Mechanical Unit Operations
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Lecture No. 18
Motion of Particles Through Fluids

Welcome to MOOCS course Mechanical unit operations. The title of this lecture is "Motion of Particles Through Fluids".

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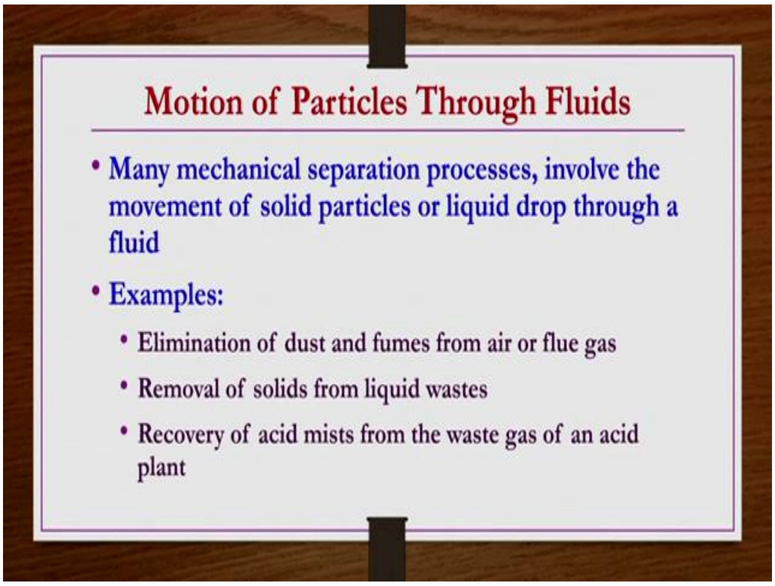

Motion of particles through fluids we have seen most of the multi-phase flows, so wherever mechanical unit operations are involved or even some unit processes are involved then we have seen that there is a kind of relative motion between solid and fluids and then that relative motion is very much important in designing this contacting equipment wherever this unit operations are taking place or unit processes are taking place.

So that relative velocity can also be represented in terms of some kind of a force balance one can do the force balance and then report it as in terms of a drag co-efficient or drag force, right. So that is what we have seen. So we have seen in the previous lecture different types of applications and then how much important is relative motion between fluids and solids especially the multi-phase flows associated with the mechanical unit operations.

We have already seen many mechanical separation processes involve the movement of solid particles or liquid drop through a fluid so there may be a relative motion between a fluid drop or a solid particle or kind of thing, in a kind of another a fluid so that is also possible not just not necessarily only solid particles sometimes liquid drops are also translating in a given fluid. So that those applications we have seen but however we see a few more applications now.

Some examples like elimination of dust and fumes from air or fume gas, then removal of solids from liquid waste, recovery of acid mists from the waste gas of an acid plant. So from the acid plant whatever the acid mists are there, there may be a few releasing them in the air as it is so that is going to be very dangerous to the environment so then what you have to do you have to separate them and then clean that air and then then only the fluid you can release into the atmosphere. So these are a few applications. We have already seen several applications.

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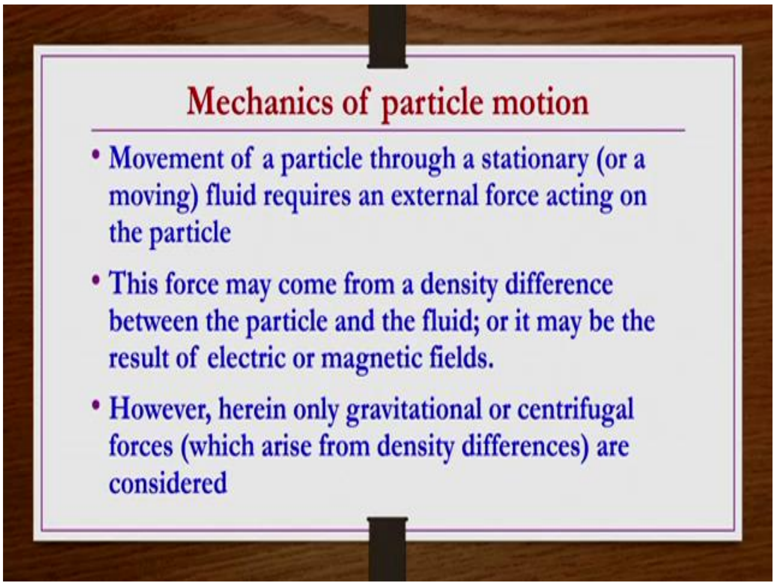

Now what we see “Mechanics of a particle motion”. So movement of particle through a stationary or a moving fluid requires an external force acting on the particle. In general, so you need to apply some kind of external force, that may be based on the gravity- external force may be gravity force or it can be a kind of centrifugal force or it can be a kind of a electric field or it can be due to their magnetic field or it can be because of some kind of pressure difference etc. So but there should be some kind of external force so that a particle can move through a stationary or a moving fluid.

This force may come from density difference between the particle and the fluid as I mentioned or it may be the result of electric or magnetic fields. So whatever the force is, that arise due to the density difference in general or maybe we can say that a gravitational force or centrifugal forces, so in this particular lecture we are considering only two two types of external forces while developing this motion of a particle through fluids. So those two types of forces are the gravitational and centrifugal forces.

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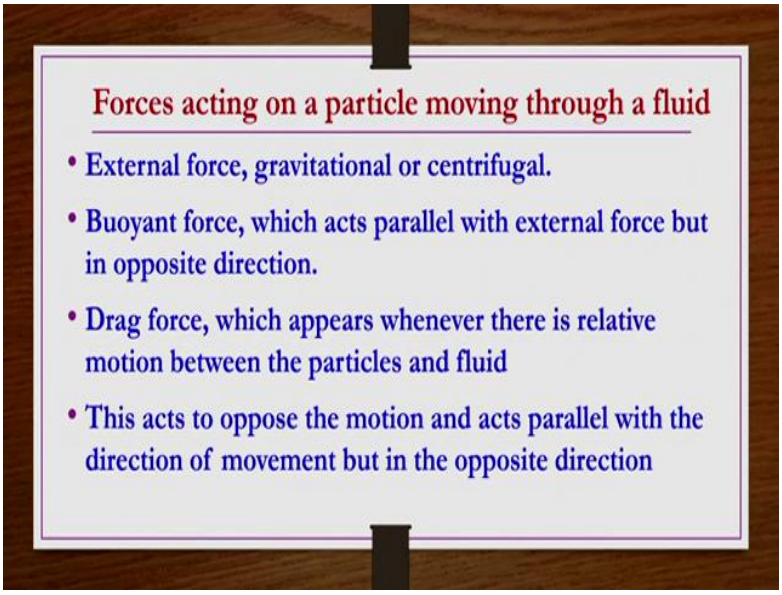

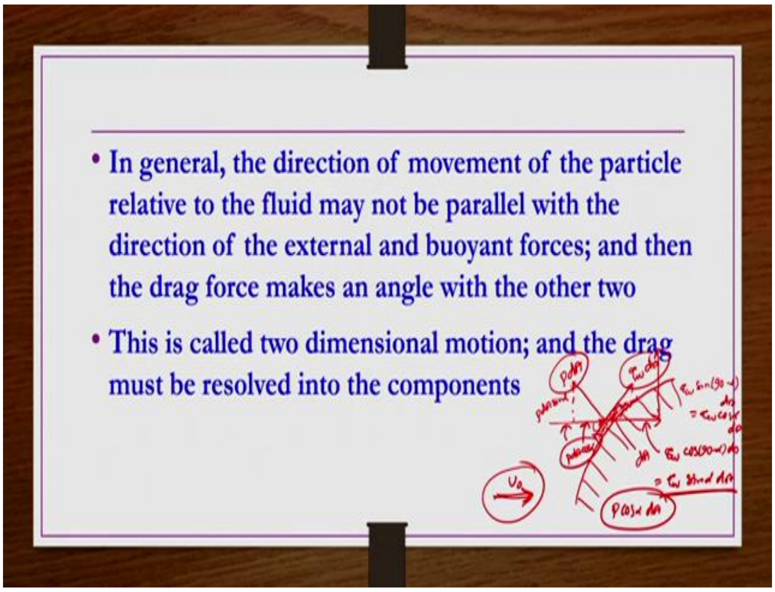

Now, “Forces acting on a particle moving through a fluid” that is what we are going to see. So what we are considering here, we are considering only external force due to the gravitational forces or centrifugal forces. Those two kind of external forces we are considering. But when a particle moving through a fluid then it may also be expecting experiencing other forces like buoyant force which acts parallel with external force but in opposite direction. Then drag force, which appears whenever there is a relative motion between the particles and fluid. And then this drag force acts to oppose the motion and acts parallel with the direction of movement but in the opposite direction, in general, if kind of a one-dimensional flow.

But, in general, direction of movement of the particle relative to the fluid may not be parallel with the direction of external and buoyant forces and then the drag force under such conditions makes an angle with the other two, so because of that one we need to find out the individual component of the drag force rather than a having a kind of total drag force, directly. So this kind of cases where drag force making an angle with the other two kind of forces is called as a kind of two dimensional force, in general, and then drag must be resolved into the components.

Like you know, individual pressure drag component you have to find out, individual friction drag component you have to find, something like that in the previous lecture when you are defining the drag forces or a developing relation for the drag forces we have taken a surface like this. So this surface like this. And then we have taken a kind of a element on the surface. So, and then fluid motion is coming in this direction, U naught. So now the drag force is the force acting on the solid object exerted by the fluid in the direction of the flow direction in the direction of the flow.

So but now the flow direction in horizontal and then this surfaces are making an angle with this surface. So if you take this angle like this so this angle something like you know making some angle so that angle based on that angle we are derive the components. So, since it is making some kind of angle with this surface, you know, what we have, we we will have in general different components. You have to find out a kind of a pressure component normal to the surface and then you have to find out a tangential component like this, so whatever the pressure force and then $\tau_w dA$, let us say this is the dA is the area of the element that we have taken and pressure force would be PdA and then shear force would be $\tau_w dA$ and they are in two different directions.

So this is the flow direction but the PdA is acting in this direction and then $\tau_w dA$ is acting in this direction. Force due to the pressure and force due to the shearing action are acting in two different directions compared to the this direction. So that is the reason, you know, this kind of flows we called it a kind of two dimensional flows and then we have to obtain their kind of the components, we have to obtain the components of these forces acting in the flow direction.

So, now in the horizontal direction we have seen that this is kind of a this force is there, so then based on the angle we have seen, so here also we have seen based on the angle how much is this you know things that force it acts acting in this direction so that also we have seen. So we have seen this tau w and then this angle is a kind of 90- α. So this this component would be τ_w sin (90-α) dA.

So, that is $\tau_w$cos α dA is in the in the other direction, in this direction that is acting whereas in this direction whatever that acting is $\tau_w$cos (90-α) dA, so this is τ_w sin α dA. So, the wall drag that is acting in the flow direction for this element is tau w sin alpha. Similarly, here if it is (90 –α) degree so then it will be alpha, so then this would be this component would be P dA sin α and then this component in the flow direction would be P dA cos α.

So this P cos α dA is going to be the form drag that is acting in the flow direction though the element is not having a kind of one dimensional flow, that is, the flow direction and then the alignment the angle that is making with the flow direction, this element they are not not same, but then it is a two dimensional flow. That is the reason it is a two dimensional flow. Under such conditions we have to find out a kind of a individual components of the drag force and then add them together to get a kind of a total drag force.

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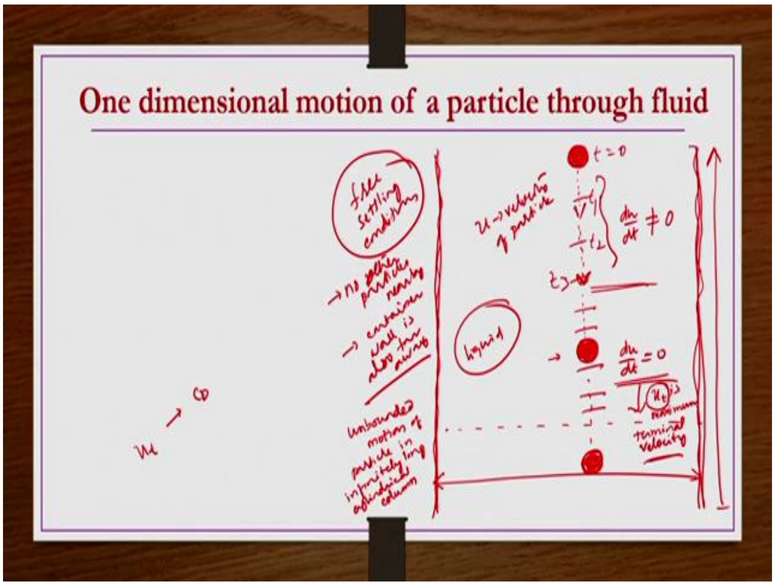

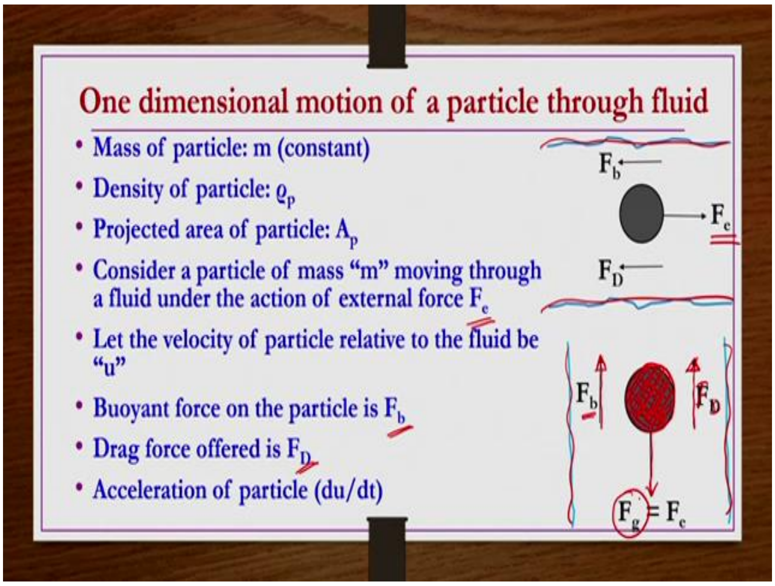

But in this lecture what we will do we will be considering only one dimensional motion of particle through fluid. So that to avoid complications and then have a kind of simplified analysis, because whenever there is a two dimensional flow it is it becomes very difficult to get the results analytically and then one may need to go for semi-analytic or semi-numerical or numerical results or by experimental approach only. So so one dimensional motion of a particle through fluid if we consider that we can do analytical analysis, simplification and then apprehend the drag forces total drag force how much it is a how much total drag force acting on a particle when it is settling in a kind of gravity motion or centrifugal motion that is what we are going to see provided the motion is one dimensional motion.

Let us consider a container, very large container. In which we are settling a particle of a small particle size. So large container we have taken in order to have a kind of unbounded motion, unbounded motion of particle in infinitely long cylindrical column. This we are trying to do because in order to have a kind of free settling conditions, in order to have a free settling conditions. So this is very much important because the analysis that we are going to do that is only for free settling condition.

How to make sure that the settling the particle settling conditions and the free settling condition you can make sure by having a unbounded flow and then no other particles nearby, nearby there are no other particles and then container wall is also far away from the particle. By taking these things, we can make sure that a kind of a the particle is settling under free settling conditions without any kind of hindrance from the neighboring particles or without any kind of hindrance from the container wall.

So now this particle it is settling. It is when it is settling, initially the acceleration of the particle, acceleration of the particle may be there. So that, it maybe settling, you know, with a certain velocity but you know $\frac{du}{dt}$. Let us say u is the velocity of a particle that is settling. When it is settling, you know, initial conditions, $\frac{du}{dt}$ is not 0 actually. But if you provide you know free settling conditions without any hindrance and then without any neighboring hindrance because of the neighboring particles or wall of container.

So eventually what happens what will happen at certain location when it reaches, a kind of a free settling conditions is achieved. So from here onwards, what you have $\frac{du}{dt} = 0$. So under the free settling conditions and then what we can expect that you can expect that $\frac{du}{dt}$ is going to be 0 and that is going to be there. You know again, not upto the entire of bottom when it reaches the bottom of the container but to some extent, some extent $\frac{du}{dt}$ remains as 0. So this is the $\frac{du}{dt}$ is 0 that means u is maximum here.

So maximum attainable velocity. So under the free settling conditions what we can have the particle maximum attainable velocity whatever is there. So that velocity we call it as Ut or terminal velocity or terminal velocity or free settling velocity that is what we call it. So this terminal velocity we are going to use this analysis. So this terminal velocity Ut is related to the drag co-efficient that is what we are going to see now, how it is related and all those things that is we are going to develop now relations.

So this is the basically the assumptions or the simplification of the flow condition that we are taking. So what we are taking? We are taking a infinitely long cylinder filled with a kind of a liquid, some kind of liquid in which you are, you are trying to settle this particle or you are trying to find out the velocity of the particle or you are trying to find out the drag experienced by this particle when it is settling in a column of liquid. So, you are taking infinitely long column large diameter and a large length large height cylindrical column you have taken and then you filled kind of liquid and this liquid you are allowing one small particle settle down.

When it is settling, so under the initial conditions settling conditions, du by dt may not be, indeed it will not be equal to 0, that is, from one location to the other location when the particles fall in the velocity will not be remain same.

Let us say from this is the t, t is equal to 0, at t is equal to t1. It may be coming here. So whatever the UN is there that is not the same by this time with (t2) U2, that is not same as the U1. So at t3 let us say particle reach to this point. So that means at t3 the whatever the velocity let us say U3 that is not the same with U2. So that means from distance and from time the particle the velocity the settling velocity is different in this region, and this is known as the non free settling region, but after certain distance what happened particle attains a kind of free settling conditions where the particle, you know the velocity is found to be independent of the time and an independent of the distance.

Whether you measured at this location, at this location, at this location or at this location as long as the free settling or terminal velocity conditions are prevailing it will be having the same velocity. So that is the reason $\frac{du}{dt} = 0$. So that means when this $\frac{du}{dt}$ becoming 0 that condition maximum velocity condition prevails that maximum velocity is known as the terminal velocity. So this is what we are going to develop the relations by the between this Ut, C_D and another kind of things, the other kind of forces that are acting that I mean like we are making we are going to make a force balance and then trying to develop these relations suitable relations.

So now the same thing so whatever we have discussed till now there is a particle but I have drawn in a kind of smaller geometry so that to include the other terms also. So now we have a container, we have a particle and then that is settling with the gravity. So the gravitational force F_g is were acting downwards here. So there will be a kind of buoyance force that will be acting opposite to the whatever the force is causing the motion. So buoyance is our acting opposite to F_g here. So that is buoyant force Fb is acting opposite to F_g. Now this particle when it is settling here, so the fluid is offering some kind of resistance.

It is not allowing to it will try to resist it will try to resist the particle motion. So that resistance is known as the drag force. So that drag force will also act opposite to the direction of the movement. So that is F_D is acting in the opposite direction to F_g as well here, so the same thing if you take a generalized one, so like in not necessarily that flow is a kind of article, once if you have a kind of flow like this and then then there is a kind of some external force F_g which is causing this particle motion in a kind of horizontal direction as shown here.

So then it is exactly the similar way, balance has to be done exactly the similar with. Only thing that F_g is replaced by F_b so what we do is start with the generalized one with F_e then for Fe we substitute F_g if you wanted to know the motion because of the gravitational force and then for Fe we substitute Fc the centrifugal if you wanted to know the motion of the particle because of the centrifugal force.

So let us assume mass of the particle m is constant whose density is ρ_P the projected area of the particle is A_p. So for simplification have shown a kind of spherical particles as a kind of circle but whichever particle you take from the project area of that particle you have to take that is let us take let us say A_p be, first generalized one and then we apply the case for the kind of spherical particles.

Now consider a particle of mass m moving through a fluid under the external force F_e, now let the velocity of particles relative to the fluid be u, then the buoyant force on the particle is F_b as I have mentioned. Drag force offered is F_D, then acceleration of particle is $\frac{du}{dt}$. Now what we try to do, we try to find out what is this F_b? What is this F_D? And then what is this F_e, then we make this balance like ($F_e - F_b - F_D$) is equal to the net force, that is $m\frac{du}{dt}$. Then we simplify this equation in order to get the free settling velocity Ut.

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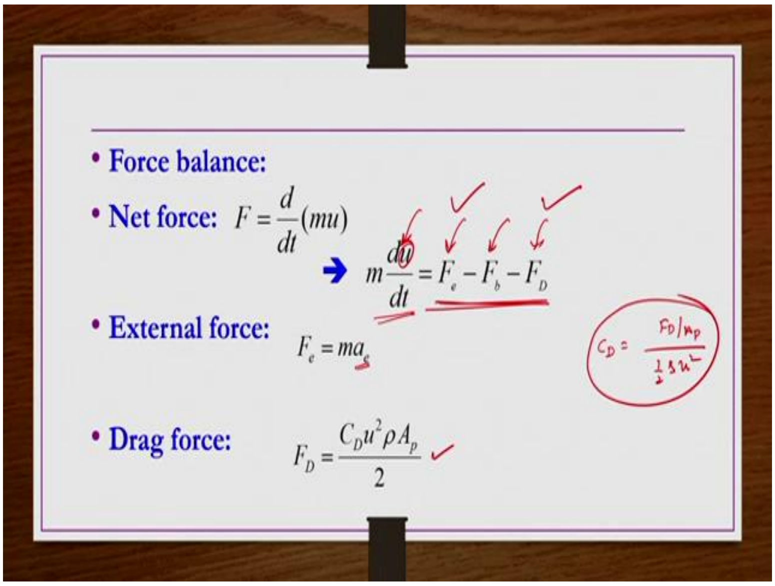

Remember all these things are for the free settling velocity condition as I explained just before. So force balance net force $F = m\frac{du}{dt}$ or $\frac{dmu}{dt}$ so in our case the mass of the particle is constant. So that means $m\frac{du}{dt} = F_e - F_b - F_D$. This is the over all force that is acting on the particle. That is external force causing the motion minus the buoyant force that is working opposite direction to the external force and then drag force which is offering resistance to the motion.

So $F_e - F_b - F_D$ that is the net force that is acting on the particle at a given instant. And then that should be crystal the $m\frac{du}{dt}$. So now we have to find out what is this Fe, what is this Fb whatever what is this FD. That is external buoyant drag forces. Individually we have to find out, we have to substitute here in order to get an expression for this u in terms of a known parameters, like $\rho mu \rho_P$ and then Dp particle diameter etc that we are we have to do, first we do for a kind of generalized case then we take a spherical particle case.

So external force if the acceleration due to the external force $F_e = ma_e$ should be external force that is quite clear and then drag force we have already seen the definition

$C_D = \frac{F_D/\mu_P}{\frac{1}{2}\rho\mu^2}$. This is what the definition of the drag coefficient that we have seen in the previous lecture. So from here F_D we can write in terms of C_D as $\frac{C_D u^2 \rho A_P}{2}$. So out of these three forces, external force and then drag force we have an expression already. Now we have to find out the expression for the buoyance force.

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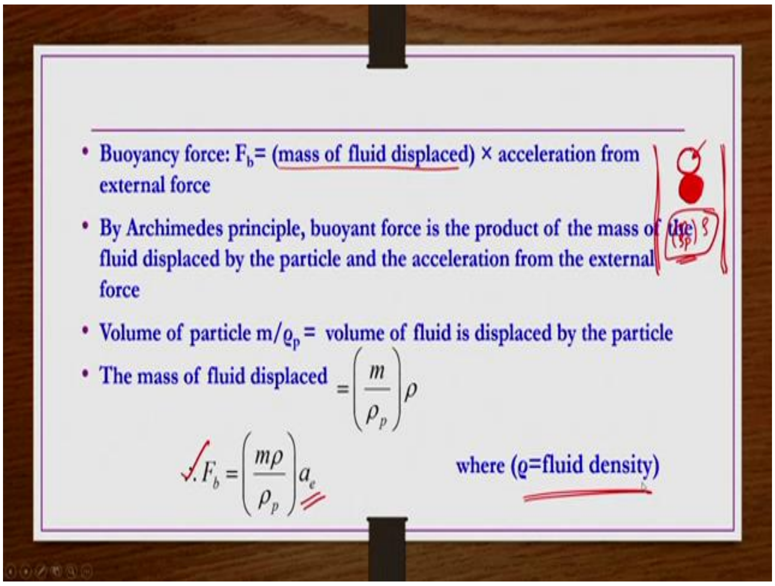

So buoyancy force is the mass of the fluid displaced multiplied by the acceleration from the external force. So let us say this particle we are having here. So, how do this is the only thing that here we have to find out mass of the fluid that is displaced. So let us say this is the particle here that we have taken, now, when it is coming here just you know underneath it like this.

So that means whatever the fluid that was here, you know initially particle was here, when it is just falling down. Let us say it has come here. So whatever the fluid that was present here this shaded portion that fluid is being replaced by particle. So whatever the volume was there here that volume is replaced by the particle. So and that volume is same as the particle value. So that is $\frac{m}{\rho_p}$. So this this is the volume of the fluid displaced by the particle. This is the volume of the fluid $\frac{m}{\rho_p}$ is the volume of the fluid displaced by the particle.

Because it is same whatever the volume of the particle is that that much volume only that is going to that much volume of the fluid only that particle is going to replace. So $\frac{m}{\rho_p}$ whatever the volume of the particle is the same volume up that much volume of the fluid is being replaced by the particle, but we know we need to know mass of the fluid displaced. So now this is the volume of the fluid displaced by the particle if this volume of the fluid if you multiplied it by the density of the fluid then you will get the mass of the fluid that is being displaced by the particle so $\left(\frac{m}{\rho_p}\right)\rho$.

This quantity if you multiplied by the ae, acceleration from external force then we have this buoyant force, this is according to the Archimedes principle. That is buoyant force is the product of the mass of the fluid displaced by the particle and the acceleration from the external force, so volume of particle $\left(\frac{m}{\rho_p}\right)$ is the volume of fluid is being displaced by the particle. So mass of the fluid displaced would be $\left(\frac{m}{\rho_p}\right)\rho$, now buoyant force should be $\left(\frac{m\rho}{\rho_p}\right)a_e$, that is acceleration from the external source.

So now we have this buoyant force also, so now if we substitute these things in the force balance equation will get the final expression for the velocity. So here this rho is nothing but the fluid density, ρ_p is nothing but the particle density. So when you substitute $m\frac{du}{dt}$ is same in the left hand side the force balance equation. This is the external force, this is the buoyant force and this is the drag force.

So that means m if you take to the right-hand side and then from these two terms if you take m common, so then you have $a_e\frac{1-\rho}{\rho_p}$ we are doing you will get something like $a_e\frac{1-\rho}{\rho_p}$ if you do LCM, so then you will get this one. So whatever this m is there here so that m that m will be cancelled and then there will be m will be coming out here.

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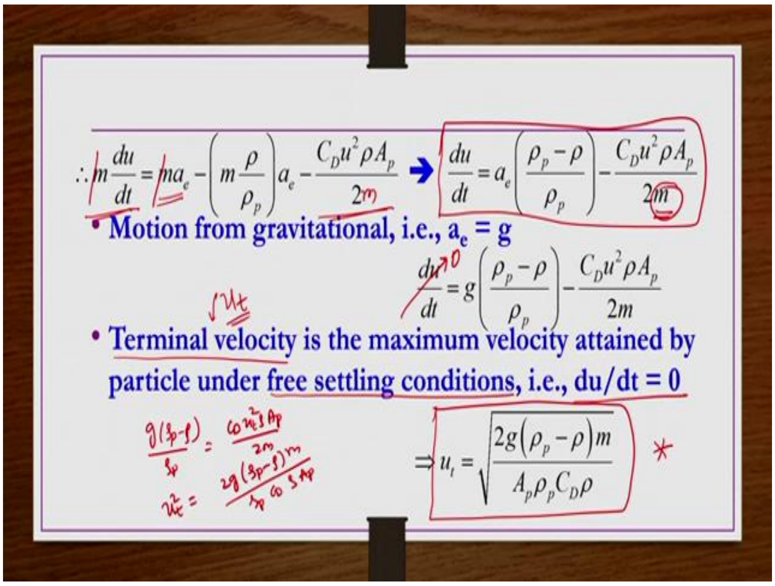

So this is this second term is remaining same only this m is coming in the denominator here. Now motion from gravitational force, this is a kind of generalized kind of expression now till now here we have not done any kind of testing. We are not done any kind of specified derivation with respect to the external field generalized external field we have taken now. Let us take external field is gravity that is the particle is settling due to the gravity. So in place of ae you should substitute g acceleration due to gravity then you will get $\frac{du}{dt} = g\left(\frac{\rho_p - \rho}{\rho_p}\right) - \frac{C_D u^2 \rho A_p}{2m}$.

Now terminal velocity as I already mentioned is the maximum velocity attained by particle under free settling conditions. So that is $\frac{du}{dt} = 0$ as I already explained. So we are trying to find out this terminal velocity U_t, This is U_t , t stands for terminal velocity, so in this equation if you make this $\frac{du}{dt}$ is equals to 0 under terminal free settling conditions then we have U_t is equals to whatever this equation, from this equation what we have $\frac{g(\rho_p - \rho)}{\rho_p} = \frac{C_D u_t^2 \rho A_p}{2m}$ and $u_t^2 = \frac{2g(\rho_p - \rho)m}{\rho_p C_D \rho A_p}$ and then if you take square root which has each side then you have Ut is equal to now this is under terminal condition

So this U should be replaced by the Ut. So this is true under the condition $\frac{du}{dt} = 0$ are when $\frac{du}{dt}$ is equal to 0 then U is nothing but Ut terminal velocity. So $u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$. So now this is the generalized terminal velocity condition for any particle, which is settling under one-dimensional conditions one-dimensional moment or the motion of the particle is one-dimensional in a kind of container and then the settling is due to the gravity. So this is the velocity expression generalized expression. It is not specific to any particle. So but only assumption that you know under free setting conditions that is terminal velocity, that is we are having here.

Then, now we take to the spherical particle case So how we do that one only this previous expression wherever that $u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$ that expression in place of Ap, we will be writing a projected area of spherical particle, that is $\frac{\pi D_p^2}{4}$ and then in place of m we will be writing $\frac{\pi D_p^3}{6\rho}$. So that the mass of the particle ρ_p , that is mass of the particle.

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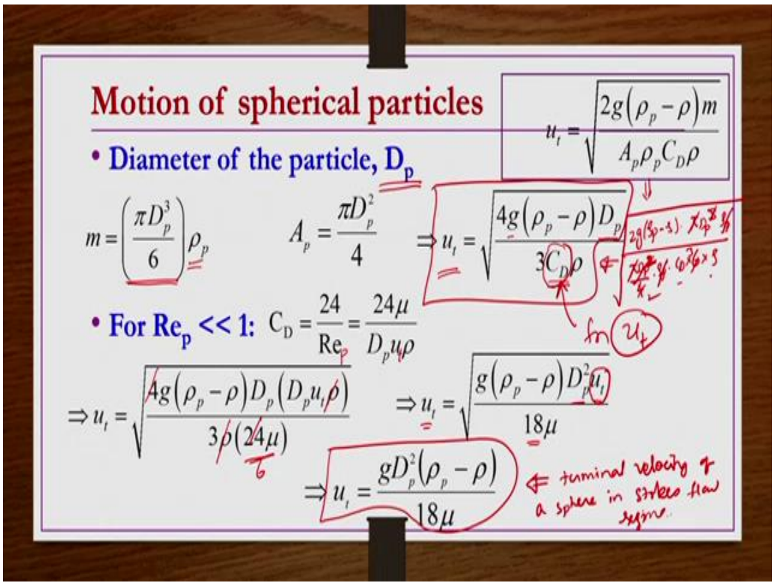

So let us say if you take the diameter of the spherical particle is D_p then this is the volume $\frac{\pi D_p^3}{6}$ is the volume of the particle. If you multiplied by ρ_p, then you will get the mass of the particle, projected area for a spherical particle is $\frac{\pi D_p^2}{4}$, then in this equation $u_t = \sqrt{\frac{4g(\rho_p-\rho)D_p}{3C_D\rho}}$ when you substitute then you will get this expression.

So here what we are doing, we are in this equation we are substituting $\sqrt{2g(\rho_p - \rho)m}$ is $\left(\frac{\pi D_p^3}{6}\right)\rho$ and then this $A_p = \frac{\pi D_p^2}{4}$ then ρ_p as it is, C_D is as it is, 6 is already there and then ρ is as it is so this rho p, this ρ_p can be cancelled out, this pi, this π can be cancelled out, this πD_p^2 and then this cube of D_p^3 can be cancelled out. So this if you can 2 2 is are , 2 3 is are so 4 by 3, you will get, so that is you will get square root of 4/3 g is as it is $(\rho_p - \rho)$ is as it is and there is D_p as it is by 3 is there, C_D is there, ρ is there, so now this is the when you substitute this one you will get this equation.

So this is what, you will get this equation, in this Ut equation you simply substitute substitute A and m, then you simplify then you get this part this expression. Now, this is the expression for the terminal velocity of this particle irrespective of which flow it is whether stokes regime or whether it is slowly settling or very fastly settling only the condition that it should be free settling condition under terminal velocity condition, so then this is valid this, this is valid irrespective of the flow regime, whether the particle is small, particle is lost, particle slowly settling or very fastly settling it is not dependent on it, it is a generalized one.

Now we have already seen there is a stokes regime then Newton's regime of the flow in the previous lecture. If the Reynolds number is very small then we call it a kind of stokes regime or be less than 1 then we call it stoke slow regime and then for that condition $C_D = \frac{24}{Re_p}$ when you substitute that $C_D = \frac{24}{Re_p}$ here then we get the kind of expression for a stokes terminal velocity or the terminal velocity for a spherical particle under stokes regime.

Then similarly for spherical particles we have that Newton's flow regime that is if Re_p greater than 1000 then C_D is equals 0.4 to 0.4 feet varies. Let us take 0.44 let us see the value substitute here then you will get settling velocity of a spherical particle under Newton's flow regime, whereas this velocity is it is it is valid for the entire range of Reynold's number from very small Reynolds number less than 1 to Reynolds number Re 1000 or even more also, only thing that corresponding C_D value one has to substitute here to get the expression.

So now but if you know the C_D value then it is fine but if you do not know C_D value C_D is again function of u or Ut so that is the reason you know, we need to further do analysis, but you can use this equation only when you know C_D. In order to get this velocity you should know C_D but again in order to know the C_D you need to know back this Ut. So they are interconnected without a one information you cannot get this value, so at least one of them should be known, if you know the u then C_D you can calculate, if you know CD then you can calculate u, so that is what we are going to see, we do step by step.

Let us take in our first case when the Reynolds number is very small Re_p is very smaller than 1 then $C_D = \frac{24}{Re_p}$ that is we have already seen. So that is $\frac{24\mu}{D_p u \rho}$. So now here this one we are going to substitute here. So this is under the terminal condition. So this u should be Ut, so now in this equation what we have done here in place of CD, we have written 24 we have to write $\frac{24\mu}{D_p u_t}$, this is we have to substitute here, when you substitute here $\frac{24\mu}{D_p u_t}$, so you have taken up numerator.

So, now you further simplify it, simply what will get you will get a kind of velocity for a terminal velocity under stokes flow regime. So here this rho, this ρ is cancelled out and then Dp and D_p^2 is there, so Ut is already as it is $g(\rho_p - \rho)$ is as it is, this 4 by 3 multiplied by 24 that you can write it as a 18 because 4 1 is are 4 6 is are 3 x 6= 18, so 18 μ as it is.

Now here again inside the square root of right side there is a Ut and then a left side is also 1 Ut. So what you do is you take square either aside so then left hand side you have $u_t^2 = \frac{gD_p^2(\rho_p-\rho)u_t}{18\mu}$. So this one Ut in the right hand side and square of the Ut in the other side would be cancelled out so that you will have $u_t = \frac{gD_p^2(\rho_p-\rho)}{18\mu}$. So this is the velocity, this is the terminal velocity under stokes flow condition for spherical particles, terminal velocity of a sphere in stokes flow regime, that is very small Reynolds number.

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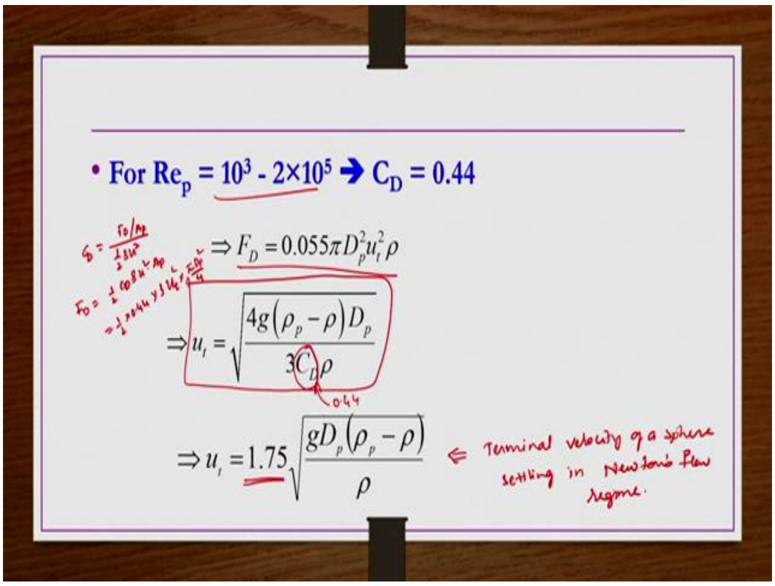

So, similarly now what we are going to we are going to do for the Newton's flow regime, for the Newton's flow regime Re_p are of 10^3 to 2 x 10^5 C_D is constant so that constant let us say 0.44, it varies between 0.44 and 0.45 slightly, but let us take 0.44 so now in this equation, whatever the F_D equation it is so that equation if you substitute you know this what we have $C_D = \frac{F_D/A_p}{\frac{1}{2}\rho u^2}$.

This is what we have, so from here $F_D = \frac{1}{2} C_D \rho u^2 A_p$, next of 0.44 you can take $C_D \rho u_t^2$ and then Ap you can take it as $\frac{\pi D_p^2}{4}$. So and then they simplify your, you simplify it then you will get $0.055\pi D_p^2 u_t^2 \rho$. This is what you get drag force, but drag force is not our interest as of now.

Our interest is to find out the motion of the particle velocity of the particle. So terminal velocity of the particle. So in this equation, this is generalized equation that we just derived here in this equation in place of C_D if you substitute 0.44 you will get, if you substitute here 0.44, then you have (4/3 x 0.44) under the square root of that all comes out to be 1.75.

So $u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$. This is the terminal velocity of a sphere settling in Newton's flow regime. So now we have developed for spherical particle in general also, we have developed the settling velocity of the particle Ut etc, we have also developed the settling velocity for a spherical particle specific to the spherical particle settling in a gravity force due to the gravity the particle is settling.

So then depending on the flow of m we have find out that terminal velocity expressions Ut for stokes flow regime as well as the Newton's flow regime indeed we have also first we have developed a generalized terminal velocity expression, then we applied for individual strokes and then you can see flow regime.

But those expression that is terminal velocity of the sphere under stokes or Newton's flow regime, you can know only if you know the Reynolds number. If the Reynolds number is less than 1 then only you can say that it that it is in stokes flow regime, and then you can apply that equation. If you if your Reynolds number is more than 10 power 3 then only you can know that it is a kind of a Newton flow regime then only corresponding terminal velocity equation you can use that we have just derived, but if you want to know the Reynolds number you need to terminal velocity then only you can use this expression, that is they are interconnected, you need to know the Reynolds number in order to use this equation.

But in order to find out the Reynolds number, you need the velocity condition. So there is a connection, they are interconnected, so you cannot calculate. So that is the reason we are going to develop a criteria which is independent of velocity term. So our independent of the Reynolds number term, so that it suggests a kind of mathematical simplification so that we can use them in our this kind of problem kind of thing.

Even without the knowledge of velocity or without the knowledge of Reynolds number we can find out the terminal velocity accordingly. So that is what we are going to do, we are trying to develop a criteria for settling regime of spherical particles without requiring the knowledge of the terminal velocity or the Reynolds number or the drag coefficient., how to do that one.

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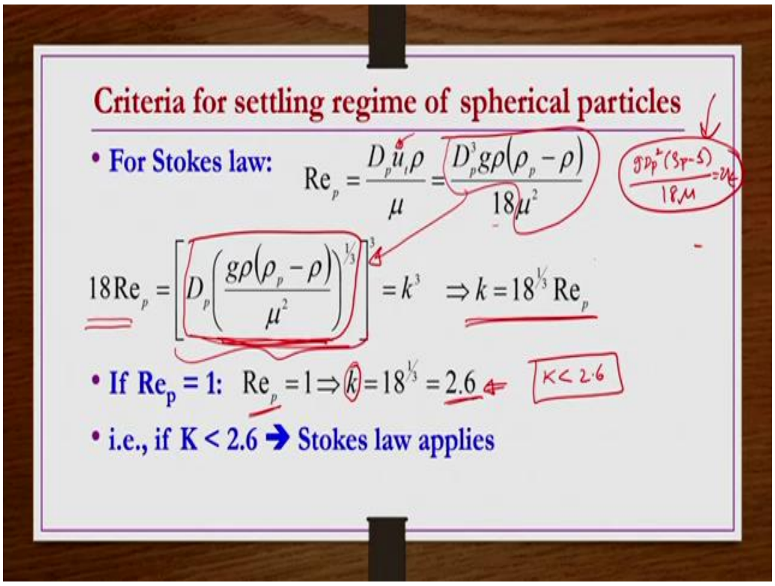

Let us see first Stokes law we have our $Re_p = \frac{D_p u_t \rho}{\mu}$. So this is the Reynolds number definition. $Re_p = \frac{D_p u_t \rho}{\mu}$, this criteria we are going to develop our two regime, stokes Stokes regime and Newton's regime and then we find out the values.

So if that value that criteria if it is matching the Stokes regime then corresponding Stokes regime equations will be used, so for that this mathematical simplification we are going to do now. This $Re_p = \frac{D_p u_t \rho}{\mu}$, ρ here this Ut under the stokes regime we have already seen that $\frac{g D_p^2 (\rho_p - \rho)}{18\mu}$ is nothing but ut and that the stokes regime.

So that we are substituting here simply $g D_p^2$ 1 Dp is already there, so Dp^3, ρ is as it is $(\rho_p - \rho)$ is here and then 18μ and then 1μ is there is $18\mu^2$. So now it is not having any

Ut value. So now this one what you do 18, you take to the left hand side from this equation and then rearrange this equation such that $D_p\left[\left(\frac{g\rho(\rho_p-\rho}{\mu^2}\right)^{1/3}\right]$. So whatever the power 1/3 to this parameter is , and then this entire whole power cube.

So that it is same as this this part so that it is same as this part. So now here this part whatever the within the square brackets part is there so that you write it as k so that if you write it as k, k is like you know $D_p\left[\left(\frac{g\rho(\rho_p-\rho}{\mu^2}\right)^{1/3}\right]$. This is going to be the k factor. So this we are taking as a kind of criteria.

Now here for the Stokes regime what we have the maximum value for our Ap we can assign as 1 under the stokes regime because Re_p less than 1 we call it as a kind of Stokes regime. So upper limit for this stokes regime, let us take as 1, so if Re_p 1 then Stokes law is applicable. So if Re_p is equals to 1 then Stokes law is applicable. So now in this equation you substitute Re_p is equals to 1. So k is equals to $18^{1/3}$ that is k is equal to 2.6. So if Re_p is maximum limit of Re_p for Stokes flow regime is 1, then corresponding k value is 2.6, maximum limit of k value for Stokes regime from here what we understand it is 2.6, that means if k less than 2.6 then Stokes regime is applicable.

That is if k less than 2.6 then Stokes law is applicable and you need to know velocity in order to calculate the Re_p, but in order to calculate the this k, you do not need velocity, so that is this this quantity, this quantity that is $D_p\left[\left(\frac{g\rho(\rho_p-\rho}{\mu^2}\right)^{1/3}\right]$ this quantity you need to find out it is only function of geometry size of the particle and then the fluid properties like, you know, ρ the density viscosity of the fluid and then density of the particle only that much you need to know, you need to know size and density of the particle and density and then viscosity of the particle you need to know, once you know, then you can find out this k value.

If this k value is less than 2.6 then you can say the Stokes law is applicable and then you can use this equation to find out Ut. So what we have done we have in general tried to escape requirement of terminal velocity because you can calculate terminal use this terminal velocity equation only when you know Reynolds number and then in order to (Reynolds), know the Reynolds number you need to know this Ut, so they are interconnected. So that is what we try to do. We try to escape of requirement of this Ut in order to do this calculations.

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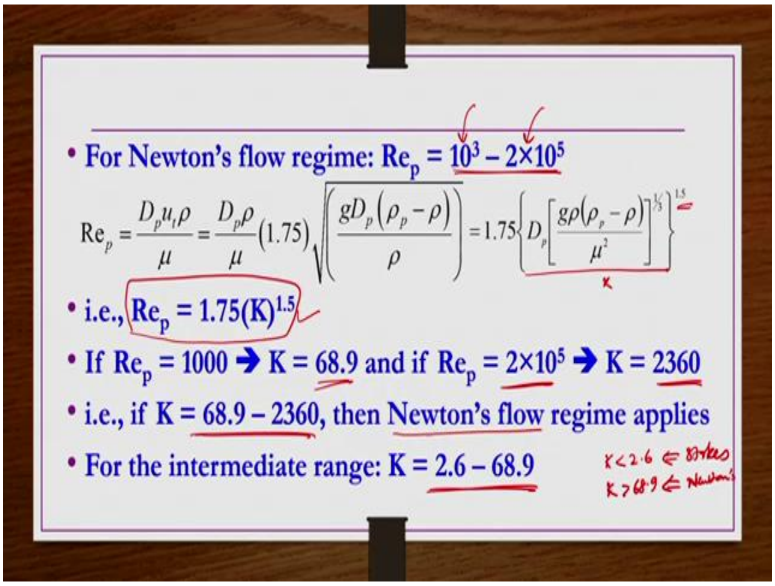

Now same thing for the Newton's flow regime Re_p is 10^3 to 2 x 10^5 , so we know that $Re_p = \frac{D_p u_t \rho}{\mu}$ so $\frac{D_p \rho}{\mu}$ is as it is, Ut under the Newton's flow regime is this one we just derived. So this is t, Ut for a Newton's flow regime, previous Ut for this, so flow regime.

So now here this equation again, if you rearrange the what you get you will get $1.75\left[D_p\left(\frac{g\rho(\rho_p-\rho)}{\rho}\right)^{1/3}\right]^{1.5}$, so this this is nothing but again your k value. So that is $1.75K^{1.5}$. So now the lower limit for the Newton's flow regime is 1000, 10^3, so lower limit of k would be 68.9. So that is here in this equation $Re_p = 1.75K^{1.5}$ here you substitute Re_p is equals to 1000 and then you calculate k, so it comes out to be 68.9.

Upper limit of Re_p for Newton's flow regime is 2x10^5, so now this you substitute 2x10^5 into here in this equation $Re_p = 1.75K^{1.5}$, then you get a value 2360. That means if your k value is in between 68.9 to 2360 then Newton's flow regime is applicable and then you can use this particular equation for your terminal velocity. $u_t = 1.75\sqrt{\frac{gD_p(\rho_p-\rho)}{\rho}}$ that you can use.

So k is less than 2.6 Stokes regime, k greater than 68.9 is a kind of Newton's regime up to 2360, so it is Newton's flow regime. So whatever between 2.6 to 68.9, it is a intermediate range, where we do not have a kind of generalized expression for the velocity. It has to be obtained calculated using the C_D value.

So what we have seen today now till now we have derived a kind of motion of particle, especially 1D one-dimensional motion of particle in an unbounded the flow conditions particle settling under free setting condition, that is without any hindrance due to the neighboring particle or without any kind of hindrance because of the container wall.

Under those conditions if a particle is settling due to the gravity, what is the expression for the terminal velocity for a generalized case generalized particle we have derived. Then we have also derived simplified that expression for a kind of spherical particle settling in the gravity force or gravity field.

Then we have developed the velocity expression for the Stokes flow regime and then Newton’s flow regime for a spherical particle settling in a gravity field. Then we have also developed the criteria because the Stokes and Newton's flow regime terminal velocity equations we can use only if you know the Reynolds number and then Reynolds number is again dependent on this terminal velocity Ut so they are interconnected. So in order to avoid the complications the interconnection of this Re_p with Ut what we have tried to do, we try to develop the criteria in terms of k.

And then that k if less than 2.6 we found it is a kind of Stokes flow regime for a spherical particle, if it is between 68.9 to 2360, then it is a kind of Newton's flow regime for a spherical particle settling in the gravity field, and then this k is in between 2.6 to 68.9 then that is a intermediate range for that range, we do not have a kind of generalized Ut expressions so that we have to calculate using the generalized Ut expressions before developing the Stokes and Newton’s flow regime whatever the Ut expression that we have used, which is function of C_D that we have to use.

In the next lecture what we will doing, will be discussing the 3 to 4 problems example problems on this free settling conditions etc under the gravity and then we try to find out how to use this equation. If you know the Reynolds number it is straightforward. If you do not know the Reynolds number how to calculate this terminal velocity those kind of things we are going to see in the next lecture.

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The references for this lecture are given here. The most of the lecture is prepared from these Unit Operations of Chemical Engineering by McCbe, Smith and Harriot book then other references we have Unit Operations of Particulates Solids, Theory and Practice by Ortiga Rivas. Some details of this lecture can also be found in this Coulson and Richardson's Chemical Engineering series second volume, the Richrdson and Harker, Transport Processes and Unit Operations, this book by Geankoplis is also involving some information about settling of particles in a fluid, then Unit Operations by Brown et al and then Introduction to Chemical Engineering by Badger and Banchero, these books are also a kind of a good reference books for the today's lecture. Thank you.