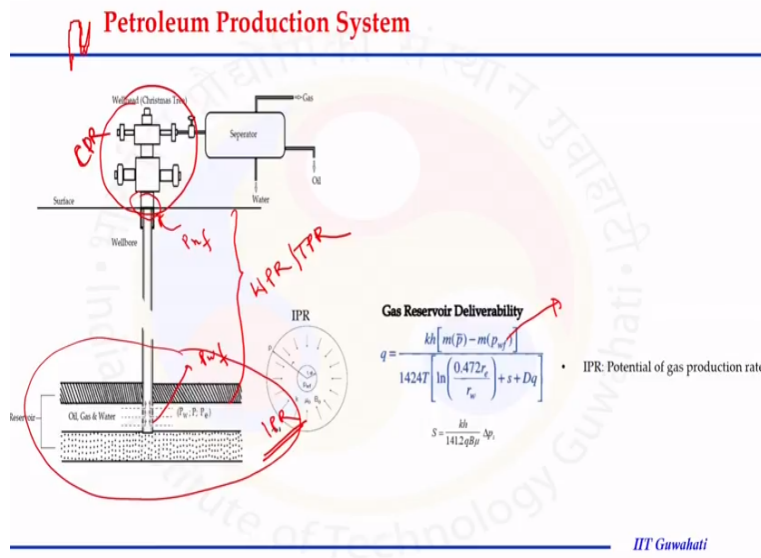


Natural Gas Engineering
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Module No # 02
Lecture No # 07
Inflow Performance Relationship (IPR) – I

Hello and welcome to the class of natural gas engineering in this lecture we will go through the conventionally natural gas production and the first thing is gas reservoir (()) (00:42) that means how the flow rate will happen what is the potential of the reservoir at what maximum rate production can be achieved from a particular reservoir or gas reservoir this is accomplished with the help of inflow performance relationship that is the topic of today's lecture.

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In flow performance relationship related this segment of our conventional petroleum production system. If we look the overall picture of production system we can divided this in 3 segment this is IPR where the flow is happening from reservoir a complex domain that is pores in the nature gases stored there in the pores region because of the permeability in the reservoir the gas is flowing from reservoir to well bore.

And the relationship developed here called the IPR next segment is WPR where from this point the pressure pwf we call it bottom hole pressure from this point the gas travels all the long distance to reach surface and the pressure is phf and the performance relation that is related pressure drop down pwf to phf with the flow rate is term in the form of WPR or TPR tubular performance relationship and WPR is well bore performance relationship.

Once the gas reaches on the top where the first thing encounters the choke device or the Christmas tree. This Christmas tree on the top that is the visible part when we visit any field is Christmas tree that Christmas tree is having few devices some are for monitoring purposes like the temperature pressure some are to control the flow rate and safety point of view like safety walls are there.

So the choke is there that is restrict the flow actually that is controlled the flow production from this particular reservoir well and the relationship here called CPR choke performance relationship. In today's class we will focus on IPR not in today and tomorrow class we will discussing in more detail about this inflow performance relationship.

If we see the mathematical expression for one condition that is called pseudo steady state condition the expressions says Q is directionally proportional to pressure difference and the form is shown here of the pressure is pseudo real gas pressure that we understood in the our properties classes how to represent pressure in the form of pseudo reduce pressure and why we should include this type of terminology in the expression will be discuss in today's class.

Overall Q is related to pressure drop + fluid and reservoir properties IPR provides the potential of gas production rate at what rate we can produce the gas from a particular well.

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Gas reservoir deliverability

- ☐ Gas reservoir deliverability is evaluated using well inflow performance relationship (IPR).
- ☐ Determination of the flow capacity of a gas well requires a relationship between the inflow gas rate and the sandface pressure or bottom pressure
- ☐ Gas production rate is a nonlinear function of pressure drawdown (reservoir pressure minus bottom hole pressure).
- ☐ IPR depends
 - ☐ Flow regime (boundary conditions)
 - ☐ Fluid properties
 - ☐ Formation properties
 - ☐ Geometry

- ☐ Compressible or incompressible
- ☐ Single phase or multiphase
- ☐ Flow behaviour
transient, steady state, pseudo steady state.
- ☐ Darcy or non-Darcy flow
- ☐ Skin effect
- ☐ Vertical well or horizontal well
- ☐ With or without Hydraulic fractures

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If we go further we see the IPR depends on the flow condition what type of flow condition are adjusting in reservoir. For example the well is being produced under steady state condition, pseudo steady state condition or the transient condition mostly it is operated that is pseudo steady state condition while steady state condition are really (()) (04:31) achieved in the reservoir or especially in gas reservoir this phenomena happens when the pressure reservoir is maintained constant by enemies.

That enemies could be by injecting addition fluid from outside to maintain the pressure or some nearby aquifers is able to maintain the reservoir pressure to the constant value and transient condition is the condition when the reservoir pressure or the bottom hole pressure is changing with respect to time we will discuss this in detail later on. Fluid properties like the fluid which is being produced from a reservoir it is a single phase fluid multiphase fluid or even in case of single phase fluid the fluid is compressible in nature or incompressible that is also that also affect the performance of IPR.

Formation properties that already mentioned reservoir is very heterogeneous system or simplicity purpose we assume certain type of the phenomena those are happening there we assume them like under the homogenous condition for example the porosity and permeability are distributed throughout the system we assume for the representation purpose or the calculation purpose the average value that we understood from our properties class how to provide the average value for the reservoir parameter.

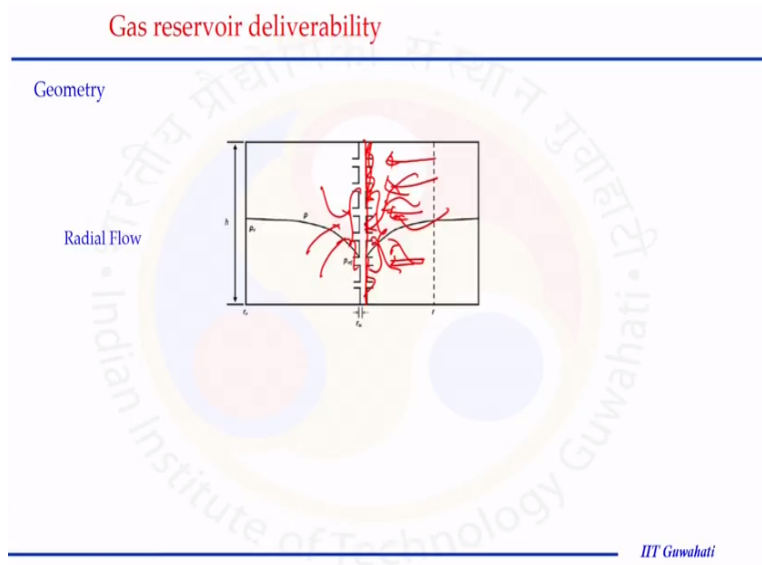
Not only the properties but the formation itself is very heterogeneous and the flow may not be happening in a uniform pattern that we will discuss. Geometry of the reservoir further what

type of the formation it is and how the flow is happening in the reservoir that will also affect the IPR performance. If we summarize quickly the compressible nature of the fluid that is the nature of natural gas will be affected we are going to deal with single phase system when it is multi-phase this system will be very complex.

And again a complex set of the equation need to be solved to represent the multi-phase system flow behavior transient steady state, pseudo steady state will be discussed. Darcy and non-Darcy effect so the flow under pseudo steady state condition or steady state condition or in transient condition could be Darcy or it could also to be non-Darcy and when it is going to turbulent region Darcy law is not applicable we have to apply some factor to account for the turbulent flow and that turbulent flow can be represented via term called non- Darcy term that will be discussed in the entire IPR discussion.

The skin effect due to some conditions the reservoir is having addition pressure drop down near the well bore and if this is happening the skin effect will play or role and we have to account that also. The well which is installed could be vertical could be horizontal or could be in the combination of that will also affect the performance of IPR and if the fractures artificial fractures are created or not created they will effect ultimately the permeability and porosity of the reservoir and if that is happening we have to account for that also.

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So let us discuss the one by one the geometric as discuss we are going to assume in our case the flow is radial but in actual reservoir is heterogeneous it may not have many any uniform flow pattern and if we see the Pie that is perforated to allow the fluid from the reservoir to well bore and if entire page on thickness or the net page on thickness that is responsible or that is contributing for this flow is being utilized and the flow is happening from all the direction of the reservoir towards this well.

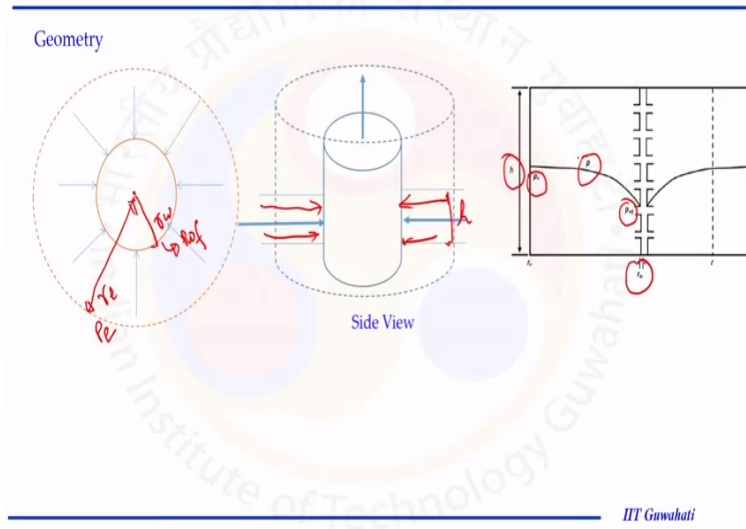
We can assume the radial flow is happening and the fluid is flowing only in the radial but if that is not the situation. For example the perforation is only one side and the fluid is just traveling from this side to reservoir well or from one direction to another direction axial it is travelling then the flow resumes could be considered under axial flow.

There might be a situation when the entire perforated region is not being used or there may be a situation when the gas is just passing through only small perforated region entire region is not perforated for example this is not perforated only this section is perforated and the flow is happening like this. So the spherical flow is happening we have to account that kind of the flow behavior in our model development.

There might be a situation when the flow is just happening from the bottom section of the pipe it means there is no perforation in the pipe the flow happening is just from the bottom of the pipe or the pipe is inserted in a gas bank where the flow is just happening from the bottom and that is called the MI spherical flow. So the flow which is happening the flow resume which is existing in the reservoir or the geometry of the reservoir or the configuration chosen to produce the gas from the reservoir will affect the IPR equation.

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Gas reservoir deliverability



Let us consider the case which we are going to study the geometry is considered as radial. So this is our center point where the well of radius r_w is installed and the fluid is being produced from a reservoir of radius r_e . The dotted line shown here for the radius r_e that means it is a very long distance from a long distance fluid is being transported towards this wellbore and the pressure here at the reservoir boundary condition is p_e while the pressure at the wellbore is p_w .

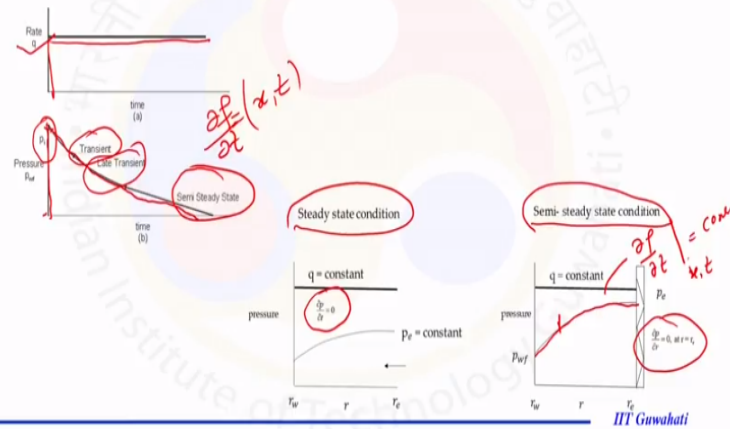
This is also called sand phase pressure and if this is happening the flow is like this if we see from this side view and this is happening from all the direction here it can also be seen but from entire page on thickness h is contributing to production. More clearly it can be seen here which we just seen in last slide this h is a page on thickness this is reservoir pressure p_e at distance r_e that is the boundary of the reservoir r_e is the radius of the wellbore and that pressure at that point is p_w .

And the p represent pressure at any particular location at particular time between r_e and r_w .

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Gas reservoir deliverability

Flow Regime



The next part is flow regime so when the well is completed production is supposed to start and when we open the well for the production at a constant flow rate q what we see the flow rate q is maintained constant q and the pressure at the well bore and the production is not happening it is either reservoir pressure P_i or P_e and when the production is start at the pressure is the wellbore start declining because of the pressure difference only the flow will happen.

So the pressure is declining bit time and achieving some pseudo steady state condition that is also called MI steady state condition. And during this time it is first going through the transient phase and later on it is related transient phase or altogether we can say in this region or at the early time of the production the condition is transient and the pressure is changing with respect to time.

In steady state condition what happens there is no change in pressure with respect to time as well as position means no reservoir information can be obtained under steady state condition because nothing is changing with respect to time and this is not a phenomena which frequently occur in the reservoir actually this is only occur when we maintain the pressure of the reservoir at a constant value by some external means.

The most common phenomena that happens in reservoir is MI steady state condition as we can see from here when the pressure is declining reaching some MI steady state condition at the

MI steady state condition the reservoir will be flowing or will be producing the gas for an most of the time and the reservoir pressure will be behaving like this.

So from P_{wf} to P_e there will be pressure gradient for that pressure gradient will be constant at a particular location change in value of p with respect to r will be constant throughout this region as well as additional boundary condition will be obtained that says pressure change with respect to position at $R_e = 0$ means there is no in flex of any fluid is happening from the boundary condition and in this condition the change in pressure with respect to time will be constant for any position x and any time t .

Here it is $\frac{dp}{dt} = 0$ under the steady state condition in transient condition your pressure change with respect to time is a function of both position x as well as time t . And these phenomena are very important when we do the gas well testing we will learn the gas well testing after the IPR where we will see how the transient condition can help us to understand more information about our reservoir what exactly happens near the well bore.

When we are doing the pressure transient test it means we are playing with our test well doing some test at understanding how the properties of reservoir are going to evaluated with the help of gas well test. Similar under the MI steady state condition we can establish the IPR relationship with the field data and that represent more accurately what exactly happening in the reservoir or in a particular reservoir.

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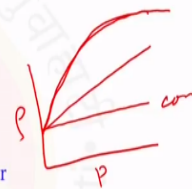
Inflow Performance Relationship (IPR Curve)

□ Material balance equation:

- ✓ Relationship between pressure drop and flow rate
- ✓ Analytical and Empirical expressions for IPR

□ Mathematical expression:

- Single phase gas flow : Type of Fluid
- Steady state & Pseudosteady state flow: Flow behavior



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Those information gas well testing will be learnt after the IPR classes if we move further we want to develop the mathematical relationship for the phenomena when q is related to pressure gradient from reservoir properties from fluid properties we understand the relationship of IPR is going to be affected because of the changes in the properties of the fluid and knowing the properties of reservoir will allow us to establish that relationship and that can be accomplished with the help of material balance equation for radial flow single gas flow from a reservoir well.

And analytical and imperial expression can be established for IPR in today's class we are going to understand the analytical expression means from fundamental theory how we can relate q with pressure difference that is adjusting in the reservoir that is responsible for flow in terms of reservoir properties as well as fluid properties it will be established under certain assumptions like the gas phase is only single phase that is gas phase types of flue will also affected.

So for example the compressibility when we are assuming our gas is compressible or in fact natural gas is compressible because it is density changes with pressure. So we will be having the relationship how the density is changing with pressure if it is not changing. If it is MI or slightly compressible it would be linear relationship when it is compressible fluid the non-linear relationship will adjust between the pressure and density and our natural gas is

compressible in the nature we have to account that from that phenomena also in our mathematical balance equation.

We will solve for steady state and pseudo steady state condition because the boundary condition those going to be applied on material balance equation will ultimately result in a mathematical expression that is represent that particular flow behavior either it is a pseudo steady state or steady state condition. We will also see the expression for transient condition.

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Gas reservoir deliverability

Pseudo Steady State: Analytical expression

$$q = \frac{kh [m(\bar{p}) - m(p_{wf})]}{1424T \left[\ln \left(\frac{0.472r_e}{r_w} \right) + s + Dq \right]}$$

- q is the gas production rate in Mscf/d, ✓
- k is the effective permeability to gas in md, ✓
- h is the thickness of pay zone in ft, ✓
- $m(p)$ is the real gas pseudopressure in psi^2/cp at the reservoir pressure p in psi, ✓
- $m(p_{wf})$ is the real gas pseudopressure in psi^2/cp at the flowing bottom hole pressure p_{wf} , ✓
- T is the reservoir temperature in R, ✓
- r_e is the radius of drainage area in ft, ✓
- r_w is wellbore radius in ft, ✓
- s is skin factor, and ✓
- D is the non-Darcy coefficient in d/Mscf. ✓

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So the big expression that is says how q is related to the parameters those are appearing in this expression q is directly proportional to permeability k . So if the reservoir is having more permeability it is allowing the fluid to flow more towards the well bore the flow rate can be achieved at a higher value. Similar if the phase one thickness is high and entire phase 1 thickness is being used as a net phase on thickness it means entire phase on is contributing towards the production the queue can be enhanced if the H is high.

And other parameters those are appearing here like s this account for the skin effect and Dq this all together accounts for the non-Darcy behavior because this mathematical equation that is shown for pseudo steady state condition has been established with the help of Darcy law. And Darcy law is applicable only for laminar flow when we phase non Darcy behavior in the flow we also have to (()) (18:45) throw some coefficient that can be accounted with the help if Dq in this equation.

Other is r_e and r_w T is also the reservoir average temperature is also in the denominator of this situation important point here is this numerical coefficient. We will see when we are developing the relationship for steady state and pseudo steady state we will see how this numerical coefficient comes in the expression. And this value depends on the unit system chosen for a particular parameter in the reservoir expression.

In this case the q is represented in Mscf of per day all these are in US field unit system and that is why the value of 1 upon 1424 is appearing in the numerical in this expression as a numerical coefficient when we are using the SI unit system the numerical coefficient will get changed. k is the effective permeability that is represented in milli Darcy f is thickness that is represented in feet.

p_p and p_{pwf} are pseudo real gas pressure and the unit associated with it is pressure is called centipoise or psi square percent centipoise in the US field unit system T in ranking r_e and r_w are in feet s as no unit in the skin factor that as no unit D is the non-Darcy coefficient and the unit is day per Mscf.

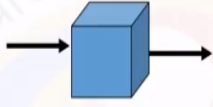
As we are going to deal with US field unit system and sometimes important to convert from other unit system to US field unit system we have to be more careful about adjusting this numerical coefficient that is appearing here and this expression is based on certain assumption like flow is single phase flow and gas is compressible homogenous reservoir is considered means the page on thickness and the porosity are not varying similar isotropic nature is the permeability or the reservoir properties are constant in all the direction.

Constant permeability was assumed flow is radial only and first it is started with the laminar and later on it has been extended for non-Darcy flow also.


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IPR Curve- Material Balance

Conservation of Mass



What goes in = What comes out + What is left behind



$q \cdot \rho |_{r+\Delta r} - q \cdot \rho |_r = \frac{\partial V \cdot \rho}{\partial t}$
 $\frac{\partial}{\partial r} [q \cdot \rho] = \frac{\partial (2\pi r h \Delta r \phi \rho)}{\partial t}$
mult. flux rate

$h = \text{constant}$
 $\phi = \text{constant}$
 $\Delta r \rightarrow 0$

Assumptions:

- Single phase flow in the reservoir
- Compressible isothermal fluid flow
- Homogeneous and isotropic reservoir system
- Constant permeability
- Radial flow only
- Laminar (Viscous flow)
- Constant Pay-zone

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So let us set up the mathematical equation how to achieve the expression which is shown in the last slide. And that can be done by applying the conservation of mass that simply says mass is conserved whatever goes in equal to whatever comes out + whatever left within the system or within the control volume or in simple way out – in = accumulation.

And under certain assumption which we are having for our system single phase flow compressible nature of the gas homogenous properties and isotropic nature of the reservoir we are going to establish the mathematical expression. So let us start with the cell balance as we do in our courses especially in the fluid mechanics class where we establish the cell balance at a distance r from the reservoir from the center of wellbore and distance or there is slab of Δr thickness we can write the in and out type of the material balance equation and you can get the mathematical model.

So let us see what we do for the mass conservation equation for this thickness Δr which is at r distance which is the mass balance equation simply says $q \cdot \rho$ at $r + \Delta r$ is volumetric flow rate this is multiply with density ρ at a condition in that is $r + \Delta r$ so from here the fluid is entering – whatever comes out is again $q \cdot \rho$ at $r =$ accumulation.

Accumulation can be represented in terms of change in volume multiply by density and this volume can be represented as $2\pi r$ multiply h and Δr is the thickness of that slab. And this is not the volume where the gas is going to store from our properties of

natural gas class we understand the volume which is being occupied or which can be occupied by the natural gas should be the actual volume multiplied by the porosity and this is the volume that will be used by the natural gas to get accumulated in that slab.

And if we assume our delta r tense to 0 we can write this in the form of derivative and that is simply comes out is $\frac{\partial}{\partial r}$ of $q \cdot \rho = 2 \pi r h \phi \frac{\partial \rho}{\partial t}$. Here you see what we assume h is constant that is why it can be taken out. We will keep writing what we are going to assume here we again assume here ϕ is also constant that is porosity so this is porosity h is phase on thickness this entire thing is the volume or occupied by the gas and this called the mass flex rate with this.

Now this equation include this q and the flow is happening through a porous media and to understand how the flow is happening in the pores media we have to go to Darcy law to establish the relationship how q is related to pressure.

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Basic Form of IPR

- Reservoir Properties: Transportation equation- Darcy's Law
- Fluid Properties: Isothermal compressibility
- Equation of state for Real gas

$q = \frac{AK}{\mu} \frac{\partial P}{\partial r}$ → permeability
C necessity of fluid

$\frac{2\pi r h}{\mu}$

$P_1 V_1 = n_1 R T_1 = \frac{P_1 q_1}{Z_1 P_1 T_1}$
 $\frac{P_1 q_1}{Z_1 P_1 T_1} = \frac{P_2 q_2}{Z_2 P_2 T_2}$
 $C = -\frac{1}{V} \frac{\partial V}{\partial P} \Big|_T$
 $C = -\frac{1}{P} \frac{\partial(PV)}{\partial P} \Big|_T$
 $C = \frac{1}{P} \frac{\partial P}{\partial P} \rightarrow C P \frac{\partial P}{\partial t} = \frac{\partial P}{\partial t}$

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And that can be done considering how reservoir properties are going to contribute for the transportation equation that can be represented by the Darcy law. Darcy law simply says for the radial flow condition $q = Ak / \mu \frac{\partial p}{\partial r}$ k is a permeability of formation which is along the fluid to travel from one point to another point or from reservoir domain to well bore.

μ is the viscosity of the fluid being produced so in our case it is natural gas is the viscosity of natural gas viscosity of let us write viscosity of fluid and A is the area that is $2\pi rh$. h is page on thickness here minus sign come when we are writing for the axial flow but in the radial flow the pressure drop is happening in the same direction when the radial is also getting reduced so the minus sign will go away. With the help of this we can modify our material balance equation or the mass balance equation.

In that equation we are having density of the fluid also and the natural gas is compressible by nature. So we have to modify that equation for that density term also that can be represented in thermal compressibility definition for a compressible fluid under isothermal condition the compressibility C can be defined as $1 - v \frac{dv}{dp}$ at isothermal condition. We know this from our properties of natural gas class change in volume with respect to pressure under isothermal condition divided by volume = compressibility of that gas.

This can be modified in the form of density we know $\rho = m/v$ if we substitute this we will get $-1/\rho \frac{d\rho}{dp}$ at T and finally we are going to get $1/\rho \frac{d\rho}{dp}$ or in other form we can write this $C \rho \frac{dp}{p}$ is equal to $\frac{d\rho}{\rho}$ and whenever we needed we can take the derivative this up to time or derivative with respect to position also of this equation to represent in our basic equation.

And when we substitute the fluid properties when we substitute the fluid properties when we substitute the reservoir properties by the Darcy law and use equation of the state for the real gas that simply says $Pv = nzRT$ or in other form $P_1 v_1 = z_1 R_1 T_1$ means the ideal gas law at one condition equal to ideal gas law sorry real gas law at one condition = real gas law.

At second condition we can replace several parameter those are going to appear in our material balance equation to get the basic form of the IPR that allow us to understand how these parameter are going to affect in equation of the state we assume 1 of the condition S standard condition and under standard condition our z_2 becomes 1 okay. So if we include these in our previous equation of the mass balance what are going to get as a mass balance equation.

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Basic Form of IPR

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left[\frac{K\rho}{\mu} \cdot r \cdot \frac{\partial p}{\partial r} \right] \right] = \phi c_p \frac{\partial p}{\partial t}$$

Max Balance eqn
p, c, μ
K
T

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Let me write this mass balance equation we are going to get $\frac{1}{r} \frac{\partial}{\partial r} \left[\frac{K\rho}{\mu} \cdot r \cdot \frac{\partial p}{\partial r} \right] = \phi c_p \frac{\partial p}{\partial t}$. So what we did we converted the equation of material balance density form to pressure form. And solving this equation we can give us the expression that is related to how Q flow rate is related to pressure but this equation is not having any analytical solution because the properties of the fluid as well as other properties like compressibility of the fluid as well as the permeability of the reservoir or the function of pressure.

And that is why this is not having an analytical solution to solve this equation we have to make other certain assumptions. So let us say what are the function of pressure is density C compressibility μ viscosity what we can assume state we can say our reservoir is isotropic we can take this k permeability outside we can assume reservoir is under isothermal condition T can also be either taken as average or just one value that can be taken out of this derivative term.

So to solve this equation we have to make certain assumptions certain approximation to account for the changes happening in the properties and then only it can be solved. Simplest part is the reservoir properties can be assumed constant but the changes in properties of the fluid because the fluid is travelling from reservoir pressure to well bore pressure it is a very high pressure job and pressure is also change in significantly the properties like compressibility and viscosity compressibility factor and viscosity will change significantly.

And to account those changes happening in these properties we have to mix certain assumptions so let us see what we can do.

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Basic Form of IPR

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left[\frac{kP}{\mu} \cdot \gamma \cdot \frac{\partial P}{\partial r} \right] \right] = \phi c P \frac{\partial P}{\partial t}$$

$\gamma = \frac{2.7 \gamma P}{2T}$
 $\frac{2P \partial P}{\partial r} = \frac{\partial P^2}{\partial r}$
 $\frac{1}{r} \left[\frac{\partial}{\partial r} \left[\frac{P}{\mu z} \cdot \gamma \frac{\partial P}{\partial r} \right] \right] = \frac{\phi \mu c}{K} \times \frac{P}{\mu z} \frac{\partial P}{\partial t}$
 $\frac{1}{r} \left[\frac{\partial}{\partial r} \left[\gamma \frac{\partial P^2}{\partial r} \right] \right] = \frac{\phi \mu c}{K} \frac{\partial P^2}{\partial t}$

We can go to literature to understand the nature of these properties so individually one property is changing for example viscosity is changing with pressure compressibility is changing what we term those are appearing in our equation like $P / \mu z$ or one upon μB_g . B_g I will explain it is volume formation factor we understand from our previous classes how to relate volume formation pressure for reservoir condition.

$B_g P / \mu z$ if the lump this parameter and see how they are going to change when the pressure is changing when allow us those can be taken out of the derivative terms or the integral terms or not if yes we can take them out or if not how we can substitute those in other form to make this equation soluble. So let us say if we see the behavior what we see when the pressure is very high region three high pressure region the properties or the lump properties $2P / \mu z$.

For example is becoming constant it is not changing the pressure so if we are having high pressure region wherever this combination of $P / \mu z$ or one upon μz is appearing in the cases where we can take them as a constant value. When the lower side or in the region 1 when the pressure is less than 2000psia we see the p changes affecting $P / \mu z$ or $1 / \mu z$

linearly it means this lump parameters changes linearly with respect to pressure and for that reason what we can assume?

We can assume this parameter is having linear relationship with p with some constant let us say a . So with the assumption we got for this reason is $2p / \mu z$ or other way one upon μz wherever it is appearing we can assume it is a linear relationship with pressure with the constant A that shows the slope of that linear relationship. When we are between 2000 and 3000 psi the relationship neither constant nor linear it is having some curvature in the nature that may vary.

This case is discuss or vary ideal condition where two relationship this lump parameter in pressure are shown in this graph there might be more complex situation. What if the situation is like that when we take the lump parameter constant or where we cannot take any linear relationship of this lump parameter with pressure?

So we should go with the mp approach and by the definition of mp approach what it does it include $P / \mu z$ in the form of mp it integrate this term or this lump parameter from base pressure to the pressure of interest and during this integration it accounts all the changes are happening in the individual parameters P and μ and z altogether and represent the true changes in the properties those are happening when the fluid is travelling from a distance from one pressure region to other pressure region.

And in by large if we can afford to do the computational integration of this lumped parameter to estimate mp it should always go with mp approach because mp approach does not assume it is a linear relationship or it is a constant relationship. You can see this mp approach can be substitute to P square approach when we see this $P / \mu z$ is constant and we do the integral we can calculate this values of viscosity and compressibility factor at average value.

And in both the cases when we are having here in this region 1 and region 3 how to calculate the value of the coefficient because pressure is changing which one i should have taken high pressure is region or low pressure region so it is approximated further in the p approximation all the parameters all the values of the fluid properties should be calculated at a average reservoir pressure or the arithmetic average reservoir pressure.

While in this reason when the linear relationship is there the average pressure should be used to calculate the value and that average should be calculated like $P_e^2 + P_{wf}^2$ by 2 and square root of this term okay. And in this reason we already integrating the term from P_v to P and S small s interval we are choosing for this integration for more accurate result we are going to get.

So now from this understanding we can see the basic IPR equation can be modified or can be linearized to make it solvable and that is we can do so when we are having this equation mass balance equation we can do the substitution for some of the parameter to get the linear form of this. One of the way we can do is let us say where we can assume the constant T is constant and γg is also constant what we can do the ρ that is still here can be represented as $2.7 \gamma g P / zT$.

Now γg is constant so we can get out another thing is we can replace this $2p \frac{dp}{dr}$ can be represented as $\frac{dp^2}{dr}$. So by doing all these things what we can do we can make this assumption or approximation of below 2000psi possible and say $P / \mu z$ or $2P / \mu z$ is linearly related to pressure now with the help of this expression and this expression we can substitute all this in this equation and finally what we are going to get is $1 / r \frac{d}{dr} \frac{p}{\mu z}$ or $\frac{dp}{dr} = \frac{f_e \mu c k P}{\mu z} \frac{dp}{\Delta t}$.

So this is in the form of p approximation if we substitute everything what we are going to get is $1 / r \frac{d}{dr} \frac{p^2}{r} = \frac{f_e \mu c}{k} \frac{dp^2}{\Delta t}$. So what we did we transform the basic IPR equation in the form of P_e^2 so this is linear in terms of P_e^2 square. Now we can solve this equation for P_e^2 similarly we can linearize that equation for m_p approach also by the definition of m_p .

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Basic Form of IPR

$$m(p) = 2 \int_{p_b}^p \frac{p}{\mu z} dp$$

$$\frac{\partial m(p)}{\partial r} = \frac{2p}{\mu z} \frac{\partial p}{\partial r}$$

$$\frac{\partial m(p)}{\partial t} = \frac{2p}{\mu z} \frac{\partial p}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial m(p)}{\partial r} \right] = \frac{\phi \mu c}{k} \frac{\partial m(p)}{\partial t}$$

We know this is a pseudo real gas pressure that = 2 p base pressure to the pressure of interest p / Mu z dp and if we do the derivative of this with respect to r we are going to guess this term 2p / Mu z del p / del r is 2p and with respect to time del mp / del T = 2p / Mu z del p / del t. So now in the equation or basic IPR equation that is in the form of p we can go with substituting these two and get the relationship in the form of mp.

And when we do that what the basic IPR equation in the form of mp we can get is 1 / r del / del r of r del mp / del r = Fe Mu c / k del mp / del t. So this is the basic IPR equation in the form of mp and seeing this equation we can say all the properties are appearing here all the parameter those are appearing in this term fe and k in the reservoir properties Mu and z are fluid properties and other fluid properties are accounted in the mp for the changes are happening.

And this Mu can be calculated at average reservoir pressure condition so this IPR equation not become linear in the form the MP and it can be solved with certain assumption that we made during the derivation of this basic IPR equation. First thing we have to apply the boundary conditions to solve this equation and for that we can go further to establish the relationship for steady state and pseudo steady state condition.

Important is here the mp approach needs computation facilities to calculate and if we cannot afford to write the computer program we have to look some other approach like p square

approximation or p approximation or in other terms we have to go to some imperial correlations that will be discussed in the next class.

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Steady State: IPR

$$\frac{\partial p}{\partial t} = 0$$

$$\frac{\partial m(p)}{\partial t} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial m(p)}{\partial r} \right) = 0$$

$$\Rightarrow r \frac{\partial m(p)}{\partial r} = \text{constant}$$

$$\frac{\partial m(p)}{\partial r} \Big|_{r=r_w} = h = \frac{2\pi r_w k}{\mu} \frac{\partial p}{\partial r} \Big|_{r=r_w}$$

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So let us see when are talking about steady state IPR what we get for the steady state condition the boundary condition we get or the initial condition we get $\frac{\partial p}{\partial t}$ is 0 for entire duration or the reservoir is under steady state condition. Similar we can write for $m(p)$ form also or p square form also $\frac{\partial m(p)}{\partial t} = 0$ just substituting this on the right hand side of our basic IPR equation and that basic IPR equation in the form of $m(p)$ is $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial m(p)}{\partial r} \right) = 0$.

So the right hand side becomes 0 for steady state condition and under steady state condition that emphasize low reservoir information can be obtained this is just a kind of hypothetical situation when we assume nothing is changing with respect to time. If we solve this with the help of Darcy law what we are going to get is we can replace some of things here when we do the first integral of this term what we are going to get $\frac{\partial}{\partial r}$ will go away and we are going to get $r \frac{\partial m(p)}{\partial r} = \text{constant}$.

And to know the value of constant we have to use the Darcy law and Darcy law simply says at $r = r_w$ is near at the wellbore the production q will be $2\pi r_w h$ this is the area permeability k viscosity μ $\frac{\partial p}{\partial r}$ at $r = r_w$. From this equation we can convert this into $m(p)$ form also that $= \frac{2\pi r_w h k}{\mu} \frac{\partial m(p)}{\partial r}$ at $r = r_w$.

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Steady State: IPR

$$c = \frac{aP}{z} \cdot \frac{1}{\pi kh}$$

$$\frac{aP}{z T_1} = \frac{aP}{z T_2} = \frac{a_{sc} P_{sc}}{T_{sc} z_{sc}} = c$$

$$r \frac{d m(r)}{dr} = T_{sc} T \left(\frac{P_{sc}}{T_{sc}} \right) \cdot \frac{1}{\pi kh}$$

$$\frac{m(r_{wf})}{m(P_e)} = \frac{a_{sc} T \left(\frac{P_{sc}}{T_{sc}} \right)}{\pi kh} \int_{r_e}^{r_w} \frac{dr}{r}$$

We can substitute in the equation and can get the value of constant and the value of constant will be $c = qp / z \cdot 1$ upon πkh . When we are putting this in the equation along with the gas law that is $qp / z T$ at condition 1 = $qp / r T$ r is cancelled out that is common in both the condition 1 and 2 even the z_2 if it is standard condition let us write is $Q_{sc}, P_{sc}, T_{sc}, Z_{sc}$.

So the P_{sc} the standard pressure 14.7 Psi T_{sc} is the standard temperature or obsolete temperature that is 520 ranking and $Z_{sc} = 1$ and if we substitute both of them in the equation for the steady state we are going to get $r \frac{d m(r)}{dr} = Q_{sc} T P_{sc} / T_{sc} \cdot 1$ upon πkh and integrating this from P_e to P_{wf} we are integrating in form of m_p so m_{pe} / m_{pwf} we can integrate this and get $Q_{sc} T$ upon $\pi kh P_{sc}$ upon T_{sc} integrating from r_e to r_w dr / r . By doing this separation of variable we can integrate this.

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Steady State: IPR

$$\begin{aligned}
 \text{if } p_e \rightarrow p_f \\
 \hookrightarrow \frac{m(p_f)}{r_w} &= \frac{\int_{r_w}^{r_e} m(p) dV}{\int_{r_w}^{r_e} dV} \rightarrow \pi r_e^2 h \phi \\
 &= \frac{2}{r_e^2} \int_{r_w}^{r_e} m(p_r) dr
 \end{aligned}$$

$r / P \rightarrow$

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And after the integration we are going to get the expression like in this form $m_{pe} - m_{pwf} = Q_{sc}$ still let us write in the form of standard condition first multiply by reservoir temperature, standard pressure, standard temperature $\ln r_e / r_w$ this $\ln r_e / r_w$ came from the integral of previous slide. We can adjust this equation in the form of Q_{sc} that says kh / T multiply by pressure difference $m_{pe} - m_{pwf}$ divided by the radius relationship $\ln r_e / r_w$ multiply by T_{sc} / P_{sc} .

I think I missed P_{ie} also there should be $p_{ie} kh$ here should also $p_{ie} kh$ and when we substitute the value for P_{ie} standard temperature standard pressure we are going to get some numerical coefficient here and other things will remain the same like kh upon T $m_{pe} - m_{pwf}$ divided by $\ln r_e / r_w$ this numerical coefficient will depend on the unit system has been chosen. So for example if US field unit system is chosen the value will be $703 \cdot 10^{-6}$.

So if US field unit system is chosen the value for this x prime will be like this $703 \cdot 10^{-6}$ and in this case the q is measured in mscf per day k is in Darcy h in feet pressure m_p is in Psi square per centipoise and the radius r_e and r_w are in feet temperature in degree ranking that is why in US field unit system the numerical coefficient comes out like this when we are measuring in SI unit system the value of this x prime will be $7.63 \cdot 10^{-7}$ and in this case the Q is measured Q_{sc} is measured in standard meter cube per day.

k is measured in milli Darcy the h in meter and mpe and mpwf both are measured in kilo Pascal square per Pascal second radius re and rw they are measured in meter and because of that unit system has been chosen in numerical coefficient will get changed and that we should be careful when we are writing this mathematical expression and we are assigning the unit for each parameter that is appearing in the expression.

Another thing could be the PE reservoir pressure can be express in the average reservoir pressure and that can be done with the help of volumetric average or taking the volumetric average of the reservoir pressure comes out as $2 / re \text{ square } rw^2 re \text{ mpr } dr$. This because this is $Pie re \text{ square } h \text{ fe}$ so by doing all this thing we can get the steady state IPR equation in the form of general pressure, average pressure or the pressure from the volumetric average.

Similar can be done for p square approach and p approach you can solve in the similar manner for this steady state condition for p square approach as well as p approach.

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Pseudo Steady State: IPR

$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial m(p)}{\partial r} \right] = \phi \mu c \frac{\partial m(p)}{\partial t}$
 $\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial m(p)}{\partial r} \right] = \frac{-2Pq}{\pi r_e^2 k h \Delta} \quad \frac{\partial P}{\partial t} = \text{constant}$
 $\frac{\partial m(p)}{\partial r} = \frac{-Pq r}{\pi r_e^2 k h \Delta} + C_1$
 $\frac{\partial P}{\partial r} \Big|_{r=r_e} = 0 \Rightarrow \frac{\partial m(p)}{\partial r} \Big|_{r=r_e} = 0$
 $\Rightarrow C_1 = \frac{Pq}{\pi k h \Delta}$

Semi-steady state condition

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Now what happens under the pseudo steady state condition if we see by the definition of pseudo steady state we are having this boundary condition or the condition will says the pressure change with respect to time is constant. So the basic IPR equation on the right hand side what we are going to get is a constant value instead of 0 that we got in steady state condition pseudo steady state condition we are going to get this relationship which is the function of

so this is function of r and t but at a particular r and t the value is constant and another condition we are having change in pressure with respect to position at r = re is 0.

Using this equation in basic form of IPR let us write again in the form of mp we are having $1/r \frac{dm(r)}{dr}$, $r \frac{dm(r)}{dr}$ or $\frac{dm(r)}{dr} = \frac{pq}{\pi k h z} \frac{1}{r}$. if we substitute using the compressibility as well as knowing the real gas law we can get the expression $1/r \frac{dm(r)}{dr}$, $r \frac{dm(r)}{dr}$ so this is left hand side that will remain constant on the other side we are going to get $-2 pq / \pi k h z r^2$ by during the substitution using the Darcy law and the compressibility of the gas we are going to get this.

Integrating first time we will get $r \frac{dm(r)}{dr} = -pq r^2 / \pi k h z + \text{constant}$ because of the integration constant can be obtained by the boundary condition because we know from this $\frac{dm(r)}{dr}$ at $r = re$ will also be 0 and putting this condition we can get the value of constant C1 will be equal to $pq / \pi k h z$ substituting this back here and integrating one more time we can get relationship for mp with respect to q and let us say how we can do that.

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Pseudo Steady State: IPR

$$\frac{r \frac{dm(r)}{dr}}{r} = \frac{-pq r^2}{\pi k h z} + \frac{pq}{\pi k h z}$$

$$\frac{dm(r)}{dr} = \int_{re}^{rw} \frac{-pq}{\pi k h z} \left[\frac{r}{r^2} + \frac{1}{r} \right] dr$$

$r_w^2 / r_e^2 \ll 1$

$$m(re) - m(rwf) = \frac{pq}{\pi k h z} \left[\ln \left(\frac{re}{r_w} - \frac{1}{2} \right) \right]$$

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So after substituting c1 we got $\frac{dm(r)}{dr} = -pq r^2 / \pi k h z + pq / \pi k h z$ integrating this from mpe to mpwf and other side from re to rw or $-pq / \pi k h z \int_{re}^{rw} \frac{1}{r} dr + pq / \pi k h z \int_{re}^{rw} \frac{1}{r^2} dr$ with respect to r and assuming the reservoir radius is too large compare to the wellbore radius that true also r_w^2 / r_e^2 will be lesser than 1.

Substituting all these things using the gas law we are going to get the expression in the form of $mpe - mpwf = pq / pie khz \ln re / rw - 1 / 2$. So this is the IPR to the form of mp for pseudo steady state condition.

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Pseudo Steady State: IPR

$$Q_{sc} = \left(X' \frac{kh}{T} \right) \left[\frac{m(p_e) - m(p_{wf})}{\ln(r_e/r_w) - 1/2} \right]$$

$P^2 \rightarrow$ approximation
 $P \rightarrow$ approximation

$$a = f(p_{wf}, p_e)$$

This equation can be adjusted to get the Q_{sc} that is equal to some numerical coefficient again that is let us write it X' again kh upon T $mpe - mpwf / \ln re / rw - 1 / 2$. So this is the same expression what has been solved earlier on the first or second slide where it says the flow rate under standard condition is function of pressure drop down this means reservoir pressure – the pressure at the sand phase and it also depends on the properties of fluid as well as reservoir.

And the X' prime again the numerical factor that depends on the unit system has been chosen and the same expression could be written in the form of p square and p square approximation can be assumed and the similar exercise can be done as we did for mp similar can be done for P approximation also and the relationship between Q and pwf and pe can be established can be form of p , p square and mp .

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IPR Curve: Analytical Expression

Different form of IPR	Steady State	Pseudo Steady State
General relationship between P & R	$p - p_{wf} = \frac{q\mu}{2\pi kh} \ln \left(\frac{r}{r_w} \right)$	$p - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln \frac{r_e}{r_w} - \frac{r^2}{2r_e^2} \right)$
Inflow equation expressed in term of $p = p_e$ at $r = r_e$	$p_e - p_{wf} = \frac{q\mu}{2\pi kh} \ln \left(\frac{r_e}{r_w} \right)$	$p_e - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln \frac{r_e}{r_w} - \frac{1}{2} \right)$
Inflow equation expressed in terms of average pressure \bar{p}	$\bar{p} - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln \frac{r_e}{r_w} - \frac{1}{2} \right)$	$\bar{p} - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln \frac{r_e}{r_w} - \frac{3}{4} \right)$

- Similar expression can be written for P² approach and m(p) approach
- The expressions can be formulated to known parameters, like formation volume factor, viscosity, etc

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So let us summarize the analytical expression those has been developed for steady state condition and pseudo steady state condition in today's class the final form here simply says how pressure difference is going to related to q for steady state here pseudo steady state here you will see additional term is appearing here this is because of the boundary condition different in pseudo steady state condition compared to steady state.

If the equation is represented in the form of reservoir pressure like p_e the pressure at reservoir boundary r_e the expression simply r will be replaced by r_e here and in the pseudo steady state condition we will get addition term because r_w / r_e replacing this r_e for r_e will cancel out. When we are taking the volumetric average at the reservoir pressure the additional term will appear in the steady state condition similarly in the pseudo steady state condition this term will get modify and compared into $-3 / 4$.

So this similar expression can be written for p square and mp approach so we solve for mp approach are just shown the expression here in the form of p similar can be done for p square approach also. The expression can be formulated to non-parameter so for example volume formation is known for a particular gas well reservoir we can substitute some of the parameter by we know volume formation factor $B_g = 0.000 ZT / P$.

If we know some of the parameters those cannot be estimated we can replace them always in the form of some other parameter those values are given us or those values can be calculated easily.

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IPR Curve: Analytical Expression- Pseudo steady state

$$q = \frac{kh}{1422T} \left(\ln \frac{r_e}{r_w} - 0.75 \right) \int_{p_{wf}}^{\bar{p}} \left(\frac{2}{\mu_g} \frac{p}{TZ} \right) dp$$

$$q = \frac{7.08(10^{-6})kh}{1422T} \left(\ln \frac{r_e}{r_w} - 0.75 \right) \int_{p_{wf}}^{\bar{p}} \left(\frac{1}{\mu_g B_g} \right) dp$$

$$q = \frac{kh m(\bar{p}_{res}) - m(p_{wf})}{1422T \left(\ln \frac{r_e}{r_w} - 0.75 + s \right)}$$

$$m(p_{wf}) = 2 \int_0^{p_{wf}} \frac{p}{\mu Z} dp$$

$$m(\bar{p}_{res}) = 2 \int_0^{\bar{p}_{res}} \frac{p}{\mu Z} dp$$

$$B_g = B = 0.00504 \frac{ZT}{p}$$

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Like here it is shown the formation factor can be used to substitute some of the values here it can be substituted here in this form and we can get the expression for q versus pressure draw down and this relationship is called IPR

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Pseudo Steady State: IPR

✓ Pressure approximation approach $q = \frac{kh[\bar{p} - p_{wf}]}{141.2 \times 10^3 \bar{\mu} \bar{z} \left[\ln \left(\frac{0.472T_e}{r_w} \right) \right]}$ $m(p) = \int_{p_b}^p \frac{2p}{\mu z} dp \approx \frac{p^2 - p_b^2}{\mu \bar{z}}$

✓ Pseudo pressure approach $q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1424T \left[\ln \left(\frac{0.472T_e}{r_w} \right) \right]}$

✓ Pressure square approach $q = \frac{kh[\bar{p}^2 - p_{wf}^2]}{1424T \bar{\mu} \bar{z} \left[\ln \left(\frac{0.472T_e}{r_w} \right) \right]}$

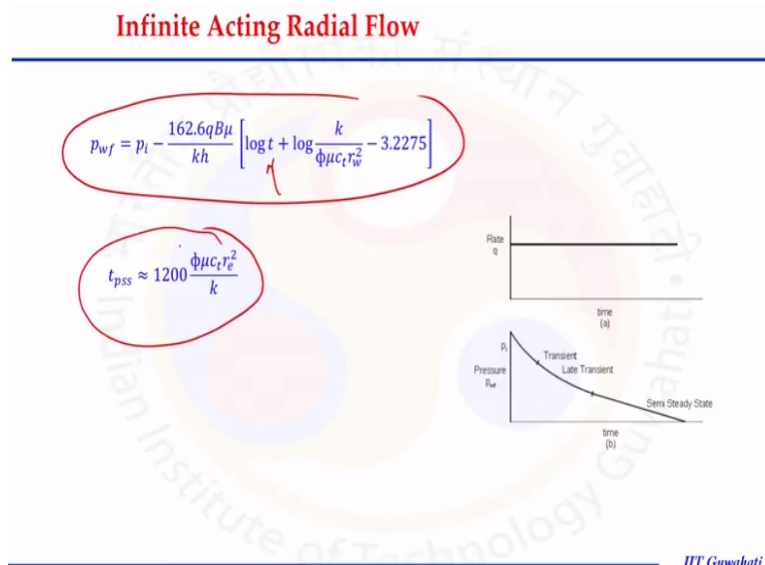
- * q is the gas production rate in Mscf/d.
- * k is the effective permeability to gas in md.
- * h is the thickness of pay zone in ft,
- * m(p) is the real gas pseudopressure in psi
- at the reservoir pressure p in psi,
- * m(p_{wf}) is the real gas pseudopressure in psi²/cp at the flowing bottom hole pressure
- * T is the reservoir temperature in R,
- * r_e is the radius of drainage area in ft,
- * r_w is wellbore radius in ft,
- * s is skin factor, and
- * D is the non-Darcy coefficient in d/Mscf.

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Here it is shown in all three form pseudo approximation, pseudo reduce pressure and its square pressure form for steady state condition you will see the expression looks in it is just some of the parameter those appears in the expression otherwise the q is related to the pressure different either in the form of p or in the form of mp or in the form of p square. So q is always related to pressure drop down,

Similar can be done for pseudo steady state condition and the usually meaning of the term those are appearing here are steady state and the unit associated for each parameter is also summarizes in this list that says the coefficient again the coefficient will depends if the same unit are used for each parameter if any of the parameter is especially the radius or the area some time reservoir area is given in acre we have to use acre for US field unit system or when we are converting the radius from acre area we have to be careful because the radius is chosen here in the formula in the form of feet.

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One more thing that is transient condition because we call it infinite up to radial flow condition is a transient condition in this condition the well is producing in such a manner where the boundaries of reservoir are not felt it actually happens near the wellbore reason and it happens mostly the early time of the production when the well just start producing the pressure at the well bore drop down to achieve the pseudo steady state condition.

And the expression here is shown here is also count for the t with t time that shows how the pressure is changing and this time to reach pseudo steady state condition can be calculated with this formula.

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Mass balance equation:

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left[\frac{kr}{\mu} \cdot r \cdot \frac{\partial p}{\partial r} \right] = \phi c_p \frac{\partial p}{\partial t} \right]$$

↓
Basic IPR for radial flow for a single phase

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left[\frac{r}{\mu} \frac{\partial p}{\partial r} \right] \right] = \frac{\phi \mu c}{k} \cdot \frac{r}{\mu} \cdot \frac{\partial p}{\partial t}$$

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left(r \frac{\partial p^2}{\partial r} \right) \right] = \frac{\phi \mu c}{k} \frac{\partial p^2}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial m(p)}{\partial r} \right] = \frac{\phi \mu c}{k} \frac{\partial m(p)}{\partial t}$$

So in summary what I can say in today's class we understand how to set up the steady state and pseudo steady state condition from the mass balance equation. So this is mass conservation equation considering the thickness of the slab is times to 0 we can get the derivative form of this using the Darcy law and properties of the fluid compressibility.

We can establish the mass balance equation in the basic form of the IPR form which is not having solution we can mix certain approximation in p form p square form or mp form to linearize the equation form and get the solution for steady state condition and pseudo state condition that we did in today's class.

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Inflow Performance Relationship (IPR)- II

- Analytical Expression

- Skin Effect
- Non-Darcy Effect

- ☐ Empirical Methods

- ☐ Simplified treatment approach- Back pressure approach
 - ☐ Laminar-inertial treatment approach- Forchheimer approach
-

In the next class what we are going to learn is further how the other parameters those are not discussed today like the skin effect and the non- Darcy effect going to add in the material balance equation and how the more complex form of the reservoir IPR will be established. The empirical correlations those are developed will also be discussed in the next class with this I would like to end my class today and see you in the next class thank you very much.