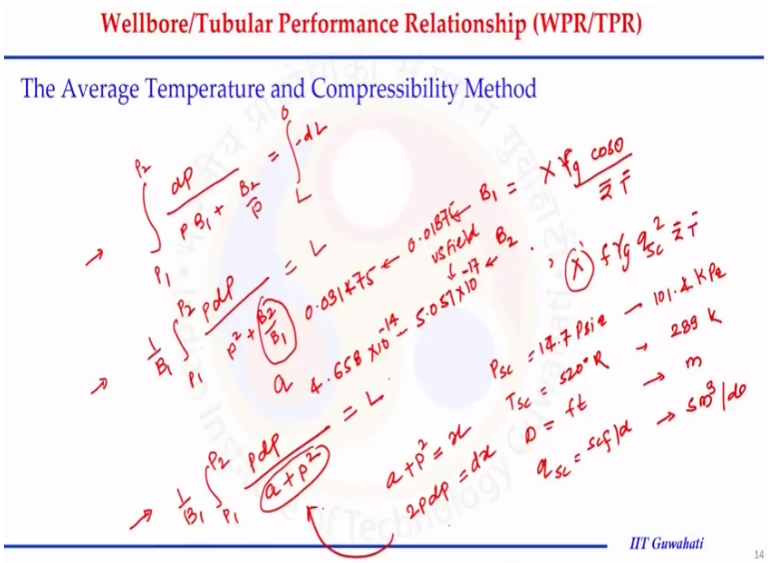

So the equation will take the form of dp / PB1 + B2 / P and on the write hand side we are still having the – dL. Now the B1 and B2 we had lump the parameter the numerical coefficient and we can write what will be the form of B1 and B2 so if we write this will be having some numerical coefficient X gamma g Cos theta by average compressibility and average temperature.

Similarly the B2 will be having some numerical coefficient f gamma g Qsc square z bar T bar and this numerical coefficient that X and we can write here X prime depend on the unit system as been chosen. So if we take filed unit system the B1 value will be 0.01875 in the US field unit system and the B2 value will be 5.057 10 to the power -17 again in US field unit system and if we convert that entire expression to SI unit system the value will be 0.031475 and for B2 in SI unit system it will be 4.628 10 to the power -14.

Depend again on the unit system chosen even for individual parameter if we are changing the value of measurement for example the diameter is changing from feet to inch the numerical coefficient will get change we understand in US field unit system the standard value for pressure is 14.7 psia we can put that value temperature is 520 degree ranking the diameter again 14 feet again Qsc in scf per day.

While in SI unit system pressure value will be measured in kilopascal and that is equal to 101.4 for the standard condition and temperature is 289 Kelvin diameter is measured in meter and standard cubic standard flow rate measured in standard meter cube per day. So depend again the unit system we are choosing the constant value B1, B2 will get change. We do not need to worry about those parameter at this moment when we are adjusting the equation finally we can care about it.

Now our equation is here that should care and how to intricate this equation from P1 0.1 to P2 or from length L to 0 we can intricate this equation. For further simplification what we can do we can take this term out like B1. So we left with P1 / P2 dp upon so we take B1 out will be having pdp p is square + B2 upon B1 and we can consider further this B2 by B1 is like A another parameter just to lump B2 by B1.

And on the right hand side by integrating this we got the value L so on the left hand side we are having 1/B1 integrating it from P1 to P2 we got pdp upon a + p square right hand side we are having L. Now to further solve this equation we can assume a + p square = some parameter some local parameter we can say x and if we differentiate this we get 2pdp = dx.

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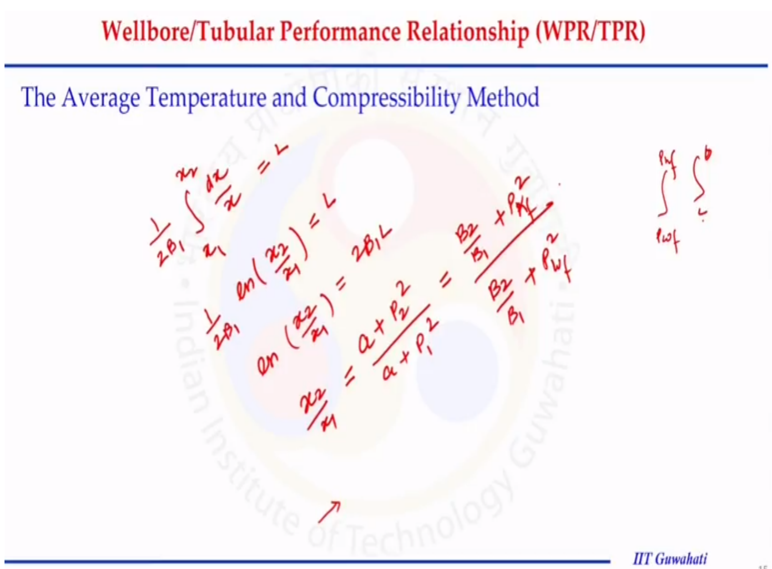

Using this we can convert this equation in the form of x and when we are doing that we will get 1 / 2B1 dx upon x integrating x from x1 to x2 and right hand side we are having L. The integration will give us 1/ 2B1 ln x2 / x1 = L by adjusting this equation we can get ln x2 upon x1 = 2B1L further x2 upon X1 that is equal to if we write in terms of pressure is a + p2 square a + p1 square.

Further A = we know B2 / B1 so we can write B2 / B1 + P2 square B2 upon B1 + P1 square now integrated L from L to 0 it means the pressure range should be at L pressure is Pwf and here it is Phf. So the P1 will become Pwf and P2 will become Phf after adjusting this equation what we are going to get is equation like this.

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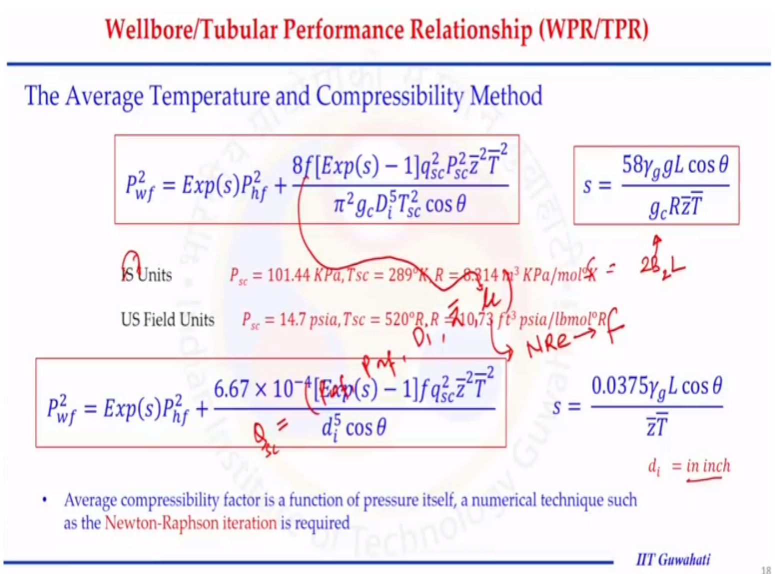

You can do the adjustments getting B1, B2 value and everything if you adjust we will get this equation this is how Q flow rate is related to Pwf Phf diameter of the pipe and the fluid properties like the compressibility as well as viscosity. Viscosity is related in the form of because to calculate the Reynolds number the viscosity and Reynolds number is required to calculate F. So the F which is appearing here is in the form the implicitly accounting for viscosity.

And the S here which is appearing here this is similar to what we got in terms of B1B2 parameter this is equivalent to 2B1L and after putting the value of B2 we are going to get the same expression here. We can understand how this equation is going to help us to establish the relationship. Now we could solve the equation, the equation which was not solvable with the assumption of constant temperature and constant compressibility the equation could be solved.

The equation had given us the relationship between the flow rate and the other parameter those are responsible for this flow like Phf, Pwf diameter compressibility and viscosity again the numerical part or the numerical coefficient appear in this equation depend on the unit system chosen to represent to given parameter I am emphasizing again the DI it is some time given in inch forms sometime it is consider in the fit unit and the adjustment should to be made accordingly.

The SI unit system this should be SI unit system where pressure temperature and value can put here and in field unit system we are having different value like pressure is measured in psia temperature in degree ranking and R is having fit Q psia per LB Mole degree ranking in field unit. If we put this in this equation we will get the final numerical coefficient that is appearing in this equation.

And this is again in the US field unit system were D is considered in inch the average compressibility factor is function of pressure itself to the problem is not solved so simply we could solve the equation. The equation which was are looking like a ordinary differential equation but not having any analytical solution we could solve and equation considering the constant temperature and constant compressibility apply the separation of very well concept and very well solution.

But here the problem with respect to compressibility is still there that is compressibility is the function of pressure that is numerical technique like a Newton Raphson method or other technique is required we can help us to perform the iteration and to find out should the value of Pwf or any parameter that is missing here should be. We will understand this with the help of an example.

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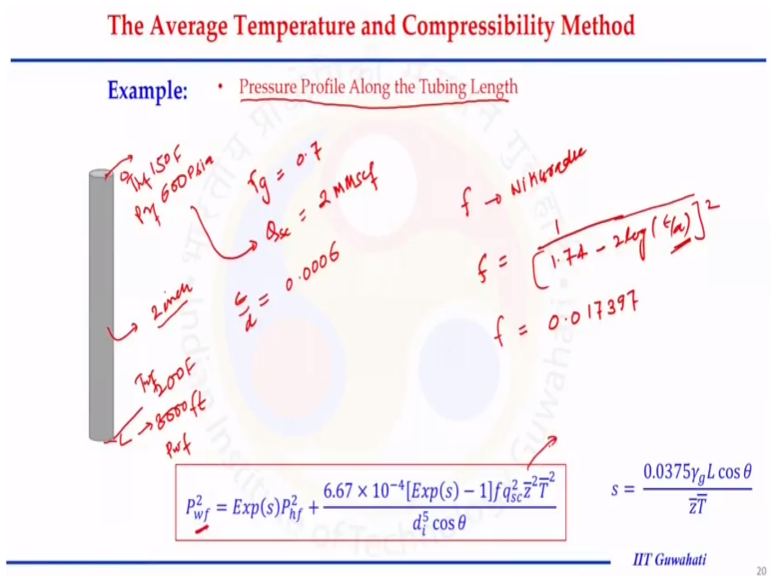

So the example for this problem (refer time: 52:40) is chosen from the both that is says that develop the pressure profile along the tubing length and some values related to this problem are given to us. So let us say this is the well bore which is spread over the lane from 0 to L and the length is given like it is deeper up to 8000 feet from where this is producing the fluid and the fluid which is produced is having the specific gravity as 0.7 and the production rate Qsc is 2mmscf.

2mmscf is 2 million standard cubic feet and the temperature at the top is given that is 150 F and at the bottom is 200F the pressure is also given at well head condition and the well head condition says pressure is set H600 psia to have this production Qsc at 2mmscf per day other things are given the diameter is given as 2 inch and the roughness of the pipe is also given f silent by d is 0.0006.

If the value is not given f silently the value is that should be assume as an standard value. Now we were ask pressure profile along the tubing lens as we are going from top to bottom what is the pressure at each section which we are going to consider or ultimately what will be the pressure of this wellbore Pwf is not given to us this is Phf is given Thf is given Twf is given. So if we look our problem or our mathematical expression where we say we have the expression with the help of average temperature and average compressibility method where Pwf is related to Phf and other parameter.

Important point comes we need to need to know the value of F we can put in any form formulate given in the literature but what we can do we can use one of the accept it and simplest form that does not need any value of Reynolds number even if it is required because we know the Q value in this problem. So let us use the value of F from the expression given by Nikuradse and that is F = 1 upon 1.74 – 2 log f salient by d and this is whole square.

See in this expression there is no Reynolds number appearing just only the roughness of the pipe and in internal diameter of the pipe or other way effectiveness of the pipe is considered. We know this value we can calculate various state value of F that will come as 0.017397 we can perform this calculation and we will see the value is coming like this. Now we know the F value we know temperature at the top and bottom we can calculate the average temperature now how to solve this problem.

How to know how the pressure is changing as we are going downside prompt surface to wellbore bottom for that we can look at the expression we can say at any particular length from the surface any particular length Pwf can be calculated we need to know the compressibility other things are known and that is where we can click the trial and error or iteration processor because compressibility depends on pressure itself at a particular location what is the pressure then only we can calculate the average pressure and that average pressure is at the compressibility can be calculated.

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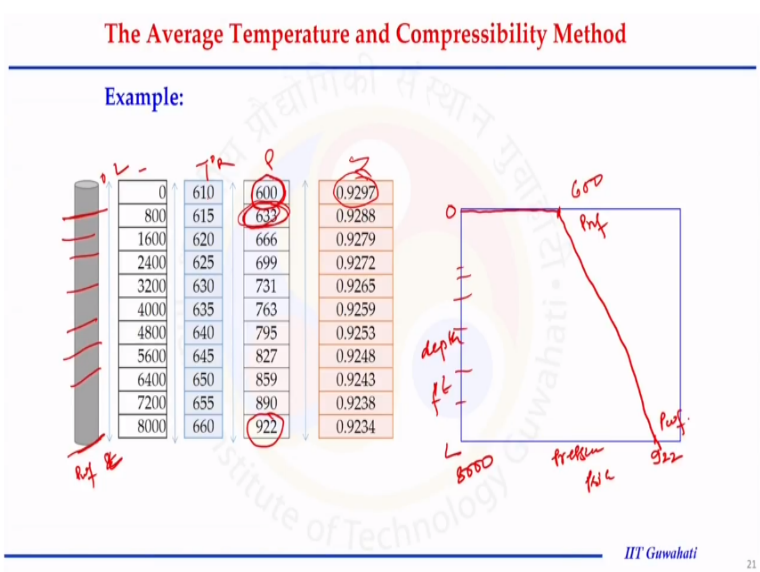

So for that purpose what we can do we can design ahh spread sheet and that spread sheet says your length this is length or this length data from 0 to L that is 8000 divided in N number of equal parts and we can choose 10 or may go up to 100 or any number 10 is something that will allow us to fast calculation we can write a mathematical program to solve this kind of the problem because every time we need to do iteration and for that purpose there should be either in excel or in Mat lab or somewhere the program that perform the Raphson method or trial and error iteration procedure.

So let us say first thing we divided this entire length from 0 to 8000 in 10 segment and we assume the temperature changes linearly with length. So the temperature data can be obtained very easily this temperature is degree ranking at the surface the temperature is 610 under the water it is 660. The next is pressure here I had shown the entire result but we know only this value 600 Psi as a Phf at the surface or at the well head the pressure is 600.

Now at this pressure at this temperature we can consider these are the average value we can calculate this compressibility factor. Now how to calculate the compressibility factor we have to go back in our natural gas properties class where we understood the methods how to calculate the compressibility factor one of the method is Brill and Becks we can use that method considering the critical temperature and critical pressure of the system that can be calculated because we know the gamma g of the system and based on that we can calculate the z.

Now this Z should be calculated at average condition so what we will do at the next level we will some value of P2 and calculate the compressibility factor. And we will see the compressibility factor calculated by perform the iteration for this psi should satisfy the function value and if it is satisfying we will get Pwf at that length. And similarly we can go down and at the end we are going to get the value of Pwf at the bottom of the well.

So now once we could do this calculation we got the length we go temperature we go pressure we can plot pressure profile as we are going down from this 0 length to L. We know at 0 surface we are having some pressure. So let us say this is depth and this is pressure range in psia and this is in feet. So we know the total length is 8000 at surface when the length is 0 we are having the pressure that is 600 and after performing the calculation we came to know at the bottom pressure is 922.

And intermediate length we can put the value of pressure and we can get the profile that is saying how the pressure is changing from Phf to Pwf and that is the way we can solve the problem for any length if the Phf and Q and other parameters are given to us. Let us see another example where we will understand in another way how we can choose this particular equation to calculate the unknown parameter or the missing parameter that is not available.

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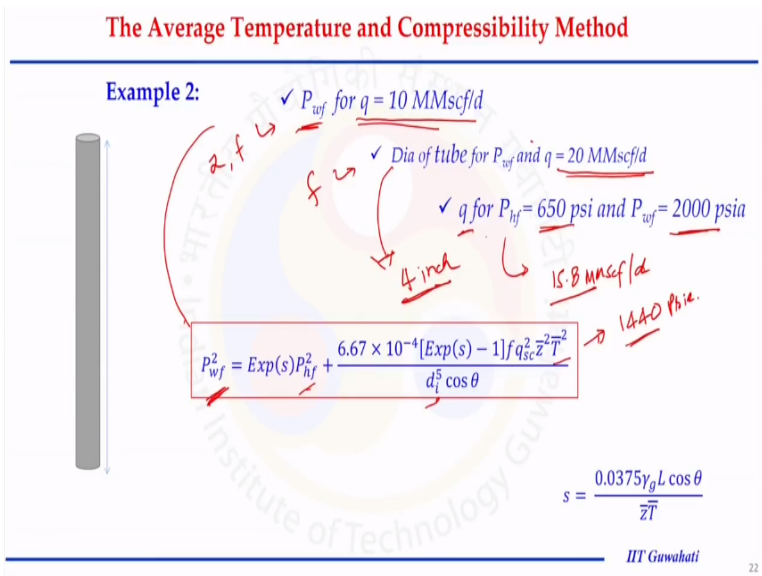

This will also help us in designing the system for example in the second case when the diameter is chosen to a system when q is known Pwf is known. And in the first case when Pwf was ask what should be the Pwf when q is given we can do the same thing we can divide this length in a N number of part perform the same iteration process as we did in the last slide and we can calculate the Pwf.

What in the other case when it is Pwf is given q is given diameter needs to be calculated again this same set of the equation can be used where Pwf is known not the diameter we can perform the iteration and the iteration is recorded it cannot be solid state iteration is required if we are going to use how to calculate the F. If F value is constant the friction value is constant we do not need to perform the iteration that is not the case we are having F that depends on the diameter or internal diameter of the tube.

So when we are going to calculate the internal diameter of the tube we have to perform the iteration with respect to F to get the optimum solution or the size of the diameter internal diameter of the cube that is going to satisfy at that Pwf at that Phf is at that flow rate can be achieved and this Pwf case is in first case we need to perform the iteration because we need to calculate the compressibility F we need to perform the iteration because we need to calculate F.

In case of third when Phf Pwf are given and we are supposed to calculate q we can use the same equation we can perform the iteration again and the iteration are record because we know the Reynolds number value or the F value if we are going to use the complex formula where we want the F is a function of Reynolds number and to calculate the Reynolds number value we need the q value that is with respect to q we need to perform the iteration.

If not then we can solve it in a much easier way and we are doing this exercise by using this equation we know several parameter those are known to us like in the first case we are having Pwf should be calculated others are given to us like average compressibility calculated by iteration processors otherwise T average diameter of TU Phf given to us. And in that case we can be perform this is left hand exercise this we will get the PWF value as 1440 while in other case when we are doing with respect to diameter and answer will be around 4 inch tube is required to achieve this flow rate when PWF whatever the Pwf is given we had chosen that.

So this PWF is used here and in case of when q is supposed to be calculated when the flow is happening between this pressure gradient we are going to get the value of q S15.8 mmscf per say we can perform the iteration and you can solve this equation considering the parameter given in the previous case. Only the parameter or the parameter can be asked to put for this solution purpose and the answer I had given this is 4 inches diameter flow rate will be 15.8 and for the first case the Pwf will be around 1440 psia.

That is where the average temperature and compressibility method can help us solving that equation even though it is an approximation is still it works and widely use in this gas industry.

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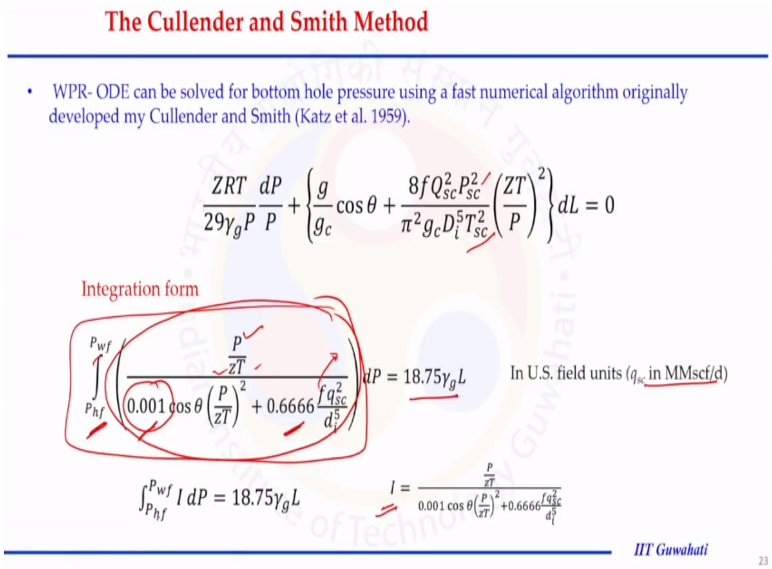

This second method that is also use in industry very frequently for such kind of calculation is Cullandar and smith method. So the wellbore performance relationship as we know is an ordinary differential equation cannot solved just by doing the simple ODE processor because compressibility and other factor depends on local condition and pressure that is changing.

Similar procedure as we understand for the previous it could not be solved but making certain approximation we can solve it Cullandar and smith also made an approximation and that approximation says why we cannot arrange this equation in the form of this where we are interrogating from Phf to Pwf and the equation is going to take this shape. This is just adjusting average related to pressure on one side and length related to another side.

We can do that arrangement and we do so the numerical coefficient appearing here depends on the unit system that has been chosen this is where the US field unit system where Qsc is taken in mmscf per day and the numerical coefficient appearing because we have put the value for the standard temperature, standard pressure and other parameter.

Now in this equation we will see PzT they are the function of local condition the compressibility at local condition temperature and pressure will also change and similarly f if f is the function of not only the roughness but also the Reynolds number it cannot be considered as constant. But altogether by arranging this way we could get integral form of equation that says if you consider this entire as I integral and we put Phf to Pwf that will be equal to right hand side.

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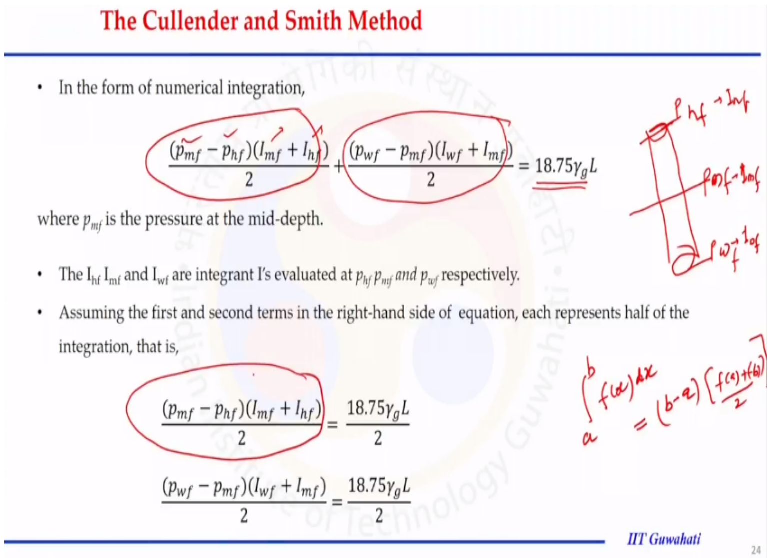

After applying the trapezoidal rule we can approximate this entire tubing length in two parts and at the midpoint let us say this is midpoint this is head point this is bottom point here the pressure is Phf or any property is Phf here it is Phf and here it is Pwf. So this is if you integrate this previous integrant this Phf we will get IHF. If we integrate and midpoint we will get IMF and similarly at the bottom it will be IWF.

And this can be done with the help of the trapezoidal rule that is simply says if you integrating any function from a to b dx that will be b – a the difference in the range function value of a function value at b divided by 2. So similarly everything has been done if you see here the entire section is divided into two parts and it is assumed to upper part and lower part are having the same we will see next.

Let us see first after applying the trapezoidal rule we got this is the difference in the limit it minus b kind of the things this is the function value of Midpoint and top point divided by 2 similar for the bottom point it can be done. So we can divide into two perform the integration and that is equal to right hand side is going same. Here it is assume like the first part and second part they are equal in terms of the value we are obtaining after performing the numerical integration.

And that is assumption says this is like this and this is half of the right hand side similarly the other term here that is half of the other hand side. If the sum both of them we are going to get the same thing. So assuming this first and second term in right hand side of equation each represent half of the integration that is give us the flexibility to have to algebraic equation those allow us to perform calculation to get the value of Pmf.

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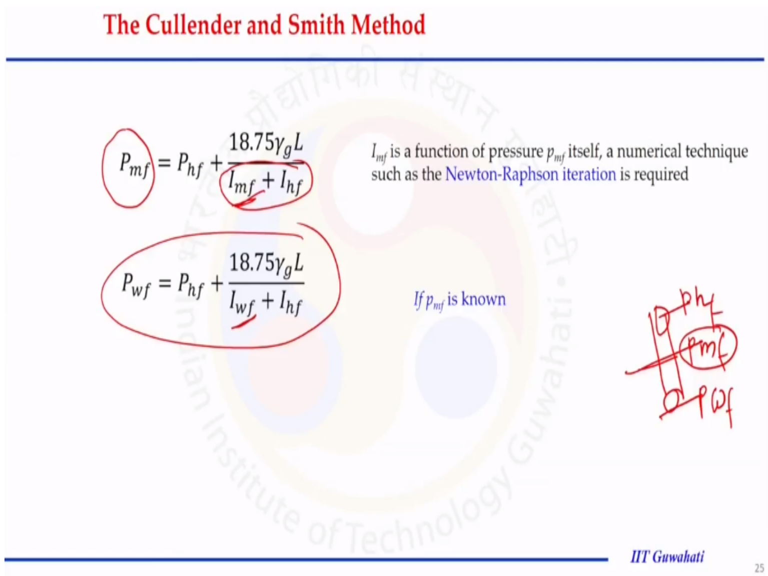

The pressure value at the middle point we already know pressure value at head and similarly we can calculate the value at bottom point but how to get this is still there is a problem because when we adjusting those previous equation in the form of Pmf we are getting Phf + 18.75 gamma g L and the summation of both the integrate here this says the IMF when we are calculating the IMF that I a function of pressure itself.

We know the pressure here Pmf then only we can perform the integration at that time that is why again the newton Raphson or any integration process is required that allow us to calculate the pressure at that condition which satisfy the interaction condition and once we know Pmf we can put in this equation and we can get Pwf this IWF is appearing here IWF is a function of Pwf we have to perform iteration again for this condition also.

But performing the Cullender and Smith method we can divided the section in two part and get this solution.

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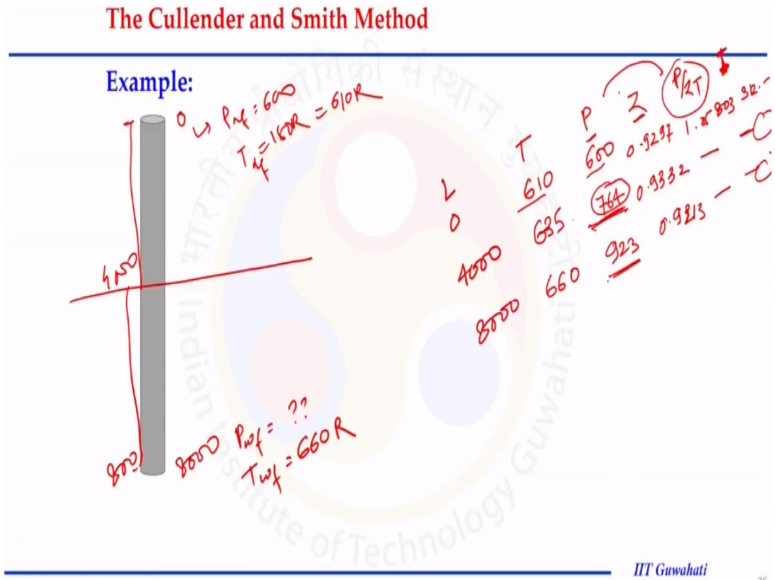

Let us understand the similar problem what we solved in the case of the average temperature and pressure method we can say the same thing where the length is spread from 0 to 8000 we know the pressure here Phf that is 600 the temperature we know here is 150R or other way 610 ranking THF and here the pressure is supposed to be calculated while the temperature is given as here is 660 R.

Now the problem is same we can divide this two parts so this is 0 to 4000 and here it is 4000 to 8000 now what we know let us say we know the length we can divide this length 0, 4000, 8000 we understand the linear approximation with respect to temperature and when we assume the linear approximation we know here it is 610 here it is 660 we can write the value of temperature and the midpoint also that will be the average of this thing.

So if I take the average this will be around 635 now it comes to pressure how the pressure is changing we know to know the pressure we have to know the integration or the integration or the integration or integral function and that integral function depend on P / zT and now we have to calculate P because to calculate this function we have to know the value of P similarly to calculate this function we need to know the Z.

And we can do this similar what we do the iteration processor for the average temperature and average pressure to know that what will be the value of pressure and temperature at the initial condition we know the pressure is 600 we can write that value here and next considering this and this value for pressure and temperature we can calculate compressibility at that point then will come out a 0.9297 we can calculate P / zT at that point 1.05803 and integration at that point is with 312 point something.

Now for next at this length we do not know what should be the value of that we can perform the iteration with respect to Z we get the value of pressure here that will come out as 764 and the value of compressibility at that will be 0.9332 at accordingly we can calculate the P / zT and integral. And further at the length of this we can calculate this value 923 will come after performing the iteration under that condition we will be having 0.9213 we can get PzT and this and we solve those algebraic equation because now we know this values IMF and IWF we will see the value of Pwf 923 and Pmf is appearing the same.

So now both the method have been discussed there are not only this two method there are several other methods those also allow us to calculate the pressure profile when the flow is happening in the wellbore. Several method are just like Cullender smith method this suggest how to divide this entire length and N number of parts not necessary those equal interval is needed for those small segment we can calculate performing the trial and error or iteration process is there the pressure that each segment as we did or the temperate and compressibility method.

And as we did for the Cullender and Smith method those we are not going to discuss in detail but there are some methods available I think cullender and smith and the average temperature and the average compressibility methods are good enough to solve our purpose.

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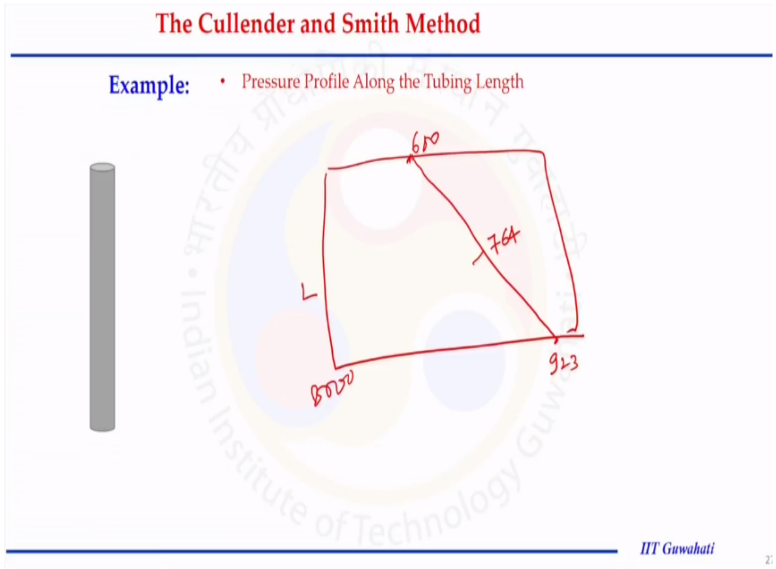

If we go further we can get with the cullender smith also the same temperature same pressure profile we can obtain with respect to L we know at this point what is the pressure 600 and when we reach up to 8000 feet we are having almost like the same here we got 923 value and at the midpoint we got the value at 764 we can draw the profile. Once we could establish this relationship we know this balance equation is functioning well it is representing what is happening in the wellbore condition.

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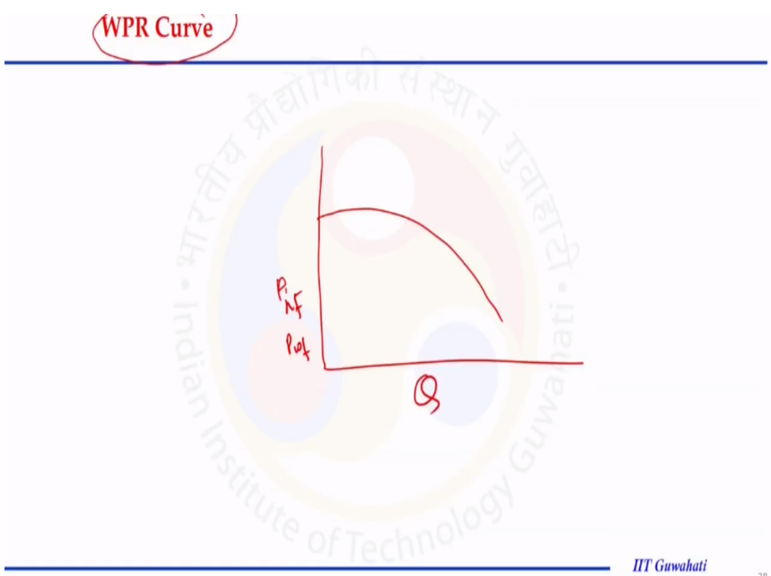

Another advantage of this is drawing the WPR curve so now we know at different Phf or different Pwf how Q is changing and knowing this curve relationship will help us to understand or the designing of WPR system we will discuss this WPR curve later on when we will be discussing about the nodal curve analysis.

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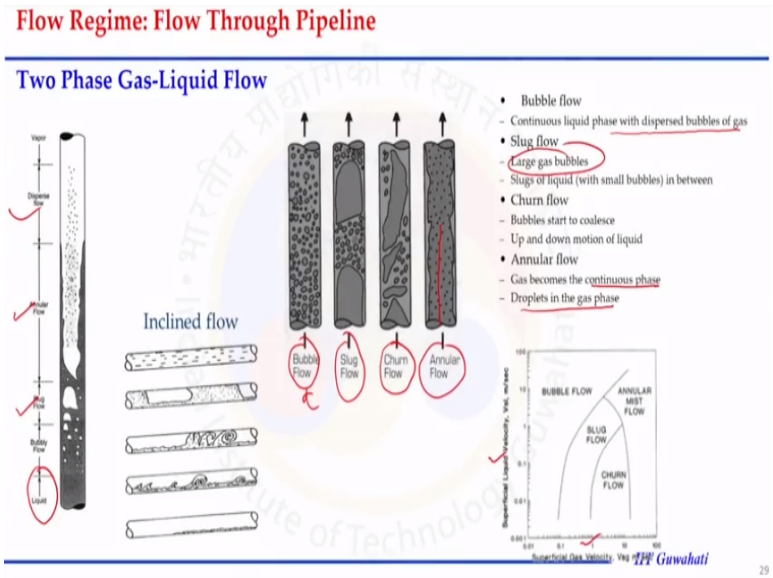

So with this what we did so far we understood how to setup the mathematical equation for WPR just considering it is happening under steady state condition there is only one single phase gas is flowing through this well but that is not the situation if we even dry gas well also produces small amount of liquid that could be water or could be oil or the condensate.

In the surface we are not seeing any liquid but during the production there might be some zones where the liquid is coming out the gas phase and getting accumulated. So when we are having the two phase system like the gas liquid flow system we may have a very complex system it is not two phase gas system that we can understand how the flow will be happening it depends on the superficial velocity of gas phase and liquid phase what is the critical velocity is the gas is having enough energy or velocity to carry the liquid accumulating in the tube to the surface or it is just getting accumulated in the system.

An depend on the relative superficial velocity or the flow regime in the system we can get different type of the two phase system and each phenomena that is happening or shown here is have been it is unique characteristics to define the system. So for example in the case where the two phase is adjusting in the system we can have a bubble flow it miss continuous liquid phase with disburse bubble of gas this kind of flow is happening in the system.

If we go further large gas bubbles are called and we are going to get the gas flow there might be possibility of join flow where the bubble start to collage where up and down portion of the liquid is happening and the annular flow where gas becomes the continuous phase and drop let in the gas have are called and we have seen the place is happening in this manner. That again depend on the relative velocity of the gas and liquid in case of the gas reservoir the gas should have enough energy to get flow otherwise the production may get seized.

This is not only the possibility there might be possibility even within a single wellbore we are getting a different zone where the bubbly flow, slush flow, Churn flow or just vapor on the top and the liquid at the bottom is happening and for that there are complex with we are not going to discuss those in detail of course depend on the implication of the tubing most of the well reservoir of the problems the well is always vertical with the theta angel is 0.

In problem we discuss consider just the vertical well if we flow is happening in the inclined flow the situation will be different the flow resume those are happening will also be different.

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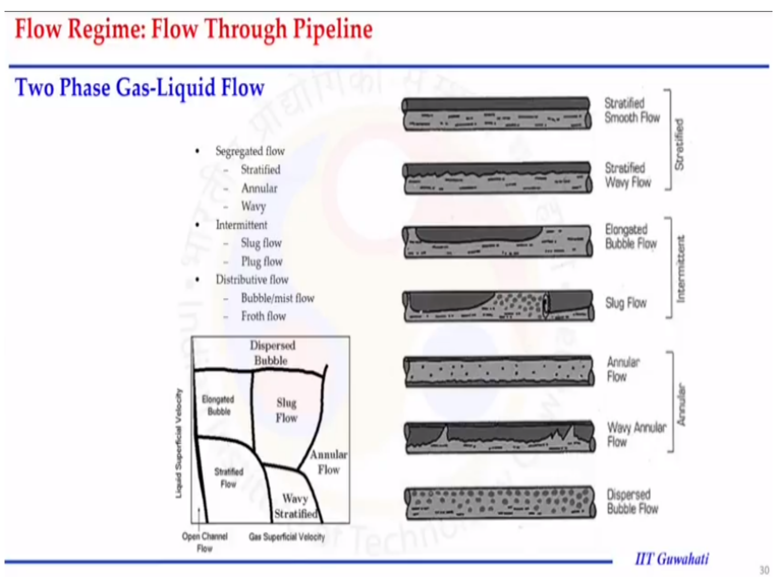

And when we are having he horizontal flow the flow regime will be completely different or the characteristic by which the flow regimes are defined will be different some of this part we will be discussing when we will be ahh dealing with transportation natural gas through horizontal pipe line or through inclination pipeline. But important point here is the flow happening in the tube depend on relative superficial velocity of gas and liquid.

The flow regime or the seepage happening between the gas and liquid characterize the system but we are keeping this phase a separate not considering in the slaves of natural gas engineering.

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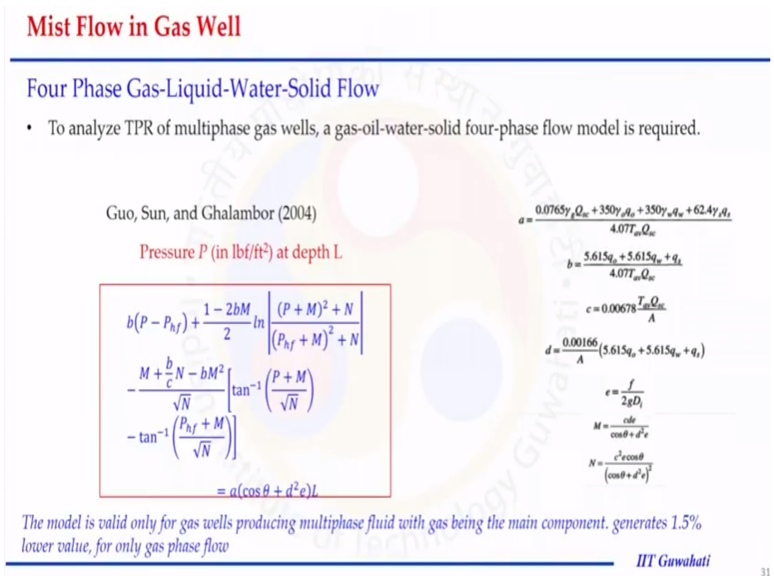

There are more complex situation may happen in the system where not only the gas and liquid but here might be a possibility that all four phases are present in the system even there might be in a small quantity but if they are present they are going to affect the system is simple equipment which we discussed is not going to help us even when we are going to two phase and the question comes when we are going to four phases how to define this system by the mathematical expression.

So Guo, Sun and Ghalambor in 2004 they had develop the expression considering all four phases are present in the system those four phases are oil, gas water and some solid particle those may be cold particles there might be just sent particles and they had arrived at very complex expression considering all four phases and in this case the situation becomes more complex. So we also not going to discuss but yes there are model adjust in the literature those allow us to understand what is happening in the tow phase system in three phase system are in four phase system.

Certain assumption has been taken to arrive those equation for example this four phase model assume the gas phase as a ideal gas and when we are considering this four phase model reducing it to just a single phase model considering on the gas flowing under that condition we see there is a difference in the result that we can get from the other and this method and the primary reason is the assumption made to generate this kind of the expression for multiphase system where it is assume gas phase reserve idea gas phase.

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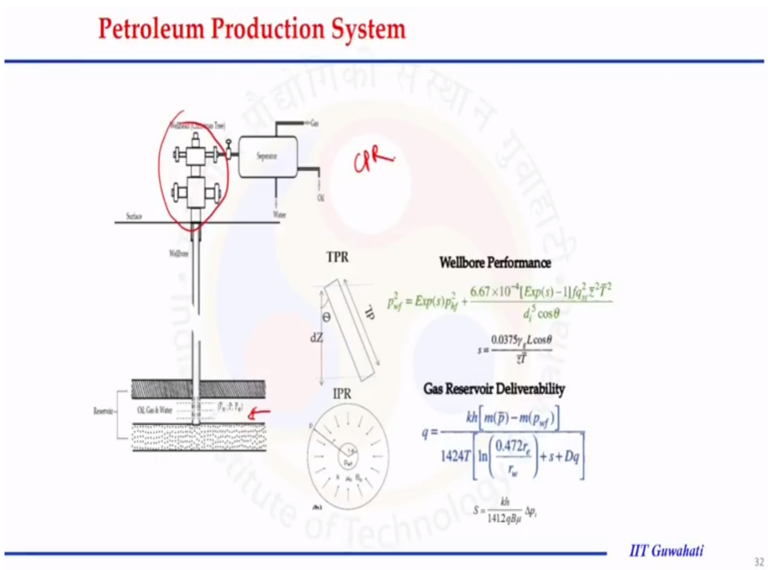

With this I would like to end WPR part let us summarize what we did so far in terms of the petroleum production system we could establish the IPR relationship and in gas reservoir deliberately concept they could see how Q the flow rate in the porous media can be related to some other parameter now we had develop today's lecture the TPR or WPR relationship and in the next lecture we are going to understand the this part where a device called choke is installed and will be having the understanding of choke performance and it is mathematical expression.

Choke is a device that is installed at well head or down hold to cause a restriction to flow a fluid thus control gas production rate. So restriction device is installed that allow us to make the restriction on the production and that is installed most of the time on the well head and some of the time in a different form like SSV sub surface safety wall in the well bore or at the bottom of the well bore with this I would like to end my today's lecture thank you very much we will meet in the next lecture thank you.