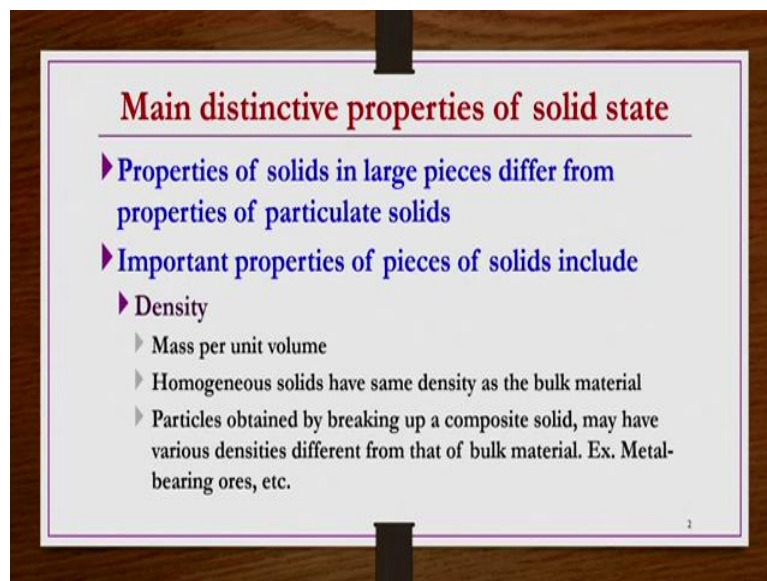


Mechanical Unit Operation
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Particle Size

Welcome to the mock course Mechanical Unit Operations, the title of this lecture is Particle size. So what we have been discussing till now is about the relevance of a particulate flows, especially in chemical engineering and then what kind of mechanical unit operations in general we have chemical engineering, in those mechanical unit operations are they really this particulate flows are useful are not, those some example some kind of classification etc. that we have in the previous lecture. So we before going into the details of this particles size we have a small recapitulation of what we have seen primarily with reference to the particulate flows in previous lecture that we see first initially first 2-3 slides

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So what we have seen that main distinctive properties of solid state so that is when we have a kind of bulk large pieces of solids so whatever the properties of those solid if you compare with the smaller sizes. Let us say those bulk size of particle you take and then you measure their physical properties like something like you know size, shape, density, hardness, tenacity, etc those thing you measure than what you do, you crush them you crush them into small-small fractions and then again for that particulate material that is smaller-smaller size particle whatever is there crushed material is there that material that solid material for that material

again you measure the same properties like you know size, shape, density, hardness, tenacity, etc.

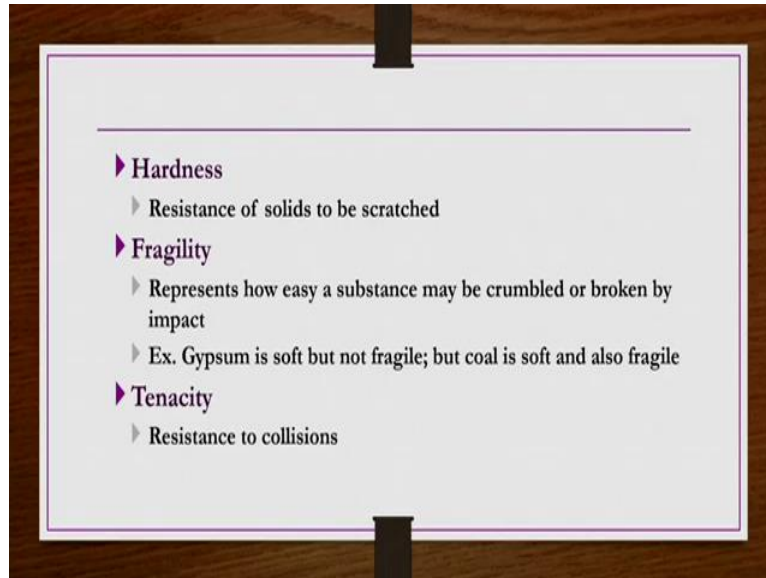
Then what you will find in general these properties are very much different from their bulk properties when you measured in a kind of larger bigger size kind of things. So that is in general happens in this kind of particulate flows. So those what are those properties which are in general different in particulate state as well as in a bulk state that we have seen, so once again we recapitulate them.

So these are in general properties of solid in large pieces differ from properties of particular solids and then important properties of pieces of solids include generally density, density we define it as mass per unit volume if the material is made up of homogeneous or the solid is a kind of homogeneous solids then the density of the bulk material as well as the particulate material obtained by crushing that bulk material will have the same density or both the densities would be same if the material is a kind of homogeneous right.

Or otherwise let us say if you take same kind of metal bearing ores in general you get from the natural resources you measure their properties, their density and then what you do after that you crush them into the small-small pieces and then you measure the density of those particulate metal bearing ores then you find the different density because the metal bearing ores it is a not kind of homogeneous solid they will be having a different kind of a composite nature.

So that is particles obtained by breaking up a composite solid, may have various densities different from that of bulk material because of this different material involved in a kind of bulk material examples as as I mentioned metal bearing ores, etc.

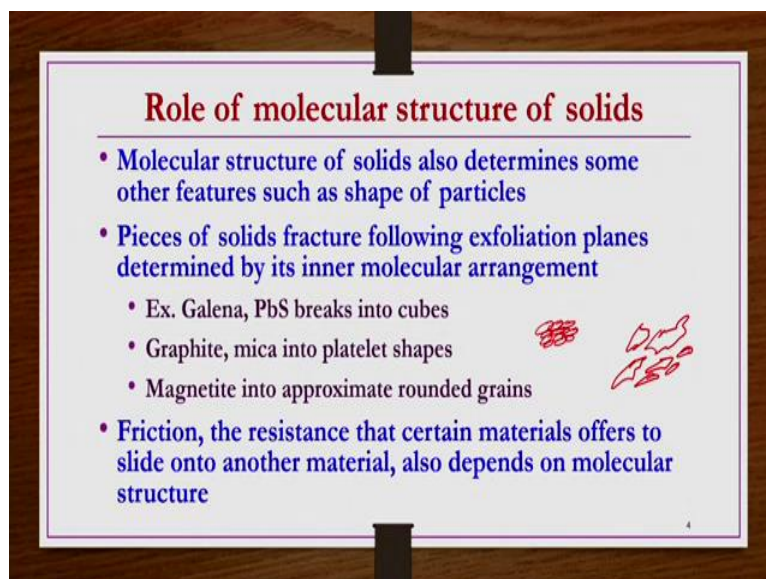
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Likewise, we have a properties like hardness which represents resistance of solids to be scratched that is different for the bulk material and then it is compared to the, whatever the so called particulate solids are there that is crushed material there.

Similarly fragility, which represents how easily a substance may be crumbled or broken by impact, so let us say what we can say in general gypsum is a soft but not fragile material whereas the coal is both soft and fragile in general. Likewise tenacity which is resistance to collisions is also different for particulate solids and the bulk material as well they cannot be same in general.

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So we have also seen, the role of molecular structure of solids in general so we have seen that same material like you know mica, graphite, etc. When you break them you always you know

get a platelet like material right. Sometimes you crush the magnetite kind of ore materials, mostly you get round shape kind of rounded particle shapes you get. Why that happens? Because inner molecular structure of the solid is different from one material to the other materials and then that is playing a kind of role in the crushed particulate solids material that we are getting.

After crushing whatever particulate material that we get, its shape in general depends on the inner molecular structure of the solid that we have right. So, molecular structures of solids also determine some other features such as shape of the particles, as I mentioned. Pieces of solid fracture following exfoliation planes determine by its inner molecular arrangement. For example, galena, PbS often breaks into almost like cubicle shapes.

Graphite, mica etc. in general when you crush them you get the platelet irregular shape particles in generally. Similarly magnetite approximate rounded grains you will get when you break these magnetite materials, because of the inner molecular structure of this material when we break them we get different shapes of these particles.

Likewise, you know what happens when the friction that is the resistance that a material offers to slide onto the other material that also depends. In general what happens let us say when you are pouring this wheat you can comfortably pour them because the wheat is offering less resistance friction for a flowing on to other wheat grains etc. So that is the reason there the friction is less so that is the reason they can comfortably flow out of container when you try to flow them out right, But when you take like a crushed mica, etc, crushed graphite material. When you try to flow out of container they flow very much difficulty flow may not be so much comfortable in general, they flow with a kind of certain difficulty level that is because of the different internal structure.

Internal structure is like a pate like structure so when this plate like material something like this are there you know this thing when they are flowing out so often what you get you find out that you know these are offering more resistance kind of thing or more friction they are offering. Whereas if you have a kind of grains like you know wheat grains or something like this so then they can easily float. So that is because their inner molecular structure is such a way that they offer less resistance while they are sliding on to the other particles right.

So this friction whatever the friction that is the resistance that a material offers to slide on to the another material is also depends on the molecular structure. Then we were discussing about

the particle characteristics, having seen that the distinctive properties of the solid state material specially in bulk state as well as in the particulate state or that is after crushing them into the smaller size particle so whatever the particulate state is there and then what is the bulk state that is bigger larger pieces are there. So we have seen that their physical properties are very much different in general especially when the solids are not homogeneous, if they are homogeneous then sometimes they may be having a kind of similar properties. But this all is in general that natural resources that we get, ores etc. they are not homogeneous so then their bulk properties are very much different from their particulate state.

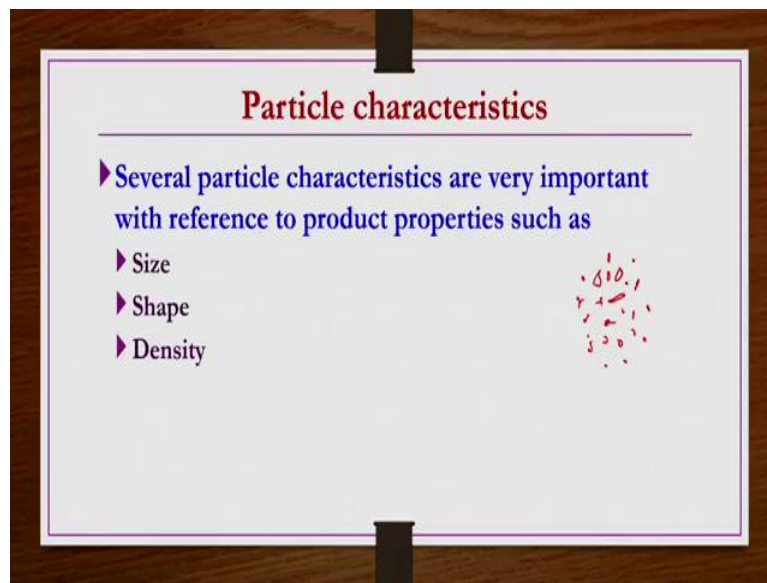
When they crush their particulate properties in the particulate state are very much different, because in general in what happens in real life application whether we are, are we treating this solid in a given unit operation or a given unit process or taking from one unit to the other unit for a unit operation or for a unit process, whatever may be the process we in general take them in a kind of smaller sizes we do not take them in a bulk size bigger size, even if we are take doing small a kinetic reaction or combustion of a particle or a coal particle whatever we take we do in a kind of smaller size particle.

So, we break them so when we break them particle that becomes very much comfortable to transport also in general though it is very difficult compared to the fluid phase or liquid gases but compare to the bulk solid phase this particulate material can be easily transported or conveyed from operation to the other operations. So the importance that I want to emphasis that in general whether unit operation or unit process or the connecting, conveying lines etc, the particulate materials or the solid material that we in general take in a kind of particulate form that is in a small crushed form.

So that is the reason what we have whatever the particle characteristic are there we need to have a kind of clear look because these properties we are going to use in general for the characterization or controlling of any of these unit operations are unit processes where this solid state materials are involved right. So that is the reason particle characteristic are very much important we cannot rely or we cannot take the bulk material property as it is for the characterization or controlling of this unit operations where this particulate materials are involved right.

So that is the reason now we are going to list out what are the particle characteristics right, and then out of which how many of them were very much important and then how to deal with that, that is what we start with.

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Several particle characteristics are very much important with respect to the product properties in general but sometimes with respect to the inlet feed material also several particle characteristic are very much important, what are this characteristic in general? They very much important one is size, size of the particle because let us say when you have a kind of big bulk material bulk solid when you break them you get them into small small particles like these small small particles like this or irregular particles and then different shapes particle all these things you get right. So now, no two particles will have same size in general, there may be exception but in general we have a when you crush this material we get millions of particles so then each and every particle will have a kind of different size and shape in general so this size is very much important one.

Then is the shape also, the shape that whatever we have that also now one particle may be of this this particular shape given here, now you can see other particles are you know other shapes in general so the shape is also going to be very much important. Most important thing that we have to see that when we crush the bulk material into the small small particles size then what happens the size is not uniform first of all and then we will not get the particles of a similar shape or same shape particles we will never get, even if the irregular shape particles we get those irregular particles will not have the same irregularity.

The degree of irregularity would be different particle to the particle so that is the reason size and shape are going to be very much essential right. Why are they going to be very much essential in general because in the transport characteristic or in the reactions where if you want where the solid material is involved or the particulate material is involved? If you want to report

the performance of a given reaction or the performance of a unit processor or the performance of a unit operation, or the performance of heat and mass transfer or the momentum transfer or the degree of reaction associated with these particles that you all are going to represent with reference to the size of the particle right.

And then unfortunately we do not have one single size so then there are many varieties of different size of particles are there so then we have to have a kind of proper analysis about the size and then in general shape is also not a similar for one particle to the other particle. We may have a wide variety of shapes of particle even irregular particle we have a difference of irregularity that is the reason we need to have some kind of characteristic, representation of size as well as the shape that is going to be very much useful in representing any of the performance of the transport phenomena of this particulate material, like transport phenomena in the sense momentum transfer when transporting from one location to other location along with the fluid, or something like that or there if there may be some kind of heat and mass transfer involved or there may be some kind of reaction is also involved. So whatever is the case so there we are going to use this kind of characteristic, size and shape of the particle in general right? So that is the reason this two things are very much essential.

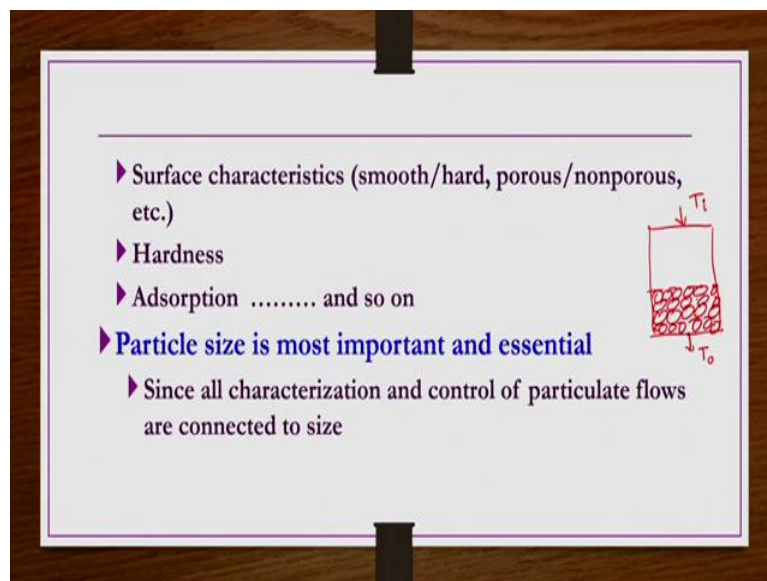
Then, density is also very much important because we have seen if the bulk material is not homogeneous then we are not going to have a kind of density particulate material of the same density right. If the material is composite material then the density of the material crushed material or particulate solids that we are going to have different density compared to the bulk density. Remember, we are just classifying the bulk material and then particulate state or particulate solids that we are calling the crust one, after crushing whatever the material that solid material is there that we are calling as a kind of particulate solid right. So for all these characteristics we are going to see for the particulate solids because we are going to use them in processes, unit processes or unit operations in the crushed form right.

So these are the three major important properties that one should be very much careful because density is also going to play role. Let us say there is a momentum and then heat transfer his occurring in a packed fluidized bed, so the density of the bed material is very much important very much essential right. So if you change the density of the material then rather taking density of particulate solid if you take the density of bulk solid then you may be ending up finding out different performance of the fluidized bed theoretically right. So or the results that you are

going to get theoretically may not be matching with the experimental values, so that is the reason density is also kind of very much crucial parameter.

So these are the when couple of examples like you know packed and fluidized bed, but any process where the solid particulates are involved so there in those processes whether unit operation or unit processes whatever the transport is occurring, whatever the reaction is occurring, these three properties whatever the size, shape and density are going to play a very vital role. So then we should be discussing in detail about these things in this and then coming lectures.

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In addition to these three properties, there are other properties are also there like you know some times surface characteristics also become important depending on the applications sometime whether the particle smooth or hard that may also show a kind of impact and the performance of this particulate transfer processes, similarly whether the material particle is a porous or non-porous that is also going to have a kind of impact.

Likewise, hardness is also a kind of important material, sometimes you know you need particle for processing without undergoing any physical change, but if that material is not having the tough strength while the process that may break down and then physical change may occur. So that again you know cause a kind of a different a prior information compare to the reliable experimental results you may whatever the a prior information that you get that may not be reliable with the experimental results if the material is breaking and then something like that

while the process is taking place because all your characteristics you are taking with respect to some kind of one particular size Okay.

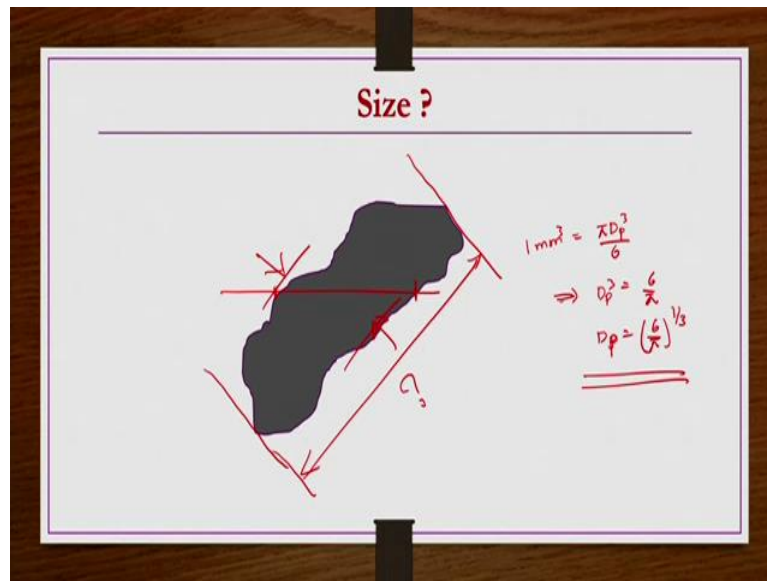
Then adsorption, adsorption characteristic of these materials especially some kind of fluids are involve along with this particulate phases than adsorption may also take place. So likewise if you keep on listing there may be several properties which may be important with respect to given operation or application but in general the size, shape and density are the three important properties that one must be careful about them.

So as I mentioned, why particle size is most essential or important because most of the characterization and control of particulate flows in general are connected to size. Let us say, in a packed bed what you have, you have a kind of packed bed where you let us say pack this column with certain kind of a packing material right. So now some fluid is coming in, some kind of process heat and mass transfer probably taking, let us take only heat transfer is taking place and then the fluid stream is coming out let us say here the temperature T_i outlet temperature T_o and then, this particulate whatever particles are there because they are at different temperature so then while this fluid is coming through this bed so some kind of this heat transfer is taking place.

So whatever the rate of heat transfer that you are going to report in general you know that you are going to report based on the size of this particle. Whatever the packing material that you have used size of that particle material that you are going to use in representing that rate of heat transfer for this example let us say. Likewise any fluid catalytic cracking reaction is taking, let us say petroleum industries what happens where the catalytic particles are there and then the crude petroleum is given inside the FCC unit, then some kind of reaction goes and then you take the different fractions of products as kerosene as one fraction and then benzene, toluene other kinds of thing or other fractions and all these kinds of things are happening in general.

But the performance, the output results you are going to report, in general you report based on the size of this catalyst the size of the particle right, so that is the reason size is a kind of a very much important this are the only two examples but any operation where these particulate solids are involved as I mentioned you know you report a performance based and the size of this particle. So that is the reason particle size is very much essential.

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Then size which size should be taken that is a big question in general when you handle particulate solids because when you take a big bulk material bulk solids, you have to crush them into the smaller sizes before taking them to the required unit operation or process unit that, those kind of things you have to do crushing size reduction in general you do it.

Let us say after size reduction, you have particles of this kind of size and arbitrary shapes but let us say one of the particle is having this particular shape, irregular shape drawn here like here so which size of the particle should you take? So we have a different dimension this is the maximum dimension right. So should we take this maximum dimension as the size of the particle or this the kind of a minimum dimension of the or minimum linear dimension of the particle, this is the maximum linear dimension of the particle, this is the minimum linear dimension of the particle, which one should you take or you bisect this material into two halves so then whatever the bisecting line is there so should you take this dimension which one you should take that is the big question, that is the big question very much important and big question one has to answer.

And then in general you do not have a single particle you, you are handling millions of particle in general in this particulate flows right, wherever the unit operations unit processes they are involved you handle them in millions right. So and then each of these particles will have a different size different shape. So how to represent what what size of the particle should you take? So coming to the number of the particles when you should take, when you take the entire material crushed material so you make a kind of fractions and then you say this particular fraction is having average diameter of this much, another fraction having the average diameter

of this much like that we can do that we are going to do in a subsequent lecture on screen analysis. Size reduction then we are doing the kind of screen analysis there we can see.

But here what we do with respect to single particle how we can represent how we can make a kind of a simpler characteristic representation of the particle size how can we do that one that is what we are going to see now here. So having seen the arbitrariness in the dimensions of a single particles then you can understand the complexities when millions of such particles are there and then different degrees of irregularities from one particle to other particle, different shapes of particles from one particle to other particle. It very much becomes essential to represent a kind of a closely related whichever is the best of it one that you have to take. There may be, now what you can do? You can do a kind of different ways of representing equivalent diameter of this particle let us say, the volume of this particle is let us say 1 mm^3 .

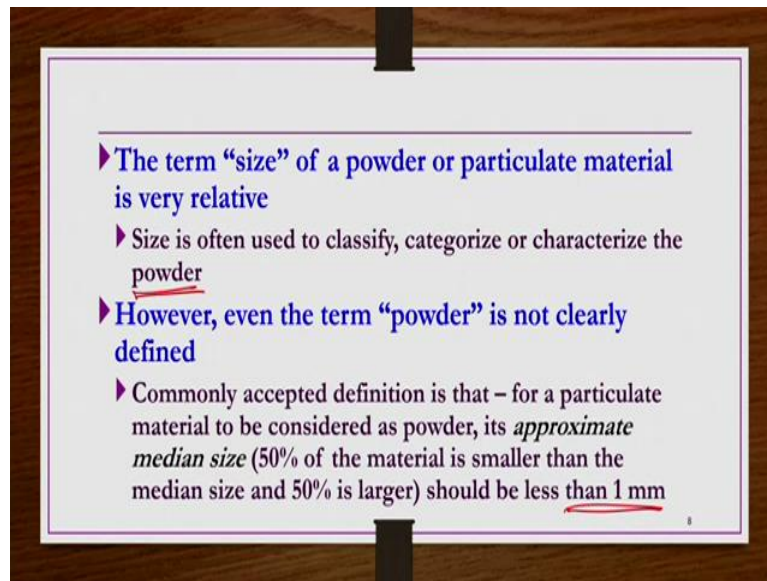
Now you represent equivalent sphere diameter that is the diameter of sphere of equal volume of volume equal to the volume of the particle, so if this is the volume of the particle you take a particle a spherical particle whose volume is equal to $\frac{\pi D^3}{6}$ so whose volume is equal to 1 mm^3 . Now what you do this is the particle its volume you measure let us say roughly 1 mm^3 . Now what you do, you wanted to represent in kind of a equivalent diameter so let us say you take a spherical particle of volume equal to the volume of this irregular particle so that $1 \text{ mm}^3 = \frac{\pi D_p^3}{6}$ so this is the volume of the spherical particle whose volume is same as the irregular particles.

So from here you get $D_p^3 = \frac{6}{\pi}$. So now the diameter of sphere of equal volume are equivalent diameter of this particle on the basis of the equal volume of sphere then you can say this particle is having equivalent diameter $D_p = \left(\frac{6}{\pi}\right)^{1/3}$ so this one representation you can do. Likewise there are N number of representation are there we have to see each and every representation and then we have to select which is best presentation for a given application and then not any kind of representation is going to be uniformly valid for all kind of application. It depends from one application to the other applications.

Let us say if we have a kind of packed and fluidized bed then volume to surface ratio whatever you is there that equivalent diameter if you take that is more reliable right. If you take kind of cyclone separators or conveying of particle add-in gases etcetera, in such cases stocks equivalent diameters are going to be very much reliable. So those equal and diameter etcetera

we are going to see in subsequent slides anyway, so the point here is that you know size how to represent size especially for irregular particles so that is we are going to see here.

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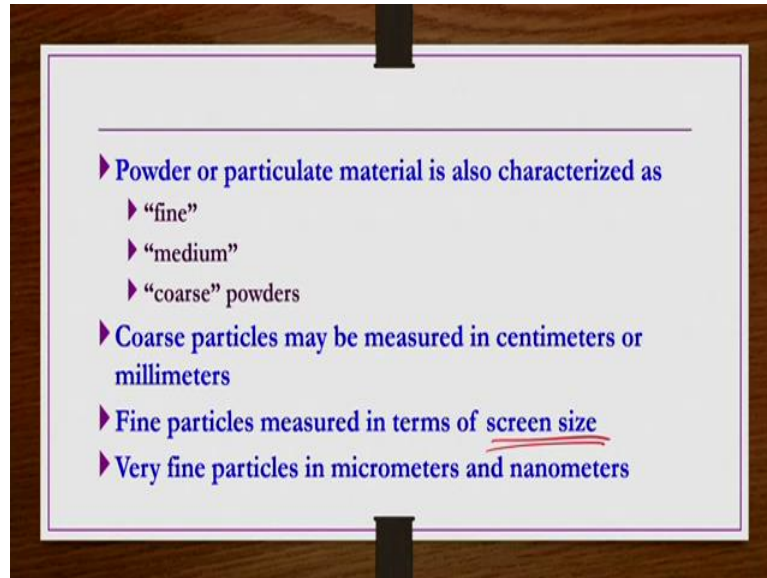
But before going this one, the term the size of a powder or a particulate material itself is very relative because I said when you make the fractions of the material one fraction may be having one size average size another fraction size may be having the another size like that different fractions of the particulate solids may of one single particulate solids you can have different sizes right. So that is, it is not possible to have a kind of exact representation or the exact representation of the powder particulate material because a different size as well as the different irregular shapes or the particle. So because of that reason we cannot have a one single value of size for a given powder or a particulate material, it is in general a kind of relative or you can say the average size of this so and so mm plus or minus this whatever the acceptable you know a deviation from that one, those kind of relative things only we give.

And then size is often as I mentioned used to classify, categorize or characterize the powder whether the powder is a cohesive or non-cohesive. In general, if the particle size of that particulate solid is very much smaller order of 10^{-3} microns, etcetera or 10^4 microns such small particle they are very cohesive in nature. So that way also particle size in general use to classify or categorize the powder right, but the powder the terminology powder itself is not properly defined.

How do you define whether a given particulate material is a powder or not? There are several definitions are there but the most often used definition is that or mostly acceptable definition is that for a particulate material to be considered as a powder its approximate median size should be less than 1 mm. Median size in the sense 50 % of the material is more than that size and then 50 % should be less than that size. If the approximate median size is less than 1 mm

then we can say that particular particulate material is a kind of powder, so this is one of the definition that is acceptable amongst the researchers working in this particulate area.

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Now powder or particulate material in general is also characterized as fine powders, medium powders, then coarse powders. These, these are a kind of relative characterization, in general coarse particles we measure in centimeters or millimeters and then fine particle in general measure in term of screen size that we are going to see in subsequent lectures anyway. Screen analysis lecture that we are going to discuss later on in detail. And then if the particles are very fine in general we report them as in terms of you know micrometers or nanometer something like that. Now size of typical powder products what we do, we take a few examples rather examples we take a few products which are in the powder form and then what is there in general size of those powder product that is what we are going to see here.

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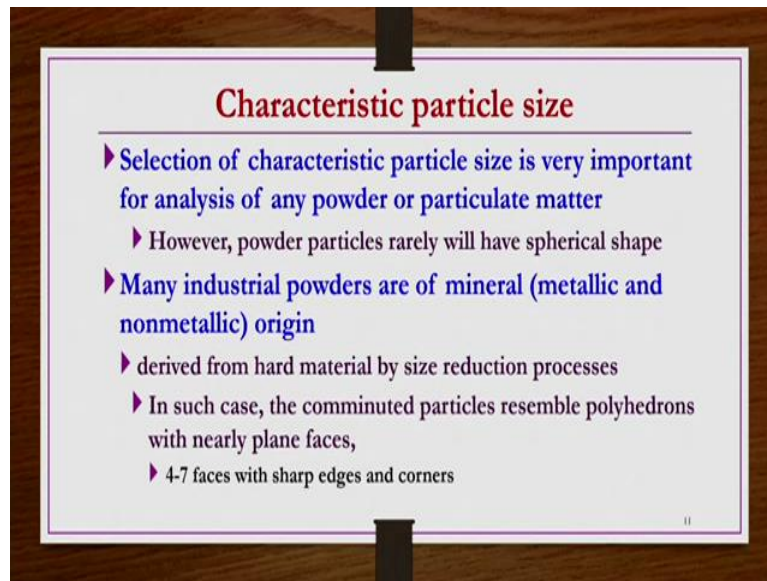
Size of typical powder products

Particle size (μm)	Typical powders	Particle size (μm)	Typical powders
10^5	Pelleted products, Crystalline industrial, chemicals	10^1	Toners, Powder metals, Ceramics
10^4	Granular fertilizers, herbicides, Fungicides Detergents	10^0	Electronic materials Photographic emulsions Magnetic and other pigments
10^3	Granulated sugars Spray dried products	10^{-1}	Organic pigments
10^2	Powdered chemicals Powdered sugar, Flour	10^{-2}	Fumed silica, Metal catalysts, Carbon blacks

What we have a let us say a pelleted products like crystalline industrial chemical etcetera they in general have the order of 10^5 micron, whereas the granular fertilizers, herbicides, fungicides in general they have order of 10^4 microns size, whereas the granulated sugars spray dried products in general have the order of 10^3 microns on the other hand, detergents may be having the order of 10^4 to 10^3 microns in general. Likewise powdered chemicals, powdered sugar, flour, etcetera have the order of 10^2 microns size in general. This is the average typical size of the material it is not of the one single particle.

So likewise other products, powder products like tonners, powder metals, ceramics etcetera, have the size 10^1 microns. Electronic materials, photographic emulsions, magnetic and other pigments in general have the 10^0 microns order of 10^0 microns, organic pigments in general have the 10^{-1} micron size whereas the metal catalyst, carbon black, etcetera, have the order of 10^{-1} to 10^{-2} microns. So these are the some of the powder products in general we see in industrial as well as the regular use so their respective sizes are given here just for kind of knowledge, to have information about the size of these powders.

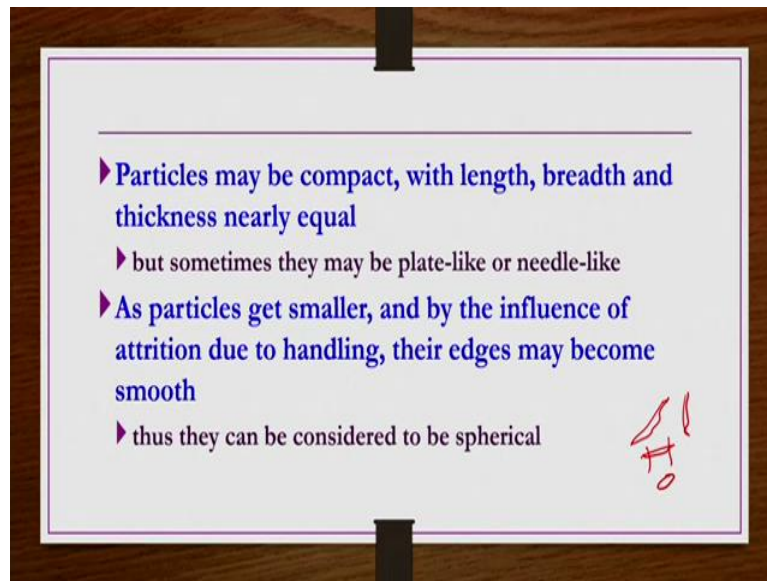
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So then characteristic particle size, we should have a kind of a characteristic particle size because it is going to be very much essential as we are going to use the characteristic size for analysis of any powder or particulate matter so wherever we are going to have the analysis control or characterization of the particulate flows there we are going to use this characteristic particle size because of that reason one must be very careful in defining this characteristic particle size because it is very much essential and important in designing any kind of particulate flow operators.

In general it assumed that the powder particle have kind of spherical shape, but in general that is not true, rarely powder particles will have a spherical shapes, whereas most of the industrial powders which are of mineral ores mineral origin they may be metallic or nonmetallic. What we do, we drive from hard material by some kind of size reduction processes like crushing grinding etcetera, so when you do this crushing, grinding, etcetera this minerals you get a small-small particles which are in general have a kind of shapes of polyhedrons of different faces 4 to 7 faces in general they have right. So in this polyhedrons may have 4 to 7 faces with sharp edges and corners, etcetera, so it is not possible to have a kind of spherical shape always, indeed spherical shape you will get rarely upon crushing mineral, crude raw material.

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Also in general particles may be compact with length, breadth and thickness nearly equal, but sometimes they may also be like plate-like or needle like particles also we get. Sometimes what happens particle gets smaller and by the influence of the attrition due to handling, while handling or taking from one container to the other containers what happens, the sharp edges whatever the sharp edges, corners, etcetera are there. In general let us say in transportation you have a something like plate-like material or needle like material something like this are there. So when they are transporting in, what happens this edges, edges may become smoother and then particles may under such condition you know one can say that they are having a kind of spherical shape but these things are occurring while in general handling them or while in the process of the transport that may be taking place right because of the smoothing etcetera that occurring because of the attrition while transporting.

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Representing size of irregular particles

• Equivalent sphere diameters: the diameter of a sphere which would have same property of the particle itself (ex. Same volume, same settling velocity, etc.)

S. No.	Name	Equivalent property of a sphere
1	Volume diameter	Volume
2	Surface diameter	Surface
3	Surface volume diameter	Surface-volume ratio
4	Drag diameter	Resistance to motion in the same fluid at the same velocity
5	Free-falling diameter	Free-falling speed in the same liquid at the same particle density
6	Stokes' diameter	Free-falling speed if Stokes' law is used
7	Sieve diameter	Passing through the same square aperture

Handwritten notes on the right side of the slide:
 - Next to Volume diameter: $\frac{4}{3}\pi r^3 = 1\text{mm}^3$
 $\Rightarrow r = \sqrt[3]{\frac{3}{4\pi}}$
 $\Rightarrow D_p = 2r$
 $\Rightarrow D_p = \sqrt[3]{\frac{6}{\pi}}$
 $\Rightarrow D_p \approx 1.108\text{mm}$
 - Next to Surface diameter: $4\pi r^2 = 1\text{mm}^2$
 $\Rightarrow r = \sqrt{\frac{1}{4\pi}}$
 $\Rightarrow D_p = 2r = \sqrt{\frac{1}{\pi}}$
 $\Rightarrow D_p \approx 0.564\text{mm}$
 - Next to Surface volume diameter: $\frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$
 $\Rightarrow r = \frac{3}{\text{ratio}}$
 $\Rightarrow D_p = \frac{6}{\text{ratio}}$
 - Next to Stokes' diameter: $\frac{6}{\mu} = \frac{D_p^2}{g}$
 $\Rightarrow D_p = \sqrt{\frac{6\mu}{g}}$
 $\Rightarrow D_p \approx 0.0001\text{m}$

So now representing size of irregular particles as I mentioned it is very much important as given one example, equivalent sphere volume diameter are the diameter of a sphere of equal volume that is what I have seen, like that there are you know different properties not only volume you can measure the surface of a particle and then what is the diameter of the particle which is having the equal surface as the particle like that one you can do. Another one is the volume surface to volume ratio you can measure this surface to the volume ratio of the particle and then what is the size of a spherical particle whose surface to volume ratio is same as a kind of a irregular particle from there you can find out the characteristics of equivalent diameter like that.

So like that there are several properties you can compare and then take the equivalent diameter right. Why we take spherical because spherical is easy to handle, in analysis also it becomes very easy to do a kind of calculation etcetera so that is the reason we take a kind of a spherical shape of the particle or equivalent sphere diameter in general we take. So equivalent sphere diameter also there are different properties several properties are there that you can compare. For example let us say equivalent property of a sphere, so that is volume of diameter that we call volume diameter or equivalent diameter of sphere where we compare the volume as I mentioned let us say one irregular particle of shape is there if its size is let us say 1mm^3 you find out a spherical particle of volume which is same as the volume of this irregular particle, so from there whatever D_p is there that is you know we call it as a diameter of sphere of equivalent volume D_p .

Like ways we can have a diameter of sphere of equal surface also, let us say you find out the you know surface of this particle as 1 mm^2 then you take πD^2 is equals to whatever 1 mm^2 then you can from πD_p^2 so then from here whatever the D_p you get so that is going to be the diameter of sphere of equal surface so that is this one. So you can compare any equivalent property of a sphere like that. You can compare volume you can compare the surface of the particle you can compare the surface to the volume ratio like that different possibilities are there right. If you compare you know surface to volume this thing then $\frac{\pi D_p^2}{\pi D_p^3 / 6}$ is equals to whatever the let us say surface to volume 1 mm inverse is there. So then from here whatever $\frac{6}{D_p}$ is 1, so from here D_p is whatever you get that may be taken as a kind of a surface volume ratio right.

So likewise you can have a surface volume ratio also. So basically in this equivalent representation equivalent sphere diameter what are we doing we are measuring either the volume or the surface or surface to volume ratio extra of a particle and then equating to a spherical particle whose volume is equal to the particle volume, or whose surface is equals to the particle surface like that we are doing in general Okay.

So equivalent sphere diameter, the diameter of a sphere which would have same property of the particle itself like you know same volume same settling velocity etcetera, we are going to take few example also how to calculate equivalent sphere diameter when we take the volume as a kind of property right. So this is how we can do, so other way is that surface volume diameter where we compare the surface volume ratio or the surface volume ratio of the particle you measure and then you find out a spherical particle of size certain size who surface to volume ratio is same as the irregular particle surface volume ratio.

Likewise drag diameter, in a flowing stream you insert this particle and then that particle may be offering some drag resistance force to the flow. So whatever the drag force is there that you measure and then at the same velocity you now insert spherical particles and then you measure the drag force. So different particles you have to insert and then see which spherical particle is offering same drag force as a irregular particles.

So corresponding size of that spherical particle should be taken as the equivalent drag diameter like that one has to do, then free-falling diameter is the one, so free-falling speed in the same liquid at the same particle density so that is you let us say you take a particle and you settle it

in the water so what is the free settling velocity of that irregular particle, that you measure then you take a different size of spherical particles, you drop them in the same conditions and then you measure the settling velocity of each spherical particles, different spherical particles you have to do with a kind of one after other one right.

So whichever the spherical particle having the same settling velocity as the settling velocity of the irregular particle then the size of the spherical particle would be taken as a kind of free-falling diameter equivalent free-falling diameter. If the free falling speed is under the stokes' law region small number Reynolds number region then that should be called as a stokes diameter. Likewise we have the sieve diameter, we take a kind of sieve where the square apertures are there, so whatever the passing through material is there we take that same aperture as the kind of same size of that particle so this anyway we are going to do in the size analysis screen analysis in the later course.

So this how one can represent the equivalent sphere diameter, equivalent sphere volume diameter this is the equivalent sphere surface diameter, this is equivalent sphere surface volume ratio diameter like that all other diameters right. So but which one is reliable that we cannot say we are going to discuss that anyway. It is going to be different from one application to the other application, we cannot generalize them.

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• Equivalent circle diameter: the diameter of a circle which would have the same property of the projected outline of the particle (ex. Projected area or perimeter)

S. No.	Name	Equivalent property of a circle
1	Projected area diameter	Projected area if the particle is resting in a stable position
2	Projected area diameter	Projected area if the particle is randomly oriented
3	Perimeter diameter	Perimeter of the outline

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Likewise equivalent circle diameter also defined, the diameter of a circle which would have the same property of the projected outline of the particle let us say projected area diameter if the projected area of the particle is resting in a stable position then, projected area diameter on

the other hand if the particle is randomly oriented then perimeter diameter that is the perimeter of the outline of the circular that we have taken so like that equivalent circle diameters are also possible. All these thing we are doing for single particle.

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• **Statistical diameter: obtained when a linear dimension is measured (usually by microscopy) parallel to a fixed direction**

S. No.	Name	Dimension measured
1	Feret's diameter ✓	Distance between two tangents on opposite sides of the particle
2	Martin's diameter ✓	Length of the line which bisects the image of the particle
3	Shear diameter ✓	Particle width obtained with an image shearing eyepiece
4	Maximum cord diameter ✓	Maximum length of a line limited by the contour of the particle

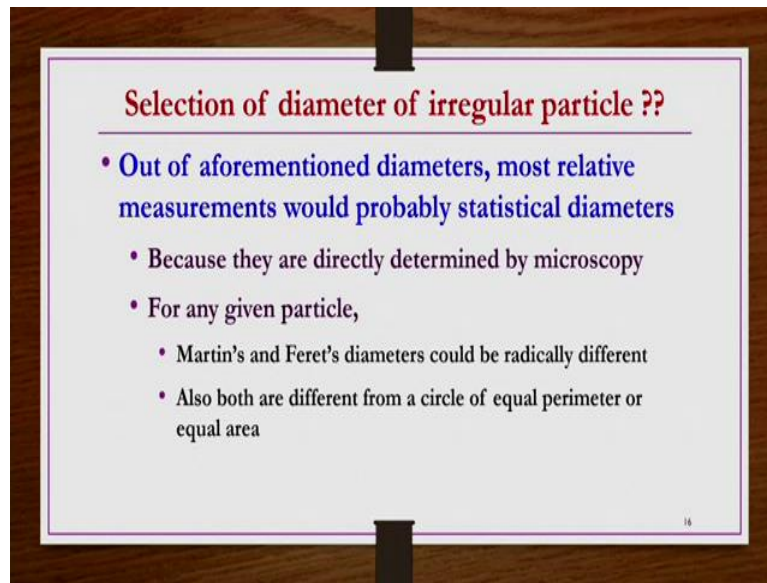
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So single particle the best one is that you know Statistical diameter because obtained these things are obtained when a linear dimension is measured usually by microscopic parallel to a fixed direction, orientation you fix and then for the fix direction you measure the linear dimension of that particle using the microscopy. So this is going to be very much reliable fine very good but this is valid only for single particle because in real life application you have a large number of particles, you cannot keep on measuring the size and shape of each and a every irregular particle using the microscopy.

In real life applications we are going to have huge number of particles so under those conditions it is not possible to find out the size of each and every particle though it is very much reliable, it is good as long as you know you are handling a single particle kind of thing. So under statistical diameter also there are different linear dimensions are possible because we are measuring parallel to your fix direction, so different linear dimensions are possible for a given irregular particle.

So one is the Feret's diameter that is the distance between two tangents on opposite sides of the particle, then Martin's diameter length of the line which bisects the image of the particle, then Shear diameter particle width obtained with an image shearing eye piece then Maximum cord diameter maximum length of the line limited by the contour of the particle.

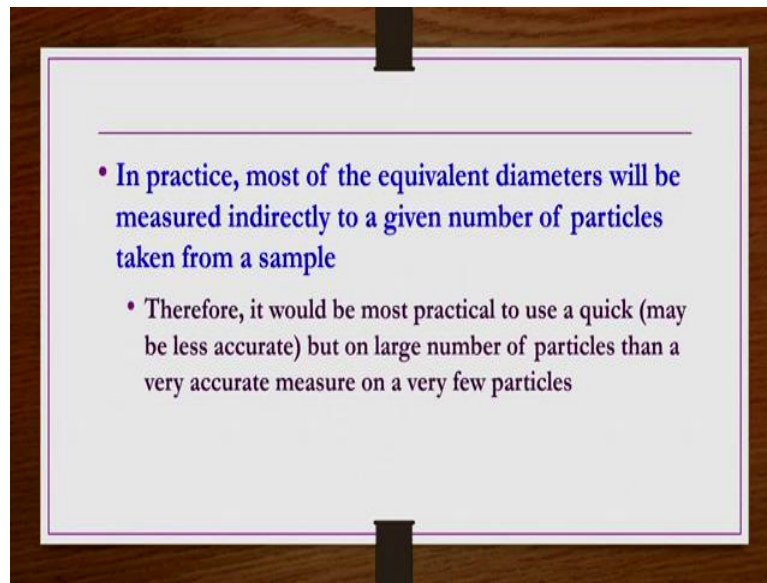
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Now, having seen these many equivalent representations how to make a selection of a diameter of a irregular particle which equivalent diameter should we use that is again big question we cannot generalize in anyway. Out of the a forementioned diameter, most relative measurements would probably statistical diameters because they are directly determined by the microscopy, this is good for single particles only few particles, but not laborious we cannot have do for huge number of particles are involved in the process.

However for any given particle Martin's and Feret's diameter whatever the statistical diameter that you find whether the Martin's diameter or a Feret's diameter they in general can be very much different from each other after all they are linear dimension of a given particles they changed from the fixed position, keeping one fixed position you get some values when you changed fixed position location direction you change then you may get the different dimensions because they are linear dimensions with parallel to fixed direction right. So Martin's and Feret's diameter could be radically different from each other that we are going to show pictorial anyway, also they are different from a circle of equal perimeter or equal area so that also we can see.

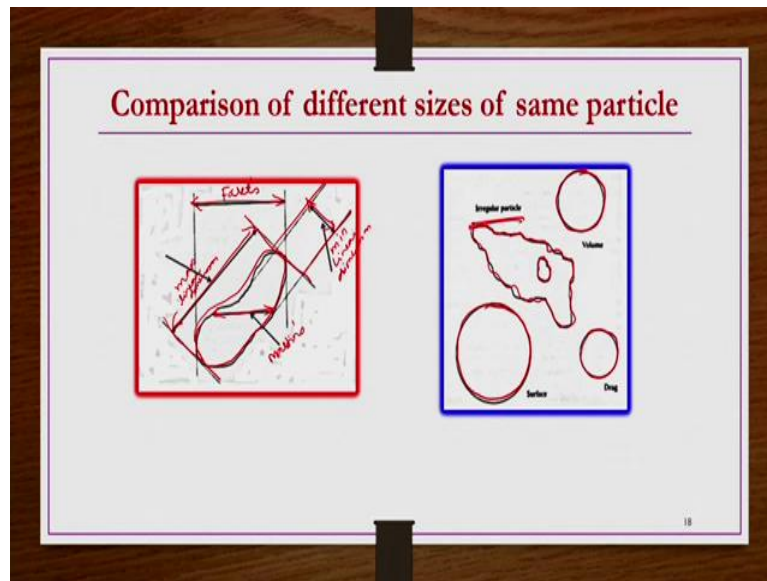
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But in practice most of the equivalent diameters will be measured indirectly to a given number of particles taken from a sample. So equivalent diameter will be measured indirectly actually we do not measure each and every particle diameter like that. So that we are going to see here this particular thing in a kind of screen analysis. Therefore it would be most practical to use a quick may be less accurate but quick on large number of particles then a very accurate measure on a very few particles, though the statistical measurements are very accurate it is not a good practice to depend on the statistical measurements especially when large numbers of particle are involved.

So when large numbers of particles are involved so it is obviously better that you may have a slightly less accurate but you wanted to have a kind of quick measurement those things that is what we are going to do in screen analysis.

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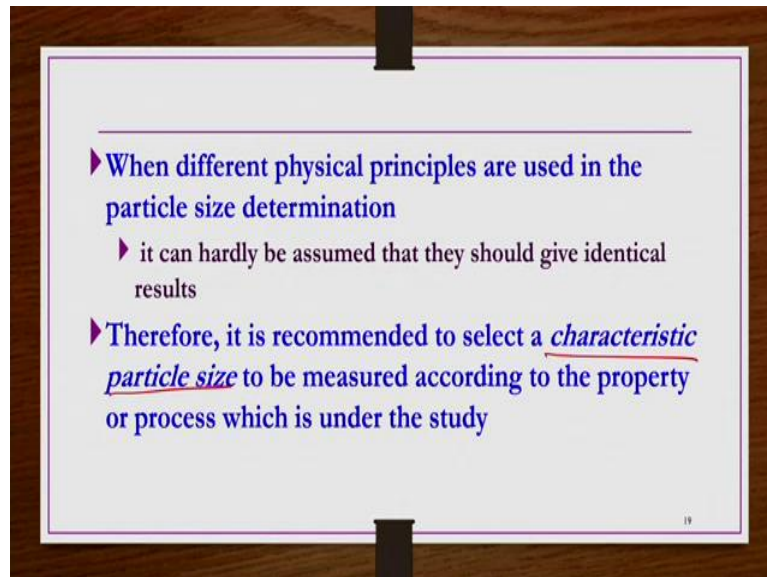
Now, why I said that you know making a selection is a very much difficult because here now this particle this is the particle shape outlining here again right, so here this is the maximum linear dimension and then this is the minimum linear dimension minimum linear dimension and then here this is maximum linear dimension, and this particular thing whatever this dimension is given this is nothing but Feret's dimension and then whatever this is one that is the bisecting the material this dimension is there that is Martin's dimension. So now you can see by statically whatever the sizes that you measure so now you can see the Feret's dimension is very much large compare to the Martin's dimension.

So, that is the reason it is a kind of very difficult to select which dimension should we taken. Or otherwise other example we take other kind of particle if your irregular particle is a having shape something like this after crushing the bulk material we get several particle one of the particles is having this kind of shape with a hole here like this. Now this irregular particle if you are going to define a diameter, diameter of a sphere of equal volume then you may get the size of this one. Then you define diameter of a sphere of equal surface then you get that is you measure the surface of this particle and then you find out a spherical particle whose surface is same as the surface of this particular then you get the bigger size like this.

Likewise, you find out the drag offered by this fluid in a flowing stream fluid stream at a given velocity so the drag in that flowing stream at the same velocity you take a spherical particles of different sizes and then find out which particle is offering the same drag as this irregular particle and then size of that spherical particle offering the same drag is like this. Now you can

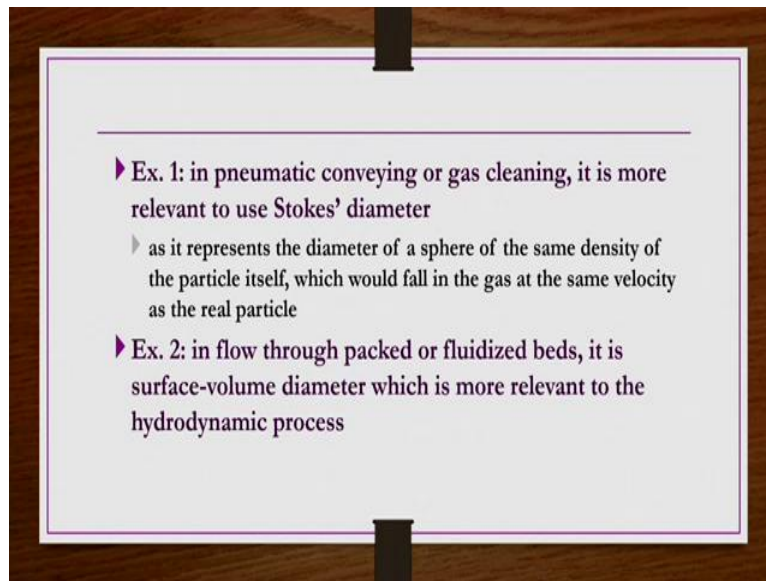
see, see all 3 equivalents three different size indeed different from each other not even close to each other. So this is just a comparison just to have a kind of difficulty of having which you know making a selection which equivalent size should be taken. So because of these things the selection should be based on the experience and the application of the process that is involved.

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So when different physical principles are used in the particle size determination that is equal volume, equal surface, equal drag, etcetera this different properties when you use principle you use to find the characteristic size then you may get different sizes as I just shown in a previous slide we get a different sizes we may not a get a kind of identical results anyway therefore, it is recommended to select a characteristic particle size according to the property or process which is under the study.

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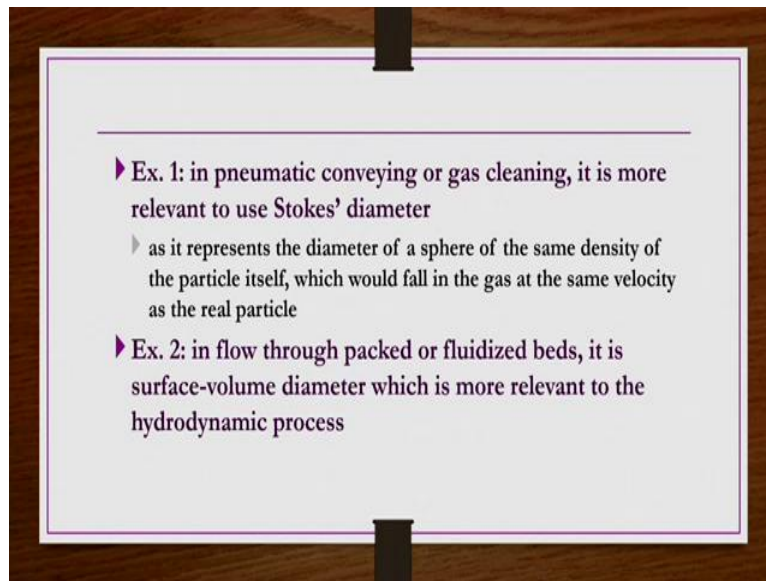


For example, if you have a pneumatic conveying or gas cleaning system, it is more relevant to use Stokes' diameter in general because the particle in this kind of a pneumatic system or gas cleaning system or particle sizes are very-very small, in general in microns right. So the settling velocity of those particles in general would be very small and then they fall under the Stokes' region. So that is the reason under those condition stokes diameter is very much reliable.

If you take another example in the flow through packed or fluidized beds, it is surface-volume diameter, which is more relevant to the hydrodynamic process. Indeed, we are going to derive this Kozeny–Carman equation for packed bed etcetera. So there we are going to use this surface volume equivalent diameter right, diameter of sphere which is having equal surface volume ratio as of the particle that is what we are going to use, in this such kind of packed and fluid as bed it is better to use surface-volume equivalent diameter.

So having listing so many characteristic sizes or the different ways of representing characteristic size of this particles you get a different results for a same particle as I shown one irregular particle the volume equivalent diameter, surface equivalent diameter, drag equivalent diameter of a sphere all three of them are having different sizes. So it is better to decide which characteristic size should be used based on the application as well as the experience of the person involved in that particular process.

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So we do some calculations sphere volume equivalent diameter calculations that is the diameter of sphere of equal volume as the particle that is what we are going to see. Now let us say if the particle shape is spherical then volume of the particle is $\pi D^3/6$, so equivalent or nominal diameter D_p we are going to find out, we are going to find out for different cases the equivalent or nominal diameter D_p which is the diameter of the sphere of equal volume.

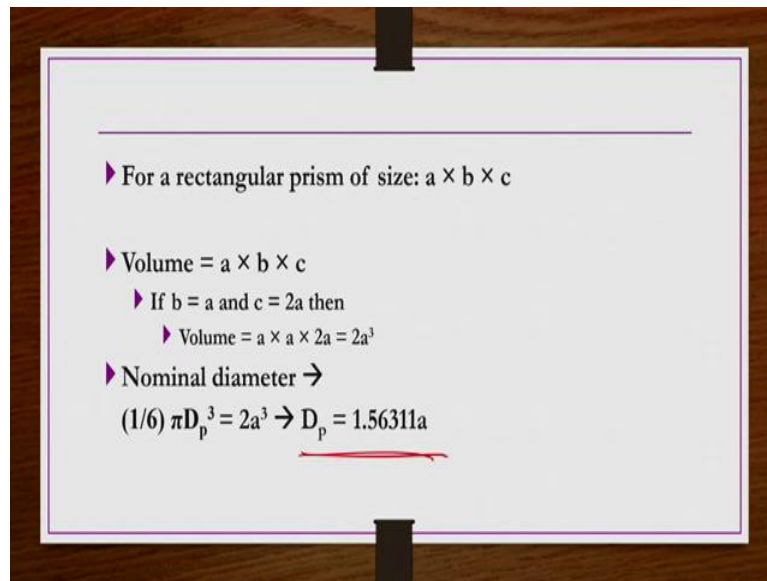
That is we are equating the volume of the particle to the volume of the spherical particle whose size is D_p and then that D_p is kind of equivalent diameter or characteristics size of the particle right. So now here $\pi D^3/6 = \pi D_p^3/6$ so that means D_p is equal to D . So since D reference material as well as the real particle both are in the spherical shape so then D_p should be equal to the D diameter of the particle of that sphere.

Let us say if you have a short cylinder where the diameter is equal to the length of the cylinder or height of the cylinder then we have the volume of the particle, that short cylindrical particle is $\pi r^2 L$ that we can write as because now here we are taking L is equals to D so $\pi D^3/4$. Now this $\pi D^3/4$ if you equate to the volume of the particle of a spherical particle of size D_p that is $\pi D_p^3/6$ then we can get D_p is equals to $(3/2)^{1/3} D$ that is short cylinders of L is equals to D where we take a short cylinders where the size is L is equals to the diameter of the particle then equivalent or nominal diameter D_p is nothing but cube root of $3/2$ times the diameter of the short cylinder that is what we get.

Likewise sort cubes if you take L is equals to D then D^3 should be the volume of the particle so D^3 when you equate it to the $\pi D_p^3/6$ then you get nominal diameter D_p is equals to 1.241

times D size of the cube. So likewise for hemispherical particle the volume of the particle is $\pi D^3/12$, if this $\pi D^3/12$ if you equate to $\pi D_p^3/6$ then D_p you will get it as 0.7937 D. Likewise, tetrahedron of size S the volume of the tetrahedron is in general $2^{0.5}$ divide by 12 S^3 and then that if you equate to the $\pi D_p^3/6$ then you will get D_p as 0.6083S. Other example like octahedron etcetera also we can do similarly.

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Similarly, if you have a rectangular prism of size $A \times B \times C$, then volume is $A \times B \times C$ and now if B is equals to A , and C is equals to 2 times A , then volume will become $2 A^3$. So we have to have a kind of spherical particle whose volume is same as $2 A^3$ so spherical particle volume is $\pi D_p^3/6$ if that is equals to $2 A^3$ whatever the D_p is there that is nominal diameter you will get $1.56311A$. A is the size of the one side of the size of the prism whatever we have taken.

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Equivalent diameter calculations of Raschig rings
 (L=D_o and D_i=0.5D_o)

▶ Volume of the ring = $\pi D_o^2 L/4 - \pi D_i^2 L/4 = \pi D_o^3 (0.25 - 0.0625) = 0.1875\pi D_o^3$

▶ Nominal diameter $\rightarrow (1/6) \pi D_p^3 = 0.1875\pi D_o^3$ 

$\rightarrow D_p = 1.0400D_o$

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Likewise, if you take Raschig rings as in general you know we have a kind of smaller cylindrical shape hollow shape like this hollow cylindrical shape something like this so whose diameter is having you know inner diameter D_i outer diameter D_o right and then length is in general L right. So if L is equals to D_o , and D_i is equal to half D_o then volume of whatever the Raschig ring is there that should be $\pi D_o^2 L/4 - \pi D_i^2 L/4$. Now you substitute L is equals to D_o and D_i is equals to $0.5 D_o$ and D_i is equals to $0.5 D_o$ you will get this one as $0.1875\pi D_o^3$. So nominal diameter $\pi D_p^3/6$ is equals to $0.1875\pi D_o^3$ where you get nominal size D_p as $1.0400 D_o$ that is nominal size of Raschig ring is approximately same as the outer diameter of the Raschig ring provided L is equal to D_o and D_i is $D_o/2$.

Likewise for given any individual particle you can measure the nominal size, these are the diameter of sphere of equal volume. Similarly you can also do diameter of sphere of equal surface to volume ratio like that you can do for any individual particles, but all these things we have done for single particle only.

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So the material provided here have been taken from these reference books McCabe, J.Smith, Ortega-Rivas, Richardson and Harker, Coulson and Richardson's chemical engineering series second volume, Geankoplis, Transport process and unit and Operations, Brown et al Unit operations, Introduction to chemical engineering Badger and Banchero, most of the data you can find out from these first two books whatever the information provided in this particular lecture most of the information is available in these first reference two books. Thank you.