Mechanical Unit Operations

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Lecture 17

Flow past Immersed Solid Objects

Welcome, to the MOOCs course mechanical unit operations. Till now we have seen the mechanical unit operations. Equipment used for a mechanical unit operations, involving only solid - solid systems something like size reduction, process of size reduction. How to make analysis of size reduction processes, what kind of equipment are available. How they work, how they are operate etc. Those kind of details we have seen. We have also seen some amount of you know details about the size enlargement.

What kind of equipment are available for the size enlargement those details we have seen. So, where basically what we have seen till now. We have seen the mechanical unit operations involving only solid - solid interaction or solid - solid systems only. We have not seen any mechanical unit operations, where solid fluid systems are involve. So, now we are going to discuss mechanical unit operations, there working principles the design etc. Involving solid and fluid systems, in a few lectures after words.

So, now we need to start discussing about the mechanical unit operations involving the solid, fluid systems. That is where the solids are also interacting with the fluids, or you know both of them are moving, or only one face moving and the other face may not be moving; or both faces moving those kind of details we have to seen. So, what happens you now there are several kind of applications? So, we see we start with some kind of list of the applications.

So, that to we have a kind of feel, why should we for a mechanical unit operations involving solid fluid systems not only just solid - solid systems. So, for that before going in to those details. Those mechanical unit operation we need to see, some fundamental details of the solid fluid systems or something like a fluid flowing through number of particles are fluid and, then solid faces are simultaneously moving and then we may be doing some kind of separations etc those kind of systems are there.

So, however having details about the single particle, you know flowing or settling in a kind of fluid or fluid of flowing along a single solid particle is going to give, some kind of information about the physics of the problem. What kind of the forces are acting at the solid interface etc. Which can be used teen kind of designing solid fluid operations or the mechanical unit operations involving solid fluid systems. So, that is the reason let us start with some basic information about flow past immersed solid objects.

Rather, directly go in to the mechanical unit operations, where solid fluid systems are involved. Because, you directly go those mechanical unit of operations you may not able to understand several details. Because, in order to understand those details, we need to have some kind of fundamental information about the flow past you know immersed solid objects, single solid objects as well as flow through pack and fluidized beds etc. We need to have some kind of basic information, then only we will be going in to details about mechanical unit operations. So, the title of this lecture is flow past immersed solid objects.

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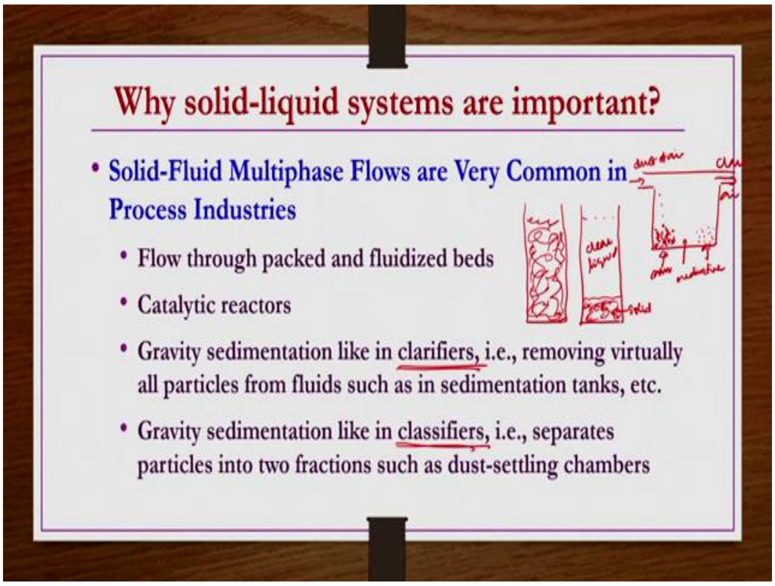

Why solid liquid systems are important? Because, you know solid fluid multiphase flows are very common in process industries, or material process industries, or chemical industries polymer industries, food industries, textile industries. Whichever kind of material processing industry you take you can find. You know this kind of solid fluid multiphase system almost kind of a ubiquitous. So, it is kind of very important to have a kind of some information about this systems.

So, what are the situations where, we have in general solid fluids systems involving? So, something like flow through packed and fluidized beds. So where you know packed beds are sometimes used to kind of heat recovery system or heat restore age system. Fluidized beds are in general used for kind of a coal combustion etc or, some kind of catalytic reactions etc.

So, for purposes in general we use so where there is a relative motion between a solid and fluid systems.

We have a filtrations etc operations in mechanic units operations, where you know we will be using the principles flow through packed fluidized beds etc. Then we have several catalytic reactions in general and chemical process industries. So, wherever this catalytic reactors are there, so solid catalytic face is interacting with the surrounding liquid face; with which it is going to undergo a kind of reaction process. So, under those conditions also under those catalytic reactors also, there is kind of relative flow between solid and fluid faces.

Then, we have a gravity sedimentation like in clarifiers actually several types of gravity sedimentations chambers are equipment are used. Which, are used for the separation of the particulate face from a kind of suspension, or in general or from a kind of some kind of sometimes they are also used in separating the particles in different fractions. So, let us say gravity sedimentations in clarifiers. Clarifiers are in general remove almost all particles.

Actually, they remove all particles from fluids, such as sedimentations tanks like you know what we have. We have a kind of tank, in which we take the suspension whichever the suspension you know having the particle and then fluid work together. So, you take them in a container or a tank like this and this you allow this particle to settle. So, what happens after some time this entirely this most of this almost all of this solids may be settling at the bottom.

Whereas, a top clear liquid may be forming whereas this solid particles, may be settling at the particles. So, this is known as the kind of sedimentation it is a kind of batch sedimentation. So, we are going to study this details also. Then, we have a gravity sedimentation like classify us. So, classify us like you know we have kind of something like that one example only I am saying. So, we have a container like this let so from the dust laden are particle laden air that comes in here like this.

So, this particles are having different size distribution, let us say whatever the particles having the bigger size, they will be having the higher settling rate. So, they try to they will be settling at the front part of this chamber like this. Well this part this I know is this gas or the air which is carrying particles. While it is flowing inside when we allow to flow this chamber. So, particles having the bigger size they will obviously having the largest settling velocity so immediately fall here.

So, particles which are having very small size, they may be having the they may be taking large time to settle and, there settling velocity may be small, then they may settling here. You know and the other extreme so, in the intermediate medium fraction may be you can find out. So, here we have a course particles, we may be settling faster. Then, we have fine particles which may be settling slowly and there is a kind of medium fraction, which may be settling you know or settling velocity may be in between of this 2 range.

And, there particles size distribution also in the same in between of this 2 ranges and then clear air may be going out from here. Depending on the performance of this chamber, of course in order to have the complete separation of this particles the size of this chamber height etc and all those things are going to be important and one has to find wall in to the design. So, we need to will be doing those design kind of things as well. So, this is one kind of other situation where there is a relative motion, between solid and fluid system and then particles are being separated.

So, this is the sedimentation tanks or something like a clarifiers, which separate the particles virtually all particles from the fluid. Classify us are a kind of equipment which separate the particle in to different size fractions or something like test settling chambers etc as discuss here. So, I am just try to give some examples so, that to have a kind of feel why should we go for this solid liquid systems or mechanical unit operation involving solid fluid systems?

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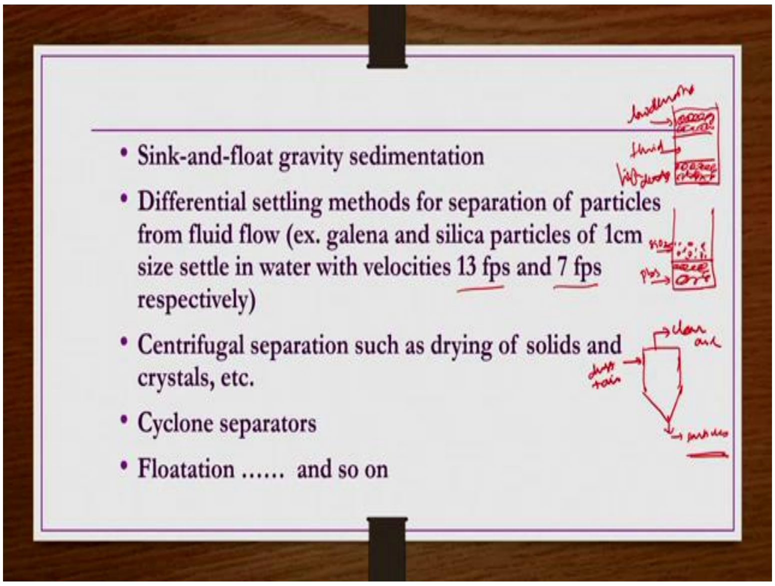

If, you see some more examples we have a sink can float gravity sedimentation. Let us say you have a kind of a particulate flow. Which are having two different types of particles. Let us say particles A type particle B type. So, there density is may be in general different or let us assume there density are very much different, from each other. One may be having very high density let us say 2000 kg/m^3. One may be having something let us say small density something like you know 500 kg/m^3.

So, then what you can do you can find out kind of a solution, solvent which is having a kind of density in between of these two. So, then when you allow this particles to flow along with this fluid, whose density is in between in the density of these two types particles. Then, what happen the heavier particles are the particles of the higher density may be settling at the bottom and then, lighter particles are the particles whose density is the smaller then the solution whatever we have taken its fluids density.

Then those particles may be suspending or, floating on the top of the liquid. So, this how you can separate out you know particulate mixture or having different types of particles of different density you like this. So, however in such kind of applications we need to find out you know in over appropriate fluid which is having density in between of this two, so that you can clearly separate. And, finding out such kind of fluid having the density in between of this kind of solid densities. The particles having high density and the particles having low density.

And, then finding out a fluid having medium density if low and high range, is a kind of bit difficult task. So, then on such conditions people also use such complex fluids, or other way such kind of systems, can also be separated by using a kind of differential settling method. What is differential settling method? You take this particle at low material or whatever you said, you try to crush it to the uniform size. Let us say you take this galena that is PBS and silica SiO_2 mixture is there.

You take that mixture and then you crush them to uniform size. Let us say 1 centimeter approximately all the particles. So, now you know are not finding a kind of fluid which is having a kind of density in between of this two. So, that you can do this you sink and flow differential settling method. So, then what you do you crush them in to kind of uniform size. Let us say 1 centimeter and then you allow this material to settle in a kind of solvent let us take water.

Then what will happen let us because the particles size now D_p is same, for both particles PBS as well as the SiO_2. Now but the density of you know PBS is higher than the SiO_2. So, what will happen the particles you know though 1 centimeter size only those particles may be settling faster, so they will be having a higher settling velocity. That is something like 13 fps and they will be settling at the bottom as a kind of almost pure PBS fraction. And, then particles which are having the silica particles though there size is also 1 cm.

There settling velocity is bit you know lower, that is 7 fps. So, because that 1 again those particles will also. Those particles may settling slower so, those particles may be forming a kind of layer on this particles on this PBS particles. So, that how you can have a kind of this kind of two fractions, so, majority of fractions in the bottom you may be having PBS. And then above it may be you having a you may be getting a separate fraction of the solids you know SiO_2 like this you can separate. This what is known as the differential settling.

So, whereas the sink and flow method, what you have you take you try to find out a fluid. So, that you know heavier particles may be settling at the earth, the particles whose density is higher may be settling at the bottom. And, then particles whose density is the smaller then the fluid, then that will be floating in the top in between we may be having a kind of fluids. So, this is the sink and float gravity sedimentation process and in the bottom one this is the kind of differential settling process.

So, this are the kind of some kind of applications we in general see. So, now we have a centrifugal separation process also, where in general we try to dry this solid and crystals. We also have the cyclone separators, in general from the process industries. Whatever the dust laden fluid, dust laden particles are there, they in general taken into a kind of a cyclone separator. Where you know this particles are being separated and then collected as a particle mess and then clear air is taken out here.

Dust plus heat so, this particles are taken in the cyclone separators and then in the cyclone separator process the sedimentation gravity sedimentation is taking place. And then there will be also a kind of a centrifugal forces also acting on this. So, cyclone separators here in the cyclone separators, we take the dust plus air or whatever the dust laden air is to that we take here. So, in this sedimentation in the cyclone separators particles will be settling and then almost clear air is taken out.

So, this the purpose of using cyclone separator. So that not to contaminate the surrounding and where on mental air with this dust particles. So, before leaving this just laden fluid form the industrial gasses influent so, they have to cleaned like this. Then, there also flotation process like this if we keep on listing, there are n number of applications. Where we can see a kind of relative motion between a solid and fluid systems. And then there may not be occurring any kind of a chemical changes there may be occurring only mechanical changes.

Mechanical or physical changes occurring and then so, those mechanical unit operations. Where the solid fluid system are involve so, one has to study some details some design process working principles equipment etc. One has to study so, that is what we are going to do in the second half of this course. So, however before going in to those mechanical unit operations let us start with some kind of basics.

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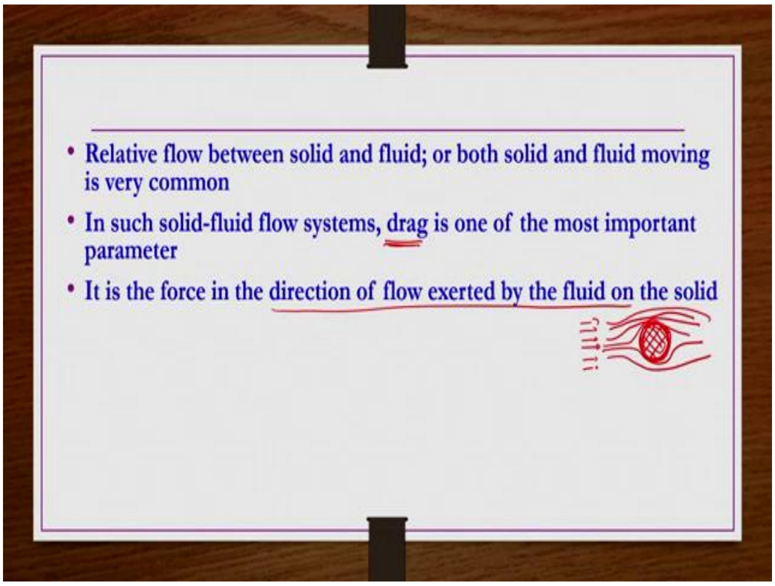

So, now we understand that the relative flow between solid and fluid, or both solid and fluid moving is a kind of very common in kind of process industries, having since several kind of applications still now. But, in such solid fluids flow systems in general. There may be several parameters involved so, drag is one of the most important parameters. So, this drag is going to be very much useful in the designing of this mechanical unit operations or, tuning the operating conditions of already existing design.

So, either of the conditions so this drag is going to have a kind of very influential role. So, let us say in this unit operations along with this motion, there is a heat transfer is also taking place. So, under such conditions there is a kind of necessity of having information about the Nusslet number(Nu) as well. If there is a mass transfer in this solid liquid system, than along with this drag there will also we need for a kind of Sherwood numbers(Sh). So, that is the reason we might have studied you know several type of correlation per Nusslet number and Sherwood number in our heat transfer courses.

So, you might also seen some kind of co relations for the drag coefficient. So, now what we do in this course we take system where only momentum transport is occurring or only there is a flow phenomena is existing there is no heat transfer or mass transfer. So, that is the region we will be discussing about the drag only. So, before going in to the drag details, you now for an individual problems. We need to have some kind of basic information about this drag, what it is drag?

So since understanding this drag is going to be very much influential parameter for the design of a given mechanical unit operation or, the multi face equipment. Where solid fluid system are interacting you know are tuning the operating parameter of such kind already existing design. This drag is going to be very important, so we need to understand some details about this drag. What is the drag? It is force in the direction of flow existed by the fluid and the solid.

Let us say we have a kind of particle and there is a kind of fluid coming in here like this. This fluid is approaching this particle so, than there is a kind of you know depending on the flow conditions of stream lines may be developing like this. So, this particle which ever we have to again on this surface of this particle, this particle may be you know experiencing some force because, of the fluid or the fluid is existing some kind of force on the solid object surface. So, that force existed by the fluid in the flow direction is known as the drag force.

So, drag force is the force exerted by the fluid on the solid object but in the flow direction. The direction should be flow direction that, is important in the direction of flow. So, drag is the force in the direction flow exerted by the fluid on the solid.

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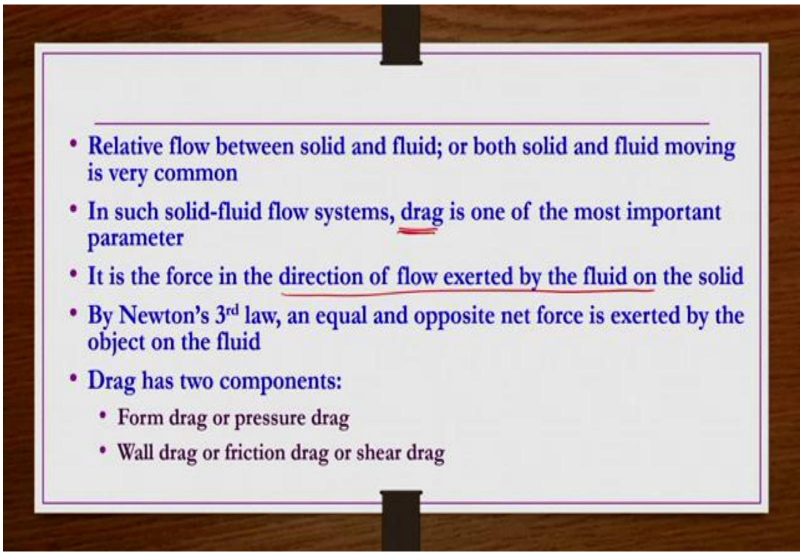

And, then we know by newton's 3rd law an equal and opposite net force is exerted by the object on the fluid. So, whatever the drag force let us say fluid is exerting a drag force of 10 newtons on the kind of solid object.

Then, the solid object would be exerting a drag force, or equal and opposite force that is minus 10N force on the fluid. So, this drag has two components one is the form drag or the pressure drag that we already know. And, there is another one wall drag, or shear drag, or fiction drag, or viscus drag. There are several terms further this you know wall drag. So, it is having 2 components the drag is having two components. Now we see how to evaluate them. So, in order to know how to evaluate them?

We have take a geometry where, both the components are existing. What does it mean by there may be some conditions or some kind of flow geometry should have may be only one of the component one drag component is dominating. Other drag component may be negligible. Some cases there may be form drag is dominating compare to the wall drag. Whereas seen some other cases wall drag may be dominating over the form drag, so that the form drag can be negligible. However since we are starting with to understand, how to evaluate this drag forces, what we do? We start with a kind of a object and which having both the components.

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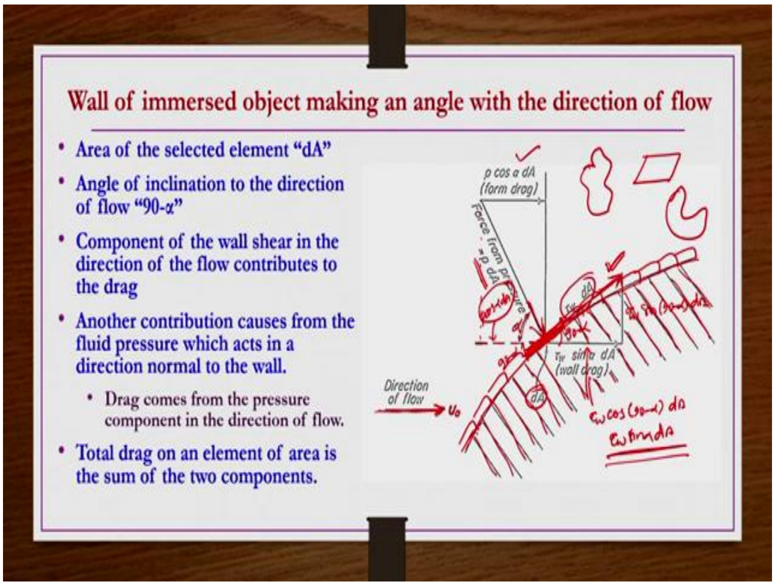

Next so let us take a solid object, where it is making an angel with the direction of flow. So, that is wall of a must object will be taking. Which is making an angle with the direction of flow. Let us say we have this geometry this the geometry that we are having like this. So, probably it is you know let us say it is kind of spiracle objector you know. An object which is having some kind of curvature. We are taking only this portion let us say so, along this solid object surface this what you do you take and element whose area is dA.

So, now this object is fixed so the one fluid is coming one Newtonian fluid is coming with velocity U now let us say. And, then this is the direction of the flow that is the horizontal direction fluid state. So, now rather on this element on this entire object. This fluid when it comes on interacting it is exerting some kind of force. That force if take its components in the flow direction, so that is the drag force. So, now but this element is making angle (90- α) with the flow attraction.

Whereas, this is so if you have this dotted line in this direction here so, this angle would be the α. Now, on this element you know it is having two components, one is the pressure component and another one is wall shear component. Pressure component would be acting normal to the surface like this. So, normal to the surface of this element. Whatever, the force there that is due to the pressure and then that force is PdA. That force is nothing but P multiplied by area of that object whatever you have seen.

So, that is PdA similarly this element is also having a kind of or experiencing a kind of shearing force. The shearing force would be in the kind of direction parallel to the orientation of the object whatever we have taken so that is this direction. So, shearing force is what shearing force is $\tau_w A$ of that whatever, the surface through which its you know acting so that is $\tau_w dA$ that is the sharing force. Now, what we understand this pressure force on this element that, we have taken that is acting in this direction like towards the elements of direction.

Whereas, the sharing force whatever is there that is working in this direction. So, but we need their components in the flow direction what is drag. Drag is the kind of force exerted by the fluid on the solid object in the flow direction. So, now we have to find out their respective component, in the flow direction that is in the horizontal direction. So, now so here let us take this pressure to force term this is the α angle. So, then what ever this components is there that should $P \sin \alpha \, dA$ and this component should be and the flow direction horizontal that should be $P \cos \alpha \, dA$.

So, $P \cos \alpha \, dA$ so, the same is represented here and the above so that to have a enough clarity picture. So, this $P \cos \alpha \, dA$ is going to be form drag or, the pressure force or due to the pressure component whatever is there. Its horizontal component because, the flow direction is horizontal now here. It is horizontal component is $P\cos\alpha\, dA$. So, that is going to be from drag and then similarly here $\tau_w dA$ this is in this direction. So, what is this angle this angle is nothing but $(90-\alpha)$.So, this is going to be $\tau_w \sin 90-\alpha.$dA and this particular component is going to be $\tau_w \cos 90-\alpha.$dA. So, $P\sin\alpha\, dA$ so, whatever the shearing force there on this particular element it's not in the horizontal direction. So, we have evaluated its component vertical and horizontal component. Since, the flow is the in the horizontal direction so, sharing force component in the horizontal direction is $P\sin\alpha\, dA$. So, this is going to be wall drag this is how we are going to evaluate this drag components for a given geometry.

Remember, if the geometry orientation changes or entire geometry changes let us say if you have a kind of geometry like this so than things may be very different. Let us say you have a kind of a geometry like this then it is going to be different. Geometry and then its orientation if you have a kind of regular particle like this. So, not only geometry if orientation is also going to have a kind of influence. So, given geometry one has to evaluate, what are their perspective pressure and then viscous dry components.

Then one has to do the calculation accordingly. So, like this you can divide this surface into the n number of elements like this. For each element you find out what is this form drag and what is the wall drag for each element and then you add them together for each element than you integrate all those quantities. Than you will get the total drag for the entire surface. Remember this things we have taken we derived $P\cos\alpha\, dA$ than $\tau_w dA\sin\alpha$ or, only for one single element having the area d A.

So, area of this selected element here is dA angle of inclination to the direction of flow is $90-\alpha$. And, then component of the wall shear in the direction of the flow contributes to the drag. Similarly, another contribution causes from the fluid pressure which acts in the direction normal to the wall pressure actually as normal to the wall. But, the drag comes from the pressure component in the direction of flow. So, the corresponding flow direction component we have to find out and then that should be taken as a kind of pressure drag. Total drag on an element of area is the sum of the two components whatever we have seen.

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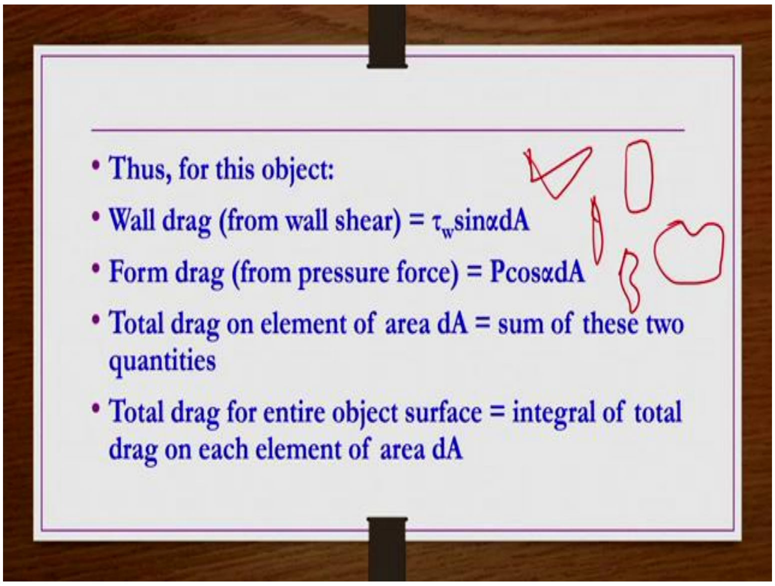

So, now for this object that we have taken wall drag that is coming from wall shear, that is nothing but $\tau_w dA \sin\alpha$. Similarly, form drag that is coming from the pressure component $P\cos\alpha\, dA$. So, the total drag on element or area dA is sum of this two quantities whatever, $\tau_w dA \sin\alpha$ d A and then $P\cos\alpha\, dA$. Similarly, total drag for the entire object surface whatever, the entire object surface we have taken so that should be integral of total drag on each element of area dA.

You have to divide the surface of the entire object into the small element and for each element you to find out what is t wall drag and what is t form drag than we have to and them together like that for individual elements you have to do. And, then you integrate for the entire surface then you will get the total drag for the entire solid object may integrating them. So, as I mentioned if you have a kind of object like this, you know your expressions may be different.

If you have kind of a object like this your expressions may be different. If you have a kind of object like this then again may be like object like this irregular particles like this. So, all of them are going to involve, in different expression for this form drag and then wall drag. So, what I mean to say whatever, this wall drag or form drag expressions derived here. There are valid only for the object and then only for the orientation inclination angle that is shown in the previous slide.

They are not unanimously true for all the objects. It is going to be different for a cube, it is going to be different for a cylinder and it is going to be different for cylinder one cylinder to the other cylinder of having different orientation to the flow direction so that is how you know they are depending on a kind of case to case.

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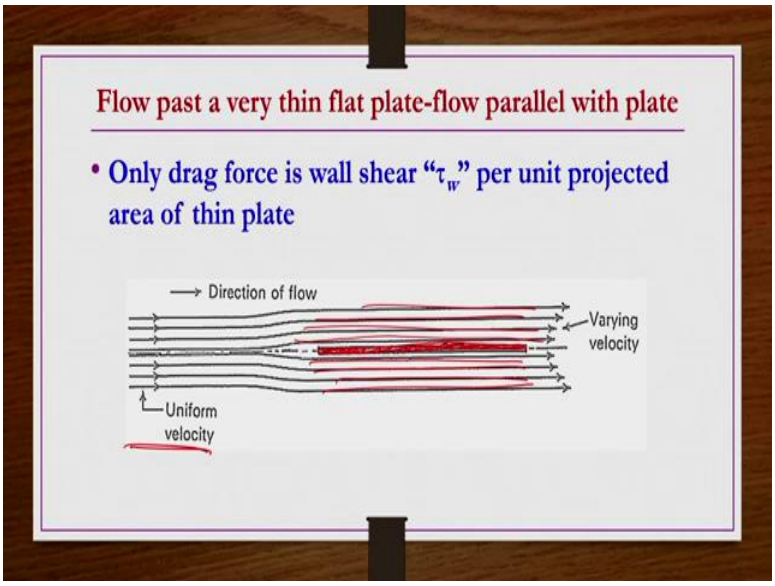

Now, we take a case to where we can see that, only one of the component may be dominating other components may be negligible. Like so let us take flow past a very thin flat plate flow parallel with the plate. So, then what happens now let us assume this is a thin flat plate that we are taking this paper sheet very thin this is very thin you can see like this very thin. So, now what you do you can have a orientation in different directions. So, let us say the flow is coming from your left to right.

So, then and if you take this orientation then drag would be different and then if you make this orientation like this it will be different drag and then if you make orientation like this and then flow is still left to right so, than the drag is going to be different. So, what we do we take a kind of case this flat but, very thin like this you know that we have taken its orientation is that now, on the flow is coming from your left to right like this, so then here what happens you know this surface this surface is only offering the resistance that is normal to the surface.

So, that is pressure drag is going to be very small whereas, this entire surface is offering kind of a sharing force. When the fluid coming here and then flowing like this. This entire surface area is going to offer some kind of a you know force or it is going to experiencing some kind of a sharing force. While this fluid is flowing in this direction so, then what kind of drag is there. So since now the same pictorial if you have we have a kind of this object like this thin plate like this.

So, now fluid is coming from left to right with a kind of uniform velocity, the direction of flow is still left to right and the horizontal direction. So, this object now as s I ay this is the flow direction so, this surface whatever the surface is going to offer or the surface whatever the force that is being you know receiving because, of the fluid flow that is going to be very small. Whereas, the force that the surface is going to receive because of the fluid motion this is that is going to be large.

So, this direction force whatever is there that is the pressure component and the surface whatever is there because of this one whatever the drag component they say that is the shearing drag. So, shearing drag wall drag is more important more larger compare to the pressure drag here. So, what we say here in such kind of flow when the fluid is coming without any distortion its smoothly going to pass like this. You know they are not the waiting much from there a stream line from there I know this the flow direction. There are not much deviating only near the surface slide deviation is there.

And after that it is going to be you know same like you know flow direction. So here what we have only drag force is wall shear per unit projected area of thin plate. So, only force that is the exact there are the whatever the fluid is exerting a force that is majorly exerting a kind of sharing force on the surface. And they it is also exerting some amount of the pressure force but, that is very small. So, we can say only drag force in such kind of geometry is a kind of a wall shear drag.

Now the same plate if you rather keeping like this, what you do you keep like this, orientation if you change that is the flow perpendicular to plates. So the plate direction and then the orientation of the plate and flow direction are perpendicular to each. So, like this so this entire surface is you know exerting kind of a receiving a kind of a drag force due to the pressure component. And then this surface whatever the surface is that very thin surface that is going to have a kind of a you know sharing component.

So, now in this orientation this pressure component because, of the more frontal area. So, the resistance whatever there because of the pressure component that is more compare to the resistance because, of the sharing component so under such conditions the geometry if you take

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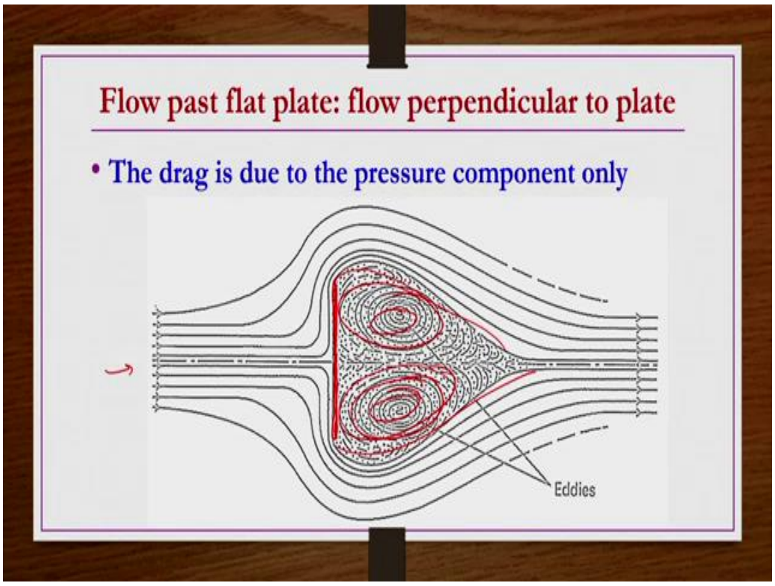

So we have a same geometry like this here and then, flow is coming in the same left to right direction and the horizontal direction. So, when it comes it offers the form of the object is offering or receiving more force. Because of the fluid and then since the object is offering some resistance to the fluid in the opposite direction. So, what happens the fluids try to escape has to jump over and then form kind of a recirculation at this something like this. So this are the recirculation and adjusting journal.

So, they are forming like this. So, under his conditions whatever the drag is there that is primarily due to the pressure component only that is form drag only. Whereas the wall drag is a kind of very small component here. So, this are the two extremes for the case of the previous case where we have consider you know both the components out there. So that is the reason when we are evaluating the expression for the from drag or the wall drag. So, we should be careful about the orientation of the geometry flow direction etc.

And then accordingly we have to derive their expressions. We cannot use one single expression unanimously or universally for all kind of objects and all kind of geometry it is not possible. So, the flow direction and then shape of the geometry orientation of geometry are going to be very having a kind of very important role in a this drag components. That is we can see the same plate if the orientation is changing from one orientation to other end orientation in one orientation pressure component is dominating other orientation you know wall shear component is dominating.

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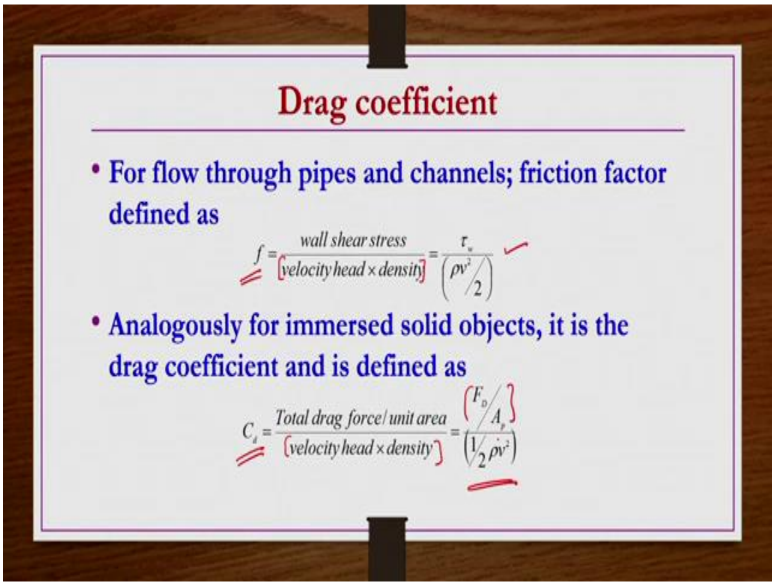

So, now how to define the drag coefficient. That so we now know this is the drag how much important how to evaluate this thing. So, then we have to evaluate some kind of expressions for the coefficient drag coefficient are something like non dimensionalised drag like friction fact that we have seen. Like inflow through pipes or channels etc. We have seen friction factor what is friction factor we have define as a kind of dimension less wall shear stress that is wall shear stress divided by the velocity head multiplied by this density.

Or if you non dimensionalised this wall shear stress by velocity head and density then we get this friction factor, that is $f = \frac{\tau_w}{\left(\frac{\rho v^2}{2}\right)}$ so, this how we have define that the friction factor for rough flow through pipe and channels in journal in our fluid mechanics course. Similar, way we can also define the drag coefficient, how to define the drag coefficient? We can similar way define the drag force per unit area because, now here in case in this drag coefficient case it’s not the only wall shear component.

But, the pressure component is also involve. So, when we add together then we have the drag force. So, drag force divided by unit area some kind of unit area and that force the drag force per unit area if you normalized non dimensionalised with same velocity and density then we get drag coefficient. So, what kind of area here we take we take the projected area we do not take a surface area or we do not take a kind of cross sectional area kind of thing we take a projected area.

Why projected area that is also we are going to see. So that is analogously for immersed solid objects it is the drag coefficient and it is defined as $C_D = \frac{F_D/A_p}{\frac{1}{2}\rho u^2}$. So, F_D/A_p is similar or analogously to tau w whatever we have taken, or similar analogously to the tau w whatever we have taken in the friction factor. And then this of $\frac{1}{2}\rho u^2$ is kind of you know velocity and density components for the non dimensionalised. So this how C_D is defined. Drag coefficient is designated by C_D friction factor is designated by A_P. So, now we see the nomenclature. What F_D what is A_PV etc that is what we are going to see now.

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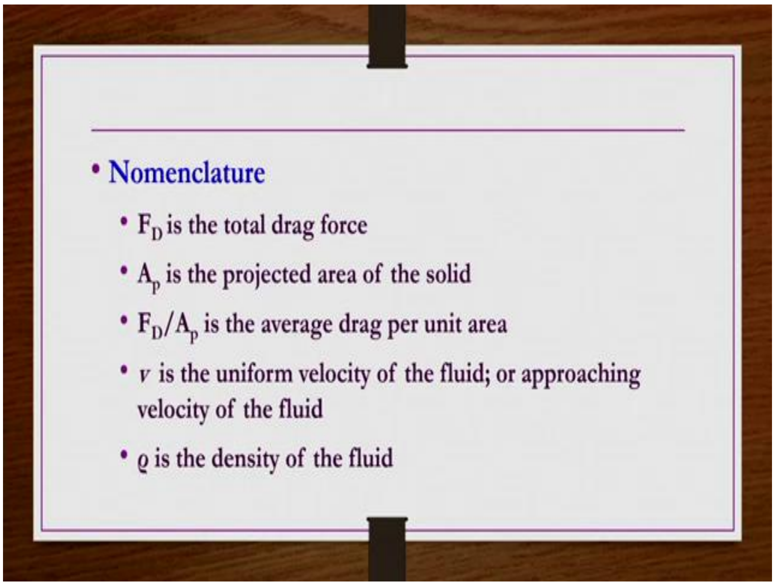

F_D is the total drag force that is you know fluid is exerting on this solid object and the A_P is the projected area of the fluid. Projected area we are going to see now and then F_D/A_p is average that per unit area of the surface and then V is the uniform velocity of the fluid or approaching velocity of the fluid. And ρ is the density of the fluid.

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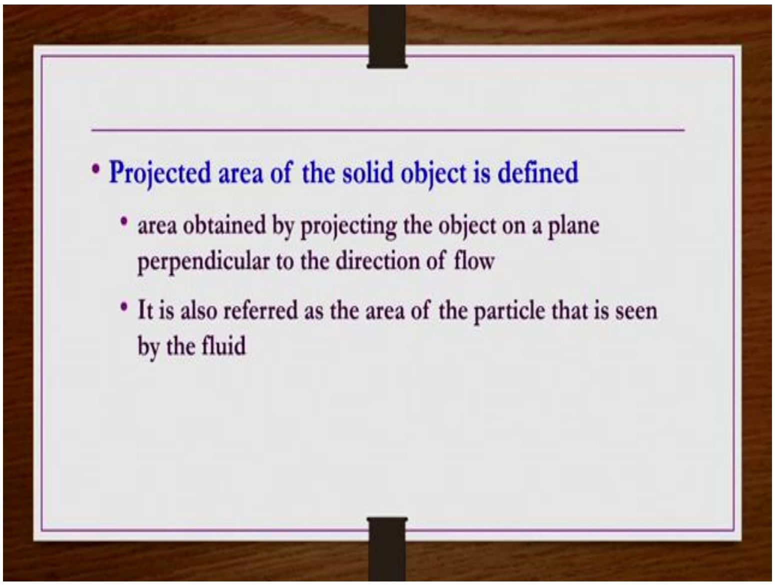

So now projected area or the solid object how to define this? It is defined as the area obtained by projecting the object on a plane perpendicular to the direction of flow. So, that is how the projected area is defined. Or, it is also referred as the area of the particle that is seen by the approaching fluid. So, now pictorial we see some examples. Now we take a projected area of a sphere that we try to calculate. So, we try to calculate now projected area for different objects.

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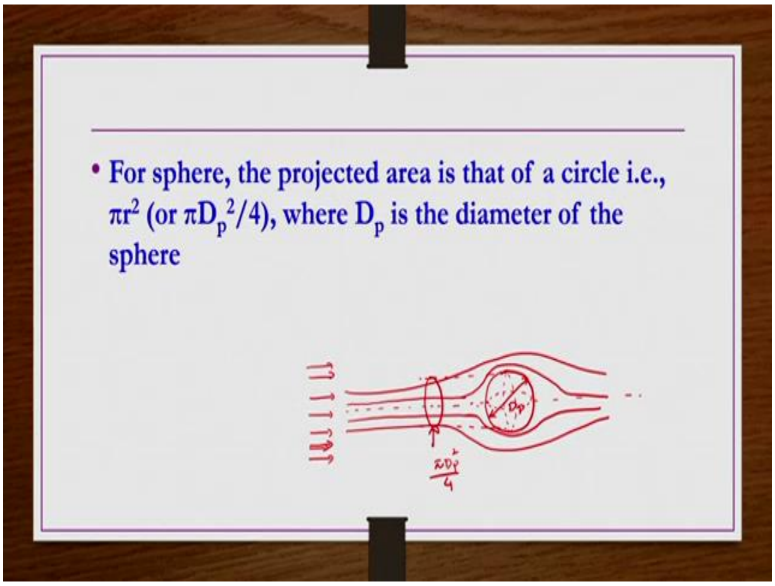

So, we take a kind of spherical object let us say like this. So, now from here the fluid element is coming or the approaching fluid is coming from this direction. So, when this flow comes here it is here depending on the holes small or large is the magnitude of this fluid. The velocity steam lines flow steam less we can have different shapes like this. So, now the projected area of this solid sphere solid spherical particle is the area that you can see from the flow direction.

What you see know let us say assume this is a kind of spherical object this is a spherical object. Let us assume this is a kind of spherical object. So, now this is the from this direction the flow is coming. So, what you see now here what you see you see a kind of circle only. So, the projected area of sphere is nothing but you know the area of the circle. That can be seen so, that area that we can see like. We you can draw there something like this so, now let us say size of this particle spherical particle is D_P.

So, whatever the circle that we see here something like this so, its area what is the area of circle is πr^2 or $\frac{\pi D^2}{4}$. This is going to be projected area for the kind of spherical particle. That is how 1 has to obtain the projected area or, simply it is the area seen by the approaching fluid elements. So, for the sphere the projected area is that of a circle that is πr^2 or $\frac{\pi D^2}{4}$ Where D_p is the diameter of the sphere.

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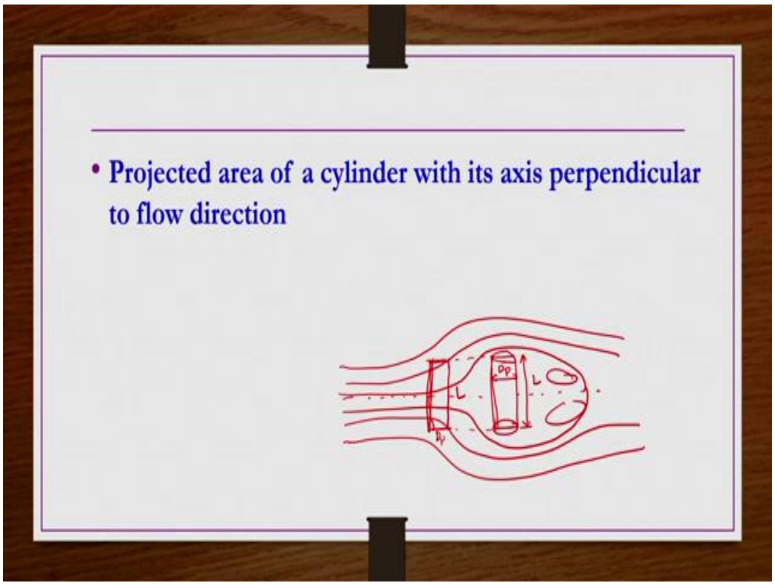

Now what we do? We take a another case projected area of a cylinder with its axis perpendicular to flow direction. So let us this is now you take it as a kind of a cylinder, cylindrical particle. Now the cylindrical particle so, in this direction and the fluid is coming. So, from here or it is assume your from there your seeing from on screen. so I am seeing from here. What you see which area you see its area of this object if you project on this surface plane so what you get. You will be getting the area of a rectangle column.

So, that is height $L \times D_p$ diameter of the cylinder whatever, that area is there that is the projected area of this cylinder. So, pictorially if you draw it here so, now this is the cylinder small cylinder that we have cylindrical particle or even cylinder whatever you call it. So, it is having length L and then diameter D_P. So now here the flow is coming either in this direction like this. That again here also depending on the flow depending on the magnitude of the velocity approaching velocity.

So there may be kind of some kind of recirculation with something like this or there may not be any recirculation with if the flow is very small. So, something like this the steam lines you can see. Now, what should be the projected area or form here when you are seeing it. You are seeing it is a kind of rectangle. When your projecting on a plane and the flow direction so, you see it is a kind of rectangular of size L and D_P.

So, what should be the projected area for cylinder with its axis perpendicular to flow direction it should be the $L \times D_p$. Similarly, you take projected area of a cylinder with its axes. This is a central axis now it is parallel to the flow direction like this. So that is now flow is coming from this side to this side like this. So, then from your screen side you know see now so, what is the area that you are going to see of this cylinder. When you projecting on the plane in the flow direction then what you see here also you can see a kind of circle. So, for this geometry also the projected area is the area of this circle that you are going to see so, that is let us say you have a cylinder like this.

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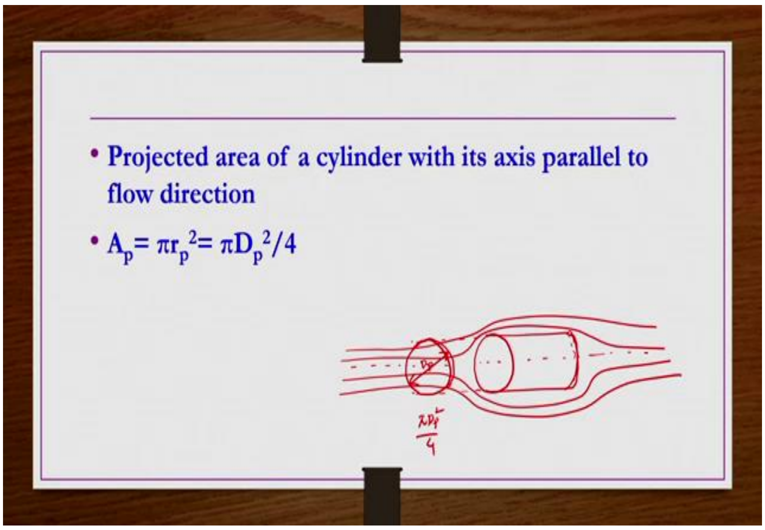

So, now this fluid elements are coming like this here. So, depending on the flow again here also there may be vortices there may not be any vortices the steam lines may be something like this. But, from the flow direction from the direction of the flow what you see you see only a kind of a circle. Whose diameter is same as the diameter of the cylinder so that is D. What is the area of the circle is again $\frac{\pi D_p^2}{4}$. So, here also the projected area is $\frac{\pi D_p^2}{4}$. So, this is how one has to discuss about this you know projected area

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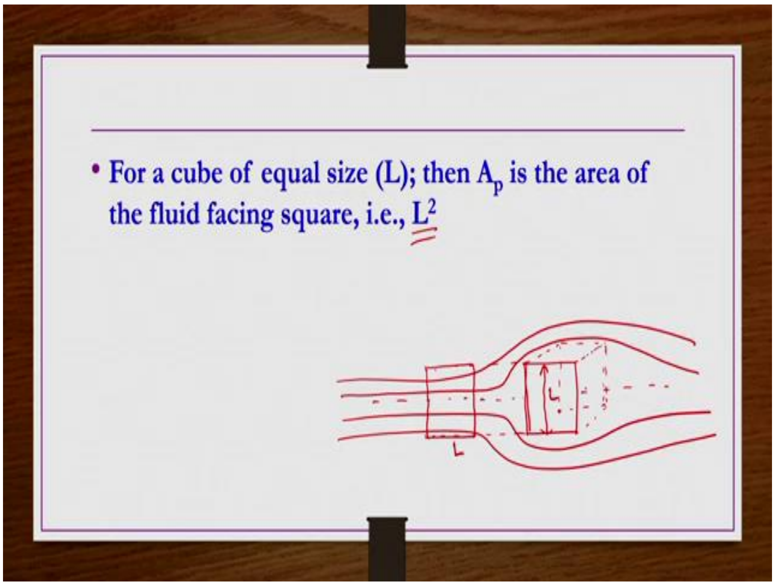

Now, let us say if you take a cube of equal size let us say you have a cube like this 2 dimensional cube i am drawing here. So, if you draw 3 dimensional something like this. So, now the size is L it is a equal size cube. So, all sizes area equal to L so the now fluid is coming something like this here and then going like this. So, when it comes depending on the flow conditions this streams lines will take a form after the geometry like this. So, but now here again if you see from the fluid side if you see what you see here you see a kind of square.

Square of shape L so, what should be the projected area for cube of equal size L than that should be the area of fluid facing square that is L^2. So, this is about the projected area. And, this projected area is different from the surface area of a you know cross section area one should not be confuse. So, in obtaining the drag coefficient the drag force is divided by the projected area of the object.

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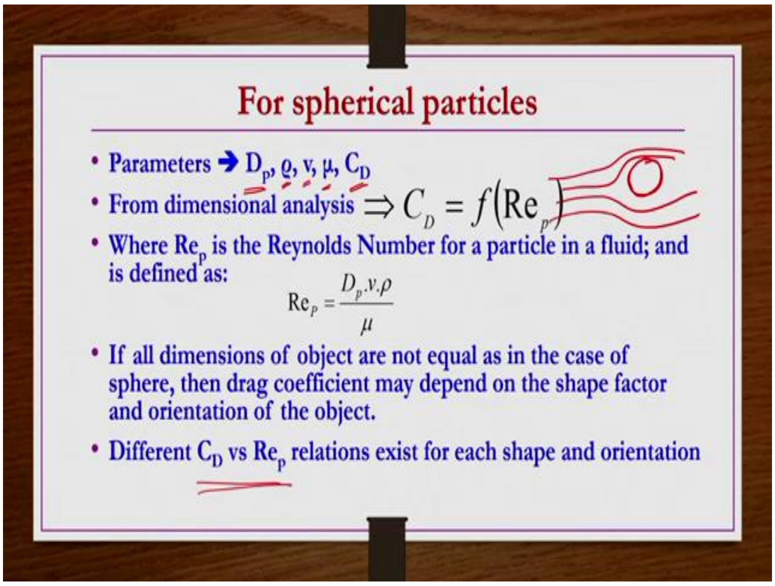

So, now we take for spherical particles for a simplicity let us start with the spherical particles. Why because the spherical particle is having the size equal D_P. The size of the particle is same in all orientation or any direction theta is equal to 0 to 360 and π is equal to 0 to 360 any direction if you see. It is having a kind of uniform size D_P so that is the reason, Simplicity that we case for the simplicity we take this as spherical particles case. So, what we have parameters in general.

Let us say for the spherical particles it is settling in a kind of newton and fluid or along this particle there is kind of Newtonian fluid flowing like this. So, what are the parameters are involve one is the diameter of the particle, then the density and the viscosity of the fluid, then the velocity of the fluid that is approaching the sphere. And then the force that is exerted by the fluid on the solid surface of the sphere in the flow direction that is drag force. So, it is dimensionalised non dimensionalised quantity is the drag coefficient.

F_D is the drag force that is the dimensional quantity C_D is the dimension less quantity that is the drag coefficient. So, this are the parameters involved there may be other parameter the surface roughness etc all those kind of thing this are primarily let we see ok smooth sphere like that. Than if you do dimensional consideration or dimensional analysis if you do. Then you what you can find two dimensional less parameters you can see here one is the drag coefficient function of you know Reynolds number drag coefficient and Reynolds number are the two dimension less parameters that we can see here.

And then what we understand from this dimensional analysis this C_D is going to be the $C_D = f(Re_p)$. So, for Re_p for a particle in fluid is defined as $Re_p = \frac{D_p \rho V}{\mu}$ that is the Reynolds number for a particle you know settling in a kind of Newtonian fluid. So, this Reynolds number is a kind of a ratio between the inertial forces to the viscus forces. It indicates the degree of you know this contributions. Right if the Reynolds number is small then we can say that the viscous force is more dominating compare to the inertial force.

So, that you know inertial forces may be neglected and then only viscus forces may be consider for the analysis. If the Reynold number is very small less than 0.1 or less than 0.01 something like that. If, the Reynolds is very large order of 10^3 or 10^5 something like that than we say the inertial forces are more dominating compare to the viscus forces. So, one can avoid the viscous forces and then consider only inertial forces for the inner analysis that is going to be that is how rational way of using the Reynolds number. For this object it is going to be $Re_p = \frac{D_p \rho V}{\mu}$. If all dimensions of object are not equal in the case of sphere, then drag coefficient may be dependent on the shape factor and orientation of the object as well. As I already mention in this drag coefficient you know the orientation of this object shape of the object is also going to have a kind of different role. The same thin plate if the orientation is changing the drag component is entirely changing from horizontal to the vertical direction orientation

If you mean the components are entirely changing their contribution are entirely becoming opposite to each other. So, that is different C_D verses Re_p relation exist per each shape and orientation of a particle. Only for spherical particle there is only one kind of correlation. But, otherwise you know one kind of C_D verses Re_p relation. Where as per the other objects cylinders cubes etc depending upon the orientation you will get the different C_D verses Re_p relations.

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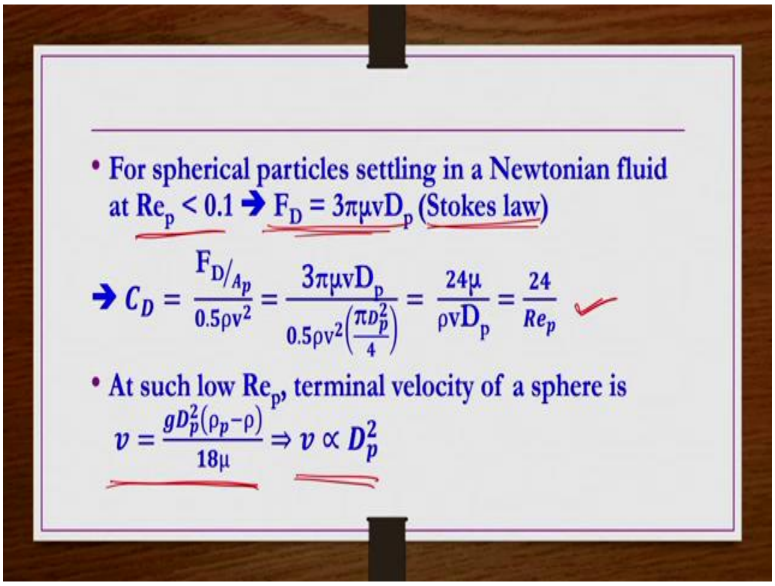

For spherical particles settling in a Newtonian fluid that is it is small Reynolds number very small Reynolds number less than 0.1 the drag force is evaluated as $F_D = 3\pi\mu D_p$ that is known as the stokes law. So, in the momentum equation what we can do the Reynolds number is very small so you can strike out all the inertial force systems. And then simplify the momentum equation so then you get the velocity profile. And, then velocity profile obtained the forces acting on the surface and then pressure forces and then viscous forces on the surface.

There you take the their component in the flow direction add them together. So, then you will get the drag force so that $F_D = 3\pi\mu D_p$. We are not going to do the derivation that derivation userly comes in to transport phenomena course part. So, if Reynolds number is small for spherical particle drag force $F_D = 3\pi\mu D_p$. It is also non as the stokes law. If you non dimensionalised this one that is if you do $\frac{F_D/A_p}{\frac{1}{2}\rho u^2}$ than you will get the Cd drag coefficient non dimensionalised quantity.

So, FD is $\frac{3\pi\mu D_p}{0.5\rho V^2\left(\frac{\pi D_p^2}{4}\right)}$. Projected area for a sphere is the area of the circle that you can see from the approaching fluid side. So, that is $\frac{\pi D_p^2}{4}$. So, than when you simplify you will get $\frac{24\mu}{D_p\rho V}$ that is $\frac{24}{Re_p}$. So, some of you may also be knowing as a for a spherical particles settling in a Newtonian fluids stokes lies nothing but $C_D = \frac{24}{Re_p}$. So that is coming from this one. At such low Re_p the terminal velocity of a spherical particle is given as $u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$

We are going to drive this relation in the next class any way. So, what we understand here if the Reynolds number is small the settling velocity or the terminal velocity of the particle V is proportional to D_p^2 or square of the particle size.

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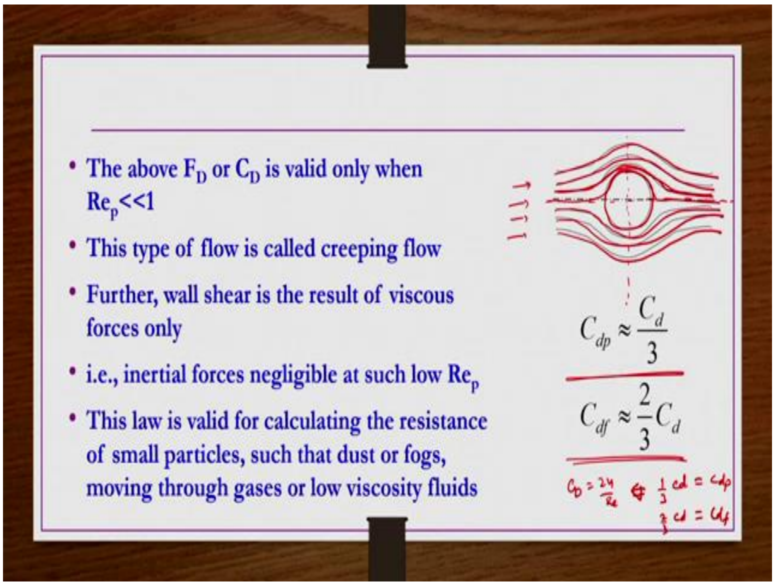

So, this above F_D or C_D is valid only when Re_p very smaller than 1 that is 1 or less than 0.1 then only it is valid. It is known as this type of flow is called as the creeping flow or stokes flow or viscous flow it is known as. So, the stream lines let us say they will be occurring like this. If you this is your solid object or spherical particle and then let us say flow is coming from this direction like this. So, there will be a kind of 4 and of cemetery we can see here so that is whatever the you know if you break this cut this surface like this.

So, whatever this stream lines you see in the left hand side the same stream lines the same path line etc that you can streams lines you can say is similarly exactly same mirror images of the each other. Also if you cut of in this direction the horizontal direction so, whatever the stream lines at the top are there so they will be exactly at the bottom also without any kind of deviation so, that is the symmetry in error of the direction left and right. That is 4 and of and top and bottom symmetry of this stream lines of we can see.

So, that is the particles are you know simply they are coming and they are climbing creeping slowly like this. and then falling down slowly without being affected by the presence of this particles. So, that is how the stream lines look for the and creeping flow conditions. Further wall shear is the result of viscous forces only here because, you know inertial forces are almost negligible. For such small Reynolds number inertial forces are almost negligible. This law is valid for calculating the resistances of small particles.

Such as you know dust or fogs moving through gases are low viscosity fluids under such conditions only when you have the particles very small as well as the viscosity is also very small. Then it happens in general the Reynolds number is going to be very small. So you have this you know Re_p is going to very small. If you evaluate the drag coefficient that C_D is equal to $\frac{24}{Re}$ whatever is there. Here you can see one third of the C_D is coming from contributing in C_{Dp} is contributing one third of the C_D. And then C_Df that is the friction drag or wall drag that is contributing two third of the total drag coefficient. So that is at the small Reynolds number under the stokes flow conditions.

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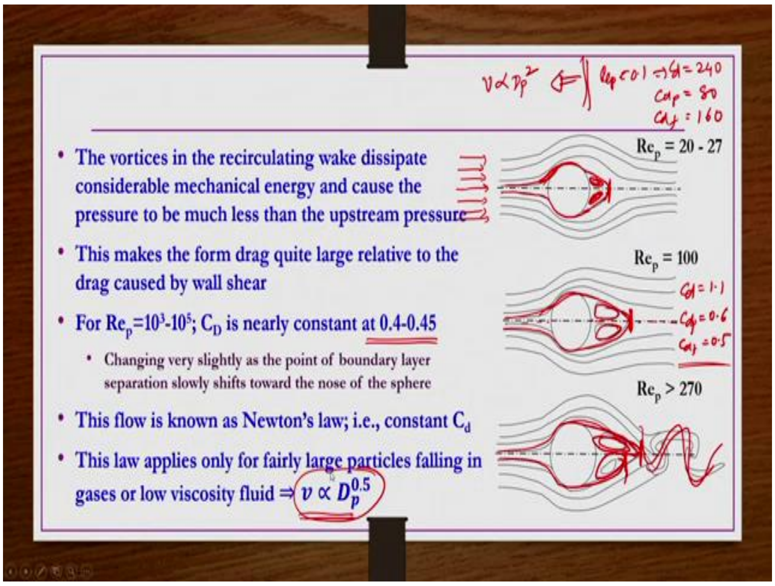

Now, what happens if the Reynolds numbers slowly increases? Reynolds number slowly increases. Let us say you go up to Reynolds number 20 something like this. So, the that is the flow velocity is increasing now the velocity of this fluid is increased. When, it is flowing fast as spherical particle. So, what happens this stream lines slowly climb up and then now here the more they are receiving more resistances because, the velocity is increased. So, because of that one there will be kind of vortices forming in this you know rear end of the particle like this.

So, there will be vortices forming like this and then this vortices are exactly same both in the top and bottom of that is the access symetrically we can see any angel that you cut you know. That you know above whatever the streams lines are there are exactly below the streams lines will also be the exactly same or the recirculation x would be exactly be the same. If you further go increase the Reynolds number what happens? This size of this vortex increases.

The gradually increases but still this you know symmetry you consider symmetry that is whatever the above the this dotted lines this excess whatever you takes so whatever the streams lines are there or the vortex shapes are there exactly the same vortex shapes you can say you can see at the bottom of that one that is known as the access symmetry of the flow pattern. But if you further increase the Reynolds number keep on increasing the Reynolds number let us say if you go beyond 270 or 250 some like this.

Then what happens the streams lines you know the vortex further may increase something like this but, there shapes are not same. So here that is in one case this lets us say here the bottom vortex shape is this much and top vortex here that shape is like this. They are not symmetric each other. Here they are exactly symmetric ending at the same point here, here also ending at the same point here. Now the top is one ending at this point here where as the bottom one is ending at this point.

So, they are not same here and then sometime if you further increase the Reynolds number what happens you know the vortex may be breaking and then kind of a non periodic or periodic vortices may be forming like this. So, this is how the flow pattern looks like for the kind of slope or sphere in general. So, this are a very few minimum information that is required to be given at UG level. There are more much more physics involved in and then reasoning and all those things are involve. So that is not going to be main part of this course anyway so we are not going into this more details. We are studying only this to see you know which is the limit is for the constant drag relations etc those kind of we are going to see.

So the vortices in the recirculating wake dissipate considerable mechanical energy and cause the pressure to be much less than the upstream pressure. So, when this you know vortex are forming the pressure and the vortex side would be much lesser than the upstream pressures. This makes the form drag quite large relative to the drag cost by the wall shear let us say it $Re_p = 0.1 C_D$is what is nothing but $\frac{24}{0.1} = 240$. So, here C_{Dp} is you one third of this one is approximately 80. C_{Df} is approximately two third of this one that is 160.

Whereas, here when Re_p is 100 C_D is approximately 1.1 and then pressure force is almost 0.6 friction drag is 0.5 so, gradually what happens what you can see its small Reynolds number pressure component is only one third as Reynolds number is increasing let us say up to Re 100 this almost 50, 50 even more than 55 to 60% of its pressure components it increased from at lower Reynolds number it is only 1/3rd now it is increased almost up to you know 55 to 60 %.

If we further go to the higher and higher Reynolds number what you can see you can see pressure components is the one quite large compare to the wall drag or the friction component. For Re_p in the ordered of 10^3 to 10^5 C_D is nearly constant between 0.4 to 0.45. That is slightly changing as the point of boundary layer separation slowly shifts towards the nose of the sphere towards the top of the sphere. So, this reason where this drag coefficient becomes constant 0.4 to 0.45 is known as the newtons law that is constant see the value.

And, under this conditions also if you derive the terminal velocity then you can find it that V is proportional to $D_p^{0.5}$and the small Reynolds number places what we have seen V terminal velocity is proportional to D_p^2under the large Reynolds number conditions what we seen here V is proportional to $D_p^{0.5}$.

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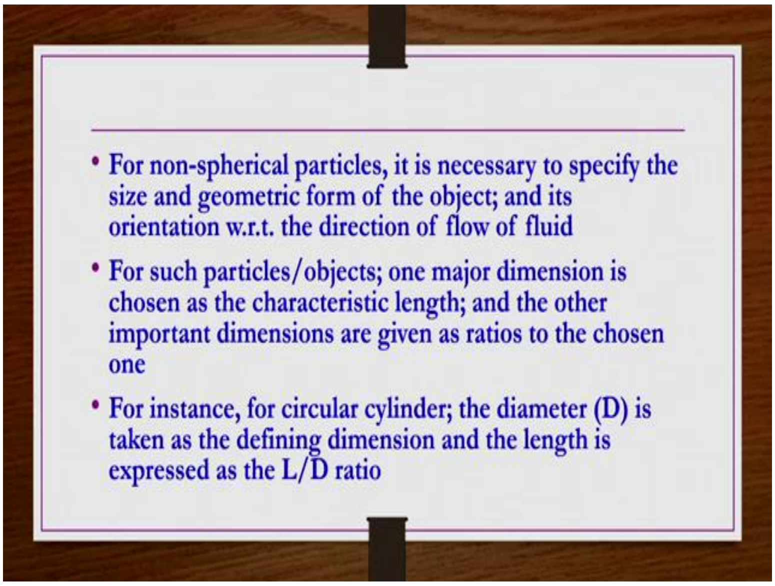

For non spherical particles, it is necessary to specify the size and geometric form of the object; and its orientation with respect to directional flow of fluid. For such particles objects one major dimension is chosen as the characteristic length; and the other important dimensions are given as ratios to the chosen one for instance, for circular cylinder, the diameter is taken as the defining dimension and the length is expressed as the L/D ratio this are the in general conventions.

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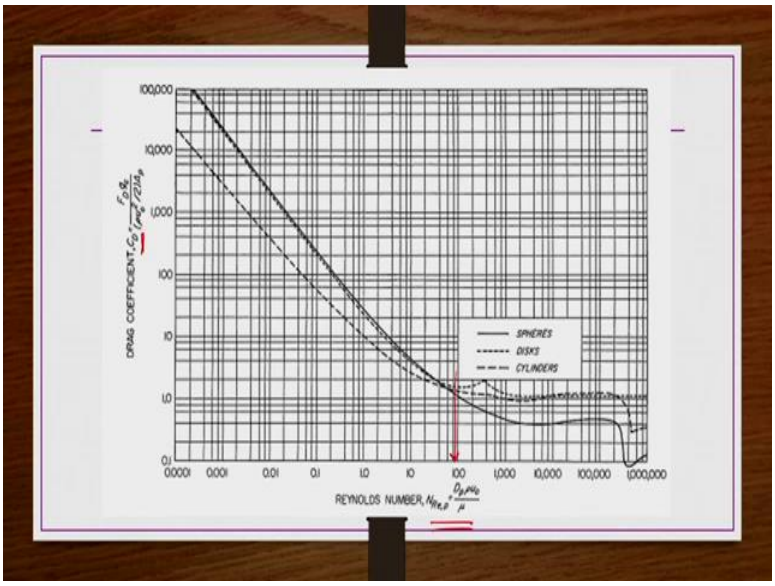

Now, let us say if you take the drag coefficient verses Reynolds number Re_p or NRe_p either way it represented. Now you can see the solid line you know that is for the spherical particles and then disc also it is shown and then cylinders also it is shown here. So, disc and then spherical particles are having almost similar drag behavior up to Re 100 or so the drag coefficient for space disc are almost same. But after that you can almost see deviation is very much higher.

Whereas, compare to the cylinders the deviation between drag coefficients of cylinder and disc and cylinder sphere is very much higher up to Reynolds number. But, after this again the difference between sphere and cylinder drag coefficient is very much higher but for disc and cylinders they are almost cylinder they are almost similar to each other. So, like this this drag co relations are available either in the I mean like a graphical form like this are in a kind of a co relations forms for a standard kind of objects like you know spheres cylinders etc disc etc or one can obtain by a numerical analysis or the experimental results anyway one can obtain it.

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So, the references the majority of the information needed almost all information presented in this lecture has been taken from this reference book McCabe smith and Harriot unit operations of chemical engineering. There, are also reference books in our courses we can one go through this unit operations of particulate solids theory and practice by Ortega rivas. Then Coulson and Richardson's chemical engineering second volume by Richardson and Harker. Then transport processes unit operations where Geankoplis. Then unit operations by Brown et al. And then introduction to chemical engineering by Badger and Banchero thank you.