Mechanical Unit Operation

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Lecture 21

Flow through Beds of Solids - 1

Welcome to the MOOCs course Mechanical Unit Operations. The title of this lecture is Flow through Beds of Solids. Why should we need to study about flow through beds of solid particles when we have already studied flow through single particle or single particle settling in gravity and centrifugal field, both the forces, both centrifugal and gravity fields we have seen? We have also seen in this section. So, then what is the point of, you know, studying such kind of flow where fluid is going through beds of solid particles.

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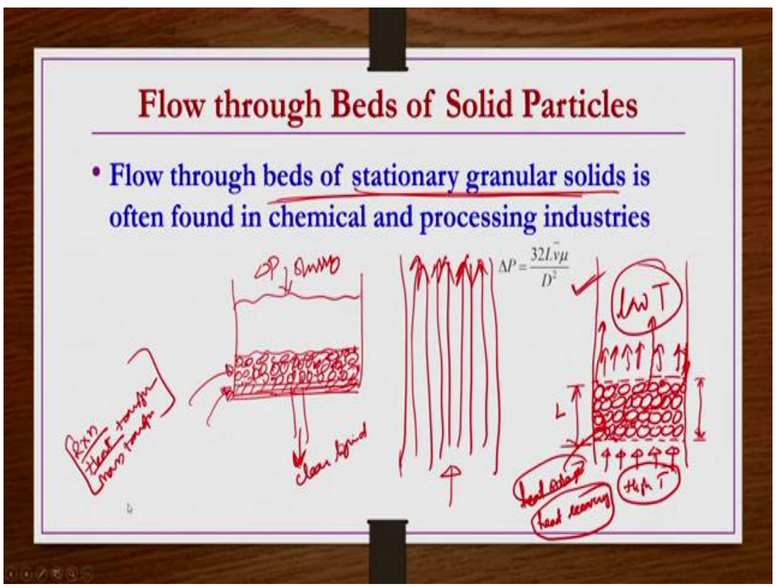

Why because, flow through beds of stationary granular solids is often found in chemical and processing industries in large number of applications. Let us say we have a kind of column in general like this and then at the bottom of the column there is a kind of perforated plate. So, this plate is having some kind of perforation, right. So, now these particles whatever the particles that you know we want we are using them as a kind of a packing and they are placed on this perforated plate.

So, that you know, particle sizes should be, you know, more than the opening of the perforation or perforation plate opening whatever these openings are there. So, that should be smaller and then particle size should be larger. So, that you know they should not fall down, right. So, then the particles are arranged like this. So, sometimes you know this top of this you know bed is also close to be taken perforations plate, right.

So, sometimes it does not, it not necessarily like you know there is a kind of a closing, you know, at the top. So, when the fluid is allowed to flow through bottom like this, you know, if this bed is packed or closed from the top then these particles will not move, there will be a kind of stationary granular solids like in a bed of stationary granular solids, right, ok. So, that means fluid flow is only taking place are, are the only one phase, single phase flow that is liquid phase is taking place.

If it is not closed at top, sometimes what happens, you know, kind of particles will raise will come to the some kind of fluidization conditions and then they will raise, and then the motion take place on, on both the phases like, you know, solid and then liquid phases. Both phases will start flowing or you know, solid are, you know, floated in a kind of floated conditions and liquids are flowing in a kind of certain high velocities, right.

So, these kind of situations are, you know, usually known as the fluid estimate that we will be discussion in, you know, in after a couple of lectures. So, let us restrict our discussions in this lecture only to the case where, you know, we have a kind of pack beds. So, that is the top of this beds, you know, are also close to kind of perforation plate something like this. So, this leads up, you know flowing through bottom and then passing through this perforated plate and then passing through this particle packing structure and then going out of the bed, right.

So, in general, you know, what we have if you have a kind of single column like this you know the fluid just flows through, and then a fluid elements will travel the distance equal to the length of the pipe or the column whatever we have taken, right. But here now part of the column is, you know, packed, you know, mostly most of the sometimes you know entire column is also packed, you know, so, let us take only this part of the packed.

So, then what happen, let us say the full fluid element coming here that may go from here like this, take this part and then go from this corner like this or the same fluid element will take this part and then go like this and then come out like this. This one may go like this, like this and then like this and then go this part like this, whatever that I have drawn. So, that means, you know it may take any, depending on the resistance offered by the neighboring particle depending on the size and shape of the neighboring particles and all that, ok.

So, that means, you know, the packing length, let us say the packing length is L. So, it is possible that, you know, particles are traveling more than this L distance, more than this L distance because of this torturous part, ok. So, that is 1 thing. So, now, coming to the applications, why should we need, you know, kind of, this kind of packed flow through packed which should be studied; there are several applications.

So, here what we are going to do, we are going to measure the kind of expression for pressure drop as a function of, you know, packing characteristics like porosity, tortuosity, etc and then particle size that has been used for the packing, shape of the particle, then the density, viscosity of the fluid, etc. In terms of measurable parameters including this L also the height of the packing. So, in terms of measurable, measurable parameters, we are going to develop an expression (press) for the pressure drag, similar way like, you know, for a pipe flow if it is a kind of, you know, laminar $\frac{\Delta P}{L} = \frac{32\mu V}{D^2}$.

$\Delta P = \frac{32\mu LV}{D^2}$ this is what we have already seen for Hagen Poiseuille equation. So, corresponding equation here what should it be, because here in the case of packing there is no kind of abstraction. So, there are simply flowing through, but here there are large number of particles are there each particle may be offering some kind of resistance to the motion of the fluid. So, the overall resistance offered by the fluid particle that should be the overall resistance experience to a different particle to pass through this kind of channel.

So, this is what we are going to see basically, right. Then coming to the applications in general what happen, so catalytic bed reactor, let us say H_2SO_4 plant what happened, so there is a SO_2. So, in general SO_2 is in H_2SO_4 have a plant case for SO_2 that will be passing through a kind of a catalytic bed. So, catalytic bed, so that is catalytic bed or catalytic reactor, so then here, so whatever the SO_2 comes out that will be reacting and in the, in the, in the presence of this catalytic bed.

So, that will be forming SO_3 and then that will be further absorbed into waters in order to produce the H_2SO_4 like that. So, for let us that kind of that is one, a one kind of example, so another example is you know drying, drying of materials you know, you know sometimes you have a kind of silica gel, etc, molecular cell, etc here in order to fulfill these drying aspects and then also we have a kind of sometime you know, this you know heat recovery system.

So, let us say high temperature fluid is coming here. So, whatever the heat that high temperature fluid is carrying that will be absorbed by this packing material here and then the fluid will be going as a kind of a low temperature or the ambient temperature fluid after passing through this bed, right. So, then whatever the heat, instead, that will be heat storage kind of things. So, this is storage, maybe sometimes when required that can be, you know, recovered further some other applications.

Let us say then when you wanted to do the recovery of the heat you can pass through your fluid which is having the low temperature and then when it pass through it may be carrying the energy from the these heat stored packing material and then maybe going out as a high temperature that is also possible. So, both heat storage and then heat recovery also this packed fluidized bed are in general list. Then we have a kind of in general, you know, filtrations like you know, when we do the filtration there is a kind of filter paper here, in general, like this at the bottom in a container, right.

So, this slurry or the suspension comes out here right and then some kind of pressure you know, delta p is applied you know, some pressure is applied on there is a kind of vacuum at the bottom either way it is possible, right. So, then when you take this slurry here, so, the particles you know, there may be settled at the bottom here like this, ok. And then clear liquid may be collected at the bottom. So, in this case is also this is in the down flow, and this can be treated as a kind of down flow packed with whatever the examples I have shown here kind of you know, up flow packed kind of things.

So, it is possible down flow, up flow both of them are possible and depending on the applications, ok. So, here these, whatever the particles the present in the suspension, they were, they are going to offer some kind of resistance once they settle on a kind of filter medium. So, this can also be taken as a kind of a example, where packed bed, go through packed where this taking place. Initially only the filter medium is resisting providing the resistance to the flow.

So, that is allowing only fluid element to pass through and then retaining the particles, so, that the separation is taking place, but gradually as the process progressing these particles may be settling on the surface of the this filter medium. So, they are also additionally adding resistance to the flow for the later coming in slurry, right. So, there will be two resistance, one resistance, but due to the filter medium and under resistance due to the settled particles, ok.

So, in the next module, we are going to viscous the filtration aspect. So, there we are going to see the equation that we are developing in this lecture or going to be useful in order to understand what is the resistance offered by these particles in designing of this filtration equipment, ok. So, and there are only a few reactions or few examples I have shown here. We have a kind of you know, some examples, we have seen there is a kind of reaction taking place, there may be a kind of heat transfer, heat recovery, there may be sometimes some kind of mass transfer also taking place, right.

So, all, all these 3 may be taking place are individually or they may be taking place together along with the momentum transfer. So, whether the reaction is taking place or heat transfer is taking place, mass transfer is also taking place in addition to the momentum transfer. So, whatever the study that we are going to do here or analysis that we are going to that is going to be there in all of these things, right. So, that is with the whatever this reaction heat and mass transfer are expected to carry out here in this packed bed they will not be possible unless there is a kind of momentum transfer.

So, there is a transfer of the momentum. So, how much pressure drop is required for a given flow rate of liquid to flow through a packed bed of having certain characteristics like in why this particle size, packing particle size, D_P, etc those kind of things. So, this D_P we are going to find out ok. So, this is the purpose that you know, we need to understand flow through beds of solid particles though we know the settling of individual particles are free settling and hindered settling of particles or suspension we have already seen. So, despite of that we need to understand flow through beds of solid particles because I have given so many examples here, ok.

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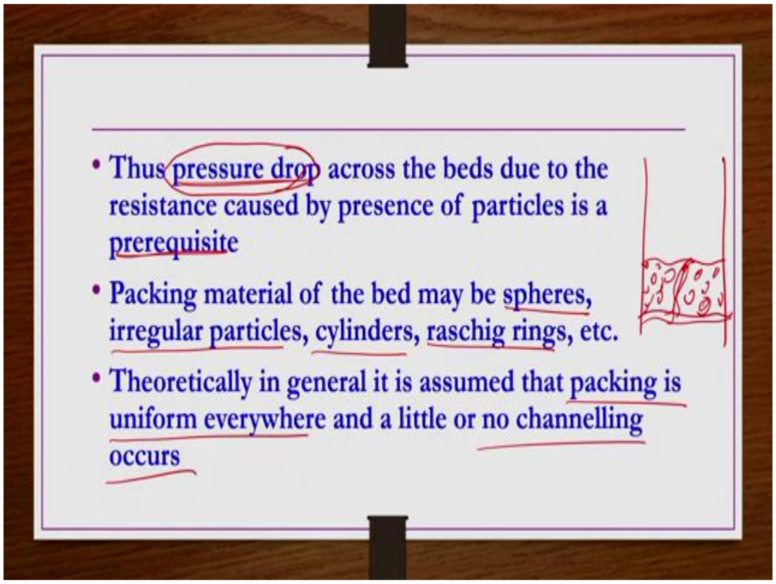

So, what we understand the pressure drop across the beds due to the resistance caused by the presence of particles is a kind of prerequisite because we have seen, you know, this example, several example, the fluid is flowing through packed bed and then when it is flowing through the bed it is offering or it is receiving so much of resistance because of the particles that are, you know, used for the packing of the column, which is forming a kind of packed bed, right. So, this particle are offering huge amount of resistance in general.

So, that is going to be very much important. Let us say you know, pressure drop for the same pressure drop you know, if you allow your fluid to flow through a single column you may be getting higher velocity something like you know, 1m/s , but under the same pressure and if you do the column with some kind of packing of let us say void is 0.4 or 0.5 and then you apply the same pressure, and then whatever the velocity that you get that will be much more lesser than the 1 meter per second of the individual column without, without any packing, right.

So, that means, you know, the velocity the average velocity in a kind of packing is also going to be very much different compared to the average velocity whatever we expect to have a kind of single column without any packing, ok. So, that is the region. So, pressure drop here is going to be very much important in designing of this, you know, equation. So, we how to find out you know, proper way how to find out this pressure drop for a given system. Further packing materials in general they can be spears, they can be irregular crushed particles, they can be cylindrical particles, small cylindrical particles, they may be small raschig rings, etc.

They can be anything usually, you know, spheres are, cylinders are crushed glass particles, etc in general use to kind of packing material on it. So, in general, you know, in reality when this bed is forming, if the particles are of uniform size and shape, then there is a possibility that whatever the packing is there is forming a kind of uniform packing is taking place, but if you have a different size particle those same shape; let us say you have a spherical particles shapes spherical but different size particle 1 mm 2 mm, and then let us say, if I remember particles if you use the packed bed is not going to be uniform in general.

And in further if both size and shape are not same if you have some spherical particles some cylindrical particles as the packing. So, it is not going to be uniform everywhere within the bed, right. Further if you have a kind of a regular particles crushed particles something like that as a kind of packing. Then the packing is going to be varying from one point to the other point, but theoretically what we assume and whatever the analysis that we are going to do we assume that the packing is uniform everywhere and there is no or little channeling occurs.

Channeling in the sense, like you know, on the bed is the let us say this is the bed. So, within this bed somewhere you have a kind of you know, these kind of channels, which is allowing the fluid element to flow through without any kind of hindrance kind of thing. So, particles are being segregated and forming a kind of channels kind of thing, right.

So, so, because of this channeling you know, the particles you know, through element maybe offering the fluid admin may be receiving a little bit less resistance, but that is not going to be useful in many cases especially. Especially, if there is a kind of heat transfer or reaction is there, ok. So, this channeling may be occurring in general in reality, but we assume that channeling is very less or almost there is no channeling. So, this is what we some basics what we are going to do about these things, ok; about the packed bed.

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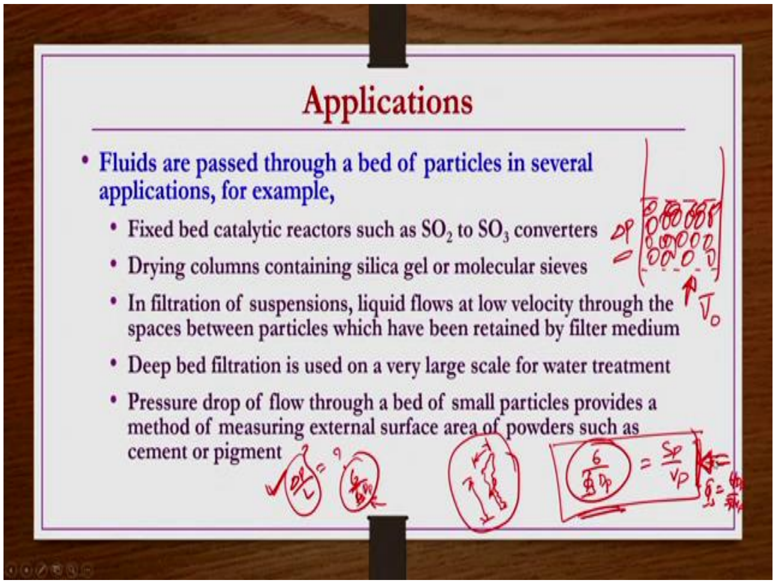

So, then coming to the applications once again fluids in general liquids or gases are passed through your bed of particles in several applications. For example, fixed bed catalytic reactors such as you know such as SO_2 to SO_3 converters. Then drying columns containing silica gel or molecular sieves. Then in filtration of suspension liquid flows at low velocity through the spaces between particles which have been retained by filter medium as explained. Then sometime deep bed filtration is also used on very large scales for water treatment, but however in such applications the, you know particles that are being settled very few particles, ok.

Pressure drop of flow through your bed of small particles provides a method of measuring external surface area of powders such as cement or pigment. So, cement or some kind of pigment particles are very small very, very small you know, you know measuring the surface area and then surface to volume ratio in general very difficult for a given sample, right. So, if you know the pressure drop, indeed we are going to obtain expression pressure drop for the laminar flow, pressure drop for the turbulent flow.

So, under such conditions we have let us say if you know the average velocity experimentally then you can know the Reynolds number them, but if you know the Reynolds number by experimentally then you can know the pressure drop experimentally, right. You can experiment it also, you can calculate the pressure drop. So, the corresponding to that pressure drop what is the velocity, etc, you can calculate and then you can use the equation that we are going to apply here.

So, then what we understand we find out something like $\frac{6}{\phi_s D_p}$ these kind of expressions are there in your equation. So, which is nothing but $\frac{S_p}{V_p}$. So, this $\frac{S_p}{V_p}$., finding $\frac{S_p}{V_p}$. is very difficult especially for very small particles in general and that has to be generally done by you know microscopic but you cannot use microscopic analysis for large number of particles and then also microscopically if you see there are different dimensions like you know ferret dimensions and Stokes dimensions, etc we have seen.

Let us say the fine particles if you see in the microscope, if you are taking having this kind of shape let us say, right. So, which dimension should you take should you take this dimension, should you take this dimension, or should you take this cross sectional dimension, which one should you take, this all we have our seen like you know in one of the previous lecture of first module. So, because of these reasons, you know, finding out this $\frac{S_p}{V_p}$. ratio in general very difficult for the very fine and in a regular particles.

So, for that you know what you can do we can find that overall $\frac{S_p}{V_p}$. from this pressure drop calculations, what we can do we can take these particles in a kind of column as a kind of packing material like this, and then and then you allow some fluid to flow non reacting, non corrosive fluid to flow through this particles which is forming the bed and then for this bed, for this velocity V_0, you know, you calculate, you know what is the pressure drop experimentally, right.

So, now, theoretically we are going to develop $\frac{\Delta P}{L}$, $\frac{-\Delta P}{L}$expression for packed bed, right. So, it is having some terms like you know $\frac{6}{\phi_s D_p}$ something like that we are going to derive. So, that means, if $\frac{\Delta P}{L}$ calculate experimentally rest everything in the right hand side is in general known, like you know, velocity of the fluid Y dash and then density of the fluid, viscosity of the fluid, size of the particle used for the packing, etc, those things are known, right.

So, size of the particle if you assume that it is not known from theoretical point of view. So, then I experimentally whatever ΔP that you get that you use and then only in the right hand side you may not be knowing $\frac{6}{\phi_s D_p}$. So, rest everything is known. So, then $\frac{6}{\phi_s D_p}$you can calculate which is nothing but $\frac{S_p}{V_p}$ for a given regular particles. So, this is how you know we can measure the surface, external surface area of powders of small size regular particles like cement and pigment by using this flow through packed bed experiments.

So, that is our possible. Indeed, this as we have seen as a kind of 1 kind of application in 1 of the earlier lectures, ok. So, this $\frac{6}{\phi_s D_p} = \frac{S_p}{V_p}$ from where are we getting we are getting from the definition of ϕ_s/ ϕ_s, $\frac{\frac{6}{\phi_s D_p}}{\frac{S_p}{V_p}}$. So, $\frac{6}{\phi_s D_p}$ is nothing but surface to volume ratio for a spherical particle of equal in diameter S_p is the surface area of the regular particle V_p is the volume of the regular particle. So, from varied from that definition it is coming this also we have seen and then we have done several problems also on these things, right.

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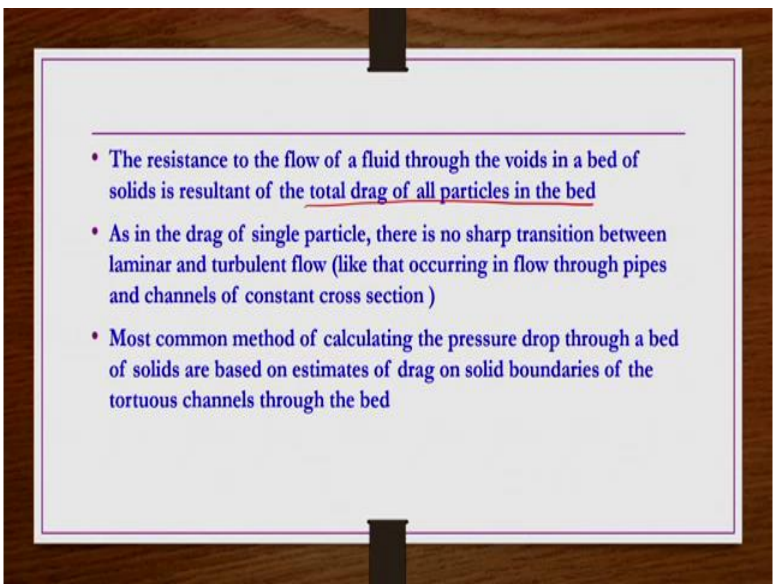

So, now, what we understand the resistance to the flow of fluid through the voids in a bed of, of solids is resultant of the total drag of all particles in the bed or the total resistance offered by all the particles. So, that is going overall resistance to the flow of fluid. But, you know, as in the drag of single particle you know there is a sharp transition between laminar and turbulent flow in general in our single particle cases we have seen like you know, if the Re <0.1 or 1 .we say that it is a kind of Stokes region or if it is a kind of a case flow through pipes then we know that you know Re_p or the Re for the kind of pipe flow case if it is less than 2100 we can say that it is a kind of laminar flow if it is more than 2100 we can say it is a kind of turbulent flow.

Such kind of sharp transitions in general are not possible in the case of flow through packed beds. In general, indeed, we know that you know from experimental it has been found that the flow through bed of particles is laminar only for small ranges. Let us say up to Re_p 5 to 10 only, if it is more than 10 it is not going to be under laminar flow conditions anyway. So, such kind of sharp transitions are not possible in the case of packed. Thus, most common method of calculating the pressure drop through a bed of solids are based on the estimates of drag and solid boundaries of the tortuous channels through the bed.

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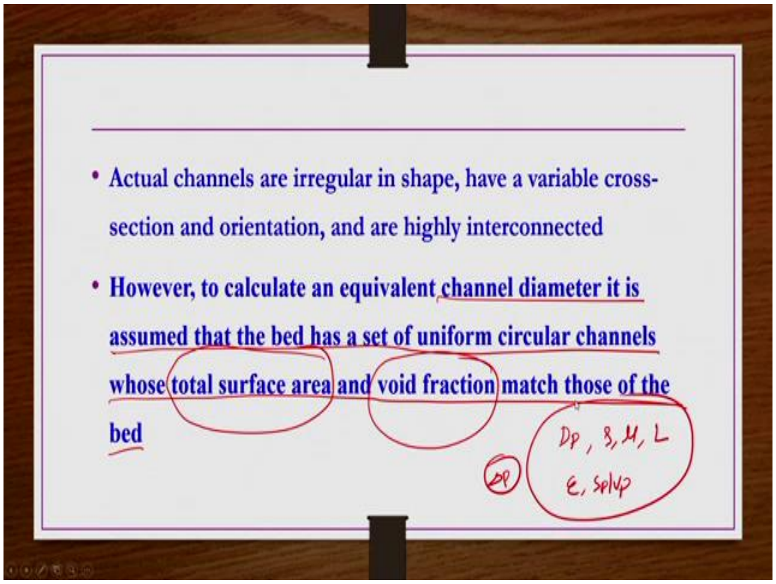

So, actual channels in general are irregular in shape, ok. We have a variable cross section and orientation and our highly interconnected porous structure in packed with in general like this. So, these particles are, you know, in general not necessarily spherical, they can be kind of you know needle like they can be, you know, crushed particles, irregularly shaped particles, they can be like anything, right. So, because of that one actual channels, you know, in general they are irregular shape.

So, this flow channel what, flow channel in the sense whatever the free area, free volume area that is available between the particles, right. So, that is very irregular, ok, and then we have a variable cross section, if it is a flow through a column. So, then the cross section is constant from the inlet to the outlet of the column through which the fluid is flowing, but if that column is packed with some kind of packing materials, so, then what we have we have a kind of cross section is varying.

So, here the cross section is different, here the cross section is different, here the cross section is different from one location to another location cross section in the sense cross section of the flow channel which is available for the fluid to flow and these channels are also highly interconnected in general, ok. So, however, we have to calculate the pressure drop. So, how to calculate pressure drop, calculate an equivalent channel diameter it is assumed that the bed has a set of uniform circular channels whose total surface area and void fraction match those of the bed, ok.

So, now, actually this why, why are we doing this flow through packed beds in general, though we have seen there are kind of catalytic reactions there is a kind of heat recovery kind of system, there is a kind of mass transfer in something like drying, etc. So, then what is the point, why are we using these things in a kind of packed bed, why not we doing in a kind of single column or individual columns like that. That is because you know, when we have this packing the surface to volume ratio for a given packing dimension it increases.

It is very much higher compared to the single columns. So, that is the more surface area you have for this transport process to occur or the mixing kind of operations to occur or whatever the momentum heat or mass transfer or reaction is taking place, that its degree of performance would be higher because of this higher surface area provided because of this package that is the reason we are doing, ok. So, now, what we understand this total surface area is the one, you know, very important one and then because of this thing this packing, etc. what we have within the void is which in the column we have a kind of void fraction.

So, these two are going to have a kind of important all along with the particle diameter, along with the density, viscosity of the fluid and then height of the packing, in measuring this ΔP, ok. So, these are the kind of things you know are playing goal here if you find out if you want to find out this D_p. So, but theoretical it is very much difficult like we know, the flow is very irregular and indeed very complex from one channel to the other for channel, from one location to the other location within the packed, packed bed because of the irregular channels, variable cross section orientation and then highly interconnected channels, etc. present in the packed beds, right. So, then how to do this one?

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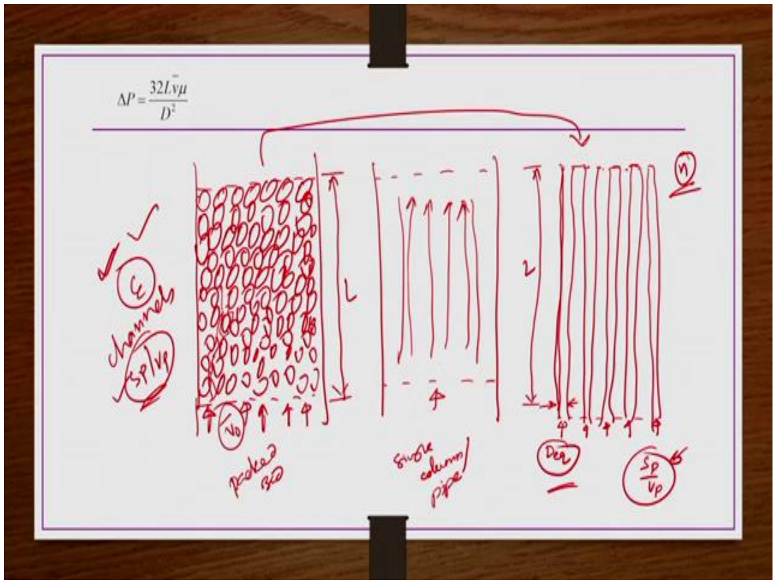

So, now, what we do we assume this bed whatever is there, you know, simplify rather than assuming, simplifying you know this is the bed packing. Let us assume, I am just making a kind of circles kind of thing. So, do not take it granted for kind of only spherical particles, the analysis that we are going to do that is going to be valid for all the particles all types of all shapes, irregular, regular, irregular, small, big particles all of them it is valid actually, ok.

So, now, what we do we how to equally represent this packed bed in a kind of you know workable geometrics kind of things. Here what we have, we have this, you know, epsilon and then this, this channel dimension that is you know, varying variable cross section, etc, right. So, that these things are there, right. So, we have to worry about the void fraction of that is, that is free space available for the fluid to flow.

What is the free space available? That is the interstitial spaces between the particles that we have one parameter, and then the surface area the other one we have to worry about, because the these two are the additional thing compared to a kind of single column if you have taken a kind of single column. This is packed bed, single column or flow through pipe kind of things. So, if you take these two cases here now, what is the difference only this package is that because of the packing this ε and then $\frac{S_p}{V_p}$ are the kind of things which are making a difference, newly being added up. So, what we do this system we have kind of we will try to generalize our simplify.

So, that we can easily do a kind of analysis and develop a kind of expression as we develop for this case like, you know, $\frac{\Delta P}{L} = \frac{32\mu V}{D^2}$ acquired like this is Hagen Pouisells equation that you know for the laminar flow through pipe the pressure drop is this one, one can derive this one and fluid mechanics course, right. So, this is the one, you know, we have like you know for now this column when we are adding some packed material or material for packing then these two things are additionally coming.

So, this system we are generalizing it, how we are generalizing now or simplifying like this, we are taking a few number of capillaries, ok. We have this capillaries like this to which the flow is taking place like this, ok. The diameter of this capillaries we take let us say D capillary, the height of the capillaries is same as the height of the packing here, ok. So, now, this n number of capillaries and you know, what we are saying there are n number of capillaries.

So, these capillaries what we are saying, you know, we are bunching together we are adding together like we are forming like you know we take n; let us say this is a capillary like n number of capillaries are there we are making a kind of bunch kind of thing. So, bunch of capillaries, ok. So, how many capillaries we do not know let us say n. So, what we are assuming that this capillaries when you are taking n number of capillaries, whatever the void space is provided that is the area internal, you know, volume of the capillary through which fluid is taking place, right for all the capillaries which we have together and then find out the fraction.

So, then that you know space the space or the void space available for this n number of capillaries is same as divided space here in the packed bed that is how we are going to do. So, we are replacing, we are getting n number of capillaries having same height as the packed bed height, right. So, those n number of capillaries are forming a kind of a bunch of capillaries and then allowing the fluid to flow through, right.

So, then the whatever the void space that is available in this bunch of capillaries that is equal to the void space of you know packed bed that epsilon that we have taken such that n number should be decided, right. Should it be 10, 100, 1000 that we do not know, indeed, we do not want to know it all. So, but theoretically this is what the one representation that we are doing that is one, and then next one is the surface to volume ratio.

Whatever the surface to volume ratio here in this number of capillaries are there, that is same as the surface to volume ratio for the packed bed, ok. So, the n number should be decided such that for the bunch of capillaries, whatever the ε and $\frac{S_p}{V_p}$ are there. So, they are same in the case of packed the bed or they should be equally equivalent to the whatever we have the packed with case, ok. So, this n, one has to find out in reality what we may be thinking, but we do not need to find out, let us assume this is n, ok.

So, this we are doing, now, here this the representation this D_{eq} is D the diameter of the capillary is equal and that is the flow channel dimension, dimension ok. The dimension which is providing the space through which the fluid is flowing in a capillary. Now, this D_{eq} we are going to find out as function of this epsilon packing particle diameter, etc. for a given system that is what we are going to do, ok.

And then here whatever D in Hagen Poiseuille equation is D still there we replaced D by D_{eq} and then V by V_{eq} replaced by the superficial velocity $\bar{V}$. So, v bar is a kind of superficial velocity of the packed bed. Superficial velocity in the sense like you know, empty tower velocity like you know this column instead. This is the same column here, this second one this, this column packed bed column is there, whatever the dimensions are of this packet bed column and then dimensions of the empty column are same, then whatever the velocity is there through the empty column under this constant same pressure drop as in packed bed that is known as the superficial velocity.

Or simply superficial velocity is a kind of empty tower velocity when there is no packing, right. But here this v should not be you know kind of, because $\bar{V_0}$ is a kind of velocity that is passing through as a superficial velocity, but the true velocity here, you know, in the packed bed the average velocity of the fluid element when it is passing through from inlet to the outlet it may be having some average velocity that average velocity v average should be taken here, right.

But we understand this $V_{avg} = \bar{V_0}\varepsilon$ here. So, those things we substitute here and then simplify this equation so, that we get the pressure drop equation for this n number of capabilities which is same as pressure drop of this packed bed. Why it is same for this packed bed because this bunch of n number of capillary whatever the characteristics are there, that is their epsilon and their $\frac{S_p}{V_p}$ they are same as this packed bed.

That is the reason they will be same, right. So, so, since these capillaries also kind of circular tubes something very fine small diameter tubes kind of thing. So, here also Hagen Poiseuille equations are valid indeed for those capillaries only this Hagen Poiseuille equation is more reliable, because of you know laminar flow conditions can be established in capillaries much easily. So, this is, this is what we are going to do now, for you know developing this expression for the pressure drop.

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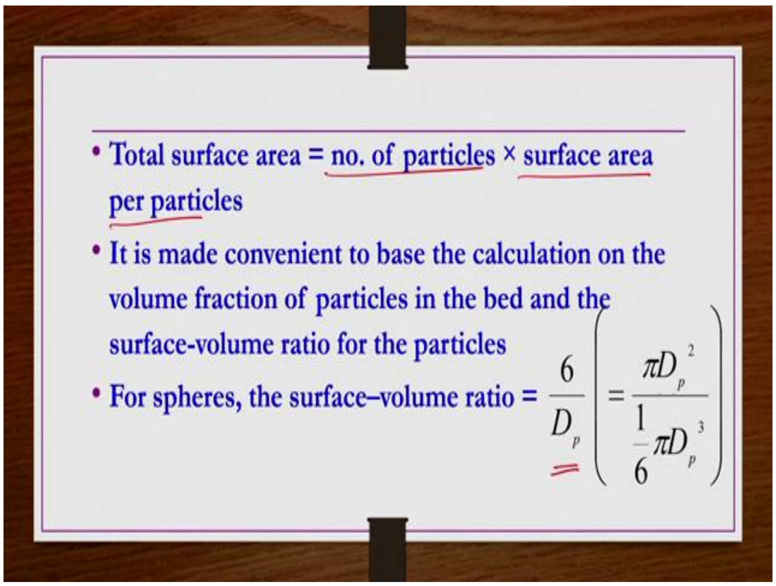

So, total surface area of the bed should be surface area per particle multiplied by total number of particles that we should do, then it is made convenient to base the calculation and the volume fraction of particles in the bed and the surface volume ratio for the particles. Why because, if you take surface to volume ratio particle then you can also bring in the information of sphericity, if you bring in the information of sphericity in equations, that means, you can also use those equation for irregular particles, ok.

So, then for spheres, the surfaced to volume ratio we know it has $\frac{6}{D_p}$ that is nothing but the surface area of this particle that is $\left(\frac{\pi D_p^2}{\frac{1}{6}\pi D_p^3}\right)$, D_p is the diameter of that particle let us say ok $\frac{6}{D_p}$. So, that is from where it is coming $\frac{6}{D_p}$ that is πD_p^2 the surface area of spherical particle divided by the $\frac{1}{6}\pi D_p^3$ that is the volume of this spherical particle, then you get surface to volume ratio first spherical particle as $\frac{6}{D_p}$. This is for the sphere. But we wanted to bring in the non sphericity or the irregular particle nature also into the consideration.

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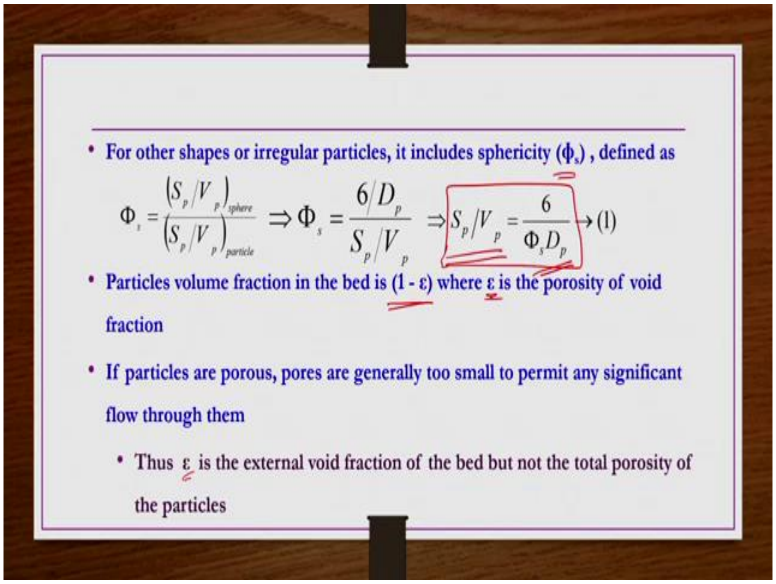

So, for other shapes or irregular particles it should also include the sphericity, which is defined as a surface to volume ratio of sphere divided by the surface to volume ratio of the particles, right. So, then from here that we have already seen that $\emptyset_s$ is equal to $\frac{S_p}{V_p}$, for sphere is nothing but $\frac{6}{D_p}$ and then for the irregular particles surface to volume ratio we do not know. So, let us keep them as $\frac{S_p}{V_p}$ that means $\frac{S_p}{V_p} = \frac{6}{\phi_s D_p}$ that means, from somehow from your calculations that you are going to do or somehow from your results if you can get the $\phi_s D_p$ information for unknown irregular particles then you can indirectly know the $\frac{S_p}{V_p}$ ratio.

Or that $\frac{6}{\phi_s D_p}$ if you do if you get $\frac{S_p}{V_p}$. That is not the primary object for some for the time being and giving us a kind of information because we have seen it is a kind of a application for finding out the surface to volume ratio of irregular material, irregular verifying materials. Then particle volume fraction and the bed is $1 - \varepsilon$ where is the epsilon is porosity of void space or the volume fraction of the empty space or the volume fraction of the liquid within the bed.

So, whatever the empty space between the particle is there, only that much space liquid can occupy that is porosity of the void fraction, ok. If particles are porous then pores are generally too small permit any significant flow through these particles. Actually, you know when we are taking the particles we are, this analysis we are assuming the particles which are used for packing irrespective of their size and shape we are assuming they are non porous, ok.

The analysis that we are doing that is non porous, but however if you even if you have some kind of porous particle their porous structure are too small in general to permit any significant flow through this internal pores. Porous, the porosity whatever we are talking about that is only for the external porous, porosity external only or the interstitial space between the particle only, not for the porous structure of the internal porous structure of the particle.

So, we are assuming indirectly that the particles are non-porous, even if they are porous the flow that is going to take place through these internal porous is going to be much more smaller than the flow that is taking place through the interstitial spaces or external porous structure, right. So, that is the reason one can safely ignore the internal pore structure of the particles in this packed beds. So, thus epsilon is the external void fraction of the bed but not the total porosity of the particles.

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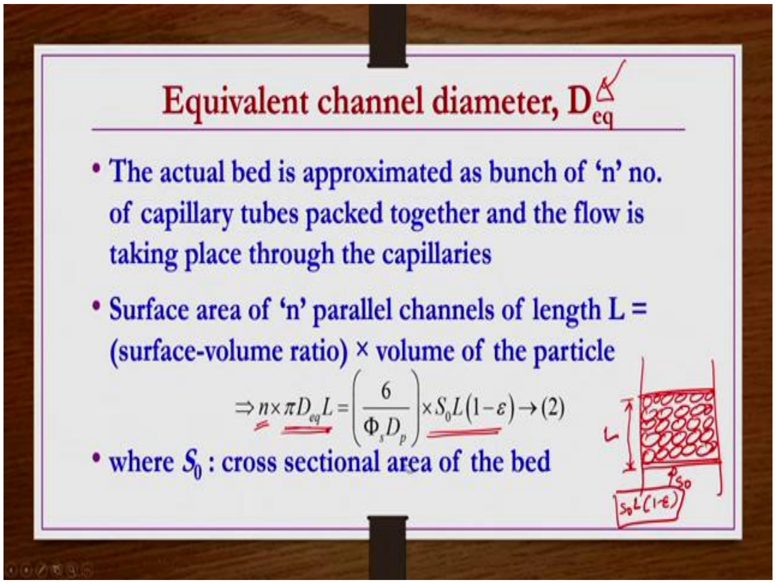

Now, we find out equivalent diameter of the capillary channels, n number of capillary channels that we are taking. So, the diameter is D_{eq} we are going to find out as function of ε are the size of the packing material, etc, ok. So, the actual bed is approximated as bunch of n number of capillary tubes packed together and the flow is taking place through the capillaries. So, that is what we assumed as I explained before.

So, the surface area of n parallel channels of length L should be the surface to volume ratio of particles multiplied by volume of the particle. So, in real packing conditions whatever the surface to volume ratio is there that if you multiply by the volume of the particles, so, then that that would be giving the total surface area of the packed, packed structure or packed bed, right. That should be equal to the surface area of one channel of Length multiplied by the total number of such channels, ok.

So, that means, if you have n number of channels or capillaries, ok, the surface area of each capillary is $\pi D_{eq} L \text{ or } \pi D L$ because they are the cylindrical capillaries, ok. So, the capillaries are you know, each of them are having you know surface area $\pi D_{eq} L$, right, and that should be multiplied by n because n number of capillaries are there. So, this is going to be total surface area for n number of capillaries of length L, ok.

So, now coming to the particle real packed bed. So, this is left hand side so it is about the capillaries, which is equivalent to our packed bed, we simplified it, ok. So, now about the packed bed, $\frac{6}{\phi_s D_p}$ is what? It is the surface to volume ratio of the particles irrespective of the size and shape, it is true for all size, all shape, regular, irregular particles, ok. This one multiplied by the volume of the particles. So, volume of the particles, how do you find?

Let us say and this is the packing here. So, these are packed between two perforated plates like this. So, the cross section area of this I know column whatever that is, is S_0, cross section area the bed is S_0 and then height of the bed is L. So, volume of the bed is going to be $S_0 L$ and then if you know, if you want to know the volume of the particles out of this volume one minus epsilon volume fraction is occupied by the particles.

Remaining is the occupied by the liquid or void space, ok. So, $S_0 L, (1-\varepsilon)$ is going to be volume of the all the particles that are present in the packed beds. So, surface to volume ratio $\frac{6}{\phi_s D_p}$ multiplying by $S_0 L, (1-\varepsilon)$, if you do you will get kind of true surface area of the packed bed that packed bed, whatever the packed bed that we have taken its true surface area of the entire bed is this one that is $\frac{6}{\phi_s D_p} \times S_0 L(1-\varepsilon)$.

So, that should be equal to the total surface area of n number of capillaries that is $n\pi D_{eq} L$. So, here S_0 is the cross section area of the bed.

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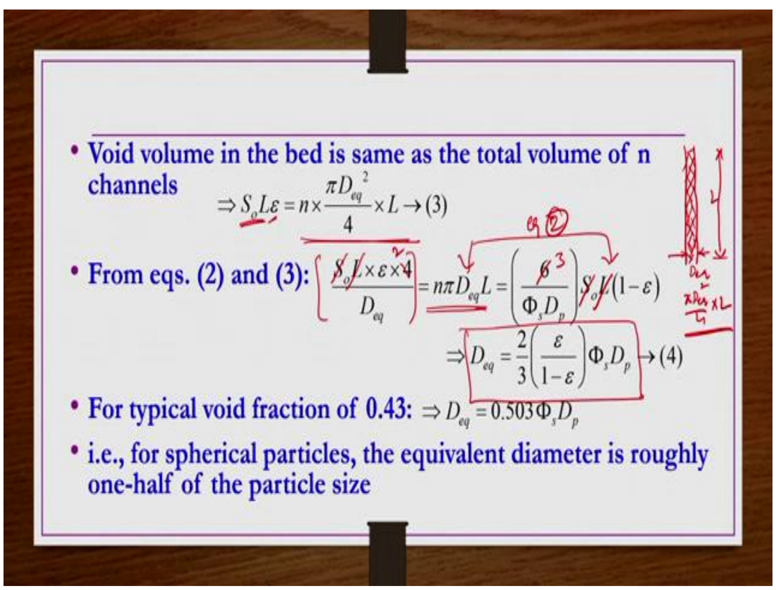

Now, void volume in the bed is same as the total volume of n channels total there are n number of channels are there whatever the volume of each, each channel is there that if you multiply it by total n number that should be equal to the existing void volume within the bed because two are equivalent now, ok. So, then when you do this one. So, similarly, as I mentioned that S_0 is the cross section area of the bed, L is the height of the bed, the height of the packing that we have done. So, S_0L is going to be volume of the bed, the entire volume of the bed which is, which is consisting of both solids and you know void space of the liquids, ok.

So, out of this total S_0L volume, only ε fraction is in a void volume. $1-\varepsilon$ fraction is solid volume. So, here we wanted to know the void volume in the bed. So, that is $S_0L \times \varepsilon$ will give you the void volume in the bed. So, that void volume should be equal to the void volume provided by n number of are bunch of n number of capillary channels. So, there capillaries you know, we have this each capillary of size you know D_{eq} diameter and then height same as the, you know, height of the bed.

So, cross section of this capillaries by $\frac{D_{eq}}{4} \times L$ if you do that is the volume of this capillary that is available for the flow. So, the fluid is flowing through. So, this volume of the capillaries now, $\frac{\pi D_{eq}}{4} \times L$ this is volume of one single channel. So, there are n number of such channels there are n number of channels so, that you know that whatever the total volume provided by this n number of channels is same as the void volume of the real packed bed.

So, $S_0L = \frac{n\pi D_{eq}}{4} \times L$. So, now, what we do, we take 4 to the left hand side and then D_{eq} also we take to the left hand side, so, $\frac{4S_0L}{D_{eq}}$. So, from here 4 we have taken left hand side and one the D equivalent also we are taking the left hand side so the remaining things $n\pi D_{eq}L$. So, this n $\pi D_{eq}L$ from just previous equation number 2 what we have find, we found it us $\frac{6}{\phi_s D_p} S_0L(1-\varepsilon)$

This is coming from equation number 2, right. So, now this is, this part if you equate to this part, then you can find it out an expression for D_{eq}, right So, that is what our target here so, that we do now, this S_0 you cancel out here and S_0 canceled L and L also you can cancel out 2 twos are, 2 threes are if you do it. So, D_{eq} you take to the other side. So, here in the left hand side you have to ε as it is ok and then from this side if you bring the $\phi_s D_p$ to the other side, so, that will be multiplied and then $1 - \varepsilon$ if you bring into the other side that will be divided.

So, D_{eq} to what we get here by this simplification, we get it us $\frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right)\phi_s D_p$ is volume equivalent diameter of the irregular particle that has been used for the packing of the column, ok. D_{eq} is the diameter of capillary and there are n such number of capillaries are there we are, which are forming a kind of bunch. So, that to have same characteristics like packed bed in terms of the void epsilon and then surface area or surface area to volume ratio, ok.

So, now, here typical void fraction of point 0.4 to 0.5 is in general possible in real of applications. Let us say if you take point 0.43, void fraction then D_{eq} you will get point $0.503\phi_s D_p$, ok. So, let us say if it is a spherical particle let us say the packing has been done with a spherical particle of diameter D_p then $D_{eq} = 0.503D_p$ because $\phi_s = 1$ first spherical particle. So, what does it mean?

If the packing is done using a spherical particle then equivalent diameter of which is required for the capillary. So, that n number of such capillaries can form a kind of you know bed having a kind of equal characteristics as a kind of true bed that we have taken, right. So, what does it mean? So, that equivalent diameter is half of the diameter of this bed, right. If the void is 0.43 and the packing is done by the spherical particle then equivalent diameter is half of the diameter of this spherical particle.

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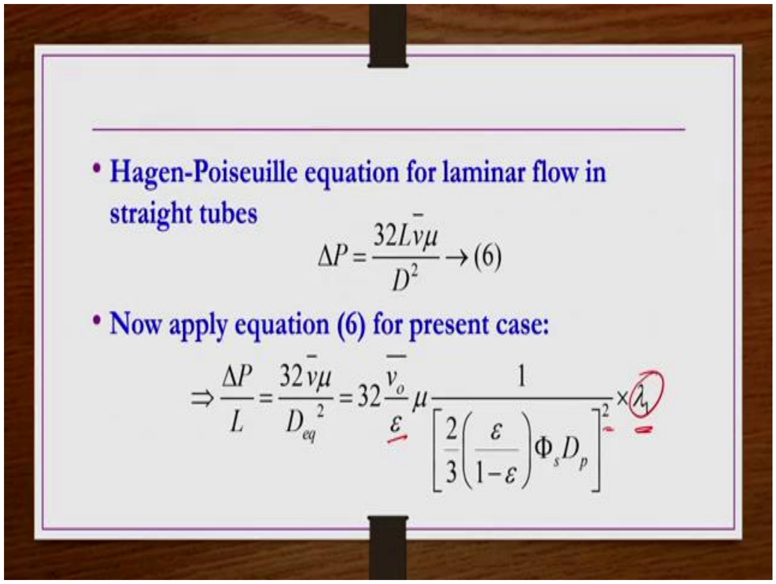

Know what we have to do? We have to find out the pressure drop calculations. So, ΔP depends on the average velocity in the channels $\bar{V}$ which is not known. Superficial velocity is known. How much you know empty tower velocity you are providing that you can know because average velocity is going to change from packing to the packing, from packing is a kind of a regular packing and in size and shape are different from you know shape of the particle is also a irregular and then sizes also be going to be different from one particle to other particle then the channels are not going to be regular, right, then, definitely the velocity is not going to be the average velocity is not going to be same from one extreme to the other extreme that we cannot know, ok.

Or you know, you can know from the experiments anyway by doing experimentally but however, you know in applications without this, this simplification theoretical analysis why are we doing because without doing a experiment we can calculate how much pressure drop is required for a packed bed of certain height and certain porosity, etc. For that purpose we are doing it, ok. So, then this v bar is proportional to superficial velocity or emptied tower velocity v naught bar and inversely proportional to the porosity.

So, that $\bar{V} = \frac{\overline{V_0}}{\varepsilon}$. So, in this equation in place of v, we replace v by $\bar{V}$ but $\frac{\overline{V_0}}{\varepsilon}$ and then D we replace by D_{eq} equivalent simply do this one. So, now, the average velocity $\bar{V}$ and D_{eq} will be expressed in terms of the measurable properties or parameters such as $\bar{V}$, D_p, ε, etc, and those things will be used in this equation. So, indeed D_{eq} we have already represented in terms of this D_p , ε and then $\bar{V}$ just now we have seen it is $\frac{\overline{V_0}}{\varepsilon}$ this information we are going to use in this Hagen Poisuellis equation.

Hagen Poisuellis equation for laminar flow in straight tubes without any packing, in straight tubes without any packing if the fluid is Newtonian and then fluid is flowing under laminar flow conditions then pressure drop is $\frac{\Delta P}{L} = \frac{32\mu V}{D^2}$ this, we already know here. So, then what we do here, we substitute $\bar{V}$ is equal to $\frac{\overline{V_0}}{\varepsilon D^2}$, we replaced by D_{eq} and simplify. So, then when we do it $\frac{\Delta P}{L} = \frac{32\mu V}{D_{eq}^2}$.

32 $\bar{V}$ is nothing but $\frac{32\overline{V_0}}{\varepsilon}\mu\frac{1}{\left[\frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right)\phi_s D_p\right]^2} \times \lambda_1$. Why it is because in straight pipes, straight tubes, etc like capillaries if there is a laminar flow, when there is no obstacle right. So, there is a fluid elements are traveling the same distance as the, the height of the pipe or the length that we have used, ok.

But in the packing the fluid element if you are packing that column with some kind of packing material some particles, etc. Then what happened is fluid element is traveling more than the height of the packing because of the tortuosity part. So, in order to bring in those information, we are taking this fraction λ_1 let us not worry about what is this λ_1 value as of now.

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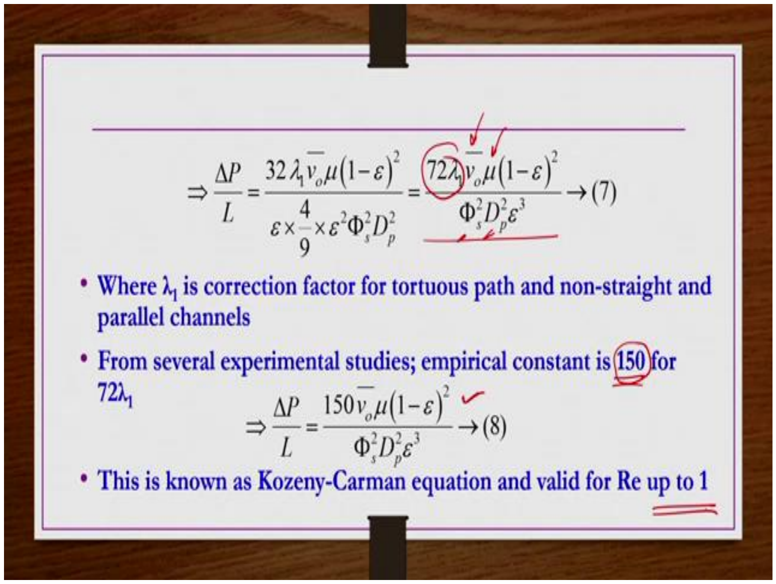

Now, this equation you rearrange and simplify it. So, then what we have $\frac{\Delta P}{L} = \frac{32\lambda_1 \overline{V_0} \mu (1-\varepsilon)^2}{\varepsilon * \frac{4}{9} * \varepsilon^2 \phi_S^2 D_p^2} = \frac{72\lambda_1 \overline{V_0} \mu (1-\varepsilon)^2}{\varepsilon^3 \phi_S^2 D_p^2}$

This is what we are having it, right. So, now here v bar is superficial velocity is known. How much velocity, superficial velocity you want to provide that you know and then viscosity of the fluid you know the voidage, void space of a bed characteristic that in general you know, right.

D_p and the volume of sphere, equivalent diameter of irregular particle you know, sphericity of the particle also you know or you can know the way the calculation that we have done in module one, right. So, then everything is known here except this one, λ_1, right, but this λ_1 is a correction factor for tortuous part and non-straight parallel to non-parallel channels in general irregular channel that is what I already mentioned, right.

But from experimental studies, empirical constant is found as 150 in place of 72 λ_1. So, like you do the experiments, right, and then try to maintain the laminar flow conditions, then if you represent your results $\frac{\Delta P}{L}$ as function of this parameters like this you will get $\frac{150\overline{V_0} \mu (1-\varepsilon)^2}{\varepsilon^3 \phi_S^2 D_p^2}$. Or you know theoretically whatever the values that you get here other than these things if you calculate.

So, that constant is coming out to be close to 150 value of experimental value, right. So, this $\frac{\Delta P}{L}$ the pressure drop, ok, pressure drop for a fluid flowing through packed bed of a height, packing height λ bed having the void space ε, then packing has been done with some kind of irregular particles. If the flow is laminar flow, then pressure drop you can calculate using this expression without doing any experimental studies, ok.

Because this has been found consistent with the experimental study, if you replace that 72 λ_1 as a kind of 150, ok. This equation is known as the Kozeny-Carman equation and valid for Re up to 1 only smaller number only it is valid. As I already mentioned in the straight pipes without any packing kind of thing we have seen that laminar flow conditions are existing up to Reynolds number of 2100 something like that, but if the same channel the tube if you have some kind of packing in between to have a kind of packed bed, so, then the flow is not going to be laminar beyond Reynolds number of 1. However, some people take approximation of Reynolds number up to 5 to 10 is also laminar, but in reality this equation is valid only up to Reynolds number of 1. So, this is for the laminar flow condition.

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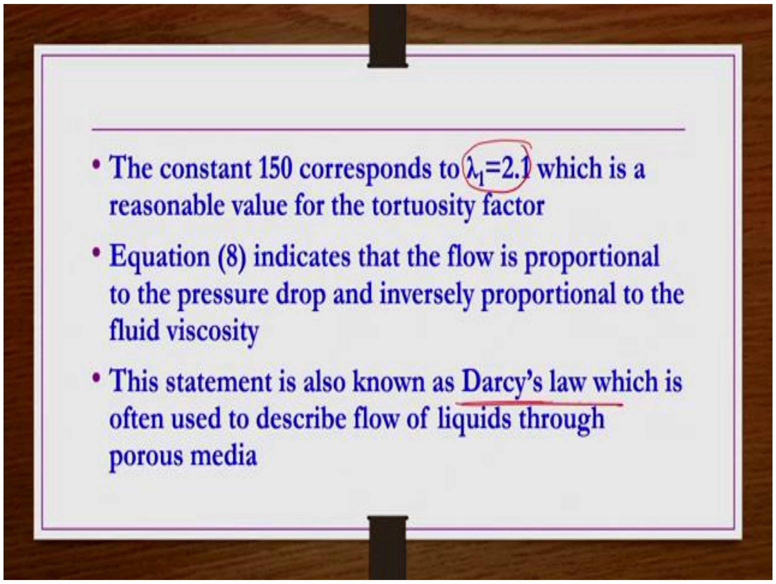

Same thing we do for kind of turbulent conditions also, but before going to do this 1, the constant 150 if you equate to the 72 λ_1, then what you get λ_1 is approximately 2.1 it is fluid element is on average traveling twice the 2.1 times the length of the packing or the height of the packing, which is reasonable value for the tortuosity factor, in general for tortuosity factors.

So, that is and that is not a kind of very blind value kind of thing. So, that is acceptable one. So, equation 8 indicates that the flow is proportional to the pressure drop and inversely proportional to the fluid viscosity. So, this statement is also known as the Darcy's law, which is often used to describe the flow of liquids through for porous media.

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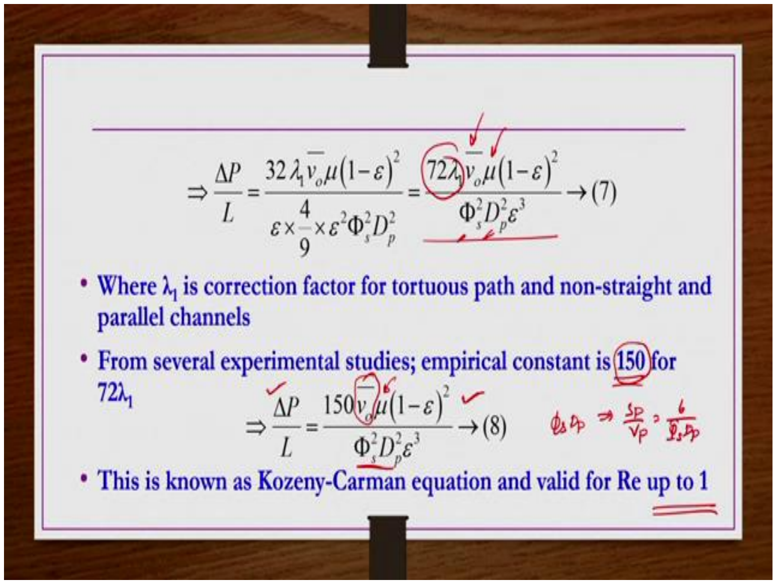

So, here we can see this v naught bar, the flow velocity, flow whatever is therefore, velocity is then that is proportional to the pressure drop and then inversely proportional to the viscosity which is consistent as it kind of Darcy's law which is used to further flow through porous media. And then other thing I was mentioning like you know, for a given pack system if you calculate $\frac{\Delta P}{L}$ experimentally, let us say if you use irregular small particle or the pigment particles, etc for which you do not you find very difficult to measure $\frac{S_p}{V_p}$ ratio you make a packed bed with those particle and an experimentally you find out what is this pressure drop, right. So, then from here you can back calculate what is $\phi_s D_p$.

So, that from here you can find out $\frac{S_p}{V_p}$ is equal to $\frac{6}{\phi_s D_p}$. So, this is the other advantage of this equation. Only thing that, you know, for the particles for which you wanted to find out $\frac{S_p}{V_p}$ ratio which is not possible or very difficult to find out through using the other conventional ways because of inherent difficulties, then, you use those irregular particles small particles as a kind of packing material and form a packed bed and do the experiments find out a ΔP, ΔP or pressure drop experimentally, ok.

So, that pressure drop you can use here in this equation and then packed calculate what is $\phi_s D_p$ because remaining things are known, experimentally how much superficial velocity you have given that you know, viscosity the of the fluid you know and then void space of the bed in general you know in general it is around 0.4 to 0.45. So, you can calculate $\phi_s D_p$from here because rest everything is known. Once $\phi_s D_p$is known if you do $\frac{6}{\phi_s D_p}$you will get the surface to volume ratio for those particles, ok.

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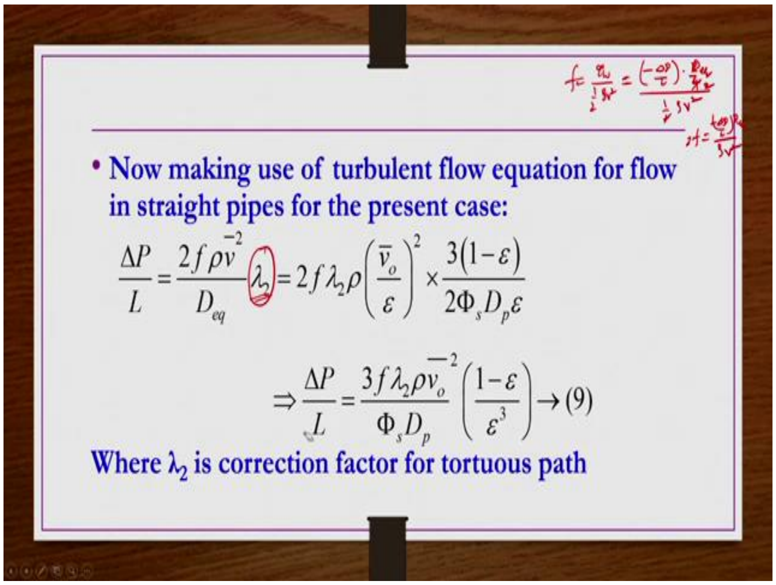

So, now, making use of turbulent flow equation for flowing straight pipes for the present case. So, what we do we have this friction factor expression $f = \frac{\tau_w}{\frac{1}{2}\rho V^2}$ for flow through pipes. So, this 1 we can write it as a kind of tau w we can write $\frac{-\Delta P}{L}$, $\frac{R}{2}$, R is the size of the capital, the pipe; $\frac{1}{2}\rho V^2$, you can do it. So, that means, there is a relation between f and $\frac{-\Delta P}{L}$.

So, that in relation if you write it, rewrite it here in this form $\frac{\Delta P}{L} = \frac{2f\rho\bar{v}^2}{D_{eq}}$ here. So, these R we can write it as $\frac{D}{4}$ or $\frac{D_{eq}}{4}$, then we have this equation. Let 2 ones are, 2 twos are 4, then we have to f is equals to $\frac{-\Delta P D_{eq}}{L\rho\bar{v}^2} = 2f$ or $\frac{-\Delta P}{L} = \frac{2f\rho\bar{v}^2}{D_{eq}}$and then data tortuosity factor further you know irregular packing that we are having in a bed, right.

So, we are not writing λ_1, we are writing λ_2 by the way because λ_1 is valid for a kind of a laminar flow conditions. So, if the flow velocity is changing the fluid element that is traveling or the tortuous path of that for a given fluid element may be different, ok. So, let us take a λ_2. So, if you are rearrange this equation like this you have $2f\lambda_2\rho\left(\frac{\overline{V_0}}{\varepsilon}\right)^2\frac{3(1-\varepsilon)}{2\phi_s D_p \varepsilon}$ So, whole square and then in place of D_{eq} you can write $\frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right)\phi_s D_p$. So, that you put it here substitute here and then simplify it. So, you will get $\frac{\Delta P}{L}=\frac{3\lambda_2 f\rho\overline{V_0}^2}{\phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$, ok. So, λ is tortuous, is a correction factor for the tortuosity or tortuous path that is existing in the packed bed.

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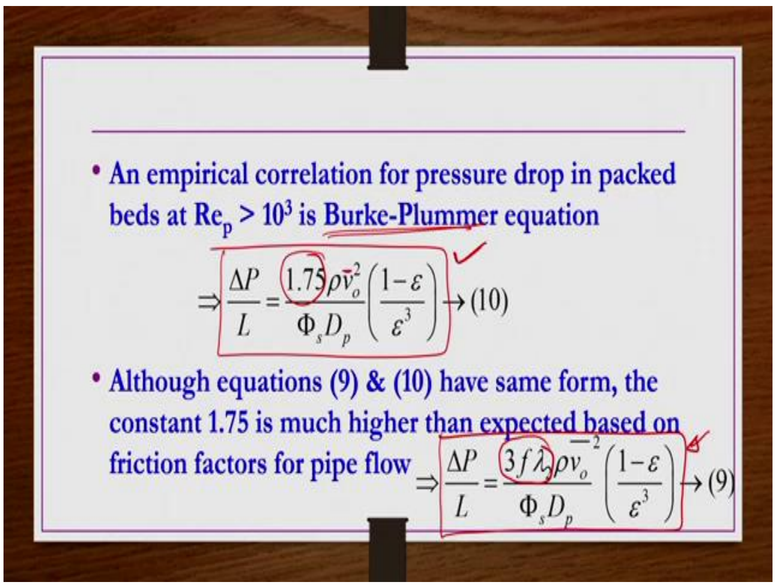

Now an empirical correlation for pressure drop in packed bed $Re_p > 10^3$ is known as the Burke-Plummer equation and it is given as this thing. This is empirical correlation developed by Burke-Plummer. So, according to them for the flow through packed beds when Reynolds number is greater than 10^3; $\frac{\Delta P}{L}=\frac{1.75\rho\overline{V_0}^2}{\phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$

Now, this equation number 10 if you compare with our equation number 9 just we derive theoretically. This is empirical correlation, equation number 10 is empirical correlation developed, developed by these people Burke and Plummer, ok, and then this is the theoretical equation that we just developed based on the theoretical information that is available. So, what we understand here this $3\lambda_2 f$ is equal to 1.75 and rest everything is same, right. So, so, although equations 9 and 10 have same form the constant 1.75 is much higher than expected value based on friction factors by flow for the pipe flow cases.

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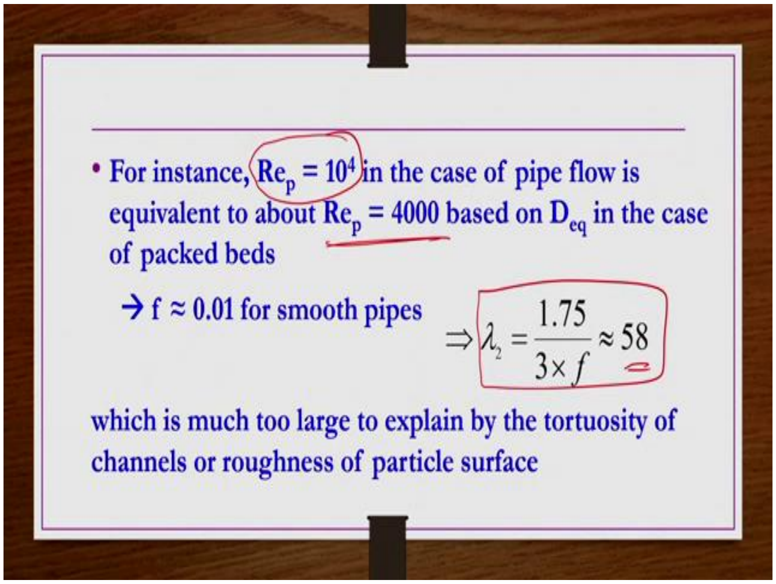

Because for the pipe flow cases, so, whatever the Reynolds number of you know 4000 is there in general for a packed bed which is equivalent to Reynolds number of 10^4 in the case of pipe flow without any packing right under such conditions f is approximately 0.01 for smooth pipes, ok. This is fanning friction factor versus Re_p for 5 flows charts are available from those charts, we can find it out that at these Reynolds number Re_p of 10^4 for pipe flow Reynolds number friction factor is approximately 0.01 for smooth pipe.

In the pipe flow where there is no packing if Re_p is 10^4, so, that is equal to 10^4. In the case of packed bed in general, ok. So, that is more than order of 10^3. So, if you use this 1, this expression in place of $3\lambda_2 f$ is equal to 1.75 you will get λ_2 is equals to 58. So, so much high value which seems to be unrealistic in general, right which is much too lost to explain why the tortuosity of channels of roughness of particle surface is you know, so, large values like you know 58 something like this. However, you know, we can use them I know with a constant 1.7 rather using you know, $3\lambda_2 f$

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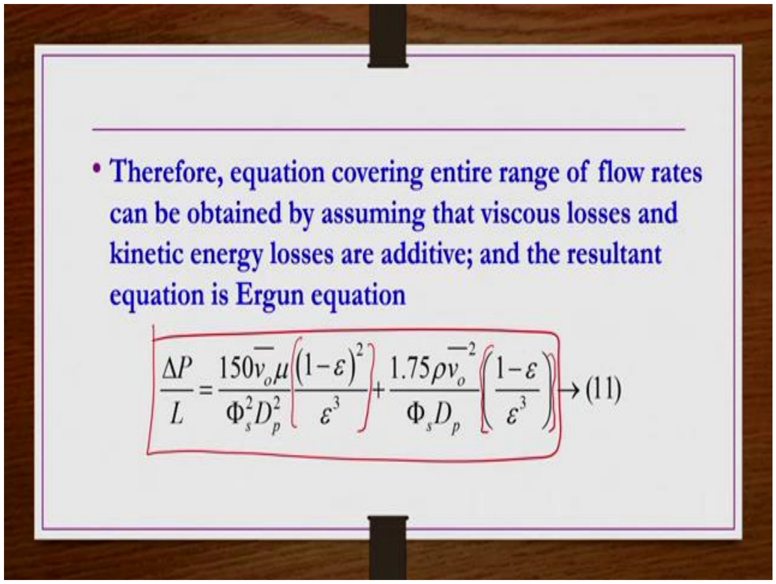

So, covering entire range of flow rates, you know 1 can obtain the pressure drop you know by assuming the viscous losses and kinetic energy is energy losses are additive then we can have a kind of Ergun equation, or the first equation whatever that you know Kozeny-Carman equation that is a $\frac{\Delta P}{L}$ is equal to 150 by that expression. So, that is you know considering only viscous effect, small Reynolds number in the sense only considering the viscous effects, ok this not considering the kinetic energy losses.

So, whereas the Burke Plummer equation that we have $-\frac{\Delta P}{L}$ is equals to 1.75 and something that expression that is only valid for only larger Reynolds number, if the Reynolds number is order of 10^3 are even more that means only conviction forces only dominates, inertial forces only dominate. So, that means, you know they are due to the whatever the losses are there because of the kinetic energy losses and then these two are the either extremes, either extremes 1 extreme and the viscous losses are for this Reynolds number of cases and another extreme is for the larger Reynolds numbers for the kinetic losses, ok or due to the kinetic losses.

So, if you assume that these are you know kind of additive kind of thing for the intermediate range, then you will get you know pressure drop by equation for the entire range of Reynolds number starting from small Reynolds number to the larger Reynolds number, ok. That equation when you add together is known as the Ergun equation and then that we can write it as $\frac{\Delta P}{L} = \frac{150\overline{V_0}\mu(1-\varepsilon)^2}{\varepsilon^3\phi_s^2 D_p^2} + \frac{1.75\rho\overline{V_0}^2}{\phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$

Now, we can see the viscous losses are having some kind of $\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$ terms and then on the kinetic losses, the second term is having $\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$ terms, ok. So, one should be clear, careful about them and then indeed we can find out the reason why it is. So, this is the Ergun equation this first part is you know Kozeny-Carman equation the second part is that Burke Plummer equation if you together we get Ergun equation which is valued for all Reynolds number small to large Reynolds number including the intermediate range of Reynolds numbers.

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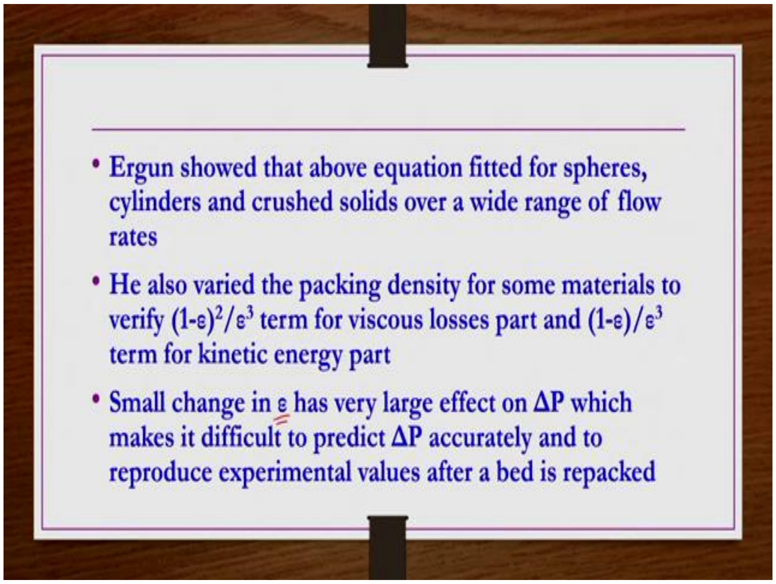

So, Ergun showed that above equation fitted for spheres, cylinders and crushed solids or a wide range flow rates, he has done several experiment for different types of packing material by wearing large range of flow rates and then he find that whatever the Ergun equation that previous equation we are seen, that is you know valid for a wide range of flow rates and in different types of packing material.

He also varied the packing density for some materials to verify why $\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$ term for viscous losses thought that is first part in the right hand side m and then why $\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$ part for the kinetic energy part that is second part that also he found. And then he found that small changes in epsilon has a you know very large effect on ΔP which makes it difficult to predict ΔP accurately and to reproduce experimental values after the bed is repacked.

Once the bed is repacked what will happen that is going to have a kind of you know this enormous void is void space is going to change, ok. It is not going to be same packing as a kind of initial packing. So, under such conditions you know, once the repacking is done, so, there would be small changes in epsilon and then definitely the ΔP is going to be strongly and being affected by this epsilon, ok. So, reproducibility of experimental results are found to be very difficult that especially when the bed is repacked.

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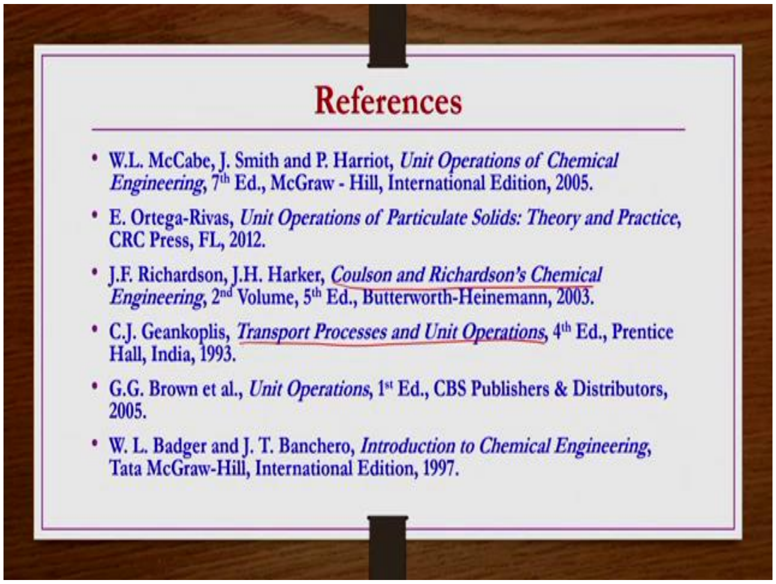

So the references the lecture presented here is primarily prepared from this reference book unit operations of chemical engineering by McCabe, Smith and Harriot. The unit operations of particulates solid theory and practice together there was Ortega-Rivas can also be a good reference book. Coulson and Richardson's chemical engineering series by Richardson and Harker, second volume, you know, is also kind of good reference for the this kind of this particularly flow through pack beds.

Some applications have been taken from this reference book transport processes and unit operations with Geankoplis for this lecture. And then, some applications are also taken from this Coulson and Richardson's chemical engineering series, second volume. Other reference books could be like, you know, better options or unit operations by Brown et al and, then introduction to chemical engineering by Badger and Banchero. Thank you.