Natural Gas Engineering
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Module No # 03
Lecture No # 11
Wellbore Performance Relationship (WPR)

Hello everyone and well back again to the class of natural gas engineering today's topic is choke performance relationship. Choke is a device which is used to restrict the flow this also called as restriction.

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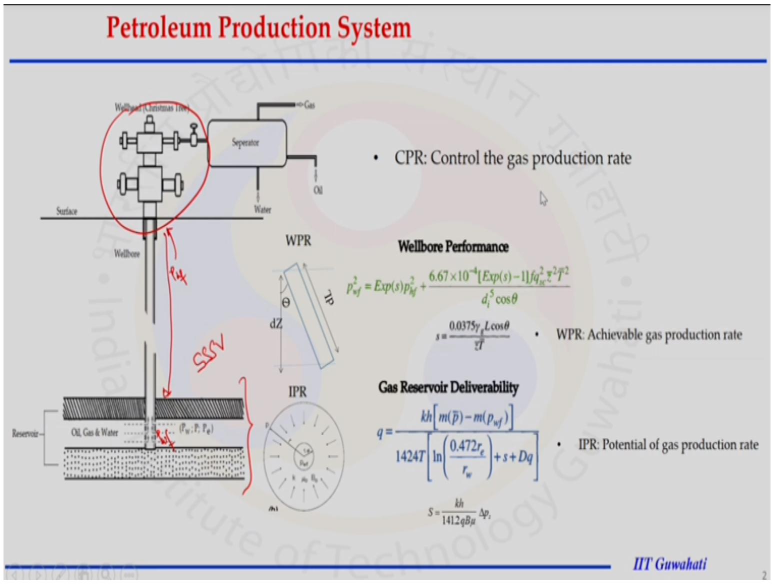

Petroleum Production System

- CPR: Control the gas production rate

Wellbore Performance

$$p_{wf}^2 = Exp(s)p_{hf}^2 + \frac{6.67\times10^{-4}[Exp(s)-1]fq_{sc}^2\bar{z}^2\bar{T}^2}{d_i^5\cos\theta}$$

$$s = \frac{0.0375\gamma_g L\cos\theta}{\bar{z}\bar{T}}$$

- WPR: Achievable gas production rate

Gas Reservoir Deliverability

$$q = \frac{kh\left[m(\bar{p}) - m(p_{wf})\right]}{1424T\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s + Dq\right]}$$

$$S = \frac{kh}{141.2qB\mu}\Delta p_s$$

- IPR: Potential of gas production rate

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So whenever we are having a well ready to flow on the top part of the well if we look our production system we can say on the top part of this well production system there is a Christmas tree and in that Christmas tree several devices are installed one of them is a choke. Choke is installed there to control the production rate other than choke there are some other devices also there to monitor like temperature, pressure, safety wall and other.

The production form of particular well can be controlled not only with the help of choke at the top but also installing self-surface safety wall somewhere in the line of this wellbore mostly at the bottom part of the wellbore if SSSV self-service safety wall is installed that also work as a choke the safety wall can also installed on the top the Christmas tree part where it also used as the safety device.

So let us see what learnt so far we had developed the relationship for our production system and we could establish the relationship for IPR how the flow is happening from reservoir to wellbore we could establish the relationship between pressure draw down and Q in terms of properties of fluid and reservoir. Further moving up we could establish the relationship between this PWF and PHF.

PHF is a pressure at the well head so the relationship called the WPR and we could establish the relationship again between this pressure difference PWF and PHF how they are related with respect to Q and the properties of the fluid and the dimension or geometry of the pipe system. In today's lecture we are going to understand CPR choke performance relation that is installed on the well head and that is controlled the gas production rate.

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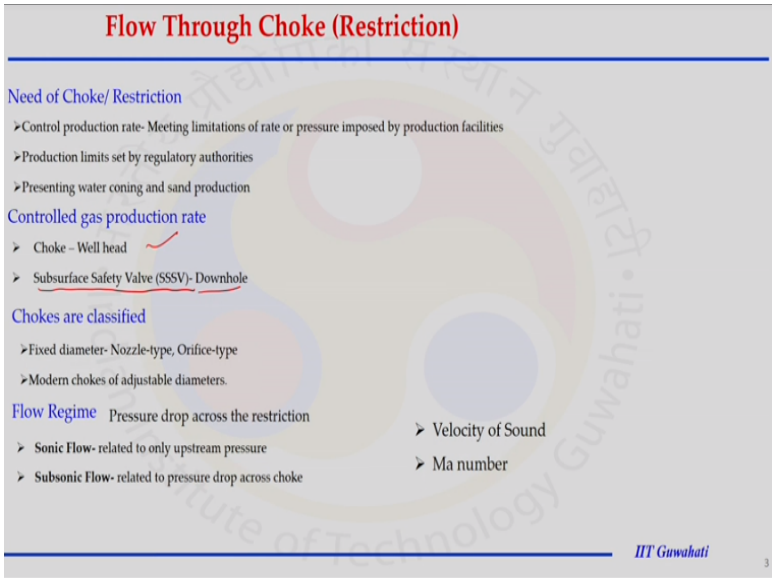

Flow Through Choke (Restriction)

Need of Choke/ Restriction

- Control production rate- Meeting limitations of rate or pressure imposed by production facilities
- Production limits set by regulatory authorities
- Presenting water coning and sand production

Controlled gas production rate

- Choke – Well head
- Subsurface Safety Valve (SSSV)- Downhole

Chokes are classified

- Fixed diameter- Nozzle-type, Orifice-type
- Modern chokes of adjustable diameters.

Flow Regime Pressure drop across the restriction

- **Sonic Flow**- related to only upstream pressure
- **Subsonic Flow**- related to pressure drop across choke

- Velocity of Sound
- Ma number

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Before we go further to understand the mathematical expression how choke device test work let us understand the need of choke and restriction choke control production rate meeting limitation of rate or pressure imposed by production facilities. So the choke is installed on the well head to control the production rate because the surface facilities is installed like this separator and other units those are designed to accommodate the particular quantity of the fluid as well as at what pressure those can be operated.

Thus choke provides the facilities for controlling the pressure as well as production rate. The production limit may be by the regulatory authority they also impose certain restriction on the production side so from a particular gas well reservoir if we are operating you have to install a choke that can meet the limitation impose by the regulatory as well as on the surface facility side. Preventing water coning and sand production this can also be achieved with the help of the installing choke at the well head or in the Christmas tree.

Control gas production rate can be as I discussed can be achieved with the choke that installed on the well head or a sub-surface safety wall that goes down whole and where we installed this as a SD or the objective of both the devices is controlling the gas production rate. There are some other ways other units also there but there was comes under the choke like the Orifice, wall and other all those comes under the choke because choke is generic name for providing a restriction in the flow.

So this device provide the restricted flow and the phenomena called restriction flow phenomena. Chokes are classified as fixed diameter or very well diameter so the fixed diameter like a nozzle type or Orifice type but now modern chokes are coming they are having the flexibility to adjust the diameter and adjusting the diameter we can adjust the flow that.

A flow regime so when the flow is happening through this choke it happens under the isentropic process because of the area is getting reduced the velocity will get increased and the flow can be sonic flow or subsonic flow happens when we are having the mac number = 1 subsonic when mac number is lesser than 1.

When mac number is greater than 1 it is a supersonic flow but that generally does not happen but by saying this mac number it means flow through a restriction can be characterized with the help of understanding the mac number or other way the velocity of mac in that media.

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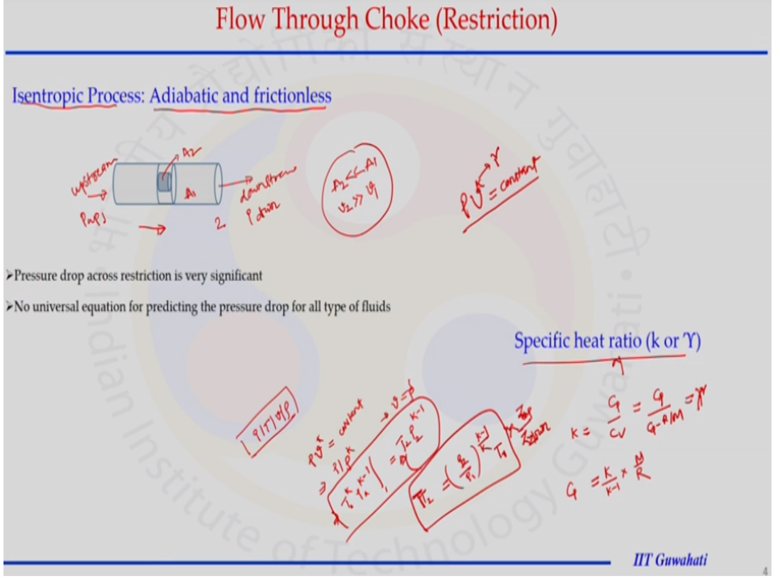

Flow Through Choke (Restriction)

Isentropic Process: Adiabatic and frictionless

- Pressure drop across restriction is very significant
- No universal equation for predicting the pressure drop for all type of fluids

Specific heat ratio (k or ϒ)

If we go further we can see how this process happens so here I have design one small pipeline which is having cross sectional area A1 and restriction within this pipe is having cross sectional area A2 flow is happening from this side to this side. So from left side to right side I can name is a upstream and this is my downstream. The pressure here is P up here is P down and the flow is happening in this direction so now if we see the restriction is imposing or the choke is imposing the restriction on the flow and by the condition of A2 is much lesser than A1 if this is the case happening our velocity V2 will be greater than V1.

So V1 the velocity on at this point 1 and this my point 2 up stream is point 1 downstream is point 2. So we understand this restriction impose by reducing the area in the pipe line we can understand the pressure drop is happening across this restriction and because of that either the sonic flow or subsonic flow is occurring this can be characterize by developing the mathematical expression which type of the flow is happening however there is no universal equation that can predict the pressure drop.

Or in other terms the flow that is happening for all types of fluid that is happening through the restriction for that we have to go to literature to understand what type of fluid is being flown from a particular restriction either is just a water, oil or it is a gas or it is mixture of all those we have to go to literature to understand that thing. However we can understand the basic mechanism that happens when the flow through restriction occurs.

So let us say in this is happening under isentropic pressure condition it means Adiabatic condition and friction less and those are from the class of fluid mechanics or especially from thermo dynamics class they understand how to covert polytrophic process to different process level like isothermal process isomeric process and similar for isentropic process. So in isentropic process the phenomena happens like P is pressure this V is specific volume.

So let us say P is the pressure this V is the specific volume to the power K is constant so the K characterize what type of flow it is if it is polytrophic process K becomes N. In some of the books K is also characterize by gamma and this gamma or K is a specific heat ratio that is property of the fluid that determine how the isentropic process is going to behave when it is flowing through the restriction.

And if we go further we can convert this from one parameter to other parameter for example we know pressure, temperature, specific volume and density they can be interchange with the help of this kind of relationship we can establish how this parameter are related at one condition to second condition. So for example we can say PV specific volume to the power K is constant and we can write this in other formal also that is PY root to the power K.

Because we know specific volume V = 1 / Rho and similar P to the power K – 1 and this is at one condition we can write similar at the second condition T2V2 to the power K – 1 if I denote this as T1 V1 and this is T2V2. So having such kind of relationship we can establish for example in this case if we know the upstream pressure, downstream pressure up stream temperature we can calculate this downstream temperature T2 with the help of expression or just converting parameter in the other form this will be T2 = P2 / P1 to the power K -1 divided by K multiply by T1.

And so having this kind of the relationship we can establish how 1 particular parameter can be estimated we will see this when we will understand how the temperature is going down when the flow through restriction is happening and what we are going to discuss is for the ideal gas the process is very complex our flow is not a ideal flow it is a real fluid where the compressibility should be included and that can be done whenever we need to understand the accuracy of the parameters to be measured otherwise the process is very complex and if we start dealing with the real gas then very complex phenomena need to be understood and need to be solved mathematically.

So for example in this case when we are understanding the temperature at down side T2 that is P2 / P1 to the power K – 1 upon K multiply by T1 and that can be corrected for the real gas just multiplying the compressibility factor Z2 upon Z1 upon Z2. So this is Z up and this is Z down to by correcting for the compressibility factor we can convert the ideal gas law to real gas law at the specific condition or a specific terminology whenever we need we can correct it.

And let us discuss about the specific heat ratio that is exactly characterize what way the isentropic process is occurring and the specific heat ratio is the ratio of CPY CV. CP and CV both are the properties of fluid and if we see they are related like CP upon CP – R by M we can convert from one form to other form and that is equal to K. I already mentioned in some of the book we will find this is gamma.

We can play with variable and can understand if I want to know this CP in terms of K this is CP is K upon K – 1 multiply by M / R. So we can convert from other again one from to other form if the situation when the specific heat ratio like from CP and CV are not known we can still calculate the value of K to characterize the isentropic process and that can be done by having one empirical formula.

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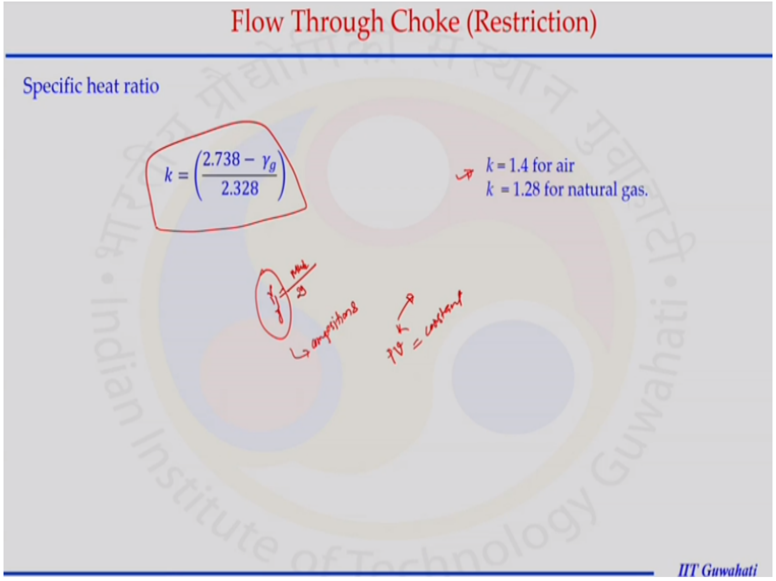

Flow Through Choke (Restriction)

Specific heat ratio

$$k = \left(\frac{2.738 - \gamma_g}{2.328}\right)$$

$k = 1.4$ for air
$k = 1.28$ for natural gas.

$\gamma_g = \frac{Mwt}{29}$

compositions

$pV^{k} = \text{constant}$

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That empirical formula implicit the specific heat ratio K = 2.738 – gamma Z divided by 2.328 where gamma Z is specific gravity of the gas and we know how calculate the specific gravity of the gas that is equal to molecular weight of gas we called it apparent molecular weight of the gas divided by the molecular weight of here that we can write either 29 or 28.97. Now knowing the gamma Z we need to know the K we can put in this expression and can get the value of K.

But how to know gamma Z we understand from our properties of natural gas from our class to calculate the gamma Z we need to know the composition of the gases compositions. So we can calculate the composition by performing the gas chromatography analysis of the gas sample we can calculate the apparent macular weight that apparent molecular weight can be used here to calculate the gamma Z.

Once gamma Z is known to me I can calculate K and once I calculate the K value I understand how my this behavior is going to be this K value now I know the numerical K value I can understand the behavior of the fluid under the isentropic condition of how temperature pressure is changing how to calculate the unknown parameter in the process. Understanding this relationship of K with gamma Z we can calculate the value of K for here that is 1.4 and for typical natural gas the value of K can be 1.28.

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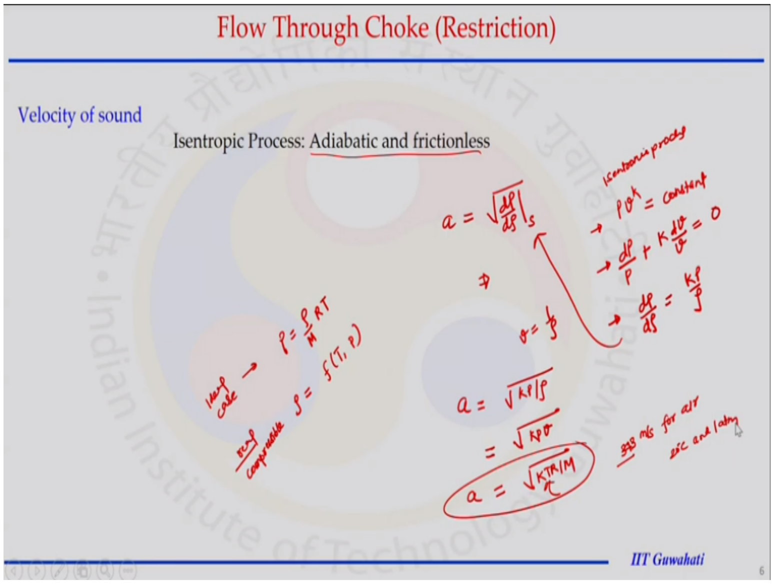

Flow Through Choke (Restriction)

Velocity of sound

Isentropic Process: Adiabatic and frictionless

$a = \sqrt{\frac{dP}{d\rho}\Big|_s}$

isentropic process

→ $P\vartheta^k = \text{constant}$

→ $\frac{dP}{P} + k\frac{d\vartheta}{\vartheta} = 0$

→ $\frac{dP}{d\rho} = \frac{kP}{\rho}$

$\vartheta = \frac{1}{\rho}$

Ideal case → $P = \frac{\rho}{M}RT$

Real compressible $\rho = f(T, P)$

$a = \sqrt{kP/\rho}$

$= \sqrt{kP\vartheta}$

$a = \sqrt{kTR/M}$

343 m/s for air 20c and 1atm

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So let us understand the velocity of sound when the fluid is flowing through this restriction we have to understand the mac number to understand the process is happening under the critical flow or sub critical flow or other way it is under the sonic flow or sub sonic flow for that we need to understand how to calculate the velocity of sound and that under the isentropic process it is adiabatic and frictionless process what we can do we can use the (()) (15:43) engineering definition of velocity of a sound in a particular media under the incentive condition means temperature and pressure condition where we are going to calculate the mac number A = under root DP/D rho under isentropic condition.

And if we are having such kind of behavior what we can do we can use again the isentropic process principle that is how the pressure temperate and volume are going to be changed under the isentropic conditions and again we are doing all this assuming the gas is ideal gas and if we do so what we can do we can say for the ideal gas my P = rho by M RT it means for a compressible flow my rho is a function of temperature and pressure.

This is for the real gas means when the gas is assume compressible and this is for the ideal gas. So let us go back to our isentropic process. In isentropic process we say P is specific volume to power K = constant and we can convert that take the derivative we can say DP / P + K del is specific volume derivative divided by specific volume = 0. And if we adjust it again using the relationship of specific volume and density we can convert this into DP / D rho = KPY rho and if we put this here what we are going to get A = under root KPY rho.

We can convert this again in another form KP multiply by specific volume or under root KTR / M. If we see here in this case the velocity of sound just a function of temperature and if we see when the here is being considered as a gas the velocity of sound in here comes out as 343 meter per second. So this is the value for here and again we have to specify at what temperature and pressure condition it is so this value is at 20% Celsius temperature and 1 atmosphere.

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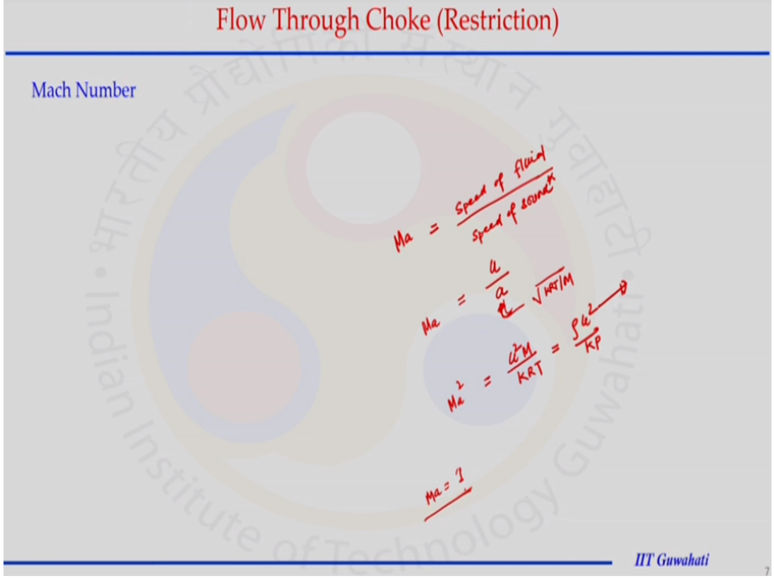

Flow Through Choke (Restriction)

Mach Number

$$Ma = \frac{\text{Speed of fluid}}{\text{speed of sound}}$$

$$Ma = \frac{u}{a} \quad \sqrt{KRT/M}$$

$$Ma^2 = \frac{u^2 M}{KRT} = \frac{\rho u^2}{KP}$$

$$Ma = 1$$

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So now we understand how to calculate the velocity of sound by using this expression now if we go further and can see how mac number is related to this velocity of sound. So by the definition of mac number is defined as speed of fluid divided by speed of sound in the same media under the same condition intuitive condition same temperature and pressure should be maintained to calculate the speed of sound in that media and if we denote speed of fluid by U and speed of sound by A we can get how my mac number is relate.

Now next come how to relate this mac number in some parameter those are easy to correlate for example the speed of sound can be from the previous slide can be replaced by under root KRT / M if we do so we get mac number square = U square M / KRT or in other form rho U square by K. So if we know value of U when the speed of fluid we can calculate the mac number and once we know the mac number we can characterize it is going to be sonic flow sub sonic or supersonic flow.

Depend on if mac number = 1 we are having sonic flow supersonic flow generally does not comes when we are having flow through the restriction. So let us see when mac number is 1 we are having sonic flow condition. The flow through restriction other than mac number it can also be characterize considering the pressure ratio that is going to be in our case we are going to characterize with the help of pressure ratio but remember the pressure ratio under critical condition or sonic condition is the condition when mac number = 1.

So when we are establishing the relationship mathematically or developing a model for job flow performance relationship we can see we can replace some terms in terms of mac number and when we consider mac number = 1 we are going to get the critical pressure condition is the condition where the flow is happening under sonic flow condition that we will discuss next.

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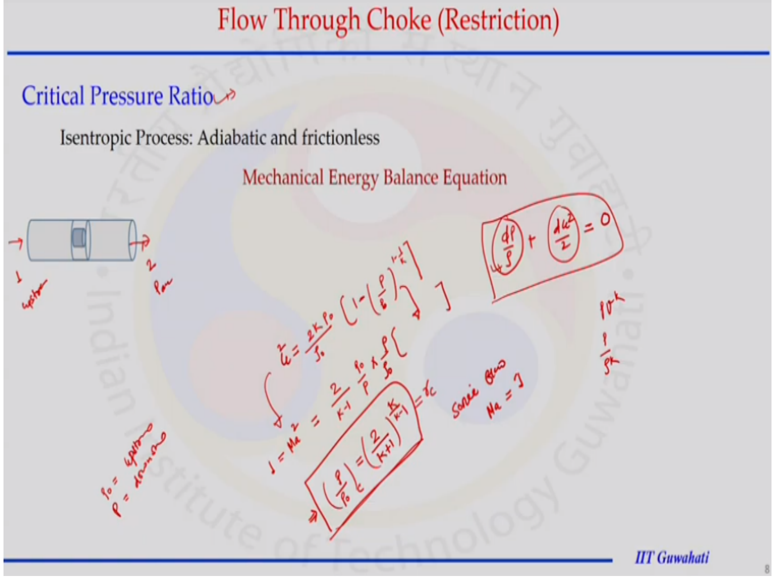

Flow Through Choke (Restriction)

Critical Pressure Ratio

Isentropic Process: Adiabatic and frictionless

Mechanical Energy Balance Equation

$\frac{dP}{\rho} + \frac{du^2}{2} = 0$

$u^2 = \frac{2kP_0}{\rho_0}\left[1-\left(\frac{P}{P_0}\right)^{\frac{k-1}{k}}\right]$

$1 = Ma^2 = \frac{2}{k-1}\frac{P_0}{P}\times\frac{\rho}{\rho_0}\left[\ \right]$

$\left(\frac{P}{P_0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = r_c$

Sonic flow

$Ma = 1$

P_0 = upstream

P = downstream

$P\rho^{-k}$

$\frac{P}{\rho^k}$

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So same process when we are going to develop the relationship for critical pressure ratio isentropic process is happening it means under the adiabatic and friction less condition there is no time for heat to go out there is no heat transfer is happening around the surrounding there is not gravity because we are considering the choke is mostly small device in a horizontal direction only and there is no friction losses is happening.

So simply the mechanical energy balance equation for the isentropic flow that is happening flow through the restriction comes out very simple form that says pressure and energy + kinetic energy = 0. So the change in pressure energy and kinetic energy are compensating each other to have this flow that is why it happens when we are having the upstream is 1 condition to downstream condition the pressure at downstream is lesser than upstream condition that is why the flow is happening and when this pressure reduction is happening velocity will go up or other way because the area is getting reduce the flow rate will also get affected.

We will see that later on let us see when we are having this pressure energy and kinetic energy we can solve this equation under certain condition applying the boundary condition and we can get the expression for how the velocity is changing from upstream to downstream we can calculate the velocity at the downstream how the pressure ratio can be calculated with this expression and similarly how we can get the mass flow rate as well as volumetric flow rate.

For all this equation we have to start with this mechanical balance equation and that is simple under isentropic condition the flow is happening and the energy balance equation is just a summation of pressure energy + kinetic energy and that summation = 0. Instead of doing the calculation here we will see later on how all these expression can be established just for solving this equation but for simplicity here when we are considering about critical pressure ratio to be calculated what we can do we can just see if assuming under condition or under real condition again we are solving for the real gas reconsidering gas is real.

So if we solve this equation mechanical balance equation what we can get? We can get the expression for velocity and if we solve for example by converting this by one form to other form again using this concept we can get the expression for U that is comes out as U square = 2KP0 / Rho 0. This is a 0 condition some base condition that could be up stream condition into 1 – PY P0 to the power 1 – 1 / K.

This the expression we got once we integrate the equation and we can put this in the mac square for where mac square = U square / A square. A is the velocity of sound under the intuitive condition if we put there we are going to get the expression 2 upon K – 1 P0 / P multiply by rho Y rho 0 and the same expression this is what we have here we can get the same expression here we can just adjust this U square to ME square and under the condition of sonic flow we will be having mac number = 1.

We can do here and say P0 is by upstream condition and P is my downstream condition if we adjust this so what we are going to get is P down / P up = 2 upon K + 1 to the power K upon K – 1. And this is the condition we got mac number is 1. So we can denote this as a critical pressure condition. It is also denoted as RC critical pressure ratio the pressure ratio at which the fluid is travelling through the restriction with the same balance the same velocity as the sound will travel under the same fluid under the similar condition.

And this critical pressure ratio will help us to understand the flow regime that is happening when the fluid is travelling through this restriction.

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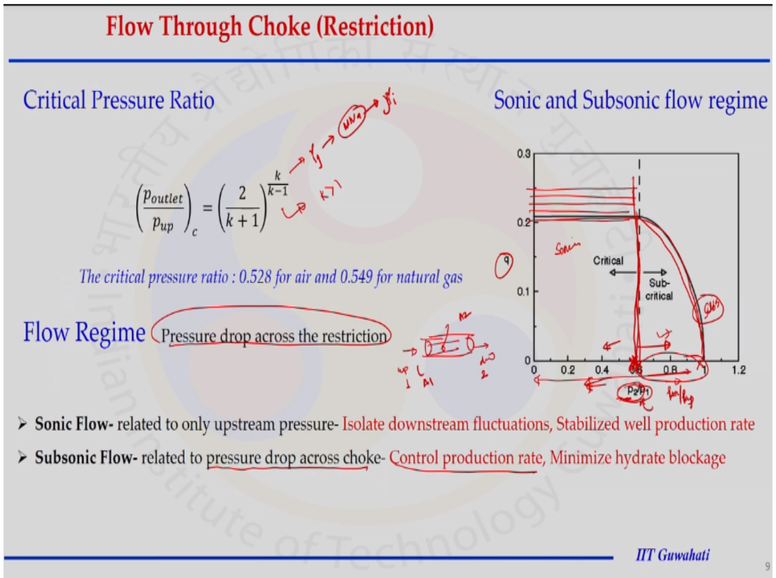

Flow Through Choke (Restriction)

Critical Pressure Ratio

$$\left(\frac{p_{outlet}}{p_{up}}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

The critical pressure ratio : 0.528 for air and 0.549 for natural gas

Sonic and Subsonic flow regime

Flow Regime

Pressure drop across the restriction

- **Sonic Flow-** related to only upstream pressure- Isolate downstream fluctuations, Stabilized well production rate
- **Subsonic Flow-** related to pressure drop across choke- Control production rate, Minimize hydrate blockage

So if we go further what we say like P outlet by P up here instead of P down I had written P outlet because under sonic condition it is not possible to calculate the P down temperature so it is P outlet divided by P up means inlet pressure through this restriction under the critical condition it comes out as 2 upon K + 1 to the power K upon K – 1.

So simply if we know the value of K for any particular fluid that can be known with the help of gamma Z or gamma Z means knowing the molecular weight of the gas that can be known by knowing the composition we can calculate K and putting this K value in this expression of critical pressure ratio we can calculate the condition where the flow will be sonic or the flow will be at a critical if we are on one side of it we will be having the sonic flow if we are on the other side it will be sub sonic flow condition.

We see form here this is P2 upon P1 if I draw the diagram again and can say this is the condition where we are having this is up stream this is downstream we say this is condition one this is condition 2 and the restriction is placed somewhere here where there is area is A2 and this area is A1. So when we are having the flow from left to right through this restriction and we plot P2/P1 is this is P down upon P up versus the flow rate Y axis is the flow rate that can be achieved.

This graph says whenever we are having the P2 = P1 means the ratio is 1 we are not having any flow, the flow is not possible when the upstream and downstream pressure are equal and the flow will happen only and only when downstream pressure is lower than the upstream pressure and that can be seen from here also this ratio will be lesser than 1.

Similar here the K value is also will always be greater than 1 because the gamma Z if we put in form of the gamma Z if we remember the previous expression if we put the value here we can see the K will always be greater than 1. Similarly the pressure ratio will always be lower than 1 whenever the flow is happening if upstream pressure is lower than the downstream pressure flow will not happen.

Now let us come to understand how this flow is happening so we are at this condition by knowing this critical pressure ratio by knowing the K value we can calculate the critical pressure ratio when we got this critical pressure ratio at this point what we can say the condition when the flow is under sonic condition this is the sonic condition and if either upstream pressure or downstream pressure is change the flow will get change.

Let us see how it is happening? So for example upstream pressure is being changed while the downstream pressure is kept constant if we are doing so what will happen the pressure ratio that was at this condition if we are increasing the upstream pressure this ratio will go down and when this ratio is going down we are on this side. While other side is possible when the downstream pressure is increased keeping the upstream pressure constant.

So when we are in this situation the flow will happen and we called it sub critical flow. Flow will happen here also and this called the sonic flow condition so we are having this critical pressure ratio here and beyond this critical pressure ratio we are having sub sonic flow and below this we are having the sonic flow condition but when we are considering this critical pressure condition the only option to increase the flow rate is increase in the upstream pressure and when we increase the upstream pressure flow will still remain the constant but at a high value.

We can increase this flow rate with increase in the pressure of upstream side under the sonic flow condition. While on the other side on subsonic flow condition what we can do we can say either change the upstream or downstream pressure we can change the flow rate and we see during the sonic flow condition flow rate is constant while under the subsonic flow condition this is have in the non-linear relationship.

And as we are changing this pressure ratio coming down, downstream to upstream ratio is getting reduced our flow rate is increasing and finally it is reaching the condition where the sonic flow will happen and further change in ratio to lower side will not affect the production rate. It happens because under the sonic condition when the fluid is travelling under the same velocity as the sound will travel and in that situation whatever is happening on the downstream side if we are changing the downstream pressure it will not be felt by the upstream side.

So the upstream is not feeling what exactly happening on the downstream side similarly whatever is happening on the upstream side will not be felt by the downstream side this is the situation when the waves of fluid or mechanical energy is traveling with the speed of sound and that is why the only option left to increase the production rate or the production of process rate is increasing the upstream pressure.

Pressure drop across the restriction will determine the low regime and sonic flow and subsonic flow can be achieved as discussed before also when we should have a sonic flow condition when we should have sub sonic flow condition is a matter of a decision because sonic flow relate only upstream pressure. So under the sonic flow condition the flow or the discharge rate or the process rate is a function of upstream pressure as I mentioned whatever happening on the downstream side is not included in the understanding of sonic flow.

It does not depend on the downstream pressure so whenever we are having a situation when we want a constant flow rate when we want downstream pressure or other condition or not affecting my flow it should always go with sonic condition and that we had seen when we were understanding the gas well testing under certain cases we want a constant flow rate that is not getting disturbed.

Because of the surface facilities like fluctuation like this separator is there, if any pressure on downstream side is getting fluctuated it is not affecting the upstream pressure or the flow rate through the restriction we want to maintain the constant flow rate and under that condition the well should be tested under the sonic flow condition or sub sonic flow condition because subsonic flow condition depends on the pressure drop across choke.

If any this is happening on the downstream side is on the separator side or some other places it will disturb the flow rate and we want to able to achieve the constant flow rate. But if the objective is to control the flow rate to regulatory flow rate we should operator the well under the subsonic flow condition because that is allow us to control the production rate as well as it allow us to minimize the hydrate block case.

What exactly happens when the flow through restriction is happening because of Joule Thomson effect the temperature also get reduced significantly and there are chances of ice block is or hydrate formation we will discuss that later on.

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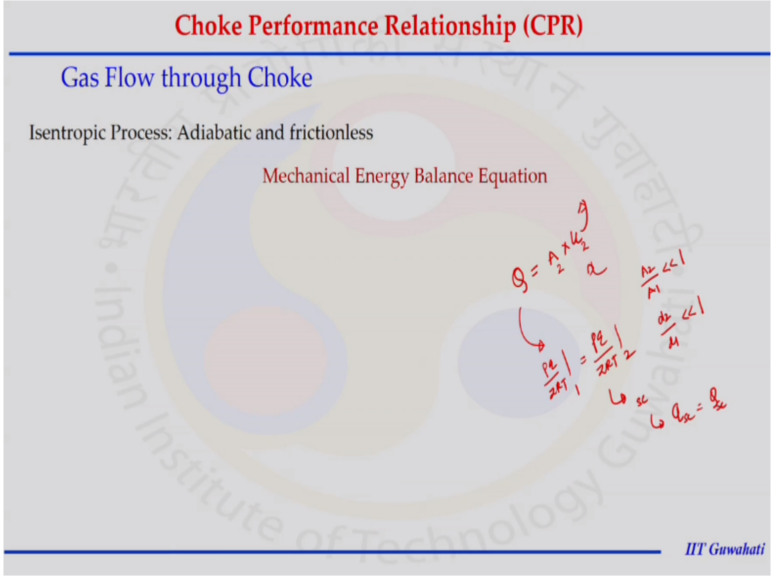

Choke Performance Relationship (CPR)

Gas Flow through Choke

Isentropic Process: Adiabatic and frictionless

Mechanical Energy Balance Equation

$Q = A_2 \times u_2$

$\frac{A_2}{A_1} << 1$

$\frac{d_2}{d_1} << 1$

$\left.\frac{PQ}{ZRT}\right|_1 = \left.\frac{PQ}{ZRT}\right|_2$

sc

$Q_{sc} = Q_{sc}$

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So the flow through choke if we want to calculate the flow through choke we are having this relationship Q is = area multiply by velocity. So if you want the discharge rate through the restriction we know the area of the restriction is A2 and we are having the velocity U2. A complex relationship will be developed even for the velocity calculation a complex relationship developed what we assume?

We assume the area of 2 is very lesser than the area of pipe line or other way D2 / D1 is lesser than 1 and we get a simplified expression again that expression depends on the velocity expression we had developed that expression for the ideal gas consideration or for the real consideration we can get the expression in for Q. That Q can again with the help of the real gas law that says PQ / ZRT at one condition can be substitute to PQ ZRT at second condition we always assume one of the condition is standard condition.

So we can get QSE or other way QSE at a standard condition it means the gas flow rate is measured considering the standard condition and for that we have to substitute standard condition or the pressure and temperature in expression.

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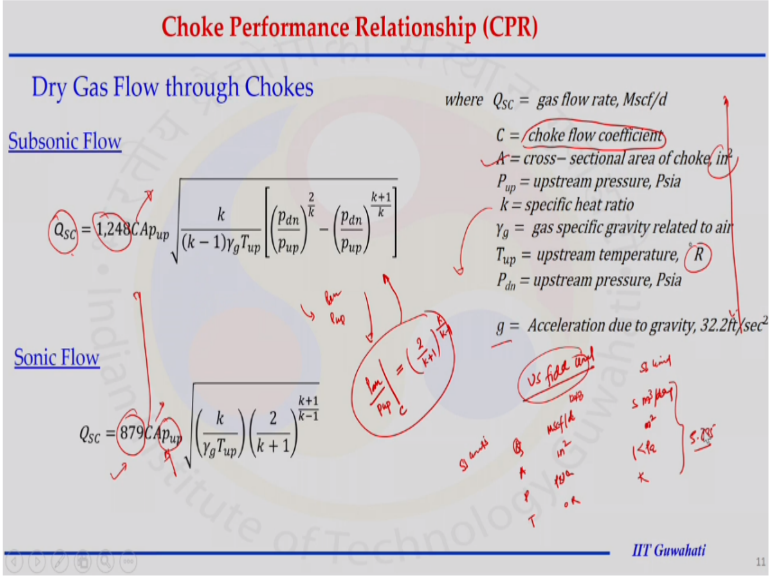

Choke Performance Relationship (CPR)

Dry Gas Flow through Chokes

Subsonic Flow

$$Q_{SC} = 1{,}248\,C A p_{up}\sqrt{\frac{k}{(k-1)\gamma_g T_{up}}\left[\left(\frac{p_{dn}}{p_{up}}\right)^{\frac{2}{k}} - \left(\frac{p_{dn}}{p_{up}}\right)^{\frac{k+1}{k}}\right]}$$

Sonic Flow

$$Q_{SC} = 879\,C A p_{up}\sqrt{\left(\frac{k}{\gamma_g T_{up}}\right)\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

where Q_{SC} = gas flow rate, Mscf/d

C = choke flow coefficient

A = cross– sectional area of choke, in^2

P_{up} = upstream pressure, Psia

k = specific heat ratio

γ_g = gas specific gravity related to air

T_{up} = upstream temperature, °R

P_{dn} = upstream pressure, Psia

g = Acceleration due to gravity, 32.2ft/sec^2

$$\left.\frac{P_{dn}}{P_{up}}\right|_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

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If we do so or we do start from mechanical energy balance equation to solve for U2 ultimately we are going to get the expression for the gas flow rate that is happening through this choke. And we consider the dry gas that is being flowed though the choke we can establish the relationship for the subsonic flow and sonic flow condition.

So subsonic flow condition as discussed already you can see the flow rate is a function of both P down and P up while in the sonic flow condition only P up is appearing in the expression other terms are here is C and that C in both the cases sonic flow and sub sonic flow the C is discharge coefficient because under the isentropic process condition when the flow through restriction is happening it is assume there is no friction losses but in actual there are friction losses happen.

We have to correct this equation for those losses in the form of discharge coefficient. So this C is called choke flow coefficient or discharge coefficient A is the area cross sectional area P up P down are the pressure on the upstream side downstream side K is specific heat ratio CP / CB G is acceleration due to gravity and all are given in the US field unit system. And when we adjust this considering like area in inch square, temperature in ranking we got this numerical coefficient here 1248.

At this numerical coefficient will get change when we are changing the presentation of NE of the parameter in this expression. Another important thing we can see the discharge rate in both the cased of sonic and sub sonic case depends directly to area, area of the restriction if the area of restriction is more, more passes rate can be achieved and upstream pressure, if upstream pressure is higher the flow rate or gas passes rate can be higher. And this sonic flow expression from supersonic flow condition can be established just putting this condition of P down by P up is actually under the critical condition.

And that critical condition and that critical condition says two upon K + 1 to the power K upon K – 1. So we substitute this expression or the condition of sonic flow in this subsonic flow equation we are going to get this expression again the numerical coefficient will depend on the unit system has been chosen to represent the other parameter and this can be change if we are using the SI unit system.

In SI unit system we say let us compare in US field unit system and SI unit system the Q is here in MSCF per day here it is standard meter cube per day. Area here it is in inches square while here it is in meter square. P here is in PSIA and here it is in kilopascal and other temperature degree ranking and here it is degree Kelvin. So if we are using the degree in US field unit system it comes around 1248 or 1240 and in US field unit system it comes out as 5.885 so depend again the unit system this can be changed here.

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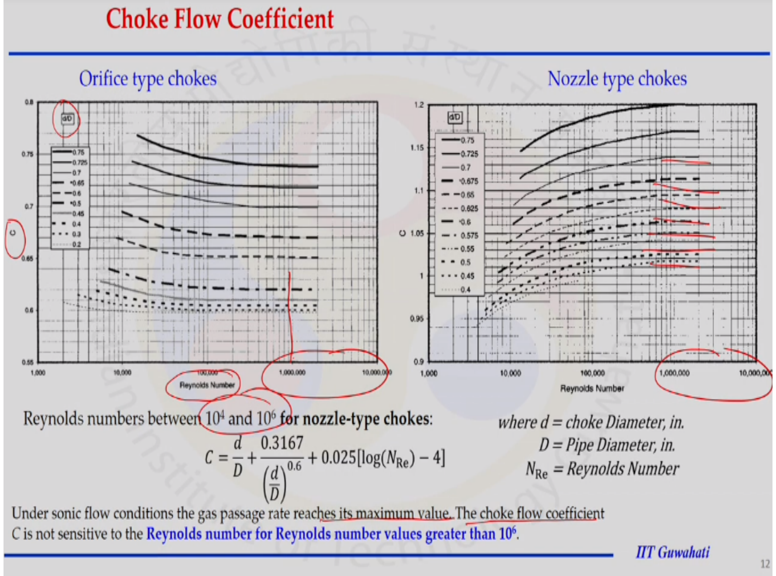

Choke Flow Coefficient

Orifice type chokes

Nozzle type chokes

Reynolds numbers between 10^4 and 10^6 **for nozzle-type chokes:**

$$C = \frac{d}{D} + \frac{0.3167}{\left(\frac{d}{D}\right)^{0.6}} + 0.025[\log(N_{Re}) - 4]$$

where d = choke Diameter, in.
D = Pipe Diameter, in.
N_{Re} = Reynolds Number

Under sonic flow conditions the gas passage rate reaches its maximum value. The choke flow coefficient C is not sensitive to the **Reynolds number for Reynolds number values greater than 10^6.**

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Important point comes here the discharge coefficient how to calculate the discharge coefficient and that discharge coefficient it depends on the type of choke is used it is either Orifice type or Nozzle type or different type whenever we are purchasing the choke it is a responsibility of the supplier or who design this choke to provide the value of discharge coefficient.

If it is not available or rough value can be estimated like 0.7 for the Orifice type and around the same value for the Nozzle type. Or other way we can calculate the accurate value of the discharge coefficient from one of this chart this chart simply says it is a relationship between the discharge coefficient C that we are supposed to calculate with respect to Reynolds number and the relationship is in terms of ratio of diameter of choke to diameter of pie DYD.

So we are having a choke which is installed in a particular pipe we know the diameter of both choke and diameter pipe we can calculate that ratio if we are on the higher side of this Reynolds number we can assume the value is almost constant we can see for all the diameter the value is becoming flat it is not a function of DYD yes but the value will get change. For example if we are having here the diameter is here we can get the value of C discharge coefficient but it becomes flat.

We can assume a constant value also or for a higher Reynolds number like between 10 to the power 4 10 to the power 6 we can use some correlation given and that relates C is a function of DYD and Reynolds number there are several correlation are given in the literature but different type of the choke as well as different Reynolds number those can be applied. For example here it is applicable for very high Reynolds number.

Under sonic flow condition the gas passes rates reaches it is maximum value the choke flow coefficient is C is not sensitive to the Reynolds number for an Reynolds number value greater than 10 to the power 6. So if the value of Reynolds number is 10 to the power 6 or more than we can assume like c is not dependent we can choose a constant value.

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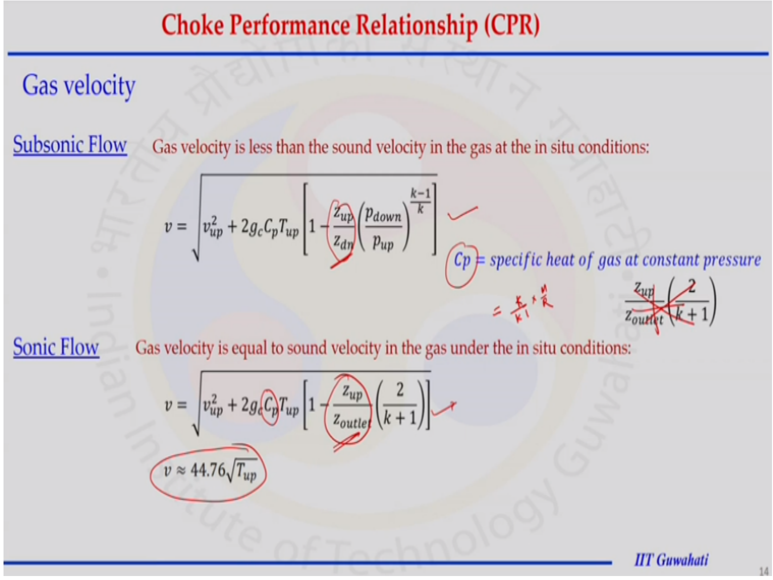

Choke Performance Relationship (CPR)

Gas velocity

Subsonic Flow Gas velocity is less than the sound velocity in the gas at the in situ conditions:

$$v = \sqrt{v_{up}^2 + 2g_c C_p T_{up}\left[1 - \frac{Z_{up}}{Z_{dn}}\left(\frac{p_{down}}{p_{up}}\right)^{\frac{k-1}{k}}\right]}$$

Cp = specific heat of gas at constant pressure

$= \frac{k}{k-1} \times \frac{R}{M}$

$\cancel{\frac{Z_{up}}{Z_{outlet}}\left(\frac{2}{k+1}\right)}$

Sonic Flow Gas velocity is equal to sound velocity in the gas under the in situ conditions:

$$v = \sqrt{v_{up}^2 + 2g_c C_p T_{up}\left[1 - \frac{Z_{up}}{Z_{outlet}}\left(\frac{2}{k+1}\right)\right]}$$

$$v \approx 44.76\sqrt{T_{up}}$$

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The gas velocity as we did for gas passes rate as did the calculation for the critical pressure ratio similar we can do for the gas velocity in fact to calculate the discharge rate we have to have the velocity and the velocity can be calculated again or can be estimated again by solving the mechanical energy equation that simply says change in pressure energy change in kinetic energy = 0 for isentropic process if we solve this under the integral sign we can get the U2 value this the velocity of the fluid and on the stream side considering the process isentropic and parameters involved.

If we do so what we are going to get instead of solving that I will just write the expression that says the sub sonic flow the gas velocity is less than the sound velocity we know about the sub sonic flow condition and the velocity can be obtained from this expression and here you can see this has been corrected for real gas for the ideal gas there is no compressibility factor but the velocity expression obtained considering the ideal gas but later on it has been corrected for the real gas including the compressibility factor.

Similar for this sonic flow condition again the condition is same we can put the critical pressure ratio condition and we can get the expression for the velocity which is happening under the sonic flow condition it means the fluid is travelling with the sound velocity and the expression can be obtained like this and that expression has been corrected again for the real gas through the compressibility factor.

Here you can see VZ down here it Z outlet we know under the sonic flow condition downstream condition either the pressure condition or the compressibility condition or consider is the out let condition. If we simplified this expression we simply get the velocity under the sonic flow condition is a function of temperature only and using this correlation and knowing the temperature of upstream side we can calculate the velocity of the fluid through the restriction.

In this expression another term is appearing CP that is CP is specific heat of gas at constant pressure is specific heat of gas can be related to K CP = K upon K – 1 multiply by M / R we can get this CP value. So specific heat of gas at constant pressure has been included in the expression of velocity. So we know the expression for the velocity we know the expression for the gas passes rate we know how to calculate the critical pressure ratio.

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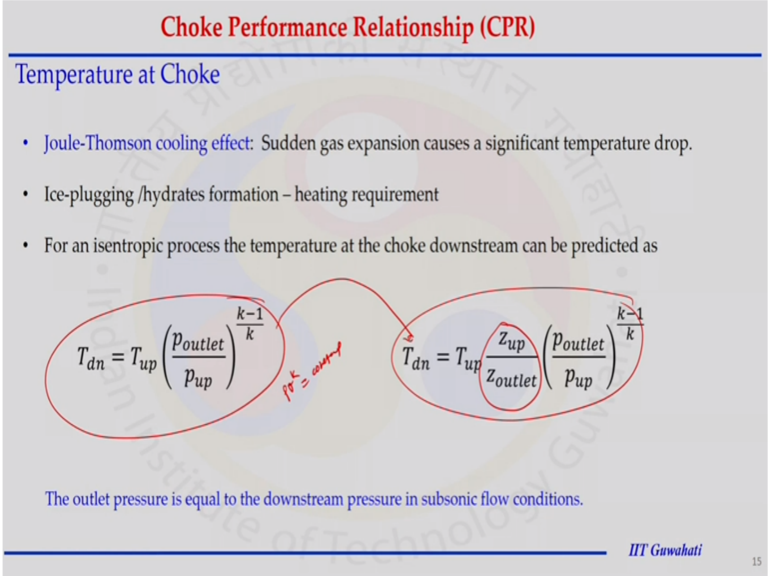

Choke Performance Relationship (CPR)

Temperature at Choke

- Joule-Thomson cooling effect: Sudden gas expansion causes a significant temperature drop.
- Ice-plugging /hydrates formation – heating requirement
- For an isentropic process the temperature at the choke downstream can be predicted as

$$\boxed{T_{dn} = T_{up}\left(\frac{p_{outlet}}{p_{up}}\right)^{\frac{k-1}{k}}}$$

$pv^k = \text{constant}$

$$\boxed{T_{dn} = T_{up}\boxed{\frac{z_{up}}{z_{outlet}}}\left(\frac{p_{outlet}}{p_{up}}\right)^{\frac{k-1}{k}}}$$

The outlet pressure is equal to the downstream pressure in subsonic flow conditions.

Now we can calculate the temperature on the downstream side and that can be done with the establishing the relationship considering isotropic process is happening and the temperature on downstream side is important because sudden pressure drop happens and because of the sudden pressure drops changes Joule Thomson cooling effect occurs that says because of sudden gas expansion significant temperature drop occurs and this significant temperature drop may reach to a situation where the ice plugging or hydrate formation are occurring and to prevent this phenomena heating may be record at the choke.

So you see sometimes the choke device or Christmas tree which is having the choke device is also equipped with the hearing element with heated to prevent this phenomena happening phenomena like either the ice plugging or the hydrate formation are happening. For an isentropic process the temperature at choke downstream can be predicted simply by considering this P P2 to the power K = constant and we did already considering one condition to other condition we can get how T down side temperature relate to T up temperature.

And considering this for a real gas we can correct it this with the help of compressibility factor evaluated of stream temperature pressure condition and downstream is under the sonic flow condition it is Z outlet condition we can combine both upstream and downstream condition in this expression to correct it for real gas law. The outlet pressure = the downstream pressure in subsonic flow condition that is why it is appearing here.

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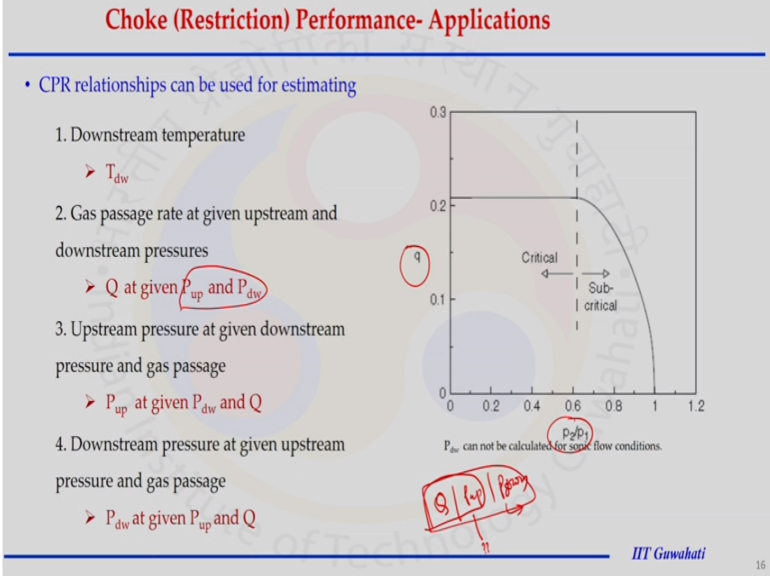

Choke (Restriction) Performance- Applications

- CPR relationships can be used for estimating
  1. Downstream temperature
     - T_{dw}
  2. Gas passage rate at given upstream and downstream pressures
     - Q at given P_{up} and P_{dw}
  3. Upstream pressure at given downstream pressure and gas passage
     - P_{up} at given P_{dw} and Q
  4. Downstream pressure at given upstream pressure and gas passage
     - P_{dw} at given P_{up} and Q

P_{dw} can not be calculated for sonic flow conditions.

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Now the application whatever the relationship we could have understand how to calculate the temperature how this sonic flow and subsonic flow condition can allow us to have the relationship between the pressure of downstream upstream the type of fluid that depends on the K value we can calculate the velocity and passes rate now the CPR relationship can be used for a estimating downstream temperature and if other two parameter are given we can calculate the third parameter.

So for example we are having the relationship between the gas passes P up and P down through this chart we can say the Q is related to P down by P up and if 1 of them is missing we can calculate the question comes for example we are having the P up and P down you want to calculate the Q the passes rate which expression we should use.

Do we know our flow is happening under the sonic condition or subsonic condition if it is happening under the sonic condition we can use the expression for sonic flow if it is happening in the sub sonic condition we can use the expression for subsonic flow. But important which one should be used that can be established by understanding of the how the flow happens through the choke.

Similar for the third application when P up was supposed to ask and Q and P down where given or under the fourth application where P down is supposed to be calculated when Q and P up are given to us we can go through one by one to understand the application CPR to estimate the unknown parameter that is appearing in this application.

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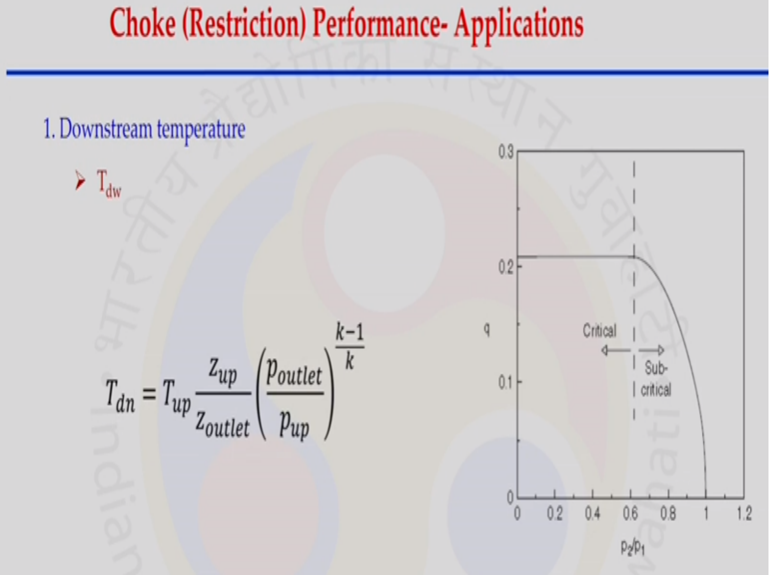

Choke (Restriction) Performance- Applications

1. Downstream temperature

- T_{dw}

$$T_{dn} = T_{up}\frac{z_{up}}{z_{outlet}}\left(\frac{p_{outlet}}{p_{up}}\right)^{\frac{k-1}{k}}$$

So like for example the downstream temperature it is very straight forward we just discussed how to calculate the downstream temperature knowing the other things.

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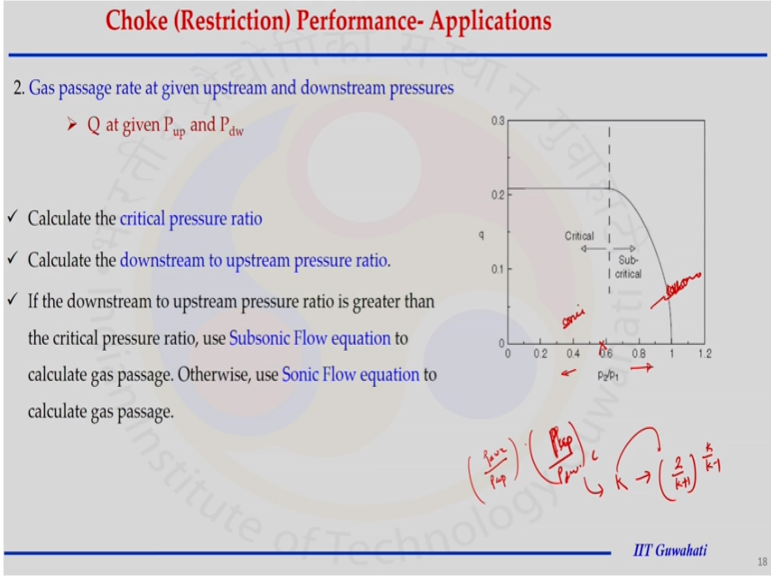

Choke (Restriction) Performance- Applications

2. Gas passage rate at given upstream and downstream pressures
   - Q at given P_{up} and P_{dw}

- ✓ Calculate the critical pressure ratio
- ✓ Calculate the downstream to upstream pressure ratio.
- ✓ If the downstream to upstream pressure ratio is greater than the critical pressure ratio, use Subsonic Flow equation to calculate gas passage. Otherwise, use Sonic Flow equation to calculate gas passage.

$$\left(\frac{P_{dw}}{P_{up}}\right) \cdot \left(\frac{P_{up}}{P_{dw}}\right)_c \hookrightarrow K \rightarrow \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$

We can go to second application if i know P up and P down and want to calculate the Q what we can do we can calculate the actual pressure ratio P down by P up and whatever the value we are getting compare this value with critical pressure condition that says P up by P down under critical condition and we know this can be calculated by knowing the K how we can calculate this.

This is equal to 2 upon K + 1 to the power upon K – 1 say if I know K I can calculate this and if the critical pressure ratio is here if the actual pressure ratio is greater than critical pressure ratio we are in the subsonic reason and if it is lesser than we are in the sonic condition. We can choose sonic condition or sub sonic condition knowing the critical pressure ratio and comparing that critical pressure ratio to actual pressure ratio.

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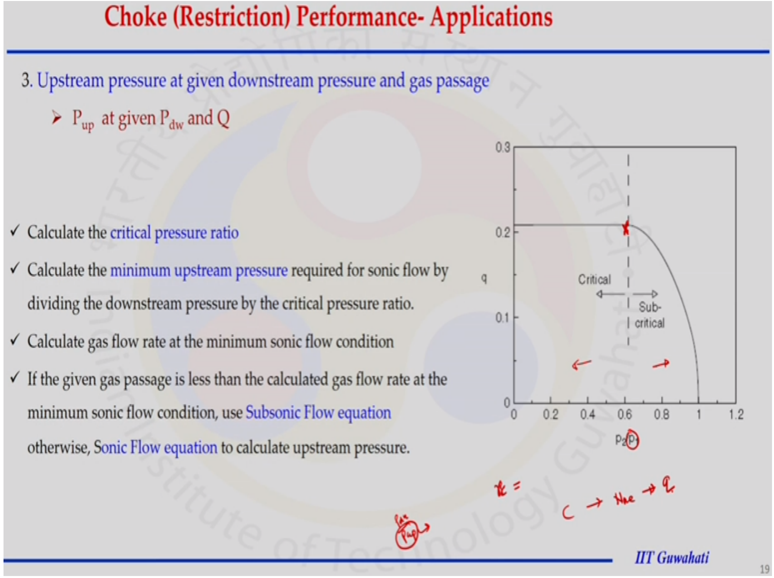

Choke (Restriction) Performance- Applications

3. Upstream pressure at given downstream pressure and gas passage

- P_{up} at given P_{dw} and Q

✓ Calculate the critical pressure ratio

✓ Calculate the minimum upstream pressure required for sonic flow by dividing the downstream pressure by the critical pressure ratio.

✓ Calculate gas flow rate at the minimum sonic flow condition

✓ If the given gas passage is less than the calculated gas flow rate at the minimum sonic flow condition, use Subsonic Flow equation otherwise, Sonic Flow equation to calculate upstream pressure.

Third application upstream pressure at given downstream pressure and gas passes. So for example we are having again the similar situation where we can calculate the critical pressure ratio RC knowing the critical pressure ratio we can decide which side we should go but we do not P up to calculate the actual pressure ratio. Actual pressure ratio means P down by P up we do not know the P up condition.

For that what we can do we can calculate this condition when the sonic condition is just at the worse is minimum upstream pressure record to have the sonic flow condition and if we calculate this of upstream pressure in this manner what we can do we can decide we are going to this sonic condition or sub sonic condition calculate the gas passes rate considering this P up condition and if the given process is less than the calculated gas flow rate at the minimum sonic condition use sub sonic flow condition other wise use the sonic flow condition.

So considering the minimum where the sonic flow will be at the worse we can decide which one should be used. But again remember in this case we have to calculate the discharge coefficient that discharge coefficient C that depends on Reynolds number and that Reynolds number depends on Q again, And we are calculating Q and that parameter depend on Q itself so we have to do certain iteration we have assume some value of Reynolds number calculate the discharge coefficient.

Calculating the discharge coefficient calculate the value of Q and then knowing the value of Q and then calculate the value of Reynolds number and see if it is with in the limit must deviation is there we can assume it in the assumption was correct otherwise we have to do the iteration to make sure the solution is correction.

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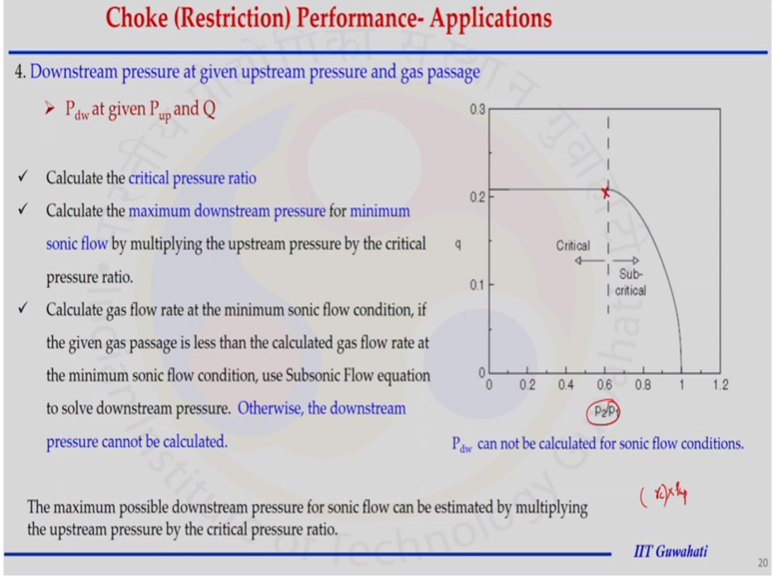

Choke (Restriction) Performance- Applications

4. Downstream pressure at given upstream pressure and gas passage

- P_{dw} at given P_{up} and Q

✓ Calculate the critical pressure ratio

✓ Calculate the maximum downstream pressure for minimum sonic flow by multiplying the upstream pressure by the critical pressure ratio.

✓ Calculate gas flow rate at the minimum sonic flow condition, if the given gas passage is less than the calculated gas flow rate at the minimum sonic flow condition, use Subsonic Flow equation to solve downstream pressure. Otherwise, the downstream pressure cannot be calculated.

P_{dw} can not be calculated for sonic flow conditions.

The maximum possible downstream pressure for sonic flow can be estimated by multiplying the upstream pressure by the critical pressure ratio.

$(r_c) \times P_{up}$

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Similar for this case also when we are supposed to calculate the downstream pressure while P up or Q are given to us what we can do we can calculate critical pressure ratio by knowing the gas composition by knowing the value of K and maximum downstream pressure or minimum sonic flow condition can be maintained like again we can calculate at this point the maximum downstream pressure that is possible for having the sonic flow condition.

If we are having more than that will go to sub sonic condition so at that condition we can calculate the sonic flow and having that sonic flow the pressure condition is downstream condition pressure at that sonic flow condition we can calculate the flow rate at the minimum sonic flow condition. If the given gas passes is less than calculated gas flow rate at the minimum sonic flow condition use sub sonic flow equation to solve the downstream pressure otherwise downstream pressure cannot be calculated.

The maximum possible downstream pressure for sonic flow can be estimated by multiplying the upstream pressure by critical reassure ratio. So we calculate the critical pressure ratio RC multiply this with P up we can get the P down under the sonic flow condition our sub sonic flow condition once we establish the relationship that is Q given is matching Q calculated through Reynolds number calculation we can use this subsonic equation that subsonic equation can give us the P down.

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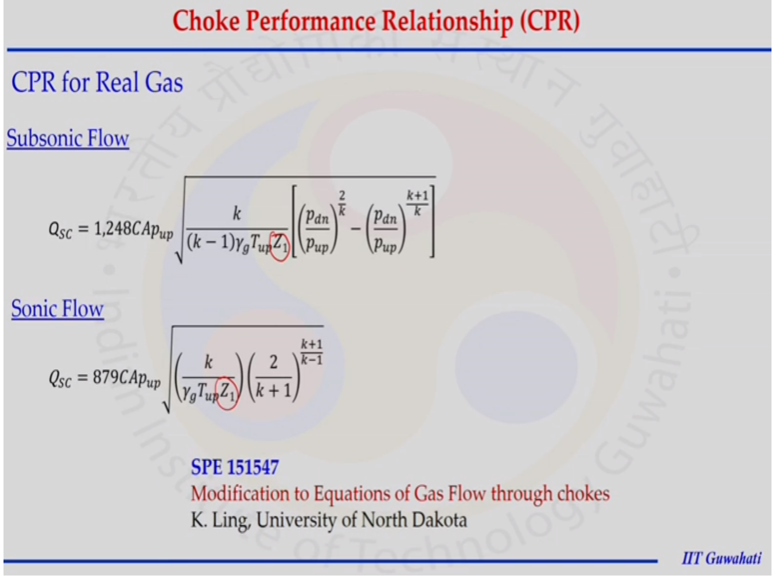

Choke Performance Relationship (CPR)

CPR for Real Gas

Subsonic Flow

$$Q_{SC} = 1{,}248CAp_{up}\sqrt{\frac{k}{(k-1)\gamma_g T_{up} Z_1}\left[\left(\frac{p_{dn}}{p_{up}}\right)^{\frac{2}{k}} - \left(\frac{p_{dn}}{p_{up}}\right)^{\frac{k+1}{k}}\right]}$$

Sonic Flow

$$Q_{SC} = 879CAp_{up}\sqrt{\left(\frac{k}{\gamma_g T_{up} Z_1}\right)\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

SPE 151547

Modification to Equations of Gas Flow through chokes

K. Ling, University of North Dakota

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CPR for real gas this as the expressions shown before they are for considering the gas is ideal gas or when we consider real gas we have to include this compressibility factor at upstream side in both the expression to account for the real gas expression and there is paper published in HP society of petroleum engineering that says like there is a more complex phenomena that exactly happens when the flow through restriction happening.

If for the real gas the compressibility factor is a function of both temperature and pressure we did into consider that even to calculate the critical pressure ratio that phenomena should be considered and more complex side of the equation should be solved but considering the accuracy recalled to understand how the flow through choking is happening and to have the estimation of the flow rate.

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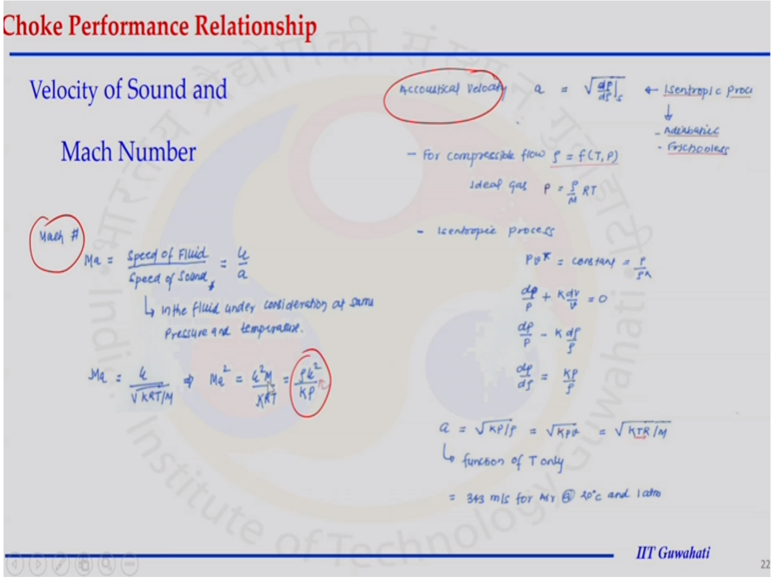

Choke Performance Relationship

Velocity of Sound and

Mach Number

Accoustical Velocity $a = \sqrt{\frac{dP}{d\rho}\Big|_s}$ ← Isentropic Proc

↓

- Adiabatic
- Frictionless

- For compressible flow $\rho = f(T, P)$

Ideal gas $P = \frac{\rho}{M} RT$

- Isentropic process

$P v^k = \text{constant} = \frac{P}{\rho^k}$

$\frac{dp}{p} + k\frac{dv}{v} = 0$

$\frac{dp}{p} - k\frac{d\rho}{\rho}$

$\frac{dp}{d\rho} = \frac{kP}{\rho}$

$a = \sqrt{kP/\rho} = \sqrt{kPv} = \sqrt{kTR/M}$

↳ function of T only

$= 343$ m/s for Air @ 20°c and 1 atm

Mach #

$Ma = \frac{\text{Speed of Fluid}}{\text{Speed of Sound}} = \frac{u}{a}$

↳ in the fluid under consideration at same pressure and temperature.

$Ma = \frac{u}{\sqrt{kRT/M}} \Rightarrow Ma^2 = \frac{u^2 M}{kRT} = \frac{\rho u^2}{kP}$

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Ideal gas assumption with some correction for the ideal gas may be good enough to go ahead. Here I have some summarizes what we did so far we had establish the relationship for the aquatics velocity considering aquatics engineering principle we could understand the mac number how it is related to the fluid velocity and the other parameters.

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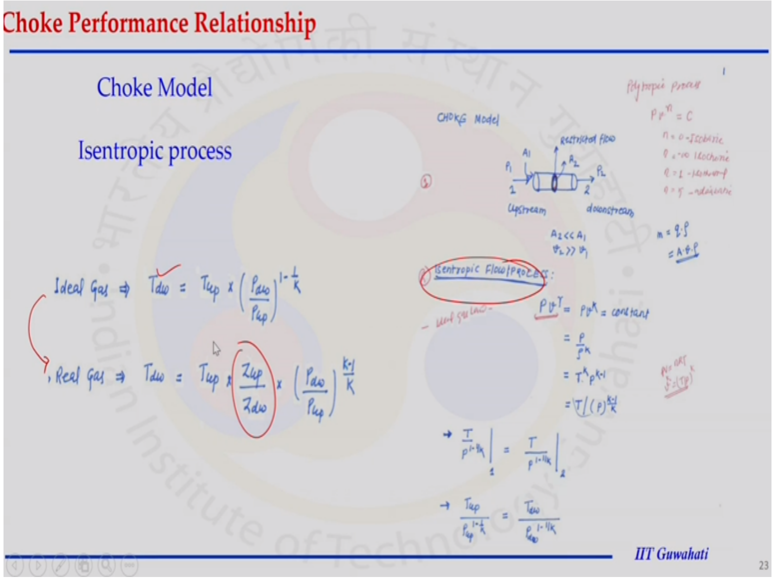

Choke Performance Relationship

Choke Model

Isentropic process

CHOKE Model

$A_2 << A_1$

$v_2 >> v_1$

Polytropic Process

$P v^n = C$

n = 0 - Isobaric

n = ∞ Isochoric

n = 1 - Isothermal

n = γ - adiabatic

$m = q \cdot \rho$

$= A \cdot v \cdot \rho$

Isentropic Flow/Process:

$P v^{\gamma} = P v^k = \text{constant}$

$= \frac{P}{\rho^k}$

$= T^k p^{k-1}$

$= T/(P)^{\frac{k-1}{k}}$

$\rightarrow \frac{T}{P^{1-1/k}}\Big|_1 = \frac{T}{P^{1-1/k}}\Big|_2$

$\rightarrow \frac{T_{up}}{P_{up}^{1-\frac{1}{k}}} = \frac{T_{dw}}{P_{dw}^{1-1/k}}$

Ideal Gas $\Rightarrow T_{dw} = T_{up} \times \left(\frac{P_{dw}}{P_{up}}\right)^{1-\frac{1}{k}}$

Real Gas $\Rightarrow T_{dw} = T_{up} \times \frac{Z_{up}}{Z_{dw}} \times \left(\frac{P_{dw}}{P_{up}}\right)^{\frac{k-1}{k}}$

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Similar the derivation has been done for the choke model isentropic process we can see here how the isentropic process can be modeled to get the temperature, pressure density and the specific volume relationship how to convert one parameter from other parameter how this can be modified for the real gas considering the compressibility factor on the ahh numerator and denominator for both upstream and downstream side.

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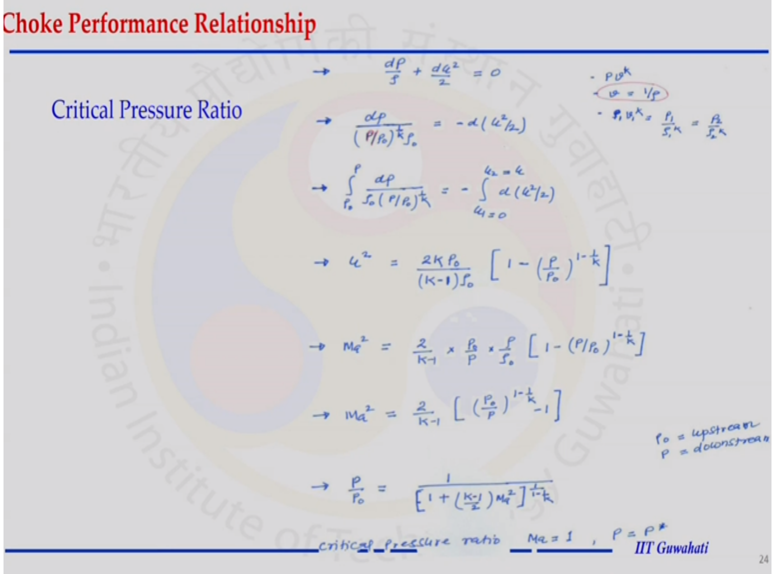

Choke Performance Relationship

Critical Pressure Ratio

$\to \frac{dp}{\rho} + \frac{du^2}{2} = 0$

$\to \frac{dp}{(p/p_0)^{\frac{1}{k}}\rho_0} = -d(u^2/2)$

$\to \int_{p_0}^{p} \frac{dp}{\rho_0 (p/p_0)^{\frac{1}{k}}} = -\int_{u_1=0}^{u_2=u} d(u^2/2)$

$\to u^2 = \frac{2kp_0}{(k-1)\rho_0}\left[1-\left(\frac{p}{p_0}\right)^{1-\frac{1}{k}}\right]$

$\to Ma^2 = \frac{2}{k-1} \times \frac{p_0}{p} \times \frac{\rho}{\rho_0}\left[1-(p/p_0)^{1-\frac{1}{k}}\right]$

$\to Ma^2 = \frac{2}{k-1}\left[\left(\frac{p_0}{p}\right)^{1-\frac{1}{k}} - 1\right]$

$\to \frac{p}{p_0} = \frac{1}{\left[1+\left(\frac{k-1}{2}\right)Ma^2\right]^{\frac{1}{1-\frac{1}{k}}}}$

- pv^k
- $v = 1/\rho$
- $p_1 v_1^k = \frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k}$

p_0 = upstream
p = downstream

critical pressure ratio $Ma = 1$, $p = p^*$

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Critical pressure ratio by solving the mechanical balance equation we can establish the critical pressure ratio condition when mac number is 1 we can get the expression for critical pressure ratio.

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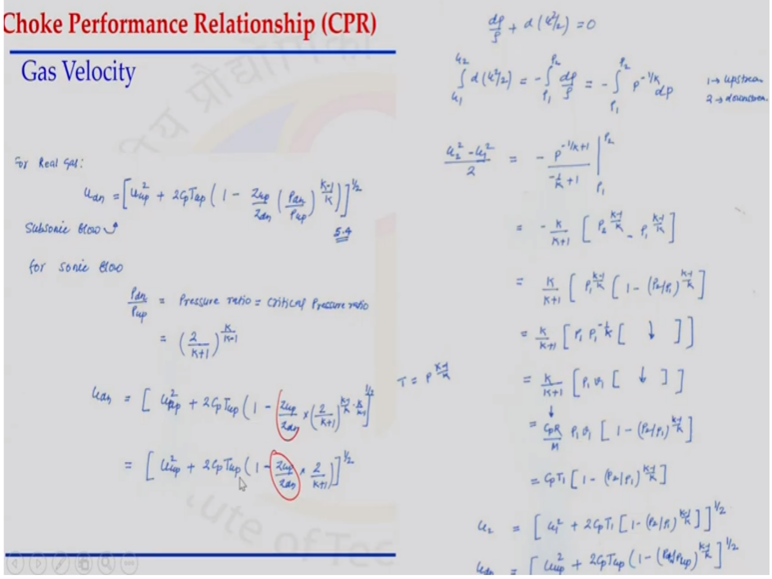

Choke Performance Relationship (CPR)

Gas Velocity

For Real gas:

$u_{dn} = \left[u_{up}^2 + 2C_pT_{up}\left(1 - \frac{z_{up}}{z_{dn}}\left(\frac{p_{dn}}{p_{up}}\right)^{\frac{k-1}{k}}\right)\right]^{1/2}$

Subsonic flow

5.4

for sonic flow

$\frac{p_{dn}}{p_{up}}$ = Pressure ratio = critical pressure ratio

$= \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$

$u_{dn} = \left[u_{up}^2 + 2C_pT_{up}\left(1 - \frac{z_{up}}{z_{dn}} \times \left(\frac{2}{k+1}\right)^{\frac{k-1}{k}\cdot\frac{k}{k-1}}\right)\right]^{1/2}$

$= \left[u_{up}^2 + 2C_pT_{up}\left(1 - \frac{z_{up}}{z_{dn}} \times \frac{2}{k+1}\right)\right]^{1/2}$

$\frac{dp}{\rho} + d(u^2/2) = 0$

$\int_{u_1}^{u_2} d(u^2/2) = -\int_{p_1}^{p_2}\frac{dp}{\rho} = -\int_{p_1}^{p_2} p^{-1/k}\,dp$

1 → upstream
2 → downstream

$\frac{u_2^2-u_1^2}{2} = -\left.\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right|_{p_1}^{p_2}$

$= -\frac{k}{k+1}\left[p_2^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}}\right]$

$= \frac{k}{k+1}\left[p_1^{\frac{k-1}{k}}\left[1-(p_2/p_1)^{\frac{k-1}{k}}\right]\right]$

$= \frac{k}{k+1}\left[p_1 p_1^{-\frac{1}{k}}\left[\ \downarrow\ \right]\right]$

$T = p^{\frac{k-1}{k}}$

$= \frac{k}{k+1}\left[p_1 v_1\left[\ \downarrow\ \right]\right]$

$\downarrow$

$= \frac{C_pR}{M}\, p_1 v_1\left[1-(p_2/p_1)^{\frac{k-1}{k}}\right]$

$= C_pT_1\left[1-(p_2/p_1)^{\frac{k-1}{k}}\right]$

$u_2 = \left[u_1^2 + 2C_pT_1\left[1-(p_2/p_1)^{\frac{k-1}{k}}\right]\right]^{1/2}$

$u_{dn} = \left[u_{up}^2 + 2C_pT_{up}\left(1-(p_{dn}/p_{up})^{\frac{k-1}{k}}\right)\right]^{1/2}$

We can get by solving this equation again all the step again I have summarize summarizes here we can get the expression for sonic flow condition and sub sonic flow condition the expression for the velocity can be established and that is again as been corrected by the compressibility factor to account for the real gases.

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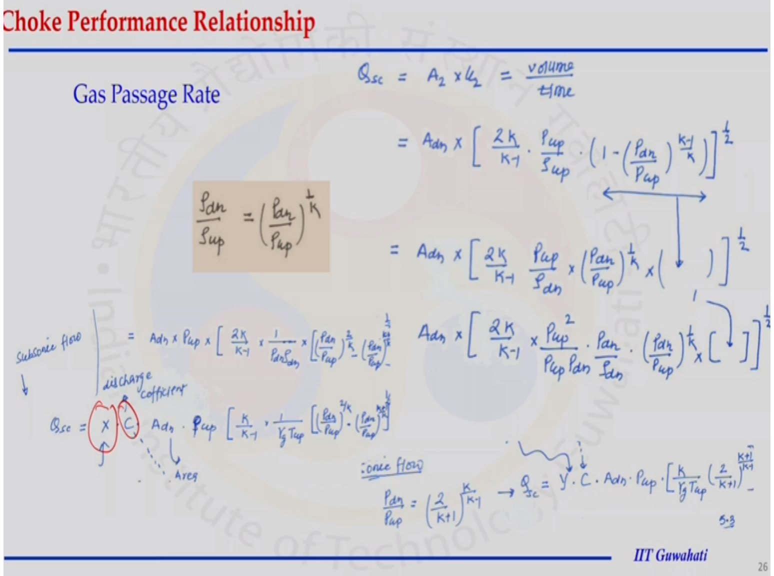

Choke Performance Relationship

Gas Passage Rate

$$Q_{sc} = A_2 \times u_2 = \frac{volume}{time}$$

$$= A_{dn} \times \left[\frac{2k}{k-1} \cdot \frac{P_{up}}{\rho_{up}} \cdot \left(1 - \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{k-1}{k}} \right) \right]^{\frac{1}{2}}$$

$$\boxed{\frac{\rho_{dn}}{\rho_{up}} = \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{1}{k}}}$$

$$= A_{dn} \times \left[\frac{2k}{k-1} \frac{P_{up}}{\rho_{dn}} \times \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{1}{k}} \times \left(\quad \right) \right]^{\frac{1}{2}}$$

$$A_{dn} \times \left[\frac{2k}{k-1} \times \frac{P_{up}^2}{P_{up} P_{dn}} \cdot \frac{P_{dn}}{\rho_{dn}} \cdot \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{1}{k}} \times \left[\quad \right] \right]^{\frac{1}{2}}$$

subsonic flow

$$= A_{dn} \times P_{up} \times \left[\frac{2k}{k-1} \times \frac{1}{P_{dn} \rho_{dn}} \times \left[\left(\frac{P_{dn}}{P_{up}} \right)^{\frac{2}{k}} - \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{k+1}{k}} \right] \right]^{\frac{1}{2}}$$

discharge coefficient

$$Q_{sc} = X \, C \, A_{dn} \cdot P_{up} \left[\frac{k}{k-1} \cdot \frac{1}{\gamma_g T_{up}} \left[\left(\frac{P_{dn}}{P_{up}} \right)^{2/k} - \left(\frac{P_{dn}}{P_{up}} \right)^{\frac{k+1}{k}} \right] \right]^{\frac{1}{2}}$$

Area

onic flow

$$\frac{P_{dn}}{P_{up}} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \rightarrow Q_{sc} = Y \cdot C \cdot A_{dn} \cdot P_{up} \cdot \left[\frac{k}{\gamma_g T_{up}} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]$$

5.3

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Similarly for gas process rate as it is mentioned it is discharged coefficient is here and the X is the numerical coefficient that will appear the numerical value will appear depend on the numerical system has been chosen the step by process has been explained here how to use the velocity expression how to convert from one form or the parameter to other form of the parameter or the set of the parameter to other set of the parameters and get the expression already that is already shown in the previous slides.

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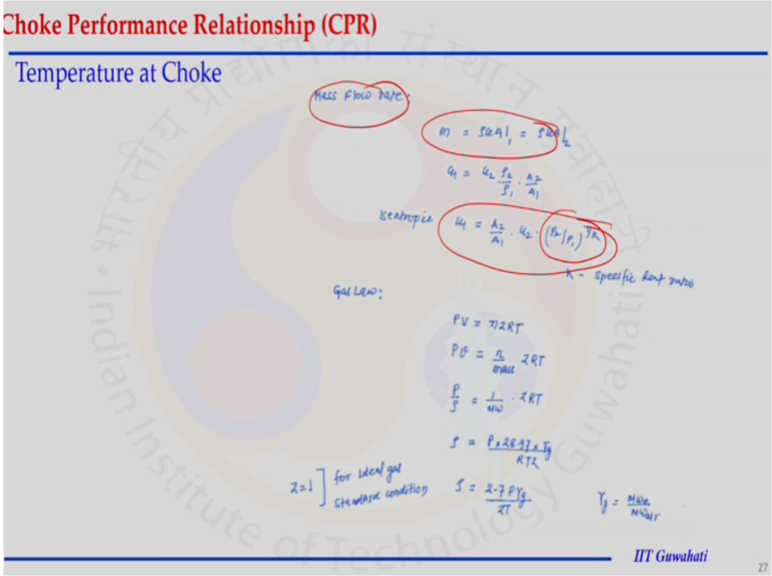

Choke Performance Relationship (CPR)

Temperature at Choke

Mass Flow rate:

$\dot{m} = \rho u A|_1 = \rho u A|_2$

$u_1 = u_2 \cdot \frac{\rho_2}{\rho_1} \cdot \frac{A_2}{A_1}$

Isentropic $u_1 = \frac{A_2}{A_1} \cdot u_2 \cdot (P_2/P_1)^{1/k}$

k - specific heat ratio

Gas Law:

$PV = nZRT$

$P\vartheta = \frac{n}{mass} ZRT$

$\frac{P}{\rho} = \frac{1}{MW} \cdot ZRT$

$\rho = \frac{P \times 28.97 \times \gamma_g}{RTZ}$

$Z=1$] for ideal gas standard condition

$\rho = \frac{2.7 P \gamma_g}{ZT}$

$\gamma_g = \frac{MW_g}{MW_{air}}$

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Similar for the temperature we can do and remember for this mass flow rate this is MUA here area is changing so we have to account for that also rho UA and this process is happening under isentropic process. So even the density need to be converted into pressure form so the isentropic process the velocity at one point was related to velocity at other point not only in terms of area but in terms of other properties like either the pressure or density or specific volume that need to be corrected.

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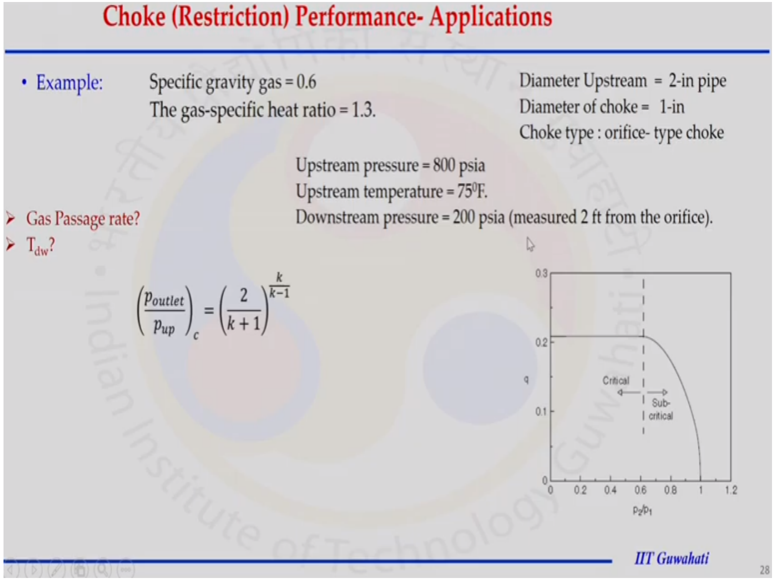

Choke (Restriction) Performance- Applications

- Example:

Specific gravity gas = 0.6
The gas-specific heat ratio = 1.3.

Diameter Upstream = 2-in pipe
Diameter of choke = 1-in
Choke type : orifice- type choke

Upstream pressure = 800 psia
Upstream temperature = 75^{0}F.
Downstream pressure = 200 psia (measured 2 ft from the orifice).

- Gas Passage rate?
- T_{dw}?

$$\left(\frac{p_{outlet}}{p_{up}}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

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So in the next class what I would like to go through with an dedicated example where certain information given to us like the properties of gas like the condition of the choke and pipe and the temperature pressure condition and we are going to calculate the missing parameter and understanding how critical pressure ratio can help us to establish the relationship how to know unknown parameter.

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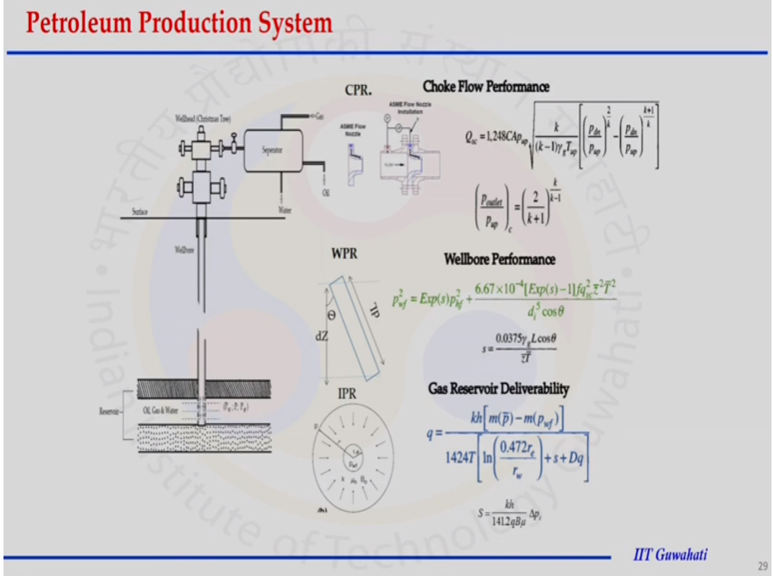

Petroleum Production System

Choke Flow Performance

$$Q_{sc} = 1{,}248CAp_{up}\sqrt{\frac{k}{(k-1)\gamma_g T_{up}}\left[\left(\frac{p_{dn}}{p_{up}}\right)^{\frac{2}{k}} - \left(\frac{p_{dn}}{p_{up}}\right)^{\frac{k+1}{k}}\right]}$$

$$\left(\frac{P_{outlet}}{P_{up}}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

Wellbore Performance

$$p_{wf}^2 = Exp(s)p_{hf}^2 + \frac{6.67\times10^{-4}[Exp(s)-1]fq_{sc}^2\bar{z}^2\bar{T}^2}{d_i^5\cos\theta}$$

$$s = \frac{0.0375\gamma_g L\cos\theta}{\bar{z}\bar{T}}$$

Gas Reservoir Deliverability

$$q = \frac{kh\left[m(\bar{p}) - m(p_{wf})\right]}{1424T\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s + Dq\right]}$$

$$S = \frac{kh}{141.2qB\mu}\Delta p_s$$

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In a summary what I could like to say we could establish so far relationship for IPR, WPR and now for the CPR this our production system is completed where all three segments IPR, WPR and CPR the flow rate and pressure draw down relationship has been established with this in the next class we will discuss once example of the CPR and the nodal analysis.

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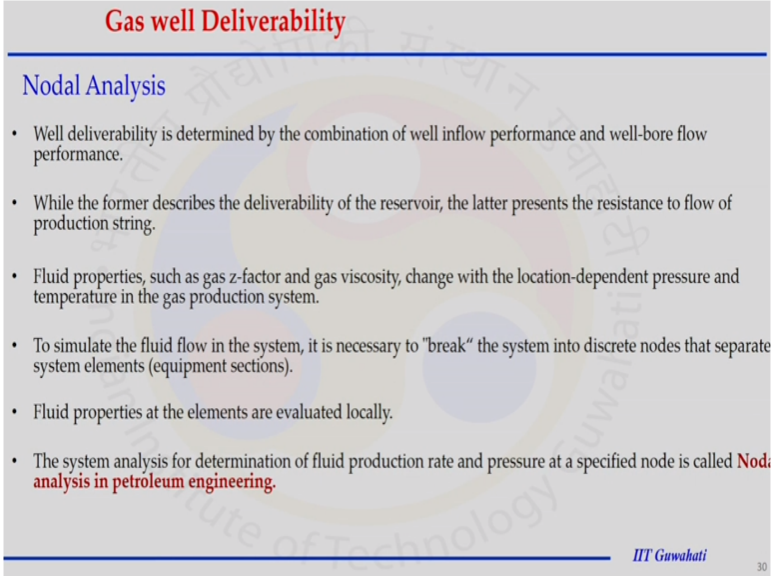

Gas well Deliverability

Nodal Analysis

- Well deliverability is determined by the combination of well inflow performance and well-bore flow performance.
- While the former describes the deliverability of the reservoir, the latter presents the resistance to flow of production string.
- Fluid properties, such as gas z-factor and gas viscosity, change with the location-dependent pressure and temperature in the gas production system.
- To simulate the fluid flow in the system, it is necessary to "break" the system into discrete nodes that separate system elements (equipment sections).
- Fluid properties at the elements are evaluated locally.
- The system analysis for determination of fluid production rate and pressure at a specified node is called **Nod analysis in petroleum engineering.**

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This nodal analysis actually a concept which says how to evaluate the properties depend on the temperature and pressure cannot be the entire system of production system breakdown in some nodes and at particular nodes the properties are calculated locally and use in the expression this is a (()) (58:43) and widely used in petroleum engineering to understand the system or to related one equation with the other equation and getting some of the parameter those are difficult to measure thank you thank you very much for watching the video we will meet in the next class thank you.