Mechanical Unit Operations
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Lecture No. 20
Motion of Particles Through Fluids-3

Welcome to the MOOCs course Mechanical Unit Operations, the title of this lecture is Motion of Particles Through Fluids part three, we have been discussing several aspects about the settling of particles through fluids both spherical and non-spherical particles, non-spherical particles generalized expressions for the terminal velocities etcetera that we have derived and then left as it is both in gravity and centrifugal field, but under the case of spherical particles we further simplified those equation for different flow regimes, Stokes flow regimes and Newton's flow regimes and then we develop their settling velocity equations.

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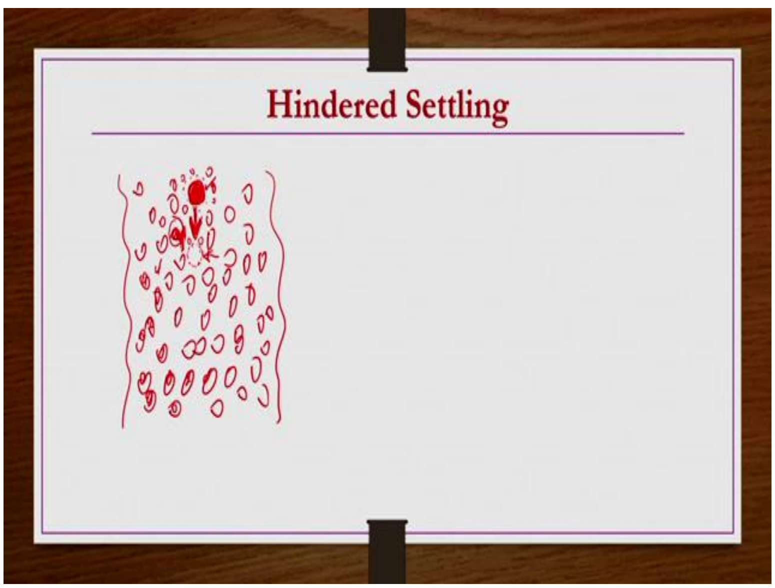

So, those things we have done for a single particle, single particle like settling in a kind of infinitely unbounded fluid. So, that is the particle is settling without any kind of hindrance or without any kind of disturbance from the neighboring particles that is in the neighborhood of the settling particle there is nothing other than the fluid medium and then container wall in which the liquid is taken, in which the particle is settling that container wall is also far away from the particle.

So, there is no influence of the container wall on the particles settling velocity because if the particle is settling in a narrow container, so, the particle velocity would also be affected by wall effects because of the wall, presence of the wall close to the particle the velocity gradients near the particle will be disturbed and accordingly the particle settling velocity will change. So, now whatever the things that we have discussed till now, in our last three lectures about the particle settling in fluid that is only for single particle settling in an unbounded infinite amount of fluid, the fluid is also kind of Newtonian fluid that is, that part only we have seen.

But, in general in reality what happens we do not have a kind of single particle system only, we have you know real life situations, large number of particles are settling or large number of particles are flowing in a kind of fluid stream or there may be a relative velocity between fluid stream and in this large number of particles, under those conditions you know, you know (how) what is the settling velocity of those number of particles, large number of particles or on what basis how those particles are being separated into the different fractions etcetera.

So, those things we cannot get that information from the single particle equation whatever we have developed so, that for that case, that we are, we are now we are trying to do you know, hindered settling we are going to discuss in this particular lecture about the hindered settling, where the large number of particles are there. So, large number of particles are settling, in a kind of fluid like this and then for the time being we assume that particles are of same shape and size. So, that developing the equation would be easier for us, let us say now, this in the particle, this is in a kind of fluid medium, we have taken a particle, let us say this is the target particle, we are trying to settle it down.

So, this is our target particle. So, now, when it is settling here, so, whatever the particles nearby are there they are affecting, they are affecting its settling velocity, because this particle, nearby particle where presence of nearby particles you know what happens the velocity gradients around these particles are at the surface of the particle, velocity gradients are strongly influenced by these particles, neighboring particles, that is the reason the settling velocity of this, this target particle in a kind of suspension is going to be very much different from the same particle settling alone in a kind of a free stream conditions or free settling conditions.

So, how much different it is, that is what we are going to see here in this lecture, and then further what happens let us say this particle has moved here. So, already there is a particle here, so, when it is coming, so, now the down coming particle may be pushing here, if assume there is a kind of there is no particle here in this region, so, when this particle is settling, coming here, now, I am just putting as a kind of dotted line, so, that to have a clarity, so, when this particle comes down and to this level, to this level, whatever the fluid is here, that is being replaced by the particle. So, that fluid what will do? That will try to move up, that will try to move up.

So, because of the you know buoyancy, so, that, that fluid raise, raise velocity of the fluid, that may be affecting the settling velocity of the neighboring particles because in the suspension, not the single, one single particle settling now, a large number of particle settling. So, so the whatever the upward motion of the fluid is there, because of the downward motion of this particle, so, this particle let us say, this target particle, first particle one when it is settling here, so, the fluid underneath here that is being replaced, that fluid what will try to do

That will try to move up because of the density differences when it is moving up so, there is a kind of upward motion of this particular liquid, so, that upward motion of the liquid is going to affect the settling velocity of the neighboring particles because neighboring particles are also settling, all these particles are settling at different rate of settling velocities, if they are different in size and shape and so so that is what it is going to happen.

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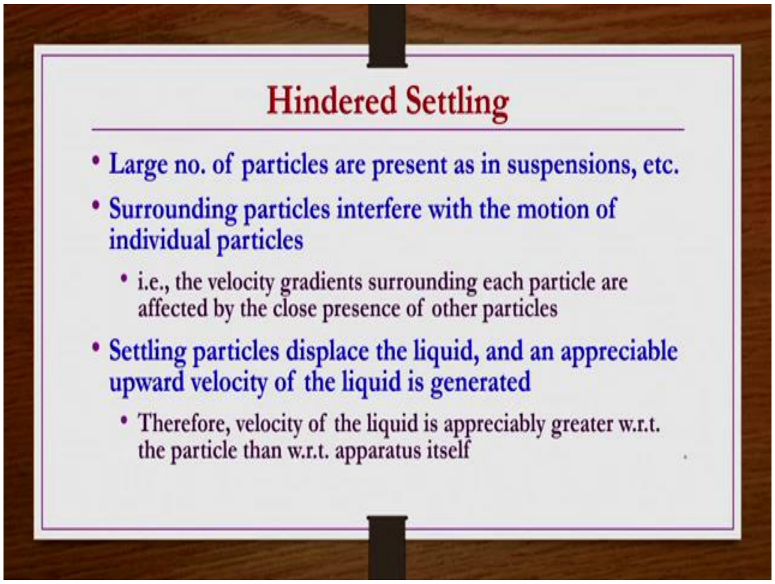

Now obviously, the circumstances whatever the settling velocity of the particle is there that is not going to be same as the settling velocity of the single particle in the same fluid. So now, what we understand? Large number of particles are present as in suspensions in general, in real life applications, it is not possible that to have a single particles as we have studied, so, surrounding particles interfere with the motion of individual particles as I explained, that is the velocity gradient surrounding each particles are affected by the close presence of other particles. So, that, that effect also we should take into consideration.

Settling particles displace the liquid and an appreciable upward velocity of the liquid is generated, so, that upward velocity of the liquid may be hindering the settling of other neighboring particles falling down, settling. So, therefore, velocity of the liquid is oppressively greater with respect to the particle than with respect to the apparatus itself, let us say if you take the relative velocity between the liquid and then container wall, so, obviously container wall they kind of 0 velocity because they are fixed kind of thing.

Now, if you take the relative velocity between the upward liquid motion and then container wall and you take the relative velocity between the upward motion of the liquid and particle then obviously the whatever the relative velocity between upward velocity of the liquid and in particle is there that will be much higher than the other case. So, for hindered flow settling velocity of particles is less than that of free settling velocity because when one single particle is settling there is no disturbance of the particle in the part, drag part.

Let us say, the particle is initially here, that is coming down here. so, it can smoothly fall down without any hindrance, but, in the case of hindered settling if there is a particle when it is coming down here, so there are particles nearby, so they may be objecting its flow so, additional resistance is there because of the neighboring particles, because of the hindrance of the neighboring particle.

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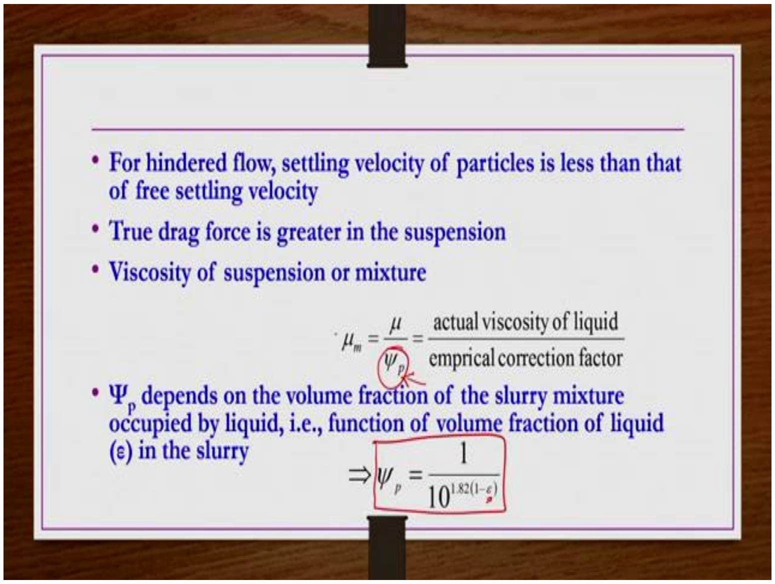

So, the settling velocity of suspensions in general is going to be much smaller compared to the free settling velocity of single particle of the same size and same density in the same fluid. So, because of this one additional resistance the drag expected or the drag experienced by the these particles, large number of particles in suspension is going to be much higher compared to the drag compared to the drag on a single particularly under free settling conditions obviously, now, here in the case of suspension, the resistance is more so, obviously, the drag would be more so, they would be expecting or experiencing more drag.

True drag force is greater in the suspension and then viscosity of suspension or mixture is also going to be different compared to the viscosity of a pure liquid here. So, because now the suspension in the suspension these particles are very very small, sometimes what happens when you when this suspensions are not settling at a kind of a appreciable velocity or they may be taking large time to settle indeed in general that they do.

So, then what happens you know the liquid becomes a kind of some other complicated, the complex liquid, complex fluid it will become, it will take a form of another complex fluid which is having a fine particles as a kind of you know suspension in that one. So, let us say initially you have taken water and then you added a kind of some amount of a particles, very fine particles and then it has become a kind of suspension, let us assume so, then that suspension is stable, it is not settling for a long time.

So, the fluid properties of that you know liquid initially it is liquid, so, it was initially having the density and viscosity of water at that temperature and pressure conditions, but now, after adding these particles, it has become a kind of suspension. So, and then these particles are not settling, they are taking long time and it is formed a kind of suspension. So, then obviously, the density and viscosity of that water is now no more be valid because its now altogether a new fluid, it has formed a new suspension in a kind of liquid form let us say. So, it is how going to have a kind of different density and viscosity.

So, those you know mixture viscosity and mixture density we have to find out so, there are several correlations or expressions are available for this you know mixture density and viscosity, we take a few which are you know, famously used in general in textbooks. So, viscosity of a suspension or a mixture is in general given by the viscosity of the actual liquid divided by some empirical correction factors Ψ_p.

So, let us say water you have taken so, water viscosity divided by there will be a kind of empirical correction factor, but this empirical correction factor it depends on the volume fraction of the liquid, now, because initially it is purely liquid only, but after adding the particles now, it is a kind of suspension. So, it is having two phases, though that phase is a kind of stable phase now, because the particles are not settling. So, then whatever the volume fraction of the liquid is there or whatever the volume fraction of the solid particles are there, fine particles are there that you can calculate.

People have find this empirical correction factor is a kind of strong function of the volume fraction of the liquid that has been taken. So, how it is a strong function? It is defined as $\Psi_p = \frac{1}{10^{1.821(1-e)}}$. So, here this epsilon is nothing but volume fraction of the liquid in the slurry. So, this suspension sometimes are kind of useful sometimes and sometimes they are kind of disadvantageous.

So, sometimes like you have the lotions etcetera or some shampoo etcetera so, those are kind of some kind of suspension, some particles, some droplets kind of things are you know suspended in a kind of different in a kind of other liquid and then that forming kind of suspension, under such conditions you do not want them to be separated for a long time at least for a year or two. So, that you know its expiry date is longer period, if all these particles are settling, so, then the shampoo would be of no use at all.

So, under such conditions it is going to be a kind of advantageous, sometimes you wanted to separate those particles so, that you can purify them in kind of you know some kind of mineral or industry etcetera. So, there is a kind of different particles are there so, along with the mud you try to wash with the, some kind of liquid let us say water and then mud is being washed out. So, but these mineral particles, if they are not settling in those washing condition then that is going to be disadvantageous. So, it depends on the application to application. So, now, we are taking you know irrespective of the applications whatever the viscosity of suspension is there that is defined like this.

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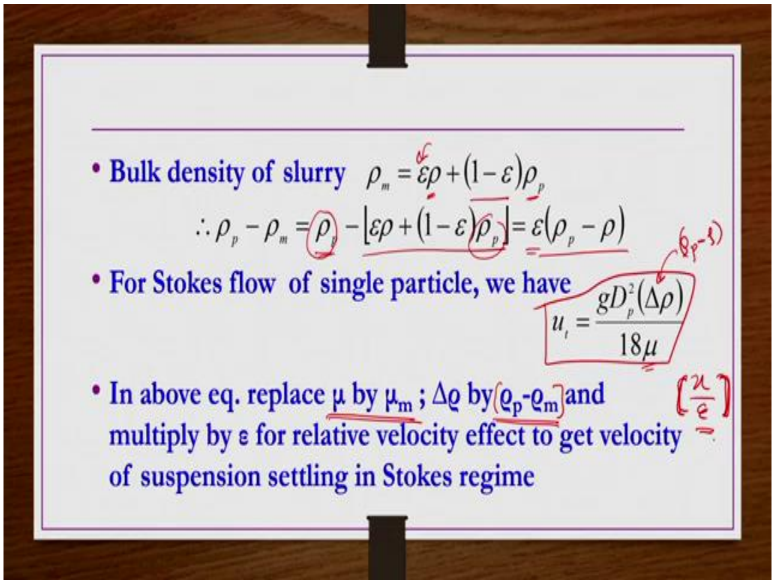

Similarly, the density of the slurry or the mixture, the suspension now, you can take as a kind of weighted average of the density of liquid and then particle, so, $\rho_m = \varepsilon\rho + (1-\varepsilon)\rho_p$, so, ρ is a kind of density of liquid and then ρ_p is a kind of a density of the particle, ε is a kind of volume fraction of the liquid phase and then $(1-\varepsilon)$ is the volume fraction of the solid particles in this suspension.

So, now, if you rearrange this equation or if you will do $(\rho_p - \rho_m) = \rho_p - [\varepsilon\rho + (1-\varepsilon)\rho_p]$ like this. So, here you know this ρ_p and then this ρ_p multiplied by 1 may be cancelled out, it will be because $1 + \rho_p$ another one is the minus ρ_p and then remaining two terms if you take the ε as a common so, you have epsilon $\rho_p - \rho$. Why are we doing this one? Because now, we are trying to develop a kind of a relation between settling velocity of suspension and free settling velocity of the single particle of the same material, same density and same size.

So, that is the reason we are going to be, we are doing all the simplification. Now, for Stokes flow of single particle we have this velocity, $u_t = \frac{gD_p^2(\Delta\rho)}{18\mu}$, in general the suspensions they take a long time to settle, the settling velocity is very very small in general for many of the suspension, so, usually the Reynolds number is going to be very small for suspensions.

So, Stokes regime can be safely used. So, however, there may be some cases where the suspension if they are not very stable, so, then the Reynolds number maybe you know more than Rep is equal to 1 but it is not going to be very large in general like you know, 1000 or you know 10^4 or 10^5, such large Reynolds numbers are not possible in the kind of suspensions in general.

So, now here if you wanted to develop settling velocity of suspension of the particles having size Dp and then in the fluid whatever the same fluid you have taken, so, then how, what should we be the settling velocity of the entire suspension? So, for that what we do, this $\Delta\rho$ that means, this Delta rho is nothing but $(\rho_p - \rho)$. So, this $(\rho_p - \rho)$ and then this mu how should we take? How should we consider it

So, that is what the question here so, for that what we do, we replace μ by μ_m that is mixture viscosity or viscosity of the suspension and then $\Delta\rho$ whatever the $\Delta\rho$ is there $(\rho_p - \rho)$ that we replaced by $(\rho_p - \rho_m)$ because mixture density, then now mixture is having a different density compared to the fluid alone ρ value.

So, whatever the particles they are settling, they are settling now, in a kind of mixtures or a kind of suspension so, that is the reason this $\rho\Delta\rho$ should be replaced by $(\rho_p - \rho_m)$ or $(\rho_p - \rho)$ in general for single particle whatever is there that should be replaced by $(\rho_p - \rho_m)$ and then this u has to be multiplied by and this right hand side expression should be multiplied by ε to the, by ε that is volume fraction of liquid phase or in general this average velocity of the in general in packed or packed without fluid is made etcetera what we have done the velocity should be divided by the ε.

So, in order to bring the effect of the narrow spaces available for the, for the fluid to move on. So, similar way, if you do here the velocity if you divide by ε or you right hand side if you multiplied by the ε so that to bring in the effect of the neighboring particles and then simplify then whatever the velocity you will get that would be the suspension velocity, that would be the suspension velocity or the hindered settling velocity of those particles.

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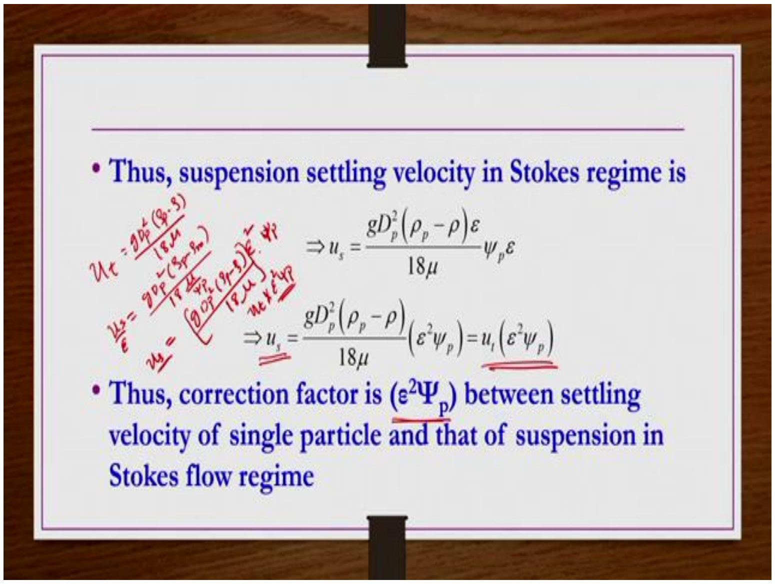

So, that we are going to do now here. So, when you do this one, you will get this expression actually, $u_s = gD_p^2\Delta\rho$ now, what we have done in this equation, this equation we rewrite here $u_t = \frac{gD_p^2(\rho_p-\rho)}{18\mu}$. So, here what we are going to do gD_p^2 we are keeping as it is $(\rho_p - \rho)$ we are replacing by $(\rho_p - \rho_m)$ and then 18μ is to be replaced by μ by Ψ_p. So, that and then this u has to be divided by the ε.

Then under such conditions whatever the velocity is there that would be the suspension velocity $u_s = gD_p^2(\rho_p - \rho_m)$ is nothing but $(\rho_p - \rho)$ multiplied by ε divided by 18μ this Ψ_p we can take numerator and then whatever the epsilon in the left hand side is there that if you take to the right hand side that would be ε^2. So, now, what we have this is nothing but $gD_p^2\Delta\rho$ that is $\frac{(\rho_p-\rho)}{18\mu}$that is nothing but u_t single particle terminal velocity this multiplied by $\varepsilon^2\Psi_p$ is going to be the suspension velocity and it says settling velocity of the same size particle in a kind of suspension. So, so, this is what we do so, then we get here.

So, now, there is a relation between the suspension velocity and then terminal velocity of single particle, why are we doing this? Because in general we have many large number of particles so, we cannot you know do a kind of individual calculation for individual size particles and all that, that becomes very difficult. So, what we try to do? We assume that the average size of the particles in the suspension is a kind of constant value, then for that constant average size of the particle Dp we calculate the single you know free settling velocity and then from there we try to multiply by the correction factor.

So, that so that to get a kind of settling velocity of suspension, it will allow us to calculate, recalculate a kind of things how much time these particles are going to take to settle down completely or to separate in two different phases, if they are completely settling then it will be separated into two phases like solids at the bottom and then liquid at the top that kind of thing.

So, that way you can, one can analyze the quality of the product that has been prepared in like if, if somebody is interested in making suspension then this us has to be very very small so, that the time for settling would be very large, very large. So, that the suspension will be stable for longer period. If someone is trying to clean the minerals etcetera then it should be as low, it should be high enough so, that it can separate into the 2 phases and then particles settled at the bottom can be taken and then taken to the next unit operations or unit processes as per the requirement.

So, that is the point of calculating this us so, that one can have a kind of estimate time how much time will it take to separate into the individual phases if it is a suspension,. So, the time has to be large, if it is a kind of separation of the particles, if you are targeting as per your applications, then it should not be very large time so, that it should be settled quickly and then separated out into two phases as per the requirements.

So, now, what we understand here, the correction factor is $\varepsilon^2\Psi_p$, if you know, the single particle free settling velocity and then if you multiply it by $\varepsilon^2\Psi_p$, then you will be having a kind of estimate or you know, a priori information about the suspension velocity.

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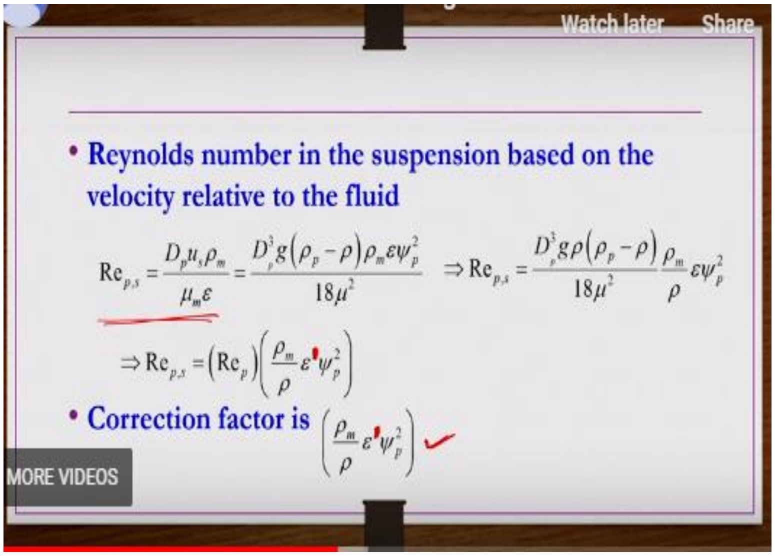

Now, similarly, Reynolds number in the suspension we have this Reynolds number if you substitute here us whatever the expression that we got us is nothing but u_t multiplied by $\varepsilon^2\Psi_p$ and then u_m also you substitute accordingly ρ_m you substitute accordingly then simplify, you will get this expression, further if you simplify and then rearrange so, you will get so, $Re_{p,s} = (Re_p)\left(\frac{\rho m}{\rho}\varepsilon\Psi_p^2\right)$, there is no square of this one. So, $Re_{p,s}$ would be $(Re_p)\left(\frac{\rho m}{\rho}\varepsilon\Psi_p^2\right)$ you will get here. So, that means the correction factor is $\left(\frac{\rho m}{\rho}\varepsilon\Psi_p^2\right)$ for the Reynolds number.

So, that is if you get the Reynolds number of a single particle that is $Re_p = \frac{D_p u_t \rho}{\mu}$ you get it, then if you multiply that 1 by $\left(\frac{\rho m}{\rho}\varepsilon\Psi_p^2\right)$ you will get the Reynolds number of the suspension. So, which is going to be much smaller compared to the Reynolds number of the single particle.

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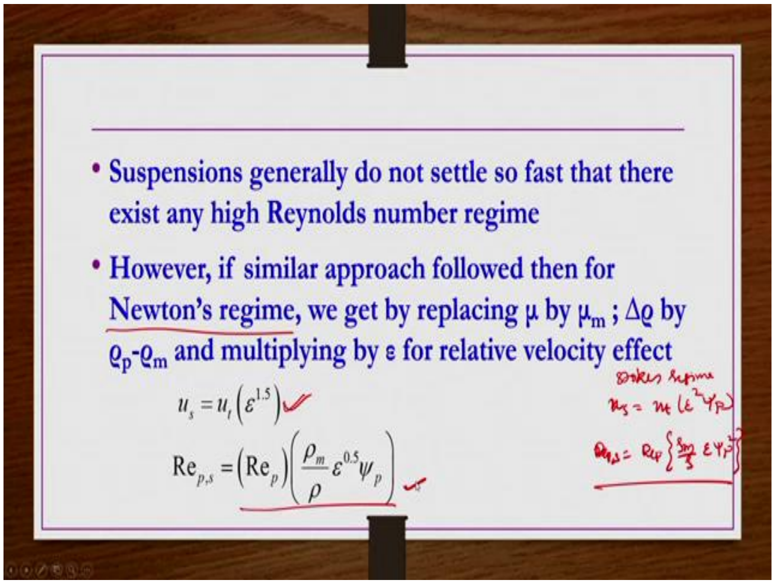

So, this is how the relations we have to develop. So, we see now, what are the relations available between these two, other than these things is there any other method available or not. So, before going to the other methods, what we do suspension generally do not settle for so, fast that there exists any high Reynolds number regime. So, similar way we can do for the Newton's flow regime also.

So, but in general as I mentioned in our Reynolds number is going to be small in general for suspension less than 0.1 or 1 in general, but there may be cases sometimes Reynolds number may be 4, 5 or maybe up to 10 maximum something like that, but in general it is not going to be as high as 10^3, 10^4 or 10^5 so, that to consider the Newton's flow regime, but however academically if you try to do similar approach whatever we have done by replacing μ by $\mu_m \frac{\Delta\rho}{\rho_p - \rho_m}$ and then multiplying ε right hand side for relative velocity effect, then you get u_s as equals to u_t multiplied by $\varepsilon^{1.5}$.

Earlier we got for Stokes regimes what we got? For Stokes regime you got $u_s = u_t(\varepsilon^2\Psi_p)$ but in the Newton's regimes we get $u_t(\varepsilon^{1.5})$ only similarly, Reynolds number also here you know what we got in the kind of Stokes regime $Re_{p,s} = (Re_p)\left(\frac{\rho_m}{\rho}\varepsilon\Psi_p^2\right)$ we got so, but here $(Re_p)\left(\frac{\rho_m}{\rho}\varepsilon^{0.5}\Psi_p\right)$ this is what we get for the Newton's flow regime.

There are other corrections are also available in terms of the particle Reynolds number they are having in this particular form us is equal to $u_t(\varepsilon^n)$ so, whatever the previous relation that were developed, we assuming that suspension is a kind of a settling only in the Newton's flow regimes and then we have developed the relation between u_s and then u_t similarly, $Re_{p,s}$ and an Re_p so, that we can get like if you have a kind of settling velocity of single particle you can know the settling velocity of the suspension itself, but sometimes it may also possible that the suspension may be settling slightly at higher velocity, slightly greater than Re_p 1 then or maybe up to Re_p is equal to 10 or 20 not higher than that one.

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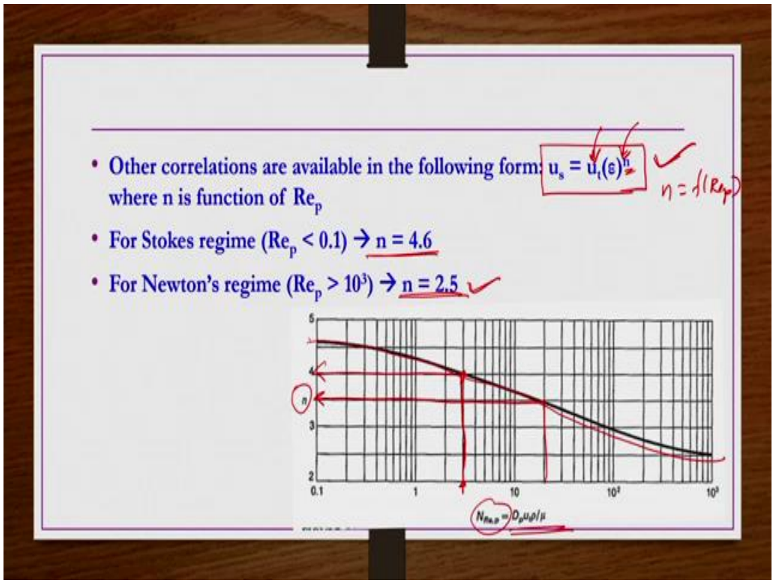

So, under such conditions obviously Stokes approximation is going to be deviating from the real situations for that reason, you know, some researchers have developed this kind of expression that $u_s = u_t(\varepsilon^n)$ and then ε is the volume fraction of the liquid. So, this n is a function of Re_p and is now function of Re_p that is what we have to find out here. So, that means in order to know this expression in order to know you know, you need to know the single particle settling velocity and then you will need to know n also, there is a kind of additional factor, epsilon for a given system is anyway known, what is the liquid fraction one can easily know.

So, what is this n as function of Re_p that we have to find out. For Stokes regimes it is in general 4.6 they are based on the experimental observation by some researchers, there is no derivation kind of this thing for this and then for Newton's regimes though it is a very rare, but let us assume like sometimes separation of the phases is also required like mineral washing as I mentioned sometimes you do not want separation of the phases, you want stable suspension like shampoos or etcetera, paste etcetera. So, for them n is going to be 4.6 that is Stokes regime, if the separation of the phases is required then that is going to be under the Newton's regime in general. So, for that case n is equal to 2.5.

So, for the intermediate values they have given n versus Re_p expression, graphical information is given, let us say if Re is 3, Re is 3 from here what we get? We get n is equals to 4, so like that if Re_p is 11 something like this, so, then n is going to be 3.5 so, like this, this way the graphically it is given like this information like this. So, now, we take couple of example problem to use these expressions and then get the required information of settling velocity of the suspension.

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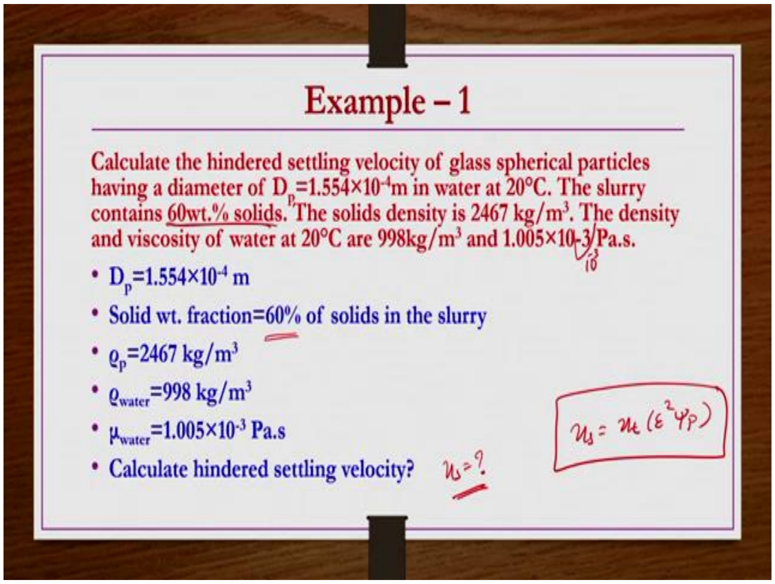

Example 1, calculate the hinder settling velocity of glass spherical particles having a diameter D_p is equals to 1.554 into 10^{-4} meter in water at 20 degrees centigrade, the slurry contains 60 weight percents solids. Solid weight percentage is given but we need ε that is liquid volume fraction, this is weight percent and that also for the solids. So, ε we have to find out, the solid density is 2467 kg/m^3, the density and viscosity of water at 20 degrees centigrade are 998 kg/m^3 and 1.005 to 10^{-3} Pascal second, 10^{-3} Pascal second here.

So, then we need to calculate the suspension velocity or hindered settling velocity of this particles having constant same size D_p is equals to 1.554 into 10^{-4} meters. Solids weight fraction is 60 percent of the solids in the slurry, ρ_p is given, ρ_{water} or ρ is also given, μ is also given we need to find out u_s, what is u_s? We will do both the approaches, first approach, what we have developed $u_s = u_t(\varepsilon^2 \Psi_p)$ this is assuming that particles are settling in a kind of Stokes regime. So, that we do then graphically we do, for the graphical approach we have to first find out the Re_p then accordingly n we have to find out.

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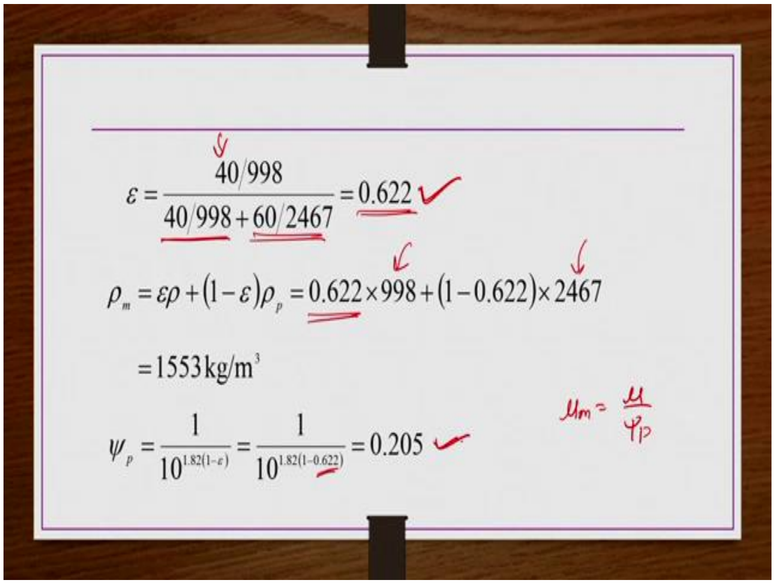

So, we see now so, ε is now, this is 40% liquid divided by the density of the liquid, divided by the 40% of liquid and then 60% of solids, if you divide respective, respectively with their densities you will get volume so, volume of the liquid divided by the total volume is going to be volume fraction of the liquid 0.622. So, 40% is liquid is there, 60% is solid are there so, 40 $/\rho$ and then $60/\rho_p$ if you do you get the volumes of liquid and then particles respectively. So, once you know those volumes, so then you can take the volume fractions.

So, that is volume of a liquid divided by the total volume that is volume of particle plus volume of liquid, you will get the volume fraction of liquid phase 0.622. Then $\rho_m = \varepsilon\rho + (1-\varepsilon)\rho_p$ that is coming out to be 1553 kg/m^3. So, $\varepsilon\rho$ we calculated this one just now, this ρ is given and then ρ_p is also given so, everything is known, so if you substitute you will get this mixture density. Similarly mixture viscosity you need to find out. So, mixture viscosity $\mu_m = \frac{\mu}{\Psi_p}$ so, the $\Psi_p = \frac{1}{10^{1.82(1-\varepsilon)}}$, epsilon is 0.622. So, Ψ_p is going to be 0.205. So, then u_s this we have got $u_t(\varepsilon^2\Psi_p)$ according to the first method.

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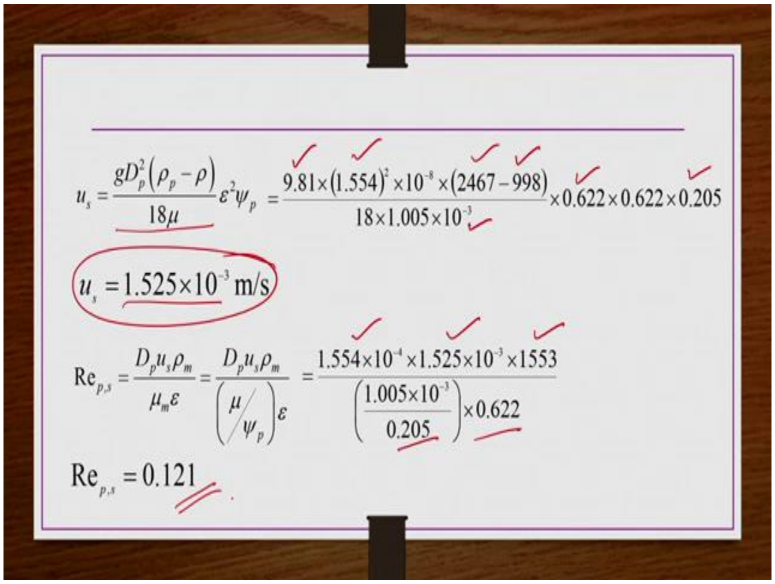

So, in this equation, we how to substitute all this information g is known Dp is given ρ_p, ρ is given, ε^2 you calculated, Ψ_p also you calculated, μ is also given here. So, if you substitute all of these values, you will get us is equal to 1.525x10^{-3} meter/second, such a small value. Though by numbers it appears like it is a kind of Stokes regime because Dp is also very small, order of 10^{-4} meters and then us is also very small, order of 10^{-3} meter/second and then viscosity is also a order of very small 10^{-3}. So, by numbers if you do $\frac{D_p u_t \rho}{\mu}$ it seems it is like a kind of Stokes regime so, it is acceptable one.

So, $Re_{p,s}$also you can similarly find out $\frac{D_p u_s \rho_m}{\mu_m \varepsilon}$. So, $\frac{D_p u_s \rho_m}{\left(\frac{\mu}{\Psi_p}\right)\varepsilon}$ as it is so, then here also now everything is known. So, Dp is given, us just you calculated, rho m also you calculated, Ψ_p calculated as 0.205, ε you calculated as 0.622 so, when you substitute all these things, you will get $Re_{p,s}$ is equals to 0.121 that is kind of Stokes regime.

Obviously, it will come Stokes regimes because whatever the u that have, that you have calculated, you calculate it assuming the Stokes regime. So, whatever the error if at all is there so, that will be incurring here in the Reynolds number also. So, if the, your velocity is whatever the Stokes assumption is true velocity is also under the Stokes regime, so, then this is also going to be true value.

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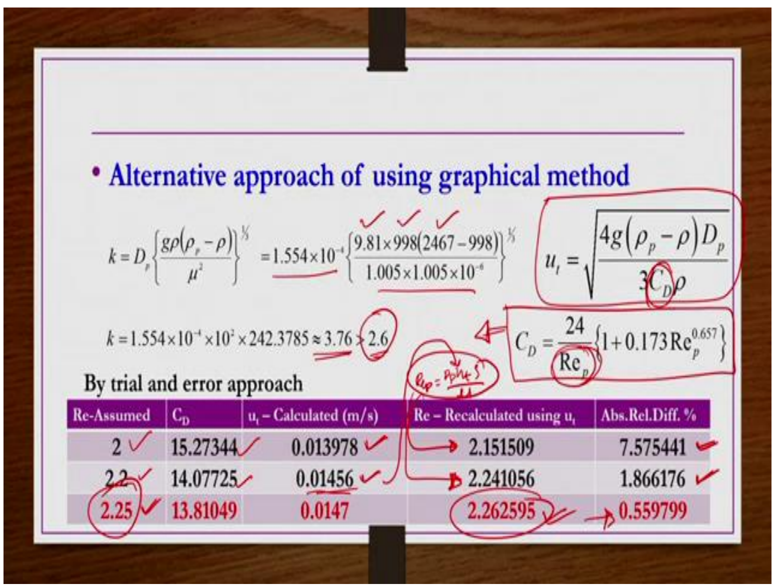

So, that you can get the true picture of the Reynolds number from the other alternative graphical approach. So, alternative graphical approach, what we are having you know, we have to find out the n value, n function of Re_p, but Re_p you can calculate only if you know u_t, Dp you know, $u_t \frac{\mu}{\rho}$ this ρ is also known, μ is also known, Dp is also known but u_t you do not know. So, since u_t you do not know you cannot calculate Re.

So, if you cannot calculate Re you cannot know $u_s = u_t \varepsilon^n$ this equation you cannot use. So, what is the alternative that we have? We have a kind of a previous lecture finding out the k factor, or the k expression that we derived, that is, $D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2}\right]^{1/3}$ if you calculate it, and then if it comes out less than 2.6 then it is a Stokes regimes.

So, then previous equations whatever we have used, they are true ones and but here n should be 4.25 for a Stokes regimes, so, that way we have to do, if it is not under Stokes regime, then we have to do a kind of trial and error kind of thing as we have done previously. So, when you substitute all this value D_p is given, g is known, ρ is given, ρ_p is also given, μ also given everything is known here.

Actually in the problem statement, the physical properties, density and viscosity of the fluid as well as the size and density of the particle are given. So, this can be known, when you calculate it, it comes out to be 3.76 slightly greater than 2.6. So, it is obviously not going to settle in a kind of a Stokes regime, if it has to settle in, suspension has to settle in the Stokes regime or the individual particle has to settle in the Stokes regime, the k value should be less than 2.6 or Re_p should be less than 1.

If the single particle is settling in Stokes regime, the suspension obviously will settle in the Stokes regime because suspension velocities in general much less compared to the single particle free settling velocity. Why because? Because of the hindrance of the neighboring particles or because of the upward liquid motion etcetera. Because of those reasons, the suspension velocity is always going to be very very smaller or hindered settling velocity of particles is going to be very small compared to the single particle free settling.

So, if single particle is not settling in Stokes regime, if single particle is settling in Stokes regime, so, then suspension is also going to settle in a Stokes regime. So, but here it is a kind of different one. So, now, since k is greater than 2.6, we have to find out the Rep before using, that Re_p versus n graph. So, it is again trial and error approach what we have to do? You have to assume some value of Rep, then calculate the C_D value using this correlation.

We can also, we can also use the C_D versus Re_p graph as we have done in the previous lecture, but now readily correlation is available you do not need to refer to the graph, it is indeed this C_D value by this correlation and C_D value from the graph would be almost close to each other, this correlation is known as the Schiller-Naumann correlation for the drag coefficient of single spherical particle settling in Newtonian fluid. So, that is experimentally matching very much well with experimental values.

So, but now trial and error approach, you do you assume one Re_p value from this correlation, you calculate C_D. So, Re_p 2 if you substitute here C_D is going to be 15.27 something and then corresponding u velocity you have to find out, corresponding u velocity so, if you assume Re is equal to 2, C_D is going to be 5.27 by substituting Re_p is equals to 2 here in this correlation, then that C_D value you can use in this u_t expression that we derived in the previous lecture so, then you will get calculated u_t value is 0.013978.

Now, using this u_t value you recalculate $Re_p = \frac{D_p u_t \rho}{\mu}$, so, you will get this Re calculated is 2.1515 and now you have to see the difference is acceptable for you or not, though they are looking close to each other. So, absolute relative difference is almost 7.5 percent between these two. So, assumed Re_p is good enough, but it is not very close to the calculated value of Re_p is not very close to the assumed value. So, let us make another assumption 2.2 then when you substitute 2.2 here in this C_D correlation, you will get C_D 14.077.

So, using the C_D 14.077 and then substituting here in this u_t expression you calculate u_t so, that is coming out to be 0.01456 meter/second, then this using this calculated u_t by substituting this u_t here in this $Re_p = \frac{D_p u_t \rho}{\mu}$ then you will get the recalculated value as 2.24 and then absolute relative difference is 1.866. So, good enough, but further if you try 2.25 similar way you can get the assumed value of you can, if you assume Re 2.25 then calculated Re is coming to be 2.26 in a similar way as other two steps we have done and then here the absolute relative difference is very much less 0.56.

So, now we can say that 2.25 is a kind of a correct Reynolds number so, the previous approach, we assume that Re_p is less than 1 and then we have used the Stokes regime settling velocity equation and develop the relations, but in reality it is not close to it is close to Stokes regimes but not falling in the Stokes regimes. So, there is some error, how much that error is there, that you can get new value of the settling velocity for the suspension or hindered settling velocity here and then compare with the previous approach.

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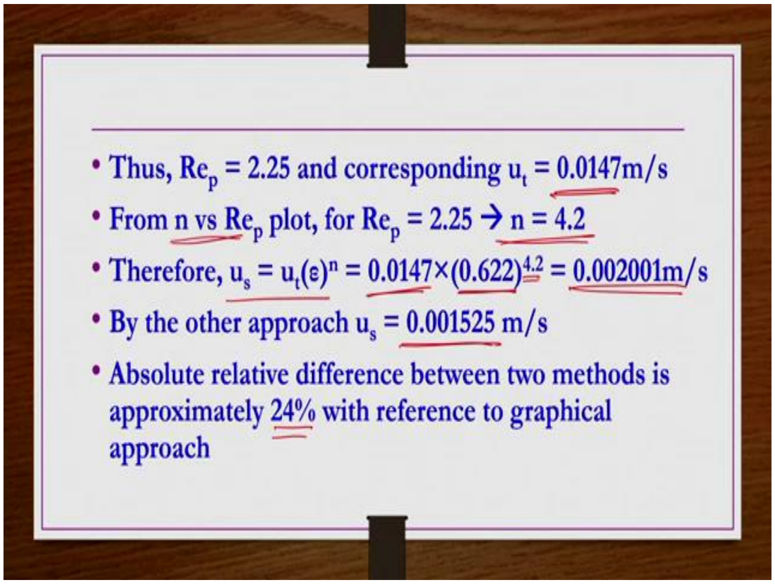

So, from this table what we get Re_p is equal to 2.25 and then u_t is going to be 0.0147 meter per second. So, now Re_p is known, so you can calculate n value. So once n value is also known from the graph so, you can try and find out, you can find out us, because u_t, ε all three are known, so from n versus Rep plot that just I have shown for Rep is equal to 2.25, n is going to be 4.2 something like that. So, then $u_s = u_t\varepsilon^n$, u_t you calculated as 0.0147 corresponding to Re_p is equal to 2.25 for single particle, ε you calculated as 0.622 and then n from the graph you are getting 4.2.

So, you can see here 0.002 meter per second is a kind of suspension settling velocity, it is slightly larger than the previous approach because previous approach we assume Stokes regime. But in reality using this Reynolds number calculation what we understand that single particle is not settling in the Stokes regime, though it is close to the Stokes regime, but it is beyond the Stokes regime, it is Re_p is equal to 2.25 so, Re if it is in increasing keeping density viscosity and size of the particle same.

So, on this velocity is obviously going to be more so, that is the reason here we get slightly larger value of the velocity and then seems to be this is more reliable than the previous one because the previous one is based on the Stokes assumptions. Whereas, in reality for this problem conditions the particles are not settling under the Stokes condition, it is slightly beyond the Stokes condition. So, the settling velocity is also slightly more than the Stokes velocity conditions. So, that is by other approach we got 0.0015 meter per second. So, if you find out absolute relative difference between two methods, it is approximately 24% with reference to the graphical approach.

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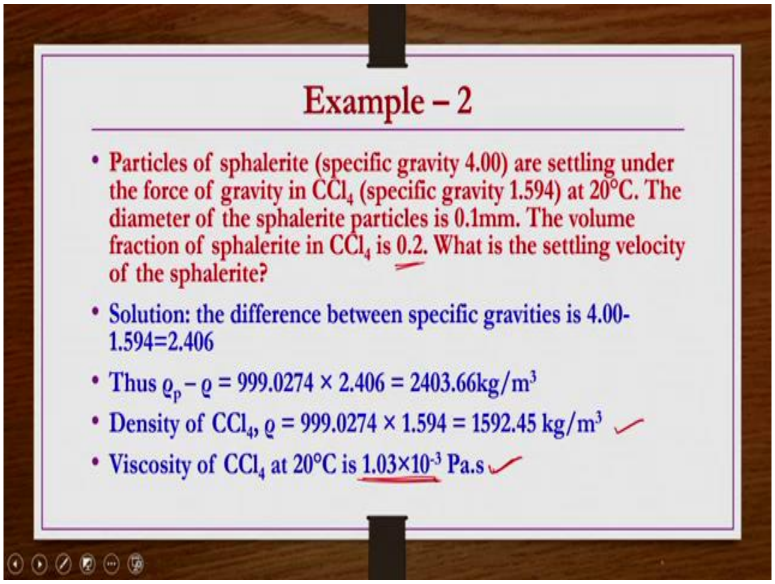

So, we take another example now, particles of sphalerite specific gravity 4 are settling under the force of gravity and CCl_4, specific gravity of CCl_4 is 1.594 and then particle specific gravity is 4 rather giving the density now, specific gravities are given so, you can now find out the densities, the diameter of the particles is 0.1 mm, the volume fraction of sphalerite is 0.2, the solids volume fraction is given that is 0.2. So, it is not weight fraction, it is volume fraction is given directly. So, if the solids weight fraction is 0.2 so, liquid volume fraction is going to be 0.8 so, ε is given 0.8 indirectly.

So, what is the settling velocity of the sphalerite, in the suspension? So, now, we have to follow the similar approach whatever we have done. So, the difference between specific gravities is 4 - 1.594 = 2.406, thus $(\rho_p - \rho)$ would be, let us say if I take 999 or 1000 also you can take and then multiply by the specific gravity. So, you will get the $(\rho_p - \rho)$ value as 2403.66 kg/ m^3.

Similarly, density of CCl_4 that is 999.2074 x 1.594 so, ρ is coming out to be 1592.45 kg/ m^3 similarly, density of the particles also we can find out and then viscosity of CCl_4 is not given, but from the standard data books reference physical and chemical properties of many of the chemicals are in general available in standard Perry's chemical engineering handbook, we can find it out it as a kind of 1.03 x10^{-3} Pa.s is the viscosity of CCl_4 at 20 degrees centigrade.

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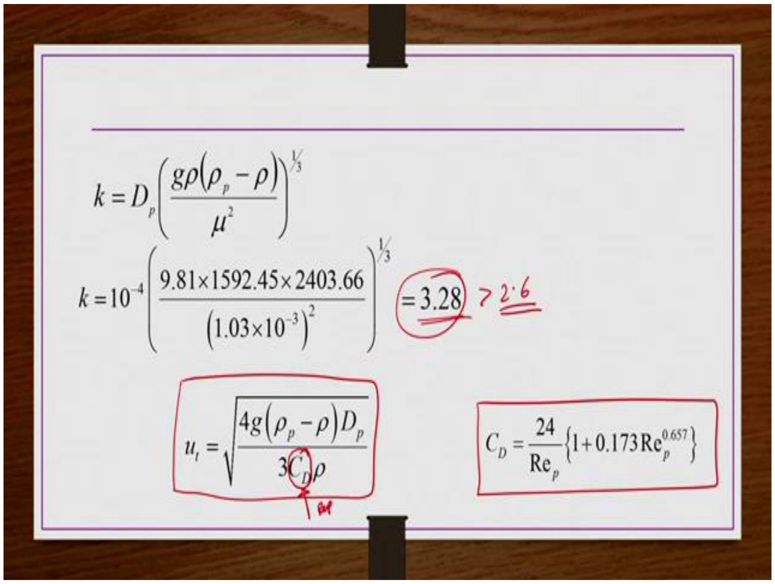

Now, we have to find out first k. So, previous example we understand that assuming Stokes regimes and then using that us is equals to ut epsilon square psi p expression is giving slight error. So, it is better is to find out what is the Reynolds number under which regime it is falling under the Stokes regime or beyond the Stokes regime then proceeding to the solution is going to be better option. So, for that first we have to find out k because ut you do not know so, Re_p you cannot know. So, if Rep you cannot know you cannot say that whether it is under the Stokes regimes or beyond the Stokes regime.

So, that is the region k we calculate which is independent of velocity or there is no velocity term in this k expression, so, that is $k = D_p \left[\frac{g\rho(\rho_p-\rho)}{\mu^2}\right]^{1/3}$ if you substitute all of them and then find out the value it is coming out to be 3.28 so, it is slightly more than 2.6. What does it mean? 2.6 k value 2.6 is upper limit for the Stokes regime. So, anything beyond 2.6 indicate that the particles are not settling in the Stokes regime though it is not too away from 2.6 value, but still it is you know beyond the Stokes regime. So, we have to find out first Re_p using the approach that that we have been finding out in the previous cases.

So, for a spherical particle, you know, $u_t = \sqrt{\frac{4g(\rho_p-\rho)D_p}{3C_D\rho}}$ this is what we know. So, now here we have to know the C_D value, in order to calculate this thing. So, for now, find out the C_D value we need to know Re_p which we do not know. So, again trial and error approach we have to follow. So, for the C_D versus Re_p relation we use the same Schiller and Naumann correlation for spherical particle settling in Newtonian fluids, that $C_D = \frac{24}{Re_p}\left(1 + 0.173Re_p^{0.657}\right)$ and then follow the exactly the same approach as we did previous lecture as well as the previous problem to find out this Reynolds number by trial and error approach.

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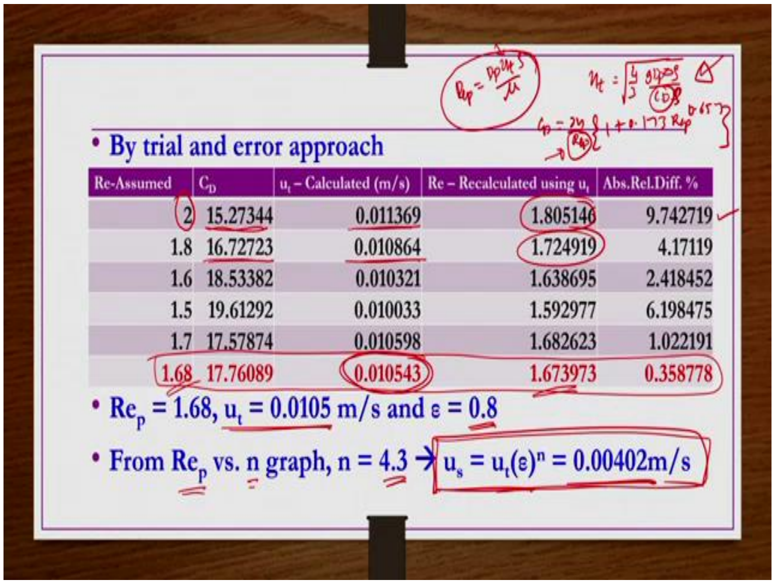

So, if you do trial and error approach, you can have the results and then if you do those calculation and tabulate you will get this information. So, if you assume Re is equals 2 then substitute here, Re is equals 2 here in the C_D correlation. So, you will get 15.27 as the C_D, this C_D value you substitute here in this expression you know then, you will get u_t is equals to 0.011369. So, then, once you have this u_t value, then you can calculate $Re_p = \frac{D_p u_t \rho}{\mu}$. So, here now u_t you calculated so, Re you can calculate, so calculated Re is coming out to the 1.805 whereas assumed one is 2. So, the relative absolute difference is around 9.74 percent.

It is a good one, but still you can improve. So, now, this Re calculated value is coming smaller than the assumed value, what you have to do? You have to go to the next Re value which is smaller than the previous Re value, previously assumed Re value. So, now here Re let us say if you take 1.8 and then substitute here in C_D correlation, then you will get C_D is equal to 16.72723 this C_D value you substitute here in this $u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$, if you substitute C_D there so, then u_t you will get 0.010864 so, now u_t you calculated.

So in the Reynolds number the expression here, you substitute that ut so, then you will get calculated Re value that is coming to be 1.724919 so, it is much improved one, the relative difference, absolute relative difference is 4.17 percent only. Further now, you take 1.6 then 1.5, 1.7 like that, if you keep doing so, you will you will find this when you take Re is equals to 1.68 assumed Re is also coming 1.6739 very close with relative absolute difference less than 0.5 percent.

So, what we can say now, the Re_p is 1.68 and then corresponding u_t whatever corresponding u is there that is u_t 0.0105 meter per second and then ε is given any way 0.8, that is solid volume fraction is given. So, 1 minus that solids volume fraction is the liquid volume fraction 0.8. So, from Re_p versus n graph, if you see for Rep 1.68 n should be close to 4.3 something like that, then u_s is equal to $u_t \varepsilon^n$, when you do this, you will get suspension velocity 0.00402 meter per second, or the hindered settling of the particles is 0.00402 meter per second.

Whereas, the free settling of single particle is 0.0105 meter per second, almost you know 50 times it is less than this value. So, you can say that suspension velocity is in general as we said very much smaller compared to the single particle free settling velocity or hindered settling velocity is going to be much much smaller compared to the free settling velocity of the same size particles in a same fluid. So, this is how we have to calculate problems associated with the hindered settling.

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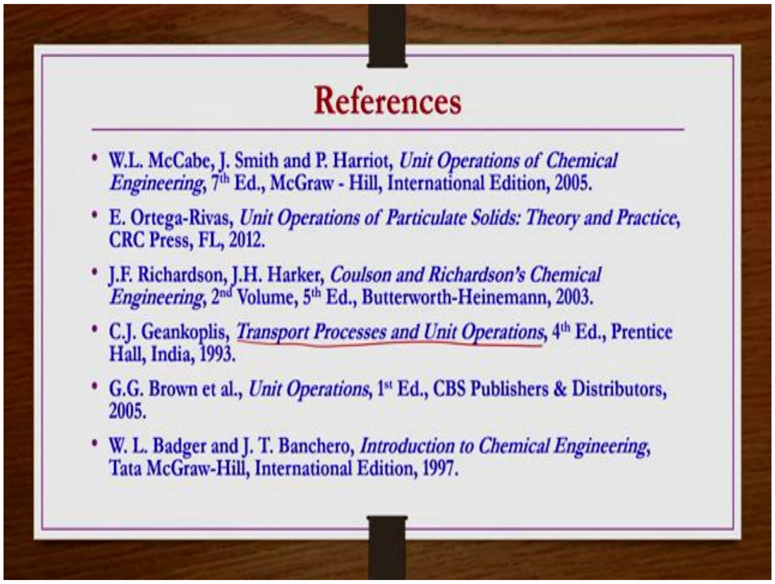

References for this lecture, the most of the lecture is prepared from this book Unit Operations of Chemical Engineering by McCabe, Smith and Harriot. Ortega-Rivas Unit Operations of Particulates Solids: Theory and Practice is the another good reference book, Coulson and Richarson's Chemical Engineering Series second volume by Richardson and Harker is also a good reference book. But the problems discussed here in this lecture are taken from this reference book, Transport Processes and Unit Operations by C.J. Geankoplis, Unit Operations by Brown et al and Introduction to chemical engineering by Badger and Banchero are also good reference books for this motion of particles through fluids especially single and hinder settling. Thank you.