Mechanical Unit Operations
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Lecture 22
Flow through Bed of Solids-2

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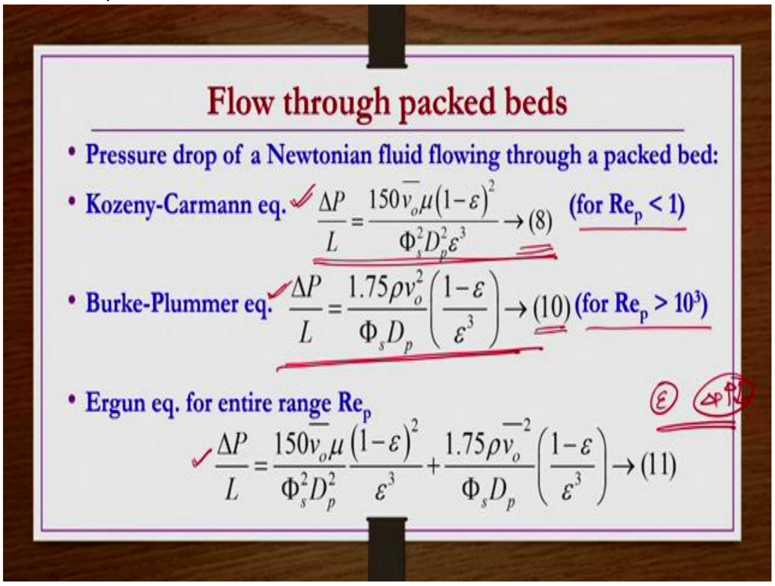

Welcome to the MOOCs course Mechanical Unit Operations. We are discussing flow through packed beds or flow through beds of solids. We have already seen how to develop pressure drop equation for different Reynolds number region that is for small Reynolds number region Re_p is less than 1 as well as the large Reynolds number regions Re_p greater than 10^3 and then we also assumed that the contribution due to the viscous losses and kinetic losses are kind of additive in total pressure drop that is occurring while a fluid is flowing through a packed bed.

We assume that fluid is a kind of a Newtonian and incompressible Newtonian fluid. So that is what we have seen. So we just have a kind of, you know summary of the equations what we have derived in the previous lecture because this lecture is a kind of a followed part of the previous lecture. So flow through packed beds, pressure drop of a Newtonian fluid flowing through a packed bed. We have seen Kozeny Carmann equation for Re_p less than 1, we have derived it as $\frac{\Delta P}{L} = \frac{150\bar{v}_O\mu(1-\varepsilon)^2}{\Phi_s^2 D_p^2 \varepsilon^3}$.

This equation for pressure drop so that is if a fluid is flowing through a packed bed but under very small flow rates or the having the Reynolds number very small, less than 1 then laminar flow conditions exist and under those laminar flow conditions the pressure drop for a fluid we can obtain by this equation. The pressure drop of a fluid flowing through a packed bed, we can obtain on this equation provide the Reynolds number is less than 1.

Because here in the right hand side everything is known. $\bar{v}_o$ is a kind of superficial velocity or empty tower velocity. μ is the viscosity of the fluid, are known in general and then ε is the voidage of the bed, so that is also known. For the given type of particle the size of the particle or the sphericity of the particle are known in general. So everything is known in the right hand side. So simply one can substitute these parameters and then find out what is this pressure drop.

Similarly, for Re_p greater than 10^3 then we have seen that Burke Plummer equation that holds very good, that is $\frac{\Delta P}{L} = \frac{1.75\rho v_o^2}{\phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$. This equation valid for the calculating the pressure drop of internal fluid flowing through a packed bed provided the Reynolds number is very large that is, you know turbulent conditions prevail within the bed.

Then what about the intermediate range of Reynolds number? Let us say the flow rate is not very small. The flow rate is not very large that the flow conditions or the Reynolds number are less than 1 or greater than 10^3 respectively. Then what we do? We assume that the contribution due to these small Reynolds number flows and high Reynolds number flows whatever the pressure drops are there, or the viscous losses, contribution due to the viscous losses in the pressure drop and then contribution due to the kinetic losses in the pressure drop are additive.

So whatever the first equation is there, that, the equation number 8 indicates, you know kind of contribution of the viscous losses in the total pressure drop that has occurred for a fluid flowing through packed bed. Similarly, equation number 10 provides a kind of information about the contribution of kinetic energy losses in the pressure drop calculation for a fluid flowing through a packed bed under high Reynolds numbers.

So but in general, there may be cases where the contribution of viscous losses and then kinetic losses both may of significant importance and both of them may be contributing to the pressure drop of the fluid flowing through this packed bed. So under such conditions how to find, we found that, you know if we add together, you know these two individual contributions then whatever the pressure drop is there, that is suitable for the entire range of Reynolds number starting from small Reynolds number to the large Reynolds number.

So this is the Ergun's equation which is valid for all ranges of Reynolds number encompassing small Reynolds number to the large Reynolds number range. Then Ergun has also found this equation is very much suitable for a different types of packing materials like spheres, cylinders, crushed materials etc and then he also found that, you know this equation valid for the wide range of the flow rates, wide range of the flow rates, small range to the larger range flow rates that is nothing but small Reynolds number to the larger Reynolds number ranges this equation holds good.

He also found that by doing several experiments small change in ε is going to show a kind of a significant change in a kind of pressure drop, whether it increasing or decreasing that depends on whether ε is increasing or decreasing. So this, even a small change like, let us say if you change the voidage from 0.44 to 0.46 it is going to show a kind of significant effect on this, pressure drop. That also Ergun has found by doing several experiments, right. Up to this part we have seen the previous lecture.

Now we try to find out the magnitude of viscous losses contributing to the pressure drop here. Similarly, magnitude of kinetic energy losses which is contributing to pressure drop of a fluid flowing through this packed bed. How to do this one? Simply whatever the pressure drop is there here, from this equation, the pressure drop if you divide by kinetic energy that is half rho v square then we get a kind of a magnitude of kinetic energy losses but in which equation should we make this adjustment? That we should do only in equation number 10 if we are looking for kinetic energy losses because equation number 10 is, you know primarily talks about only the kinetic energy losses contribution in the pressure drop.

So if you wanted to find out the magnitude of the viscous losses in this pressure drop contribution, in this pressure drop then you know, the pressure drop should be divided by the viscous forces per unit area and then equation that should be used that is the equation number 8 because equation number 8 talks about the, you know contribution of the viscous losses in the pressure drop calculations for the Newtonian fluid going through packed bed here. So that is what we are going to do now here.

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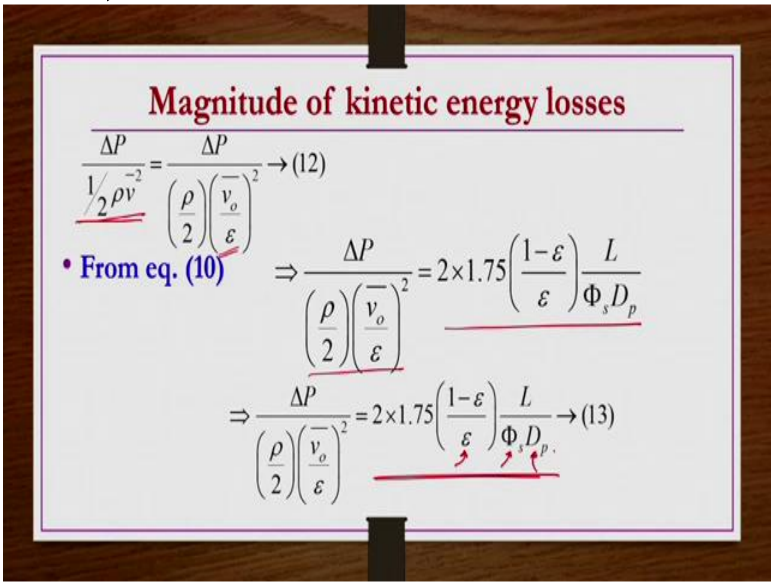

So let us start with the kinetic energy losses. So whatever the ΔP is there that we are dividing by $\frac{1}{2}\rho\bar{v}^2$, v is average velocity within the bed, v bar here we have taken, designation, notation $\bar{v}$, it is not v^{-2} it is $\bar{v}$, so this $\bar{v}$ is nothing but the average velocity of the fluid within the bed.

That is not known but we found that this $\bar{v}$ is nothing but $\bar{v}$, $\frac{\bar{v}_0}{\varepsilon}$that is the superficial velocity or empty tower velocity or under this pressure drop condition without packing if you allow the fluid whatever the velocity is existing, average velocity is existing in the empty tower that velocity is $\bar{v}_0$ so that is in general known. And then ε is kind of a voidage of the bed. So everything is known.

So now what we do in equation number 10 that is the pressure drop at high Reynolds number that is Burke Plummer equation that we did, on that equation we rearrange such that on both the side that equation is we divide by $\left(\frac{\rho}{2}\right)\left(\frac{\bar{v}_0}{\varepsilon}\right)^2$ then we get the magnitude of the kinetic energy losses in the pressure drop calculation for the flow of Newtonian fluids through packed beds. When we do this one, we get in the right hand side, $2 \times 1.75\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{L}{\phi_s D_p}$.

So this right hand side, this equation everything is known. Voidage is in general known for a given bed. Sphericity of the particles in general known and then equivalent diameter of the, volume equivalent diameter of the particles are known for a given system.

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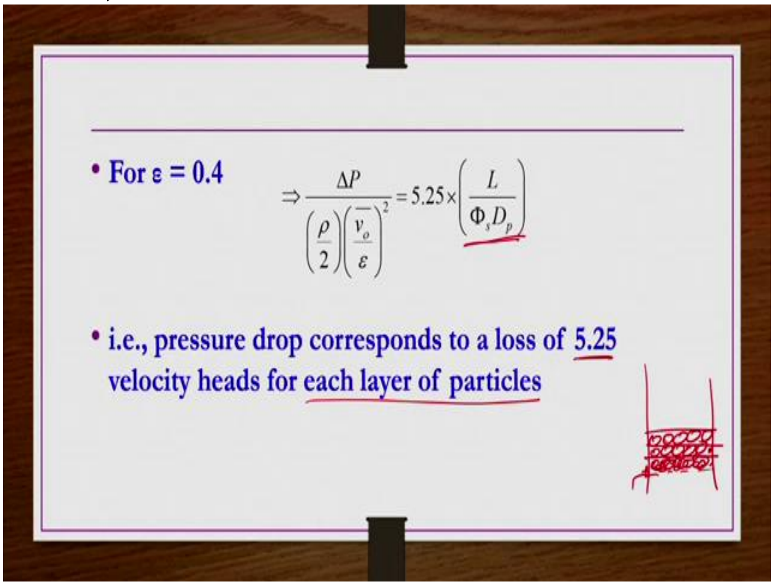

Let us say if you take a voidage 0.4 and then if you calculate what is the right hand side term, you will get right hand side term of the previous equation, $5.25 \times \left(\frac{L}{\phi_s D_p}\right)$. What is $\frac{L}{\phi_s D_p}$ indicates L is the height of the packing. D_p is the, you know the size of the particle, the equivalent size of the particle or volume equivalence sphere diameter of the particle, that is known for a spherical particle D_p is nothing but the diameter of the particle. For a short cylinders D_p should be, you know equivalent diameter we have to find out for short cylinder and then multiply by its sphericity 0.874 that is what we have to do.

This indicates you know the pressure drop you know corresponds to a loss of 5.25 velocity heads for each layer of particles whatever this $\frac{L}{\phi_s D_p}$ is indicating the layer of particles. So the pressure drop under these conditions when ε is 0.4 it corresponds to a loss of 5.25 times the velocity heads for each layer of particles.

You have a kind of, you know bed like this. There is a perforated plate, you know and then different particles are there like a kind of, you know layers, kind of particulate and they are, if it is spherical particles you can see like, you know of spherical particles and all of them are of same size you can easily say that these particles are, one particle, another they are forming a kind of layer.

So each layer is going to offer, let us say this is one layer, so this one layer, this two layer, this is second layer let us say, third layer if I write you know so each layer of particle, you know per, you know whatever the packed bed height that has been taken, that is offering 5.25 times the velocity head, you know, of a pressure drop. That is what is meant by pressure drop response to a loss of 5.25 times velocity heads for each layer of particles.

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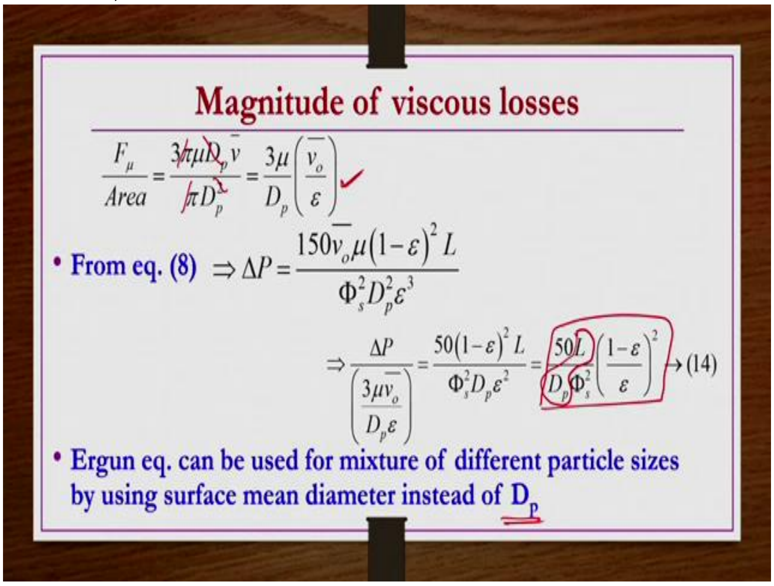

Now similarly we try to find out magnitude of viscous losses. Viscous losses what? if viscous forces are there, the viscous forces whatever are there divided by area if you do, so you will get the, you know viscous losses per area. This particular term if you, the pressure drop if you divide by this particular term, then you will get the, kind of you know magnitude of the viscous losses or the contribution of the viscous losses in the overall pressure drop of, you know fluid flowing through packed bed.

What is this Fμ, this viscous force? We have already seen by Stokes' drag on a single particle whatever the Stokes' drag is there or the drag force in the Stokes' limit is there, that is nothing but the viscous force, because that is the only force acting under the small Reynolds number range and that is the viscous force. So that is $3\pi\mu D$ that is what and then, the area of the particle that is pie let us say if you take Dp^2.

That if you do you get this term like you know π and π is cancelled out here, and then $\frac{3\mu}{D_p}$ so 1 D_p and then square of this D_p is cancelled out, so $\frac{3\mu}{D_p}$, $\bar{v}$ is nothing but $\frac{\bar{v}_0}{\varepsilon}$. So from the Kozeny

Carman equation whatever is the $\frac{\Delta P}{L} = 150\pi^2 D_p^2$ and all that term is there, from that equation if you divide by this particular term then you will get the magnitude of viscous forces.

Why that particular equation Kozeny Carman equation, because that is the equation showing the pressure drop due to the viscous losses only. No other contributions are included in that equation and then indeed we wanted to know the magnitude of viscous losses. We do not want to find out, you know, include the other terms here.

Then from equation 8 we have this $\Delta P = \frac{150\bar{v}_0\mu(1-\varepsilon)^2 L}{\phi_s^2 D_p^2 \varepsilon^3}$. So this is the Kozeny Carman equation. Now both side this equation if you divide by $\frac{3\mu\bar{v}_0}{D_p\varepsilon}$ then you will get this particular term. So this indicates a kind of a magnitude of viscous losses, you know for the, for one layer of particles. So that is for one layer of particles the pressure drop is going to be, you know $\frac{50L}{D_p\phi_s^2}\left(\frac{1-\varepsilon}{\varepsilon}\right)^2$ times the viscous forces.

So that is what it indicates. But this here also we have $\frac{L}{D_p}$ so that indicates per layer of particles. So this is about magnitude of viscous losses and then kinetic losses. So now these equations we have developed for a generalized kind of cases of particles are having same size and shape. And their size and shape we brought into the picture by including both ϕ_s as well as the equivalent sphere volume equivalent diameter D_p^2 we have included here.

But in general you may have a kind of variety of particles, you know, different types of mixtures of particles like you know, a few particles may be spherical, a few particles may be hemispherical, a few particles may be, you know short cylinders, a few particles may be raschig rings like that you may have different fractions. So under such conditions also these equations are valid especially Ergun equation is valid as he has already tried different types of particles.

So but under such conditions what you have to do, you have to find out the surface mean diameter of the particle for this mixture of, you know particles the way that we have done you know previously in one of our lectures in screen analysis. There for a mixture of particles we have find, we found different types of, you know mean diameters, surface mean diameter one has to take here for that case and then same equation can be used exactly the similar way here. So Ergun equation can be used for mixture of different particle sizes by using surface mean diameter instead of D_p, simply that is it.

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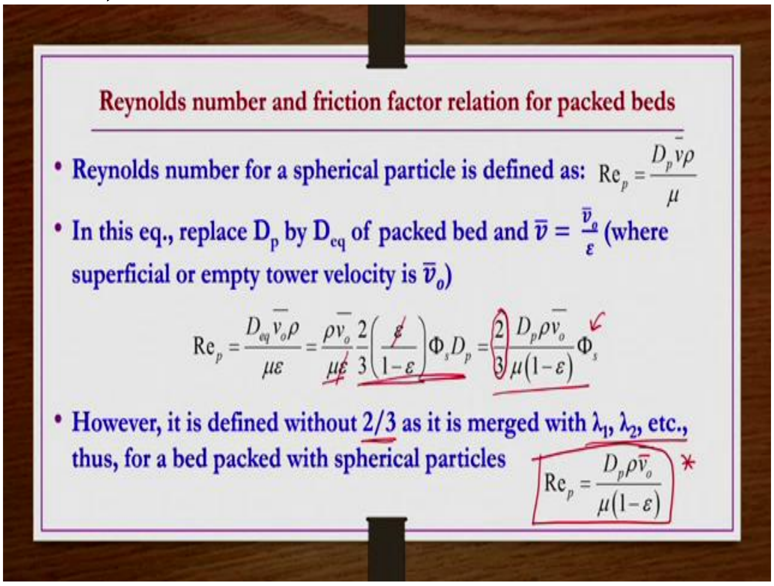

So now here, you know the pressure drop equations whatever, the equation that we have derived here, we derived in Kozeny Carman equation, we have derived in Burke Plummer equation and then Ergun equation we have written so all these equations are in a kind of dimensional form.

But in general in transport phenomena, momentum transfer especially it is beneficial to write the final solutions in terms of dimensionless parameters. So what we do, the pressure we try to write in a kind of dimensionless friction factor and then remaining terms we write such a way that you know some kind of terms like Reynolds number etc that will come into the picture. In the dimensional analysis or dimensional consideration if you do this equation you will get 2 dimensionless parameters.

One is the friction factor for a packed bed; another is a kind of Reynolds number. So before going to represent this Ergun equation in terms of those dimensionless parameters we try to write what are those dimensionless parameters for a packed bed. For a single particle Reynolds number we know that $\frac{D_p \bar{v} \rho}{\mu}$. But if you have a packed bed packed with different types of particles then what is the Reynolds number? Is it the same one or different one? That we define.

Similarly, for friction factor, for flow through empty column we know that tau w by half rho v square, that we know. But is it the same thing here in packed bed when we have the column or tower is filled with some kind of packing? That is what we are going to see first and then generalize this second equation in terms of those friction factor and Reynolds number because that will be easier way of using this equation for calculations, and as well as there could be, serve as a kind of generalized purpose rather than specific to some cases only.

So Reynolds number and friction factor relation for packed beds, so as I said for a single particle we know Reynolds number and then for single empty column without packing we know the friction factor. We try to find the expression for a packed bed or a column packed with a few particles, few types of particles. For single particle, especially for spherical particle Reynolds number we know it as $Re_p = \frac{D_p \bar{v} \rho}{\mu}$, $\bar{v}$ is nothing but the average velocity, ρ is the density of the fluid, μ is the viscosity of the fluid, D_p is the diameter of the spherical particle.

Now in this equation D_p we replace by D_{eq} and $\bar{v}$ we replace by $\bar{v}$, $\frac{\bar{v}_0}{\varepsilon}$ in order to bring in the effect of the packed bed characteristics in this Reynolds number. Then what will happen? We will have $Re_p = D_{eq}$; $\bar{v}$ is nothing but $\frac{\bar{v}_0}{\varepsilon}$, so $\frac{D_{eq} \bar{v}_0 \rho}{\mu \varepsilon}$ we get. So $\frac{\rho \bar{v}_0}{\mu \varepsilon}$ we keep as it is here, then next level what we do? We write D equivalent expression.

D equivalent expression yesterday and in previous lecture we have derived it as $\frac{2}{3}\left(\frac{\varepsilon}{1-\varepsilon}\right) \phi_s D_p$. So when you write this one, this equation you will get $\frac{2}{3} \frac{D_p \bar{v}_0 \rho}{\mu(1-\varepsilon)} \phi_s$. So this ε and this ε is cancelled out. So for spherical particle especially the ϕ_s is equal to 1.

Then this 2/3 we in general we do not include because they are in general merged with λ_1, λ_2 etc those kind of constants, so without taking this 2/3 constants and then for, if you assume that particles are spherical particles then Reynolds number is these things, $\frac{D_p \bar{v}_0 \rho}{\mu(1-\varepsilon)}$ is the Reynolds number for a packed bed packed with spherical particles. If it is packed with non-spherical particle then ϕ_s will also come into the picture.

Only the 2/3 will not be there because those constants in general we have the tortuosity factors λ_1, λ_2 etc those things we are having so in those things we mix up and then we write only the one, you know Reynolds number because this is what we get if you do the dimensional analysis or dimensional consideration if you do and then try to find out what are the dimensionless parameters Re_p you are going to get this one. You are not going to get anything is multiplied by those kind of thing so either of the reason that 2/3 can be taken off.

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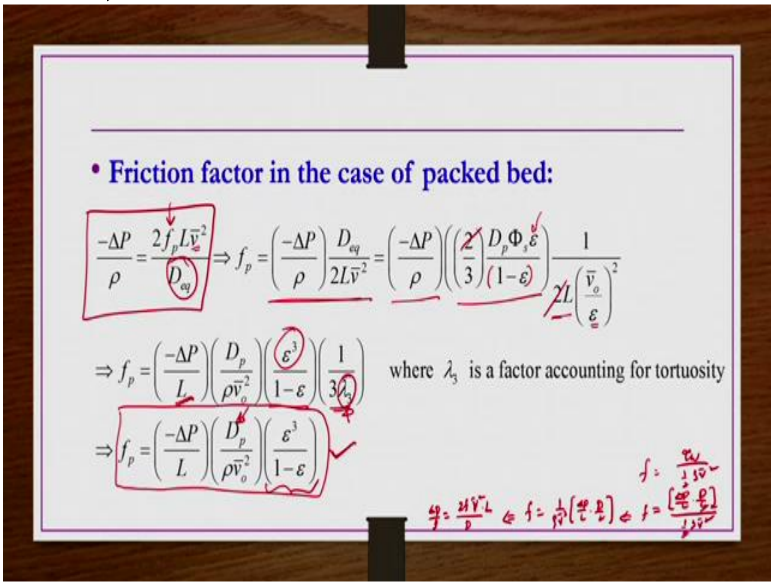

Now similarly what we do, friction factor, friction factor in the case of packed bed. Before that what we do friction factor in empty tower what we have, we have $\frac{\tau_w}{\frac{1}{2}\rho\bar{v}^2}$. And then τ_w is nothing but $\frac{\Delta P}{L}, \frac{D}{4}$ is nothing but τ_w for a kind of, you know Newtonian fluid flowing through an empty column and then divided by this $\frac{1}{2}\rho\bar{v}^2$ is as it is.

Then here what we get is $\frac{\Delta P}{\rho}$, if you rearrange this equation you will get here from this equation, from this equation if you rearrange $\frac{\Delta P}{L}$ is equals to half, or if you rearrange this equation, f is equals to 2 one's are, 2 two's are, so f is equals to $\frac{1}{\rho\bar{v}^2}$, this multiplied by $\frac{\Delta P}{L}\frac{D}{2}$, you will get. Further you get here $\frac{\Delta P}{\rho}$, if you keep one side so then $\frac{2f\bar{v}^2}{D} \times L$. So that is what you will get.

$\frac{\Delta P}{\rho} = \frac{2fL\bar{v}^2}{D}$. Here now what we do, for f we are writing f_p, p stands for packed bed, in order to just differentiate between the friction factor for the case of empty column or empty pipe and then friction factor with column with having a kind of packed material. So this f_p is for the packed bed and then D is replaced by D equivalent. Similarly, $\bar{v}$ is also replaced by $\frac{\bar{v}_0}{\varepsilon}$. So that is what we are going to do.

From here we wanted to find out friction factor so let us keep P only one side, remaining terms if you write $\left(\frac{-\Delta P}{\rho}\right)\left(\frac{D_{eq}}{2L\bar{v}^2}\right)$ so $\left(\frac{-\Delta P}{\rho}\right)$ is as it is. D equivalent we found it as $\left(\frac{2}{3}\right)\left(\frac{D_p \phi_s \varepsilon}{(1-\varepsilon)}\right)$ in one of the previous lecture, okay and then 1/2 L is as it is and then $\bar{v}$ is like, you know, $\left(\frac{\bar{v}_0}{\varepsilon}\right)^2$ okay because there is a square for $\bar{v}$ so then here also we have the whole square term.

So now this equation, now this 2 and this 2 can be cancelled out. So here $\frac{\Delta P}{L}$ we can write it as one term, and the $\frac{D_p}{\rho \bar{v}_0^2}$, we can write one term, here there is one ε and then here there is ε^2 so we club together, so we get ε^3 and then $(1-\varepsilon)$ is as it is, and then this 1/3 is as it is, but there is a kind of tortuosity factor.

So let us consider one factor λ_3, in order to bring the tortuosity factor because this L is the packing length, height of the packing but the fluid element is traveling more than the packing length because of tortuous paths. So because of that one, lambda, one factor should come. Already we have taken λ_1, λ_2 for previous derivation so let us take λ_3, so, but however this, you know, when you do the dimensional consideration you take only the parameters.

We do not take the constants etc in the picture, or those can be clubbed with the other kind of the constants of the final equation that we are going to derive. So without $\frac{1}{3\lambda_3}$ it has to be defined because of the dimensions considerations, we take only the parameters, we do not take the constants. So here friction factor in the case of packed bed what we get $f_p = \left(\frac{-\Delta P}{L}\right)\left(\frac{D_p}{\rho \bar{v}_0^2}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right)$. This is what we get.

So now this is the additional term coming because of the, you know packing material and then this part is also coming because of the packing material. So, yeah anyway so now what we do this Ergun equation we non-dimensionalize so that Ergun equation can be written in terms of this f_p friction factor or f versus Re_p or Re that is what we are going to do now.

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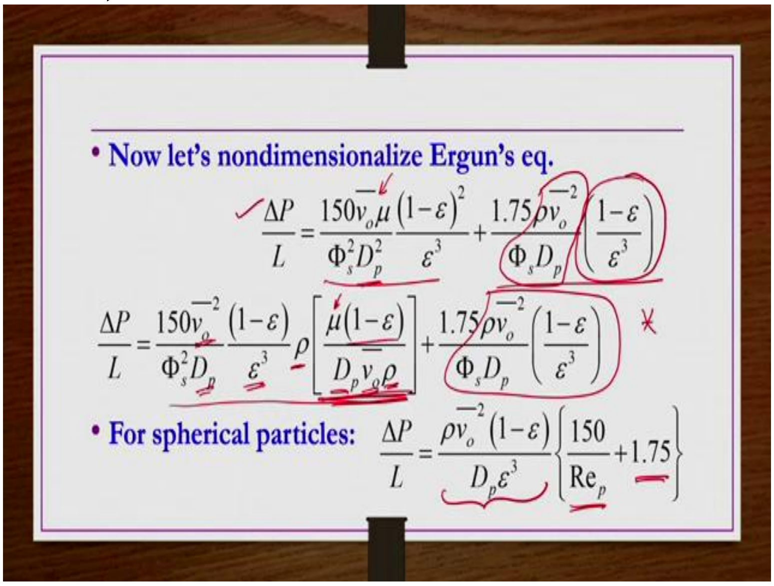

So this is the Ergun's equation that we had seen which is having the additive contribution of both viscous losses and then kinetic losses. So this is the viscous losses part. This is the kinetic losses part and then this is the total overall pressure drop. So overall pressure drop is a, having the additive contribution of the viscous losses and then kinetic losses.

Now this equation we are going to rearrange. Rather rearranging what we do, we take this particular term like you know, write in the first term whatever the terms we are having. So we rearrange such a way that you know this $\frac{\rho\bar{v}_0^2}{\phi_s D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$.

This we keep here in the right hand side also, so here so that is the reason we have here, you know $\frac{\rho\bar{v}_0^2(1-\varepsilon)}{{\phi_s}^2 {D_p}^2}$ is as it is, so whatever the μ is there, so that we are writing outside in the parenthesis, extra terms we are writing here in this parenthesis after taking out these, these rounded terms, whatever these terms are there we wanted to have common in both the terms.

So ρ is required here which is not there so we are multiplying 1 ρ and then dividing 1 ρ here. And then out of $\frac{(1-\varepsilon)^2}{\varepsilon^3}$, $\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$ we are keeping here so $(1-\varepsilon)$ is remaining here and then out of Dp^2, one D_p is taken out so one D_p is remaining here, and then here in first term initially we have $\bar{v}_0$ only but we want $\bar{v}_0^2$ so one $\bar{v}_0$ term is here dividing so that it will be balanced. So this is the additional term we are getting.

So now why are we doing these things? So this particular term whatever is rounded here, this term will be taken common, so that we have this 150 multiplied by this term and then if you multiply that will be $\frac{\mu(1-\varepsilon)}{D_p \bar{v}_0 \rho} + 1.75$. What is this term in this square parenthesis? It is nothing but $\frac{1}{Re_p}$, it is nothing but $\frac{1}{Re_p}$, that is, you know Reynolds number for the case of packed bed.

So for the spherical particles we just strike out these phi s terms for the simplicity we take the spherical particle otherwise this D_p should be multiplied by the ϕ_s then, so $\left(\frac{\Delta P}{L}\right)$ in the right hand side $\frac{\rho \bar{v}_0^2 (1-\varepsilon)}{D_p \varepsilon^3}$ we are taking common after taking ϕ_s is equal to 1, that is for spherical conditions. So from the first part we have $150 \times \frac{\mu(1-\varepsilon)}{D_p \bar{v}_0 \rho}$. So that is nothing but $\frac{1}{Re_p}$. So we can write 150/D_p and then this part is anyway taken common so only 1.75 is remaining.

So now what we do? We take this particular term whatever the term that we have taken common. That we will take to the left hand side and combined with the pressure drop term to see what we will get.

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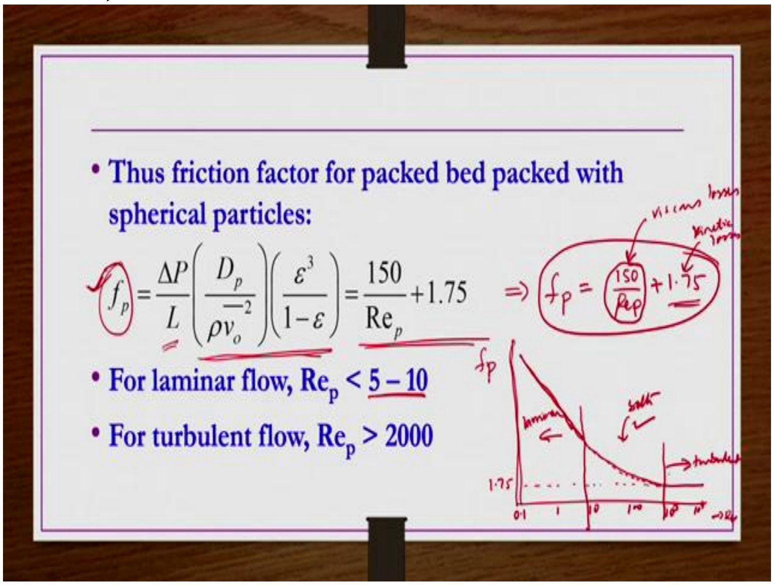

So right hand side we are having only $\frac{150}{Re_p} + 1.75$ and then whatever the factor manipulation term was there that we have taken to the right hand side and then combined with $\left(\frac{\Delta P}{L}\right)$, already existing $\left(\frac{\Delta P}{L}\right)$. So if you see the friction factor derivation that we just derived, $\left(\frac{\Delta P}{L}\right)\left(\frac{D_p}{\rho \bar{v}_0^2}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right)$ is nothing but friction factor.

So that is what we understand here, from here so friction factor for packed bed is nothing but 150 by Reynolds number for packed bed plus 1.75. This is what we get. And then this equation is having 2 contributions again. The first term, the first contribution in the right hand side is nothing but the viscous losses. So whatever this term is there, that is going to have influence if the Reynolds number is small and then second part is known as the kinetic losses or the contribution due to the kinetic losses and this part is going to have an influence or the dominance or dominant role when the Reynolds number is very large, or the flow is very large, so that is for laminar flow Re_p should be less than 5 to 10 in general.

Basically it should be less than 1 by default but people have found by several experimental studies it can be extendable even up to 5 to 10, Re_p 5 to 10 also it is going to be laminar flow and under such condition f can be directly taken as 150 by Re_p, not necessary to have 1.75, even if it is added that is going to have very small difference.

Similarly, turbulent flow condition, when Re_p is greater than 2000 then fp is equals to 1.75 only, that is a constant value, so that is not going to change in general. So whatever the first part, viscous part contribution also if you add for larger Reynolds numbers so that is going to be very small value to show any influence on the friction factor value, or from there whatever the pressure drop that you calculate.

So that means fp versus Rep if you plot, let us say .1, 1, 10, 100, 10 cube, 10 power 4 like this if you have then so it should be like something like this. For small Reynolds number it is going to be linear something like this up to Re 10 something like this and then for large Reynolds numbers it is going to be constant like this corresponding to value of you know 1.75, and then in-between Reynolds number it is going to have curved shape like this.

So this is in general, valid for viscous part of laminar flow and this is in general, this part is valid for the turbulent flow or larger Reynolds number flows where only kinetic losses are predominant. Here both contributions are going to have a significance. So this is how you know we have to do the, you know conversion of pressure drop information in terms of, you know friction factor and then Reynolds number for given equation. Now what we do, we take a few example problem to see how to make use this equations in order to know pressure drop of a Newtonian fluid having packed bed having certain voidage.

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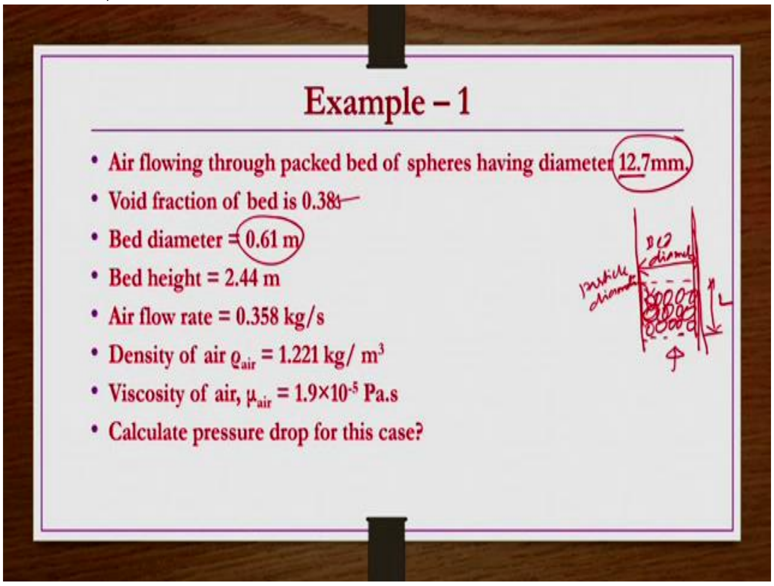

So example one, air flowing through packed bed of spheres having diameter 12.7 mm, so spherical, you know there is a packed bed which is packed with a kind of spherical particles having diameter 12.7 mm. Void fraction that has formed because of this packing is 0.38. Bed diameter, so it is 12.7 mm is the particle diameter and then 0.61 is the bed diameter. So see this diameter, the column is there, so the column in which you are doing this packing kind of thing.

So this is nothing but bed diameter and this is nothing but particle diameter. Our particles are having certain diameters. Assume all the particles are having same size and shapes. Spherical shape is given and then size is 12.7 mm that is also given. Bed height, the L, the packing height let us say you pack up to this part, so the packing height is L. It is not the length of the entire column, it is only the packing height. 2.44 mm is the packing height and then air is flowing through this bed at 0.358 kg per second.

It is not given volumetric flow rate, it is mass rate is given and density of air is 1.21 kg per meter cube. And then density of viscous, and then viscosity of air is 1.9 x10^{-5} Pa.s so calculate what is the pressure drop.

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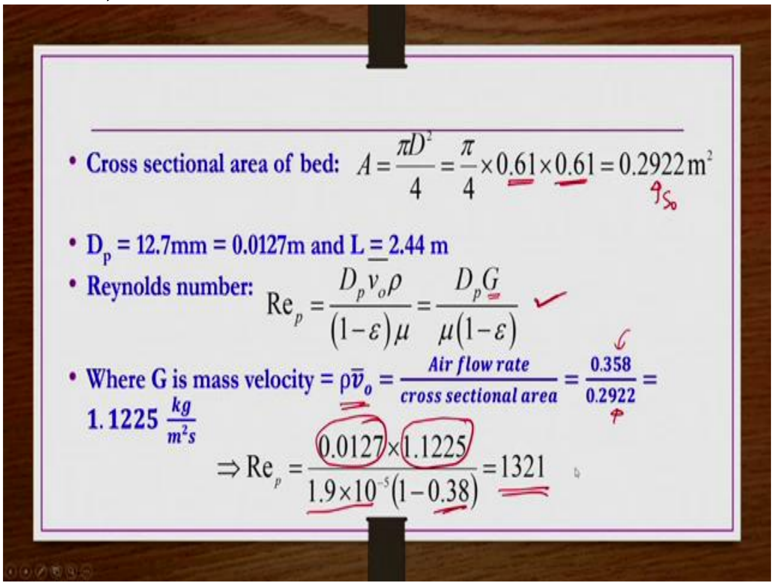

Very simple you have to find out the Reynolds number first of all, whether it is under the small region or the high flow region so that we can use only that particular part. So first actually you know in order to know the Reynolds number you need to know the velocity, superficial velocity at least $\bar{v}_0$ you know, you need to know. But it is not given. Rather it is given the mass velocity. If the mass velocity, if you multiply by the cross sectional area then what will, I mean if you have the density, if the density is multiplied by the cross-section area then you will get the mass velocity = $\rho\bar{v}_0$.

So for that we need this cross-section area of the bed, $\frac{\pi D^2}{4}$, D is now, this is the, you know column diameter. I am not writing D_p. It is not particle diameter. It is for the bed, the column, whatever the column that has been taken for packing that column diameter is 0.61 meters. So it is having the cross-section area of 0.2922 m^2. It can be written as S naught also because in our derivation this we have represented as S_0. So D_p is given as 12.7 mm and then height of the packing is 2.4 meters.

So the Reynolds number $Re_p = \frac{D_p \rho \bar{v}_0}{(1-\varepsilon)\mu}$for the packed bed, we have just derived it. So here $\rho\bar{v}_0$ if you multiply together then you can write it as a kind of mass velocity. So that means, because v naught bar is not given, mass rate of the air is given. So from there you can find out, you know somehow this mass velocity. So that is the reason we have written the Reynolds number in terms of G that is mass velocity is nothing but $\rho\bar{v}_0$.

So that should be nothing but air flow rate divided by the cross-sectional area. So whatever the air flow rate is there, if you divide by this cross-sectional area then you will get mass velocity in kg/m^2.s. So this if we use here in this equation Reynolds number then you will get 1321 as the Reynolds number. The air flow rate is given as 0.358 kg per second and then cross-sectional area of the bed that we have obtained it as 0.2922-m^2. So the mass velocity G is going to be 1.1225 kg/m^2.s. So that you substitute here in the Reynolds number here. This is the mass velocity. This is the diameter of the particle and this is the viscosity and 0.38 is ε. So Reynolds number is coming to be 1321.

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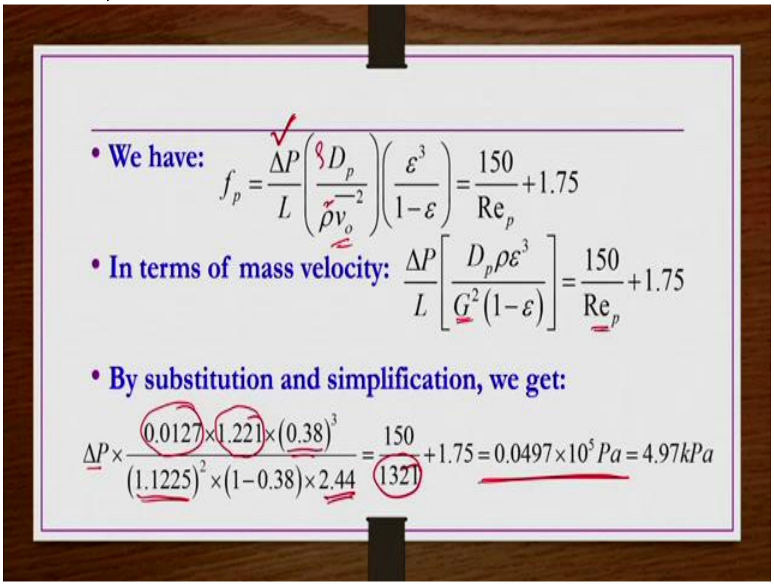

So now we have this equation $f_p = \frac{150}{Re_p} + 1.75$. We need to find out ΔP. We were not asked to find what is the friction factor, okay. So in terms of mass velocity this we have to rearrange this equation. Why, because here also we have this v bar term which is, $\bar{v}_0$ term which is not known. So what we do, you multiply by ρ and then divide by ρ so that in the denominator we have $\rho^2 \bar{v}_0^2$ so that we can write G^2.

That we can write as G^2 so then we get $\left(\frac{\Delta P}{L}\right)\left[\frac{D_p \rho \varepsilon^3}{G^2(1-\varepsilon)}\right] = \frac{150}{Re_p} + 1.75$. So now here everything is known. Here everything is known. G also we have calculated. Rest everything is given. Re_p we have already calculated. So we substitute here.

So $\left(\frac{\Delta P}{L}\right)$ is 2.44 meter. D_p is 0.0127 meters and then ρ is given as 1.221 kg per $meter^3$, epsilon is given as 0.38. G we calculated as 1.1225. Re_p we calculated as 1321. So when we substitute all these things and simplify you will get 0.0497×10^5 Pascals that is nothing but 4.97 kPa.

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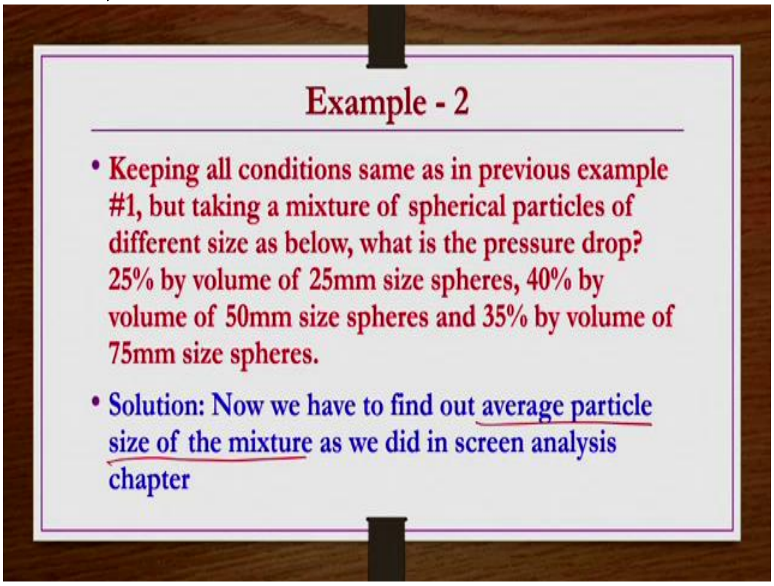

So now what we do? We take another example problem. So here what we do, we have seen like in a, you know in one of the statements while we deriving the, after derivation of the Ergun equation that Ergun equation can also be used for the mixture of particles, only thing that if you are using that equation for mixture of particles then D_p has to be replaced by the surface mean diameter. So that, on that basis we are going to do this problem now.

So keeping all conditions same as in previous example number one that we just solved but taking a mixture of spherical particles of different size as below then what is the pressure drop? In the previous example problem spherical particles were only taken but all of these particles were having same size. Now also the spherical shape is there but the different fractions are there. 25% by volume 25 mm size spheres are there, 40% by volume 50 mm size spheres are there and then 35% by volume 75 mm size spheres are there. So then you, what you have to do, solution, the screen analysis. We have to find out average particle size of the mixture as we did in the screen analysis chapter.

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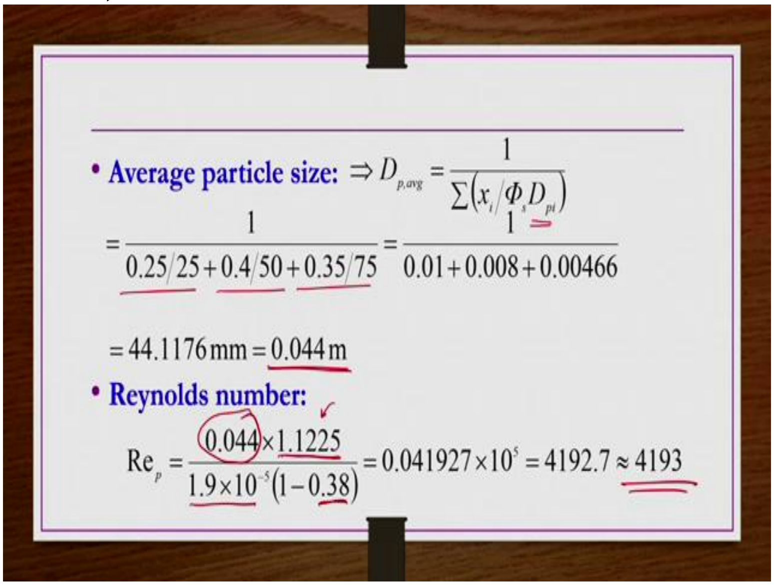

So when we do it the average particle size D_p average, especially surface mean diameter if you name it, it is $\frac{1}{\Sigma(x_i/\phi_s D_{pi})}$is, you know spherical particles different size you know 25 mm, 40 mm like that it is given, you know. 25 mm, 50 mm and then 75 mm, ϕ_s is 1 because all particles are spherical shape only here also, x_i is also given, you know 0.25, 0.4 and 0.35 fractions.

When you substitute all these value, 0.25 by 25 mm, 0.4 by 50 mm and then 0.35 by 75 mm, add them together and then take the reciprocal. Then you will get 44.1176 mm which is nothing but 0.044 m. So corresponding to this D_p value what is the Reynolds number that we can find out. So when we do this one, D_p is 0.044, in this case we have not taken rho v bar but we have calculated G because $\bar{v}_0$ is not given. G is, we calculated as 1.1225 in the previous problem, same thing we have to use.

Viscosity whatever is, previous problem is, same thing we have to use here. voidage 0.38 given in the previous problem, same thing we are using here. Except D_p rest all the parameters conditions are same. So when you substitute these values here in the Reynolds number definition then you will get Reynolds number is approximately 4193.

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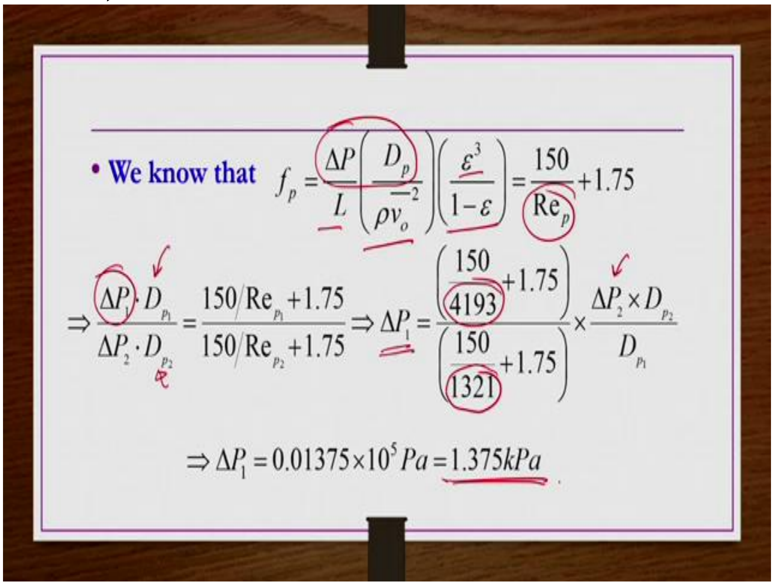

Then we have seen that $f_p = \frac{150}{Re_p} + 1.75$. So what we can see here $\Delta P D_p$ is there and all these are same in both the cases, previous case as well as this case. Only this $\Delta P D_p$ is changing and then this Re_p is changing. Rest everything is same.

So we can write it as 2 cases, case 1 previous problem, case 2 this case our present problem. So then we can write $\frac{\Delta P_1 D_{p1}}{\Delta P_2 D_{p2}} = \frac{\frac{150}{Re_{p1}} + 1.75}{\frac{150}{Re_{p2}} + 1.75}$ that we can write so that to reduce our calculations. When we do this one, so ΔP_1 let us take present case because P_1 I have used 4193 in the calculations.

So ΔP_1 would be present case here so you will get only ΔP_1 is not known, D_{p1} we just find it out as a 44 mm something like this. Previous problem Re_p is 1321. This problem Re_p is 4193. So then we substitute D_{p2} whatever that 4.97 kilo Pascal previous problem and then you do the calculation. It is going to be 1.375 kilo Pascal.

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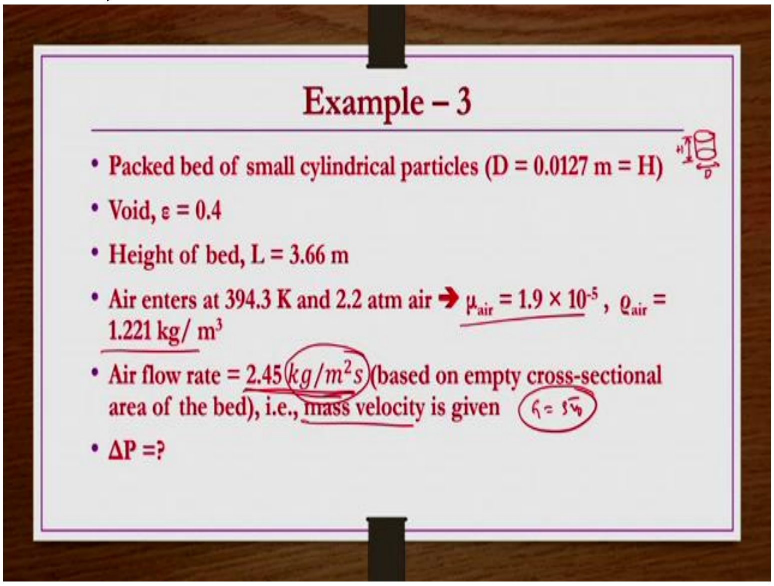

Now we take one more example problem. Let us take packed bed of small cylindrical particles. The previous two problems we have taken spherical particles though one case we know all particles are same size but second problem all particles spherical shape but there are different mixtures, mixtures of different sizes like you know 25, 50 and 75 mm like that. So now here we are taking in this example problem small particles so small particles, small cylindrical particle in the sense that L should be equals to height of the cylindrical particle, so something like this. This we have already seen.

If you take this one as the kind of cylindrical particle and if it is the diameter of the short cylindrical particle and then this is the height of the short cylindrical particle. If H is equals to D or they are very close to each other, then we can say those particles are say kind of short cylindrical particle. So packing material is now short cylindrical particles. Voidage is 0.4, height of the bed is 3.6 meters and then here also air enters at certain temperature and pressure corresponding viscosity 1.9 x 10^{-5} and then corresponding density 1.221 kg/m^3 for the air at this temperature and pressure conditions of the bed.

And then here also the air flow rate is given. Air flow rate is given in kg per meter square second. That is based on empty cross-section area of the bed. So that means mass velocity is given, so that is G equals to rho v naught bar is given. Rather you know v naught bar, you know G, rather mass rate G is given. So then what is the pressure drop? First what we have to do, we have to find out the sphere volume equivalent diameter D_p for this short cylinders what we have to find out what is ϕ_s. If you remember ϕ_s directly we can use for the short cylinder ϕ_s values here.

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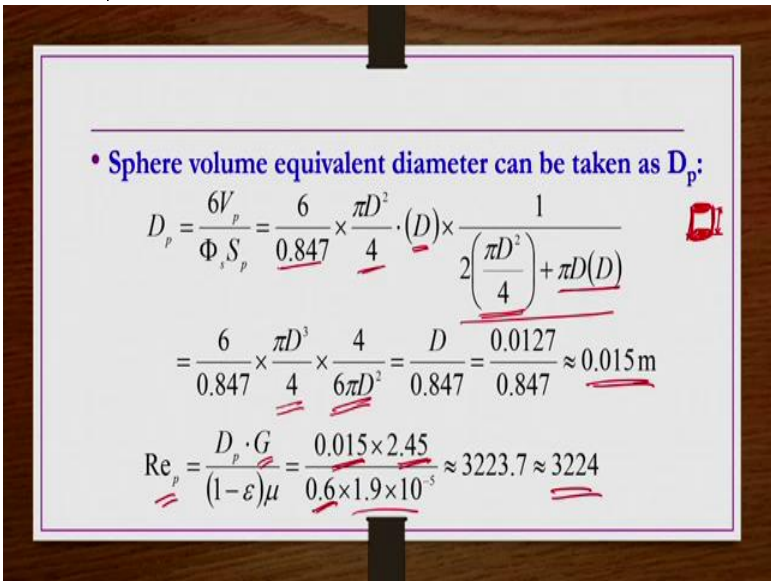

Sphere volume equivalent diameter can be taken as D_p here and then $D_p = \frac{6V_p}{\phi_s S_p}$. Vp is nothing but volume of the particle, S_p is nothing but the surface area of the particle. Now particle is a short cylinder, and then ϕ_s is the sphericity of the short cylinder. For short cylinders sphericity we have already found it as a kind of 0.847, in chapter 1. Now this short cylinder the volume if you wanted to find out you have to find out the cross-section area of you know this thing and then multiplied this cross-section area and then multiplied by height H. But now H is equals to D.

So we can write $(D).\frac{\pi D^2}{4}$, this is the, $\frac{\pi D^2}{4}$ is the kind of cross-section area of the circular or the diameter based on the, you know cross-section of the cylinder and then multiplied by height of the cylinder that will give the volume of the cylinder. Now height of the cylinder is equals to the diameter of the cylinder that is D. And then 1 by, surface area is the, you know surface area of this cylindrical, the circular cross-section this one plus surface area of the, you know remaining cylindrical section.

So circular section area, surface area is you know $\frac{\pi D^2}{4}$ and there are 2 these, you know circular sections are there so that should be multiplied by 2 plus you know surface area of the cylinder is π D H, so H is now D, so that is π D . So this also we have done previously but still we are doing once again, okay so then we substitute here we know this $\frac{\pi D^3}{4}$ as a kind of volume of the, you know short cylindrical particle. And then $6\pi D^2$ as a kind of surface area of small cylindrical particle then substitute D values here.

Then we get 0.015 meters as the sphere volume equivalent diameter or 15 mm as Dp here. Now using this D_p if you find out the $Re_p = \frac{D_p G}{(1-\varepsilon)\mu}$ because G is given, it is, rho is given but $\bar{v}_0$ is not given. So rho into $\bar{v}_0$ is nothing but G mass velocity which is given as 2.45. That you can use here. D_p just now you find it out, is 0.015 meters, 0.4 is the voidage. So $(1-\varepsilon)$ is .6 and then 1.9 x 10^{-5} Pascal second is the viscosity so Reynolds number is coming approximately 3224.

So once this Reynolds number is known then you can use $f = \frac{150}{Re_p} + 1.75$ equation and then find out f and from there we can find out ΔP. Or directly you can use the ΔP equation in the Ergun's equation form. Either way you can do it.

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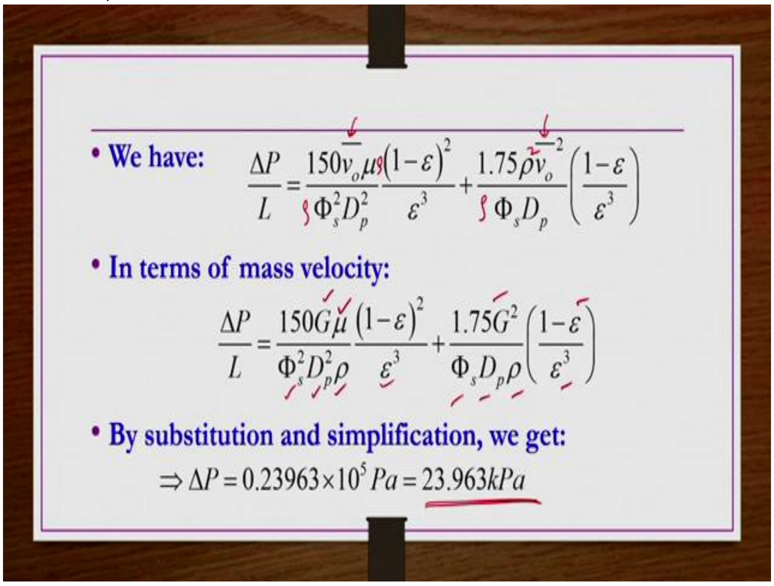

So since previous 2 problems we have done in terms of you know friction factors then you calculated ΔP but what we tried to do here, we tried to do the same thing directly using the Ergun equation so that to have a practice for the different way. So this is the, you know Ergun's equation. So here also this $\bar{v}_0$ is not known. Both the terms are having, you know $\bar{v}_0$.

So what we do, we multiply wherever $\bar{v}_0$ is there by density so that we can have a kind of, you know here $\rho\bar{v}_0$ has a G and then ρ^2 and then you have ρ here so that here you have G, in the second term you can have G^2. Then $\frac{\Delta P}{L} = \frac{150G\mu(1-\varepsilon)^2}{\phi_s^2 D_p^2 \rho\varepsilon^3} + \frac{1.75G^2}{\phi_s D_p \rho}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$.

Now if you substitute all these values G is known, μ is known, ϕ_s is known, D_p you calculated, ρ is known, ε is known so everything is known here in this right hand side equation, if you substitute all the values here and simplify you will get ΔP as 23.963 kilo Pascals.

So this is about the flow through packed beds, especially for Newtonian fluids going through packed beds made of different types of particles. So how to develop the equations for small Reynolds number, large Reynolds number we have seen and then combining together we have formed the equations, Ergun equation which is valid for the entire range of Reynolds number and that information the pressure drop versus measurable parameters of the packed bed system that information, that are Ergun equation we have converted in terms of the friction of pressure drop or f versus Re_p. That we have found out as $f_p = \frac{150}{Re_p} + 1.75$

Then we have taken some problems, you know so that gives a kind of most, most of the information about flow of a Newtonian fluid flowing through a packed bed, okay. Now next lecture what we do, we will discussing about the flow of the Newtonian fluid through a fluidized bed.

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The reference for this lecture is primarily the, for this especially packed bed lectures, we are referring the unit operations of chemical engineering by McCabe, Smith and Harriott book, then other reference books are Unit Operations of Particulate Solids: Theory and Practice by Ortega-Rivas, Coulson and Richardson's Chemical Engineering Second volume by Richardson and Harker is also a kind of good reference book. This Transport Processes and Unit Operations by Geankoplis is having several examples of problems which can be helpful for the students. Other reference books are Unit Operations by Brown et al and then Introduction to Chemical Engineering by Badger and Banchero. Thank you.