

Mechanical Unit Operations
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Lecture 23
Fluidization

Welcome to the MOOCs course Mechanical Unit Operations. The title of this particular lecture is Fluidization. So till now, what we have seen, we have seen flow through single particles. When a single particle is settling what is the settling velocity. Similarly, for such kind of system is large number of particles are there something like suspension so, what is the hindered settling we have seen.

We have also seen last two classes flow through packed bed that is a column is there and then that column is packed with some kind of particles, then the particle bed is constrained both at the bottom then top. Then the flow is taking place through bottom or the top. So, then what is the pressure drops? The pressure drop for that particular fluid flowing through that kind of packed bed.

So, those calculations we have already seen, we have also seen some kind of example problems. All right. So, the pressure drop under such conditions like a flow through packed bed weight is nothing but a kind of total drag, the total resistance offered by the particles for a fluid to flow through such kind of packed beds. Now, in this particular lecture what we do, we have a same kind of packing, we have a column, we have a kind of packing. But the packing is now is not constrained at the top, is not constrained at the top and then fluid is flowing through bottom to top in upward flow direction.

So, then what kind of flow characteristics will be there? How the packing structure will change? Obviously, the packing structure whatever the packing is there that is not constrained. So, because of the increased flow rate if there is a possibility that the packing structure of it will change and then how this packing structure will change if the fluid is a kind of gas or if the fluid is a kind of liquid? That is gas solid system as well as the liquid solid system we are going to see.

However, the packing structure that is going to be changing with respect to increase to superficial velocity of the fluid, those characteristics are different for liquid solid system as well as the gas solid system. Despite of that one, if the velocity is sufficiently low enough then both liquid solid system or gas solid system they will be behaving similarly,

they will be behaving qualitatively similarly. So, that is the reason. Let us start with a kind of a general characteristic of a gas solid and then a liquid solid system and then go into the fluidization conditions.

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General Characteristics of Gas-Solid and Liquid-Solid Systems

Downward flow through beds of particles

- No relative movement occurs between the particles except in the case of unstable initial packing
- In laminar regions $\Delta P \propto \bar{v}_0$
- In turbulence regions $\Delta P \propto \bar{v}_0^2$

$$\frac{\Delta P}{L} = \frac{150(\bar{v}_0)\mu(1-\epsilon)^2}{\Phi_s^2 D_p^2 \epsilon^3} + \frac{1.75\rho\bar{v}_0^2(1-\epsilon)}{\Phi_s D_p \epsilon^3}$$

Handwritten notes on the slide: 'laminar' under the first term and 'turbulent' under the second term of the equation.

Let us say a general characteristics of gas solid and then liquid solid system and under such conditions we take a downward flow through beds of particles. Okay, what we have? We have the kind of column as I mentioned already, right. So, this column is packed with some kind of particle backing material. So, we have different types of packing materials also those things also we have seen.

Now, let us say you take one type of particular packing material and then you pack it, okay. Right. Now, you do your kind of downward flow here, okay. First let us start with the kind of downward flow. So, now anyway at the bottom we should have a kind of perforated plate, okay. So, now, in the downward flow what we do, we do not take any kind of you know, constraint confining the packing at the top.

So, despite it is not confined at the top, the bed is not confined are not packed at the top, since the flow is coming downward whatever the flow it is, even if it is high flow rate is there that is not going to change the packing structure that is not going to change the packing structure substantially so, which can alter the pressure drop. Since the packing structure is not changing, so obviously, the pressure drop is not going to change.

So, whatever the pressure drop versus velocity or the flow rate relations are there for packed bed those like you know Kozeny Carman equation or Blake Plummer equation or you know

Ergun equation those we have those we have derived, they will be valid here. So, why they are valid here because there is no relative motion between the particles here if the flow is downward, okay. So, there will not be any relative movement occurs between the particles except in the case of unstable initial packing.

Of course, initial packing itself if we hold on loosely and there is a white space available, which can be by arranging some kind of fluid is coming at high velocity. So, such kind of small rearrangement will be there anyway. But if you take initial compact packing, so there will not be any kind of rearrangement of the packing, if the flow is downward, if the flow is downward. So, under such conditions there will not be any relative moment between the particles.

So, there will not be any kind of rearrangement of the packing. So, under such conditions whatever the Ergun equations are there they are valid without any kind of difficulty. Right. So, in Laminar regions, we have already seen from the Kozeny Carman equation pressure drop is proportional to the superficial velocity \bar{v}_0 . Similarly, under the turbulence regions also we have seen that the pressure drop is proportional to the superficial velocity square that is \bar{v}_0^2 . So, that is coming out from this equation. Ergun equation that we have developed which is having two contributions.

The right hand side, the first term is a kind of viscous contributions are the contribution due to the laminar flow conditions. And then second term that is because of the turbulent or you know kinetic losses contributions, right. So, from the first part laminar conditions we can see that pressure drop is proportional to \bar{v}_0 . And then the second case, turbulent regions pressure drop is proportional to the \bar{v}_0^2 , okay.

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Upward flow through beds of particles

- Frictional pressure drop should be same as of downward case if
 - bed is constrained at the top
 - flow is such that the structure of bed is not disturbed by flow
- By gradually increasing the flow rate, the point at which drag on particles becomes equal to their buoyant weight
 - Bed rearrangement should occur within the bed, then expands in order to offer less resistance to flow

$\Delta P = (F_g - F_b) / S_p$

Now, let us say same packing what we do, we take the same packing we take a column, now we pack it with a certain kind of you know packed material you know spherical particle or short cylinders or raschig ring or crushed glasses or whatever it regular particles you wanted to take, you take and then pack them. Now, here also, we will not have any kind of you know constrained, closing the top, confining the bed at the top that we are not going to have here also.

But now here what we are doing in this case we are taking the flow, upward flow, we are taking the flow upward flow and then assume this column is sufficiently long enough something like this. Okay that is there is a kind of free board also, if it is a kind of this area is a kind of packing area. So, this free space whatever is there that is known as the free board, okay. Okay.

So, under such conditions whatever the upward flow through beds of particles, the friction and pressure drop, whatever the pressure drop that we have calculated through Ergun equation that would be same here also as in the case of downward flow case, if the bed is constrained at the top. If the bed is constrained, so, whatever the high velocity also you give the particles are not able to escape from the packing area because now the bed is completely packed. On the top also it is packed, though there is a free board area.

So, if you constraint at the top then also it will not be, the packing structure is not going to change, the packing structure is not going to change. So, the pressure drop would be the same as in the case of whatever the pressure drop that you get in the case of downward flow

through these beds of particles or pressure drops should also be same as in the case of downward case, if the flow is such that the structure of bed is not disturbed by the flow.

If the flow is not such high the kinetic energy associated with the flow of the fluid is not so, high that it is not going to disturb the packing structure then also the pressure drop in the case of a upward flow upward flow through beds of particle would be same as in the case of downward flow through the bed of the particle. So, under these two conditions it will be same whether upward flow or downward flow, the pressure drop would be the same.

But however gradually increasing the flow rate. The point at which the drag on particles becomes equal to their buoyant weight you know then what happens bed rearrangement should occur within the bed then expands in order to offer less resistance to flow. So, under such conditions what happens you know the pressure drop is increasing, right? Here is increasing with increasing the superficial velocity whether upward flow or downward flow as long as the pack with conditions are prevailing right.

So, but what happens you know, if you do not have the constraint of the bed at the top and then you gradually increase the velocity what happens some kind of rearrangement of the particle packing structure takes place and then bed starts expanding, bed starts expanding and then pressure drop what happens after that point you know it will become almost like a kind of independent of the superficial velocity.

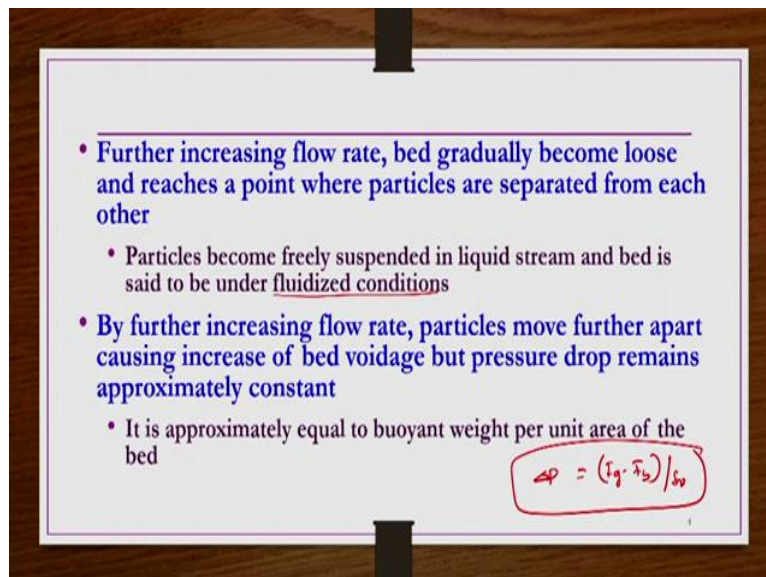
Why because? When these particles are moving away from each other the packing is becoming looser and then bed is being expanded. Then you know the drag, drag on particles whatever is there that becomes equal to the buoyant weight of the bed. So, then what happens here you know whatever the buoyant weight that is you know gravity force post due to the gravity minus post due to the buoyant is there that should be balanced where they drag force right.

So, so, this divided by the let us say you take the cross section area of the bed and then cross section area is not so, then that should be equal to do the drag force per unit area. So, that you know in the case of pressure drop that you know in the case of packed bed that drag whatever is that we are referring as a pressure drop, right. Pressure drop are the resistance offered by all the particle for this fluid to flow through this packing is you know that drag, the total resistances represented by the pressure drop here.

So, when you when the particles move away from each other the packing becomes looser and then starts expanding. At that point, at the point where the expansions start takes place, the drag on particles becomes equal to their buoyant weight. So, that is under such conditions this would be happen and then this is going to be constant after this point onwards, after this expansion taking place onwards you know the drag whatever is there that is going to be start I mean like less, bed is going to offer less resistance to the fluid because now particles are moving away from each other so, the void space is increasing.

So, initially whatever the linearly increasing or initial whatever the increasing pressure drop with increasing flow rate condition instead that will come to the rest and then from that point onwards the pressure drop is going to remain kind of constant.

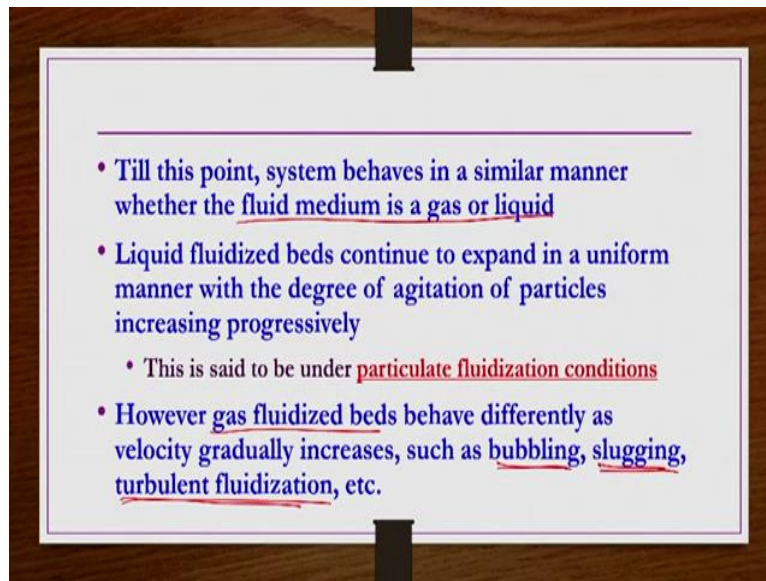
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Further increasing the flow rate, bed gradually become loose and reaches a point where the particles are separated from each other. And then particles become freely suspended in liquid stream and then bed is said to be under fluid as to conditions. The same thing we are going to see pictorial in the next slide anyway.

By further increasing flow rate particles move further apart causing increase of bed voidage but pressure dropped remains approximately constant. That is, it is approximately equal to the buoyant weight per unit area of the bed, per unit area of the bed that is $\frac{(F_g - F_b)}{S_p}$ that should be equals to the ΔP , right. This is going to be constant this constant pressure drop, this going to be maintained from that point onwards, right.

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So, till this point, you know the system behaves in a similar manner whether the fluid medium is gas or liquid. As I mentioned already, you know depending on the fluid medium that is being used to flow through this packed structure whether it is a liquid or gas the different characteristics may be obtained at higher velocities especially at higher velocities. But till this point, the point where the fluidization takes place or the point at which the buoyant weight of the bed is balanced by the overall resistance offered by the particles are the drag on all the particles, till that point you know the system is going to behave quite similar way whether the fluidizing medium is a kind of fluid or kind of liquid, it does not matter.

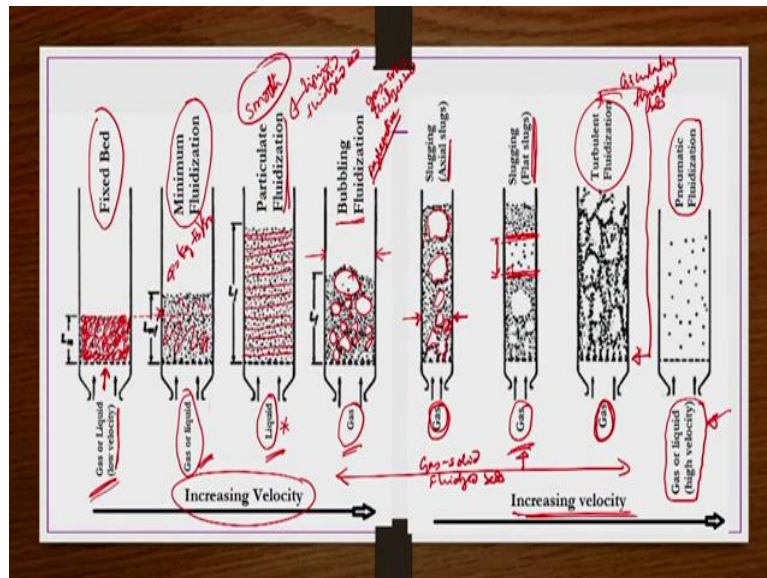
So, that is till the point where the fluidization starts are till the point where the fixed weight conditions perishes and then from there fluidization point starts that breakthrough points till that point fluid weather the solid system or liquid solid system it is going to behave similarly. So, after that, if you have a liquid solid system, or liquid fluidized beds which you are having, so, the bed is going to expand in a more uniform way with a degree of agitation of particles increasing progressively.

So, after this you know, fluidization beginning or incipient point of fluidization you know, when you further increase the velocity if the fluid is a kind of liquid then a kind of a uniform expansion of debate takes place and then it kind of you know constant to bed density can be find at any location within the fluidized area within the bed okay.

So, that is known as a kind of particulate fluidization, our smooth fluidization kind of thing. Right, pictorially we are going to see. But however, after this point of the fluidization are in

after this incipient point of the fluidization, if the fluid medium is a kind of gas, if the fluid medium is a kind of gas, or gas fluidized beds if you are having then that bed is going to behave very differently with increasing the velocity. It may have something kind of bubbling fluidization, it may have a kind of slugging fluidization. It may also have a kind of a turbulent fluidization, etc. So, how they look like pictorially that we are going to see here now.

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So, now see the first case is a kind of case where we are taking the fluid medium whether you know gas or liquid. So, now here what we are taking with increasing velocity the extra erection increasing velocity, this is shown here, drawn here. So, what does it mean? What does it mean? So, velocity increasing. So, that means, the first figure the velocity is low. But second figure the velocity is slightly higher than the first figure, and then third figure the velocity is further higher, and then fourth figure here the velocity is going to be highest amongst the all four cases. That is what mean be here these are increasing velocity, okay.

Now, the first picture what we have, we have a kind of a packed bed. So, this is the kind of packed bed that we are having, or a kind of column is packed with some kind of packing materials, some kind of packing material. And then at the bottom, we have a kind of perforated plate here, we have a kind of perforated plate here. The perforation openings are you know smaller than the average size of the particles you know so, that the particle should not fall down through the perforation.

And now, we gradually giving start allowing the fluid to flow upward here like this, right. So, as long as the velocity is low, as long as the velocity is low, the gas or liquid, whether you

use our gas or liquid to flow through this bed, this bed is not going to expand, it is not going to expand or, its packing structure is not going to change. If the packing structure is not going to change the pressure drop versus the velocity or the flow rate relation whatever is there that we developed through the Ergun equation's that will be hold here also. And in these conditions we know as we call it as a fixed bed.

Remember here, the top of the bed is not constrained. The top of the bed is not constrained despite of that one, the particles are not escaping from this area, the packing structure is not going to be disturbed because of this incoming the velocity flow rate. Because incoming velocity or incoming flow rate of the fluid that is not having a sufficiently large kinetic energy that it is going to disturb the packing structure.

So, if the packing structure is not disturbed. So, the behavior is going to be same as a kind of packed bed or fixed bed, okay. Now, what we do we increase the velocity slightly now we increase the velocity slightly so that what happens you know particles move from each other when the velocity has increased the kinetic energy of the fluid medium that is sufficiently high enough that it is disturbing the packing structure.

Now, that it is disturbing the packing structure and now we can see the height of this bed is increased. Height of the bed initially it was up to this point, right now it has expanded because now particles are moving from each other and the packing is becoming loose and then kind of voidage is increasing here. And then, why it is increasing gradually up to certain point where the drag total drag whatever is there or the total resistance is there that is balanced by the buoyant weight of the bed. So, that is under this condition $\frac{(F_g - F_b)}{S_p} = \Delta P$, right.

So, until that point, this is what the bed would be slightly like this, it will not be exactly kind of fixed bed. But the structure wise it looks like a uniformly packed structure but uniformly expanded packing, expanded packing. Initially, it was up to this location only. Now, it is slightly expanded but it is still uniform, okay.

So, this condition is known as the minimum fluidization condition or where the particles sufficiently apart from each other and then point at which this buoyant weight of bed is balanced by the total resistance offered by all the particles, okay. So, this behavior also whatever the minimum fluidization behavior that we are calling and calling that is going to be same with the fluid medium is gas or liquid, okay. Only the numbers will change, only the

number will change depending on the liquid or gas, right. But the behavior would be the same, behavior would be same.

Now, what does not mean by numbers will change? What is the minimum fluidization velocity at which the bed expand start fluidization? If you take the gas, if you take a liquid keeping all other parameters same like you know particle diameter voidage, etc. So, then that minimum fluidization maybe slightly different because the density of a gas and then density of a liquid that I have been used they will be different. So, that numbers would be different but the flow patterns that you get they will be like this only, okay.

Now, further you increase the flow rate, but you take now upward flow of the liquid now we are taking only liquid, from this point onwards we are separating liquid gas systems, right. So, further if you increase the velocity, increase the superficial velocity of the fluid and then fluid if it is liquid then bed is going to be expand uniformly, expand uniformly what you can see here whatever the expanded bed, the minimum fluidization condition is there the same structure is the here also in the third picture also. Exactly the same structure is there but the expansion is more, right.

The uniform expansion is taking place and then expansion is more that is the difference only here. And then you can see here this at any location in the bed you take the density of the bed it is same actually that means at any location, at any location within the bed if you if you try to calculate what is the volume fraction of the solids and then what is the volume fraction of the liquid, it is not going to change much from one location to other location.

That is the reason this fluidization is so smooth, some books this fluidization is also referred as smooth fluidization but particulate fluidization is more appropriate kind of terminology that you find in many books. This you will get in case of you know of fluidizing medium is liquid at high velocities, high velocities but it should not be very high. If it is very high then particles may be rapidly moving away from each other and then they may also come out of the column those sections we are having anyway, right.

So, now at moderate to larger velocity, if the incoming fluid is a kind of gas that is gas superficial velocities moderate to large that is sufficiently are sufficiently away from the minimum fluidization velocity then what we see bed is behaving quite differently compared to the liquid solid system or the this is kind of a this we can say as liquid solid fluidized bed

right which is under the kind of pneumatic, which is under a kind of particular fluidizing condition.

But now if you have a kind of gas solid fluidized bed and then the superficial velocity of the gas is sufficiently larger than the minimum fluidization velocity we can have a kind of bubbling fluidization bed that is what mean by you know we have a some kind of voids or bubbles kind of things are forming here like that I am arrowing like this. And then these voids or bubbles are almost you know pre of the particle, within these whites or bubbles there are no particles in general are very less particles in general, you can say, such kind of voids or bubbles or formed that is the reason these kind of behavior if you find such fluidization of gas solid fluid as a bed is known as the bubbling fluidization.

Some books, you also find the terminology for this as aggregative fluidization, because earlier it was believed these particles are agglomerating and then forming a kind of cluster agglomerates kind of thing leaving you know more space for this gas to form a kind of voids or bubbles like that. But I know, subsequently there were no reasons found that you know particles are being you know, forming kind of aggregates. So, bubbling fluidization is a kind of more appropriate terminology.

Okay. Now, what we do, we take one more case few more figures, where we have an increasing velocity only further increasing velocity, right and then what we do further increasing velocity. But the fluid whatever you take that you take it now gas and then the column diameter you make a smaller one or you take a slightly narrower column compared to the these sections here.

Whatever the first four figures the column diameter is sufficiently large compared to the fifth figure here you know, slugging fluidization. Here the column diameter is small, if you have a narrow column and then packing is there and then velocity the fluid medium that you are passing through this narrow packed column, the fluid is a kind of gas and then that superficial velocity, velocity of gas is very high then most of the cross section of the column maybe you know occupied by the slugs.

The slugs are forming like you know, initially bubbles are formed these bubbles are formed coming together and then forming kind of bigger slugs and then these bigger slugs they become so, big that you know they are sometimes you know, they they almost you know occupy the entire cross section of the particles leaving only a little space for the particles.

They occupy entire cross section of the bed column leaving only little space for the particles here like this.

Now, you can see that what I mean, most of the column is occupied by bigger slugs. So, that is the reason these you know, these things are known as a kind of slugging fluidization and in such kind of slugs whatever shown here they are known as the axial slugs. Now, if you take the same narrow column same packing, but the further you increase the you know superficial velocity of the gas here also we are taking gas in this case you know sixth figure then also axial slugs will form.

But this axial slugs are you know kind of flat you know more or less kind of flat, having only a few particles you know occupied within the axial slug area. So, this is also slugging fluidization, but the slugs are kind of flat slugs because the slugs are having you know entirely being the you know the cross section area of the column for some heights like this and they are more or less kind of flat they are more or less kind of flat. So, that is the reason this this fluidization is known as the slugging fluidization. But the slugs are flat slugs, right.

Now, further increase the superficial velocity of the gas, we are still taking this you know you know gas solid fluid as beds. So, further increasing the gas superficial velocity the bubbles whatever the bubbles or bubble formed and particles will be high velocity kind of turbulent conditions will prevail inside the bed and then sometimes even the particles go out of the free board and go out of the column. So, such particles are in general you know collected at the top by some means and then circulated back to the bed.

So, they are also known as the circulating fluidized bed, okay. So, circulating fluidized bed that is known as you know this particle recirculation is there here. So, like this, okay. So, these kind of beds if you do not have a kind of you know the circulation of the particles then we call them as a kind of turbulent fluidization, if you capture those particles and send them back as a kind of recycle to the bed again. So, then we you can call him as a kind of circulating fluidized beds.

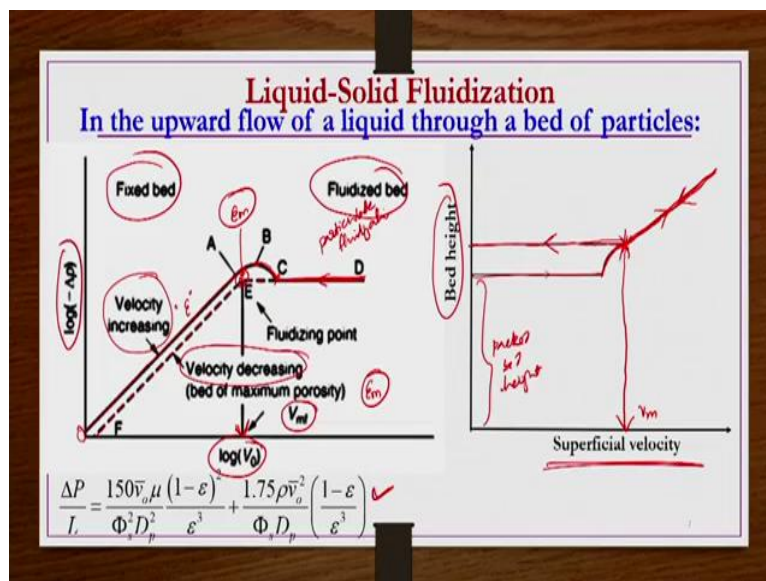
Okay. So, so, now, you can see here up to this point first two figures whether the fluidizing medium gas are liquid the behavior is same, right. Then if you have a kind of liquid solid fluidized bed then it will be like you know particular fluidization kind of thing. But if you have a kind of solid gas solid fluidized bed then the behavior may be quite different from the

liquid solid particulate fluidization, you can see different types of behavior, you can have if you are a fluid medium is a kind of gas, right.

Then if you further increase the velocity to very large velocities, very high velocities then particles are almost far away from each other I kind of you know particularly conditions you know particle transport kind of thing will take place because of the pneumatic conditions that are going to prevail because of the high velocities and then such fluidized bed the fluidized bed under such conditions is known as the pneumatic transport or pneumatic transport or pneumatic fluidization. Okay.

So, this kind of pneumatic fluidization may take place where do you have gas or liquid fluidizing medium but the velocity has to be very high. So, this is about the flow patterns of you know different types of gas, solid, as well as the liquid solid fluidization that we have. Now, what we do, the pressure drop versus the flow rate behavior we see for a liquid solid systems, okay liquid solid systems.

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So, liquid solid fluidization, if we have the upward flow of a liquid through a bed of particles then we can have the pressure drop versus flow rate or velocity on log will occur, then what we have, we have a kind of representation like this. So, if the velocity is small okay initially, velocity we are gradually increasing from no velocity or zero velocity. So, if the velocity gradually increased like this, you know, but the velocity is small increasing but velocity is small then we can have a kind of you know increasing pressure drop with increasing, you

know, velocity conditions up to point A because up to this point you know the velocity is small that it is not going to change the packing structure.

So, since it is not going to change the packing structure, it is going to behave as a kind of fixed bed, the conditions whatever the fixed bed conditions are there, they will be very much applicable. Since for the fixed bed we already know that $\frac{\Delta P}{L}$ is given by this equation by Ergun equation from this equation we can say that the pressure drop increases with increasing the velocity. So, up to this point here like this.

Now, but further if you increase the velocity what will happen, a kind of bed expansion will take place. So, the slope will start diminishing and then maximum pressure will be attained at point B. Further if you increase the velocity of the liquid what will happen you know particles will move from each other and then they will be under kind of expanded fluidized but expanded fluidized conditions like you know the particles are away from each other and then the drag force is balanced balanced to by the buoyant weight of the bed until this point C where the pressure drop slightly decreasing from point B to from point B, okay.

This decrement is usually very small and then it is very difficult to trace out experimentally. Right after reaching this point C, if you further increase the velocity gradually what will happen The pressure drop is you know going to remain constant because at this point C, at this point C the buoyant weight of the bed is balanced by the drag the total resistance offered by the particles. So, the pressure drop is going to be remained constant from this point onwards, right.

So, after this A point you know it is offering slightly less resistance and then that less resistance is going to remain constant independent of the velocity from C point onwards. So, this is a kind of fluidized conditions and then we are talking especially for liquid solid fluidization so, it is we can see as a kind of particulate fluidization. So, where the bed uniformly expands and then bed density is going to be same at different location within the column wherever expanded column that you have taken within the expanded bid wherever you take it. So, the pressure is going to be same, okay.

Now, if you wanted to decrease the velocity, you have seen, you have gone up to the particulate fluidization conditions then what you are doing? You are decreasing the velocity rather further increasing the velocity, so that the particles go out of the bed and then the fluidization go into the pneumatic transport region. So, let us not increase the velocity to such

high velocity because the particles are moving out going out of the column we wanted to stay within the column because we wanted to track back the pressure drop versus velocity curve while decreasing the velocity, right.

So, when you attain the particle fluidization conditions and then you found that the pressure drop is going is independent of the velocity, then you are decreasing this velocity again here. So, gradually, when you decrease it, so, the CD curve now will be back to DEC and then from C it is not going to form, taking this B, A part rather it will gradually further decrease up to point C. Why, because now here one initially we have the compact package, initially we have the compact packing with voidage ϵ Now we have fluidized bed then we have fluidized bed by increasing the velocity gradually, right.

So particulate fluidization has a situation has arrived. Now we are decreasing the velocity. So the particles are settling down the particles are settling down and there will be a point all the particles are touching to each other, all the particles are touching to each other and then further decreasing the velocity the particle rearrangement is not going to be so high that you know the compact packing you will be getting as it was kind of initial packing, you are not going to get the compact packing as initial packing, right. Because it has the bed has disturbed. Initial compact packing we do usually by you know vibrating, shaking, etc. right.

So, once this point E has reached then it is particles are most like you know resting on to each other not only just touching they are resting on to each other as a kind of bed has formed by you know particles resting on to each other kind of thing. And then you know further decreasing the velocity they are not going to rest or comparison the bed is not going to compress again, right. So, but again the particles are resting on to each other what, what happening you know it is having a kind of a fixed bed condition has arrived. Fixed bed condition has arrived but the packing is not as compact as initial compact. So, the voidage is more, the bed expands and slightly expanded and voidage is more, right.

If the bed is expanded slightly. So, the voidage is more. So, obviously if voidage is more the pressure drop is going to be less. That is the region from this E point onwards also when you decrease the flow rate or velocity the pressure drop is decreasing with the velocity, but it is not following initial OA curve. But it is following this EF curve like this. Because now the bed because of the fluidization, it has taken place and then by decreasing the velocity particles are settling and rest touching each other and then resting on to each other. But that resting on to each other is not providing the initial compact packing kind of thing.

So, but it is having slightly different you know voidage that is slightly higher voidage or we can say this is the maximum voidage possible under fixed bed conditions, under fixed bed conditions whatever the higher voidage is there at pointly that is the maximum voidage possible under fixed bed conditions are the minimum voidage that is possible under fluidized bed condition. So, because after that one if you increase the fluid velocity, the voidage is going to increase right.

So, that is the region this point, the point E is known as the point of fluidization or incipient point of fluidization. And then, corresponding velocity we know we call it as a kind of minimum fluidization velocity. And then that maximum voidage is known as the voidage at the minimum fluidization conditions epsilon. So, remember epsilon m is a kind of maximum voidage possible under the packed bed conditions and then it is minimum voidage possible under the fluidized bed conditions and this is the point that occurs at this velocity v_m that is the minimum fluidization velocity.

So, below this minimum fluidization velocity bed behaves as a kind of a fixed bed or packed bed and then beyond this minimum fluidization velocity, the fluidization will take place and then bed behaves as a kind of a fluidized bed, right. So, now, initially you increase the velocity, you decrease it. So then while increasing the velocity zero, A, B, C, D curve you formed for this pressure drop versus flow rate. But when you decrease the velocity you found this curve D, C, E, F like that, they are not matching only CD line is same while decreasing or increasing the velocity.

So, this increasing velocity curves are the solid line curves, decreasing velocity curves are kind of in dotted line curves. Okay. But now again, if you after decreasing it, again if you increase it, will it follow that OA curve or FE curve? So, initially, the the packing is not as kind of initial compact packing it is going to follow this EF, FE curve then E to C then C to D curve, this curve only it is going to follow it is not going to trace back to zero a basic curve at any conditions, once it has been fluidized or once the initial pack compact packing has been disturbed, we will not be able to trace back this O, A, Believe, C for any flow conditions. Okay.

So, now the same thing we can see for the case of you know bed height also. Bed height versus superficial velocity if you see So, this bed initially what happens you know initial compact height whatever bed height was there you know initial compact packed bed height or initial packing height whatever compact packing you have taken that you know it will be

there up to certain velocity it will not be disturbed. But after a certain velocity what will happen you know this will you know take this curve like curve like this because you know this is the reason where the expansion of bed is taking place and then buoyant weight of the bed is balanced by the overall assistance offered by all the particles and then after this point you know if you further increase the velocity the bed height is going to increase like this. Okay.

Now, you are you know fluidized did sufficiently you can see the fluidized conditions, okay proper fluidized condition not just expanded minimum fluidized conditions. After reaching that sufficiently high fluidized condition you are decreasing the velocity. When you decrease the velocity you can see that bed height you know gradually decreases with the decreasing superficial velocity.

But after this point, it is not going to come back like this the previous curve but it is going to be constant like this because at this, by the time velocity decreases to this point the particles are settled and they are resting on to each other and then further compact packing of those particularly is not going to take place. So, that packing height is slightly higher than the initial packing height.

So, while decreasing the velocity the height would be slightly higher and then because of this slightly higher packing height, the voidage is going to be higher. So, the pressure drop is going to be lower as we have seen in the previous curve. So, if you wanted to find out this you know superficial velocity from bed height versus superficial velocity curve, the velocity corresponding to this point is known as the minimum superficial velocity.

Okay. So, this is about the liquid solid fluidization. Now, we are going to do the minimum fluidization velocity calculations and then we have seen this minimum fluidization of part up to the minimum fluidization conditions whether it is a gas or liquid fluid medium that we are using, the behavior is going to same. So, the analysis equation development is going to be same for gas solid or liquid solid fluidized beds. Okay.

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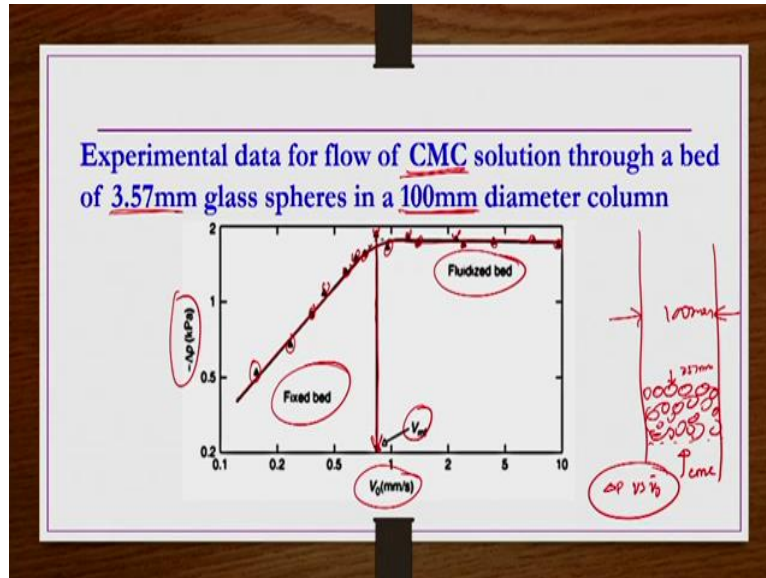
- linear relation can be obtained between pressure gradient and superficial velocity on log coordinates up to the point A where expansion of the bed starts to occur
- Then the slope of the curve gradually diminishes as the bed expands
- As fluid velocity gradually increased, pressure drop passes through a maximum value at point B and then falls slightly up to point C and then eventually attains a nearly constant value independent of liquid velocity (and follows CD line)

- If velocity of liquid is reduced again, the bed contracts until it reaches the condition where particles are just resting in contact with one another (up to point E)
- At point E, it has maximum stable voidage for a fixed bed of particles
- Up to point E, no further change in voidage of the bed occurs as velocity is reduced provided it is not shaken or vibrated
- The pressure drop (EF) in this re-formed packed bed shall be less than that in the original bed at the same velocity

- If the liquid velocity were now to be increased again, new curve (EF) would normally be re-traced and slope will suddenly drop to zero at the fluidizing point E
- In an idealized fluidized bed, pressure drop corresponding to ECD line is equal to buoyant weight of the bed per unit area
- However, in practice, deviations from this value may be observed due to channelling and interlocking of particles
- Point B is situated above CD because the frictional forces between the particles must be overcome before rearrangement of particles can occur

So, whatever I explained in these figures have been given as a kind of notes here, for the convenient of the students, right.

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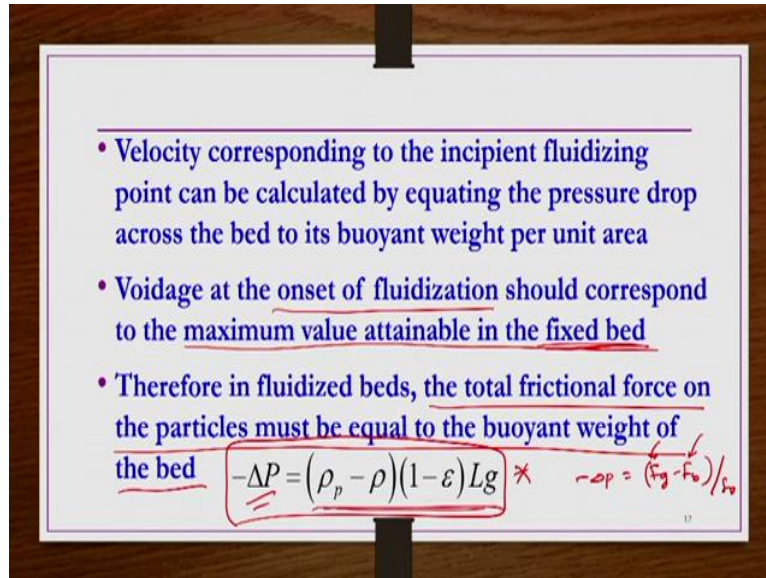
So, now we see some experimental results. Pressure drop versus the flow rate for a CMC solution flowing through a column having you know 100 mm diameter with packing 3.57 mm glass spheres. So, they have a kind of 100 mm column, in this column you know at the bottom there is a perforated plate and then this glass spheres are packed to certain initial height like this. Compact packing was done using this glass particles, 3.57 mm, right. And then through the bottom they allowed CMC solution to flow through up and then exponentially what is the ΔP versus \bar{v}_0 a superficial velocity is here.

So, that you know captured experimentally they have calculated and then put in pictorially like this. So, now, we can see to certain velocity the these points are in experimental points actually, all these things are experimental points. So, they are slightly scattered. But if you dry kind of curve fitting lines or, up to certain velocity it is gradually increasing and then there is a diminishing of slope and then pressure drop slightly increasing up to point C and then after this pressure drop is almost remaining kind of same, constant.

So, the point here up to this whatever is there you know, that point the velocity corresponding to this point from where the pressure drop is becoming constant onwards that velocity is known as the minimum fluidization velocity, below the minimum fluidization velocity bed behaves as a fixed bed where the pressure drop increases with the flow rate. After the or beyond the minimum fluidization velocity, the bed behaves as a fluidized bed where the

pressure drop is remains constant or the pressure drop remains independent of this superficial velocity of the fluid. Okay.

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So, velocity corresponding to the incipient fluidizing point can be calculated by equating the pressure drop across the beds to its buoyant weight per unit area and then voidage at the onset of the fluidization our incipient point of the fluidization should correspond to the maximum value attainable in the fixed bed conditions, okay, in the fixed bed conditions. Therefore, in the fluidized bed, the total frictional force on the particles must be equal to the buoyant weight of the bed.

When you do these things, you will get this expression we are going to derive this one anyway. This is coming simply by you know $\frac{(F_g - F_b)}{S_p} = -\Delta P$, you substitute this buoyant force and gravity force simplify then you will get it. So, in Ergun equation in place of ΔP is simply you know you read this equation so, that those that equation whatever the simplified equation is that will be valid for fluidized bed conditions, okay.

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Minimum fluidization velocity of gas-solid and liquid-solid fluidization

- Pressure drop across the bed = (gravitation force - buoyant force)/unit cross-sectional area

$$\Delta p = \left[\frac{mg - g\rho \frac{m}{\rho_p}}{S_0} \right]$$

$$\Delta p = \frac{mg}{\rho_p} (\rho_p - \rho) / S_0$$

$$\Delta p = (1 - \varepsilon) L S_0 g (\rho_p - \rho) / S_0$$

$$\frac{\Delta p}{L} = (1 - \varepsilon) g (\rho_p - \rho) \quad \text{--- (1)}$$

So, minimum fluidization velocity of gas-solid and liquid-solid fluidization. Remember up to the fluidization incipient fluidization are the minimum fluidization conditions whether the fluidizing medium is a gas or liquid it is going to be here kind of a similar way. So, whatever the minimum fluidization velocity calculations are there, they are going to be same whether the fluid is a medium is a gas or liquid, okay.

So, pressure drop across the bed should be equated to the gravitation force minus buoyant force per unit cross section area. So, this is the gravity force, this is the buoyant force divided

by the cross section S not. So, from these two terms $\frac{mg}{\rho_p}$ if you take common $\frac{mg}{\rho_p} \times \frac{(\rho_p - \rho)}{S_0}$.

Now, $\frac{m}{\rho_p}$ is what? $\frac{m}{\rho_p}$ is nothing but the volume of the particles within the bed. Volume of the particles, all the particles within the bed, okay.

So, that you can calculate by you know $LS_0(1 - \varepsilon)$ because this is the bed initial bed that you are having. So, let us say all this is in a kind of you know packed conditions. So, if this is the initial packing height L and the cross section area of this bed is S not, okay. So, what is the volume total volume of the bed, bed volume, bed volume is going to LS_0 . This bed is consist of particles and then void spaces, right.

So, then volume of particles, all the particles would be what $S_0L(1 - \varepsilon)$ where ε is the void space or the volume fraction of the void space is epsilon. So, $1 - \varepsilon$ is the volume fraction of the particles within the bed. So, volume fraction of the particles multiplied where the volume of the bed if you do you get volume of the particles, volume of the particles within the bed.

So, that we are substituting $LS_0(1 - \varepsilon)$. This S_0 is cancelled S_0 . L you can take to the left hand side. So, you have $\frac{\Delta P}{L}$ is equal to $(1 - \varepsilon)g\Delta\rho$, right.

So, now, this equation we are going to use an Ergun's equation in order to find out in order to find out you know minimum fluidization velocity. Or the point at which are the velocity when you are gradually increasing, there would be a certain velocity at which the pressure drop would be equal to this one and after that it is going to be constant. So, corresponding velocity we can call it as a minimum fluidization velocity \bar{v}_{om} and then corresponding void we can call it epsilon margin, okay.

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• **At this minimum fluidization condition**

- Voidage is minimum voidage $\rightarrow \varepsilon = \varepsilon_m$ *but more voidage under fixed bed condition*
- Velocity is minimum fluidization velocity $\rightarrow \bar{v}_o = \bar{v}_{om}$

• **By substituting equation (1) in Ergun's equation**

$$\Rightarrow g(1 - \varepsilon_m)(\rho_p - \rho) = \frac{150\mu\bar{v}_{om}(1 - \varepsilon_m)}{\Phi_s^2 D_p^2 \varepsilon_m^3} + \frac{1.75\rho\bar{v}_{om}^2(1 - \varepsilon_m)}{\Phi_s D_p \varepsilon_m^3} \Rightarrow \bar{v}_{om} = ?$$

$$\Rightarrow g(\rho_p - \rho) = \frac{150\mu\bar{v}_{om}(1 - \varepsilon_m)}{\Phi_s^2 D_p^2 \varepsilon_m^3} + \frac{1.75\rho\bar{v}_{om}^2}{\Phi_s D_p \varepsilon_m^3} \rightarrow (2) \Rightarrow \bar{v}_{om} = ?$$

So, at this minimum fluidization condition, so, wherever the minimum fluidization conditions appear so, the $\frac{\Delta P}{L}$ is going to be $(1 - \varepsilon)g\Delta\rho$. Okay. So, that is known as the minimum fluidization condition or the velocity at which $\frac{\Delta P}{L} = (1 - \varepsilon)g\Delta\rho$ has just derived. So, that is the minimum fluidization point and then that monument fluidization condition d voidage is going to be maximum attainable voidage under packed bed condition so, that we are writing ε_m

Initial packing is epsilon but the initial voidage of the bed is ε initial compact packing when you do, the voidage is whatever the void volume fraction is there that is ε . But after you know fluidization as taking place, so particles are loosely packs and started expanding where the pressure drop is balanced by the buoyant weight of the bed, the fluidization has increased

slightly and then that is the maximum voidage attainable under the packed conditions. So, that we are writing ε_m

And then velocity is minimum fluidization velocity where \bar{v}_0 , we are Replacing by \bar{v}_{om} , right or \bar{v}_{om} . So, this these two Replace in the Ergun equation. ε we Replaced by ε_m and then \bar{v}_0 we Replace by \bar{v}_{om} and then in the Ergun equation $\frac{\Delta P}{L}$ we Replace by $(1 - \varepsilon)g\Delta\rho$. So, this is nothing but $\frac{\Delta P}{L}$ of our Ergun equation and then this is the right hand side thumb as it is but what only thing that we have done here velocity, superficial velocity we Replaced by the minimum fluidization velocity.

And then voidage we replaced by the, you know, maximum voidage at minimum fluidization conditions. So, this is the minimum voidage under the fluidized conditions but maximum voidage under fixed bed conditions. Or the trader point from where the fixed bed is going to become fluid as bed that point voidage is the maximum for the packed bed conditions and minimum for the fluidized bed conditions, minimum for the fluidized bed condition, maximum for the packed bed condition. So, that ε_m be Replaced.

So, in the Ergun equation $\frac{\Delta P}{L}$, we Replaced by $g(1 - \varepsilon_m)(\rho_p - \rho)$ as just we derived and then in the right hand side everywhere you know velocity, we Replaced by the minimum fluidization velocity and then epsilon we Replaced by ε_m , okay. That is the only change we have done.

So, this equation one can solve to get the minimum fluidization equation, It is a kind of quadratic equation that you get when you substitute all the properties like you know voidage, gravity, densities and then packing size, etc. shape factor etc. all those things when you substitute, you will be having a quadratic equation for minimum fluoridation velocity that you solve then you get the minimum fluidization velocity.

We are going to do one example problem also. So, this now what we do, from this equation this $(1 - \varepsilon_m)$ if you cancel out the other side from both sides, so, then you have this further simplified equation, okay. From here also you can get the minimum fluidization velocity indeed both of them are same actually, they are not different.

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• For $Re_p < 1$: $\Rightarrow g(\rho_p - \rho) \approx \frac{150 \mu \bar{v}_{om} (1 - \epsilon)}{\Phi_s^2 D_p^2 \epsilon_m^3}$ + viscous losses / kinetic losses

$\Rightarrow \bar{v}_{om} \approx \frac{g(\rho_p - \rho)}{150 \mu} \left(\frac{\epsilon_m^3}{1 - \epsilon_m} \right) \Phi_s^2 D_p^2 \rightarrow (3)$

• Empirical relations based experimental studies indicate that $\Rightarrow \bar{v}_{om} \propto D_p^{<2}$ (slightly less than 2) and minimum fluidization velocity is not quite inversely proportional to viscosity

So, now, in the case of single particle conditions, a single particle settling in a column of fluid if we know the Reynolds number, so, then we have a kind of you know expression for a free settling velocity of the that particular particle. If we know that the particle settling under the Stokes regime then we have a kind of free settling velocity for that particle under Stokes regime.

Similarly, if we know that particle is settling under the Newton's flow regime then we have a kind of separate settling velocity equation under the Newton flow regime. So, we try to do similar kind of thing if we have a kind of flow conditions, laminar flow conditions, small Reynolds number conditions where you know only viscous losses are contributing more and then kinetic losses are negligible or very small compared to the viscous losses then we can develop an expression for the kind of a minimum fluidization velocity.

Similarly, we can do if the flow is very high and then viscous losses are going to be very small then compared to the kinetic losses, so, then considering only kinetic losses part, we are going to develop another equation further minimum fluidization velocity valid for you know high Reynolds number flows. So, for Re_p less than one.

So, the equation number two the previous equation left hand side is as it is, right hand said we are taking only first term which is contributing to the viscous losses which is contributing to the viscous losses because whatever the kinetic losses terms are their second term 1.75, etc. that is, you know very small zero very small that it can be neglected compared to the viscous losses. So, we take only first time in the left hand side. It is exactly the similar way what we

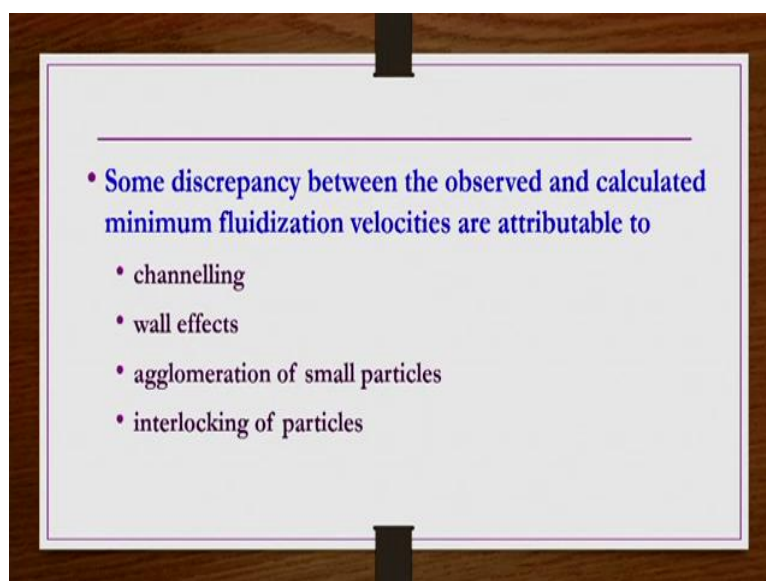
have done for the packed bed conditions, when we are deriving you know you know magnitude of viscous losses and kinetic losses, etc.

So, we take only viscous losses for small Reynolds number because they are dominating one. So, that means, in the right hand side we have to take only first term and then you simplify it then you will get a kind of expression for the minimum fluidization velocity has given me this equation number three. Remember this equation is valid only if the Reynolds number is small or Re_p less than one.

Empirical relations based on experimental studies indicated that you know this \bar{v}_{om} what we are seeing here, it is proportional to D_p^2 from theoretically what you find minimum fluidization is proportional to D_p^2 , but experimentally it is founded \bar{v}_{om} are the minimum fluidization is you know proportional to the D_p to the power of slightly less than two, not proportional to two but slightly less than two. That is some deviations are from the experiments in general possible.

So, those deviations are this is one deviation and another deviation what we see here from this you know equation number three that minimum fluidization velocity is inversely proportional to the viscosity of the fluid under you know the smaller Reynolds number of conditions. But experimentally it has it has been found that it is not you know quite inversely proportional to viscosity. It is not quite inversely proportional to viscosity, okay.

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So, some discrepancies between the observed and calculated minimum fluidization velocities are in general possible. Some of the discrepancies we have seen that they are not quite proportional to the minimum fluidization velocity is not quite proportional to the D_p^2 , it is proportional to D_p power slightly less than two and then it is not quite inversely proportional to viscosity experimentally. But theoretically, it has been found that minimum fluidization is inversely proportional to the viscosity of the fluid such kind of discrepancies may be there, they are in general attributable to something like channeling may occur during the fluidization conditions.

There may be some kind of wall effects, you know, there may be some kind of agglomeration of particles also depending on the size of the particles, some particles may be agglomerating, then also it may be possible. Sometimes if you have a kind of needle a kind of particle, so, then interlocking of particles may also take place. So, then again under such conditions fluidization behavior may be slightly different than observed one. So, because of these reasons, there may be some kind of discrepancies.

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• In case of free settling of spherical particle at low Re_p :

$$\Rightarrow u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu} \quad \checkmark \quad \rightarrow \div \text{ans}$$

• Ratio between terminal and minimum fluidization velocities:

$$\frac{u_t}{\bar{v}_{om}} = \frac{gD_p^2(\rho_p - \rho)}{18\mu} \times \frac{150\mu(1 - \epsilon_m)}{g(\rho_p - \rho)\Phi_s^2 D_p^2 \epsilon_m^3}$$

$\epsilon_m \approx 0.45$

$$\frac{u_t}{\bar{v}_{om}} = \frac{8.33(1 - \epsilon_m)}{\Phi_s^2 \epsilon_m^3} \rightarrow (4)$$

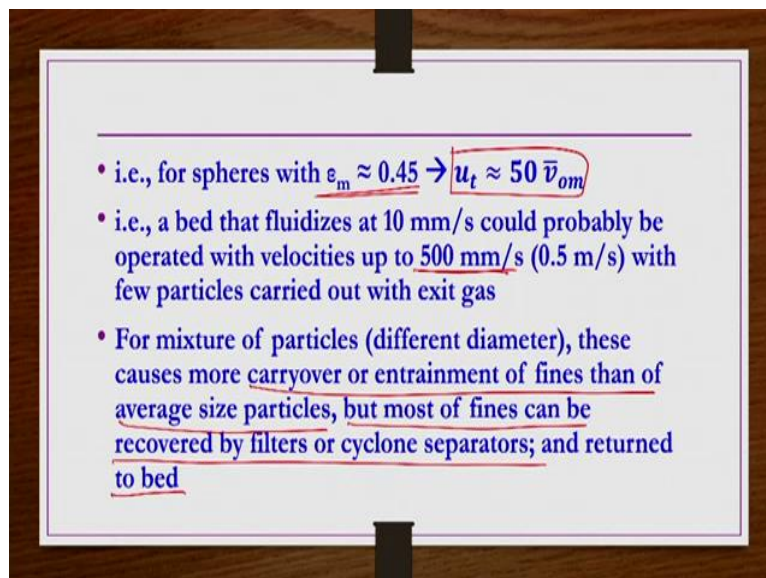
Now, what we do, we try to see the free settling velocity of a single particle and then compare with the kind of minimum fluidization velocity for smaller Reynolds number conditioned. So, small Reynolds number conditions when Re_p is less than one we have already seen for spherical particle free settling velocity is nothing $\frac{gD_p^2(\rho_p - \rho)}{18\mu}$.

So, now, this equation, what we do we take the ratio between this terminal velocity and minimum fluidization velocity that is u_t by \bar{v}_{om} . So, this equation divided by equation number this one divided by equation number three, that is just previously derived equation if you do so, $\frac{u_t}{\bar{v}_{om}}$, so this is u_t free settling velocity of single particle and then divided by \bar{v}_{om} is nothing but in $\frac{gD_p^2\Delta\rho\phi_s^2\epsilon_m^3}{150\mu(1-\epsilon_m)}$, right. So, $(1 - \epsilon_m)$ actually. Right.

So, now, here g you can cancel out, D_p^2 , D_p^2 you can cancel out and then let us say we are taking spherical particles, let us not worry about the shape factor. So, μ and then μ is also cancelled out. And then $\Delta\rho$, $\Delta\rho$ is can be cancelled out. So, what we are having? We are having $(1 - \epsilon_m)$ by ϵ_m^3 multiplied by this $\frac{150}{18}$ that is 8.33, that is what we get here.

So, now, in general if the packing is done by the spherical particles So, then voidage does is in general .45 divided at minimum fluidization condition is approximately .45 if you do the packing using spherical particles. So, under such conditions if you take ϵ_m is equals to approximately .45 and then substitute in previous equation, the privation equation is this equation. In this equation, if you substitute ϵ_m is equals to 0.45 which is mostly accepted value for packing with spherical particle then you can get this ratio as kind of 50.

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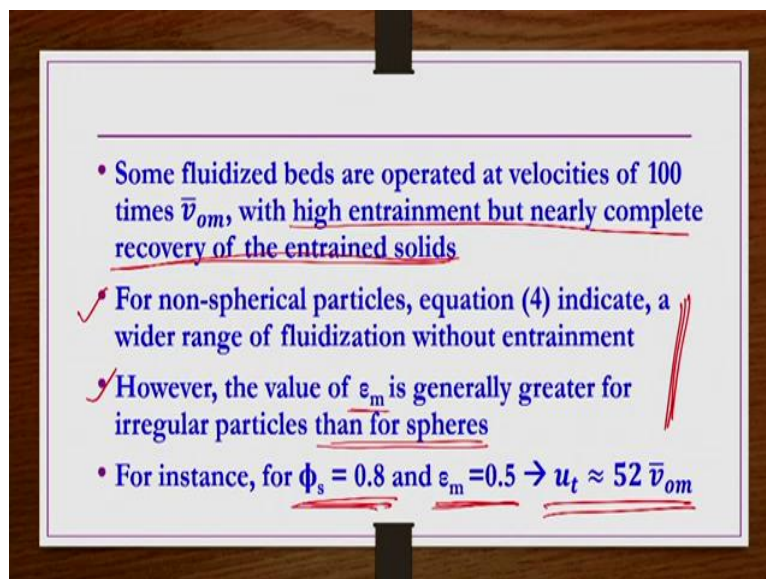
That means, resettling velocities almost 50 times the minimum fluidization velocity. So, if your velocity, whatever you are going to provide for the you know minimum fluidization to take place at least minimum fluidization to take place. So, it has to be at least 50 times the free settling velocity of the single particle. You find out what is the free settling velocity of

single single particle u_t and then you multiply by 50 whatever the value that you get that much velocity, minimum velocity you should provide so that to have a kind of minimum fluidized the conditions are fluidization with a few particle going out.

So, that is the reason we evaluated this kind of ratio. That is the reason we elevated this kind of ratio, okay. So, now, that is a bed that fluidize as a 10 mm/s could probably be operated with velocities up to 50 times higher that is 500 mm/s, point five m/s with few particles carried out with exit gas, okay. For mixture of particles different diameter, this causes more carryover our entrainment of fines than of every size particles in general.

Nut most of fines can be recovered by filters are cyclones separators; and returned to the bed. If you have a single shaped particles that is fine, but if you have the different types of particle mixture, then you know it is possible that some particles having smaller size and then depending on the shape, they may be easily carried over under these kind of you know velocities whatever you are operating based on the spherical particle calculations. Right. So, but however, they can be collected and then circulated back to the bed.

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So, fluidized beds are operated at velocities of 100 times, minimum fluidization velocity with high entrainment, but nearly complete recovery of the entrained solids. Sometimes you know, if you are not worried about the entrainment of the solids, because you are sufficiently able to recover them and then send back to the bed so, that you can have a circulating fluidized bed, so under such conditions some fluidized beds are even you know operated at 100 times the

minimum fluidization velocity provided the flow is under the laminar flow conditions provided the flow is under the laminar conditions.

This is all remember we are doing under the laminar flow conditions, okay. Once develop the equations, minimum fluidization equation generalize the equation, after that this discussion we are doing for small Reynolds number we are going to do for higher Reynolds number of cases also that you do not need to give such high number of times the minimum fluidization velocity in general, okay that we are going to see any way. For non-spherical particles whatever the equation flow that we develop indicated a wider range of fluidization without any entrainment. However, the value of ϵ_m is generally greater for irregular particles than for spheres.

These are all; these observation two points are you know in general very experimental observations. So, for instance, if you have a non-spherical particle with the shape factor 0.8 and then minimum voidage at the minimum fluidization conditions epsilon m then $\frac{u_t}{\bar{v}_{om}}$ you are going to be 52, slightly more than this spherical particle kind of case, where by using just epsilon m .45 if you use this spherical particles this ratio we got it as 50. But now, here if the irregular particle you use and then slightly more ideas you use then also it is coming out to be 52 slightly higher than the spherical particle conditions.

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For $Re_p > 10^3$:

$$g(\rho_p - \rho) = \frac{1.75 \rho \bar{v}_{om}^2}{\Phi_s D_p \epsilon_m^3} \Rightarrow \bar{v}_{om} = \left\{ \frac{g(\rho_p - \rho) \Phi_s D_p \epsilon_m^3}{1.75 \rho} \right\}^{1/2} \rightarrow (5)$$

$$\frac{u_t}{\bar{v}_{om}} = 1.75 \left[\frac{g D_p (\rho_p - \rho)}{\rho} \right]^{1/2} \left[\frac{1.75 \rho}{g D_p (\rho_p - \rho) \epsilon_m^3} \right]^{1/2}$$

Handwritten notes: $u_t = 1.75 \sqrt{\frac{g D_p (\rho_p - \rho)}{\rho}}$ and $Re_p > 10^3$

$$\Rightarrow \frac{u_t}{\bar{v}_{om}} \approx \frac{2.32}{\epsilon_m^{3/2}} \rightarrow (6) \text{ i.e., for spheres with } \epsilon_m \approx 0.45 \rightarrow u_t \approx 7.7 \bar{v}_{om}$$

So, now, we take for higher Reynolds number conditions, okay. High Reynolds number conditions, the kinetic losses are important. So, right hand side of the Ergun equation we are

taking only the second term, only second term and then left hand item ΔP we are replacing by $g\Delta\rho$ by left hand side $\frac{\Delta P}{L}$ we are replacing by $g\Delta\rho$ multiplied by $(1 - \epsilon_m)$.

So, there seems to be one minus epsilon m in the right hand side also that epsilon m one minus epsilon m left hand side and right hand side is cancelled out. Or whatever the equation two that we derive left hand side we are keeping as it is, right hand side we are taking the term associated with the larger Reynolds number that is the second term only we are taking. This is valid for large Reynolds number 10^3 or more.

So, from here what you get minimum fluidization velocity is nothing but $\left[\frac{gD_p^2\Delta\rho\phi_s^2\epsilon_m^3}{1.75\rho}\right]^{1/2}$. This is what we are going to get. Now, here also what we do, we take a free settling velocity of single particle but under higher Reynolds number conditions that is Newton's flow regime we know the settling velocity of the single particles Newton's flow regime we know this is Newton's regime that is Re_p is greater than 10^3 . Then for single particle free settling velocity, we got it like this.

Now, here also we take the ratio between the free settling will last and minimum fluidization velocity under high Reynolds number flow conditions then we have this one here, okay. So, both the powers are you know one by two. So, here gD_p here gD_p can be cancelled here ρ , here ρ can be cancelled at. Here $\Delta\rho$ $\Delta\rho$, here $\Delta\rho$ can be cancelled out. So, what we have $1.75\sqrt{\frac{1.75}{\epsilon_m^3}}$. So, that is coming out to me $\frac{u_t}{\bar{v}_{om}}$ is approximately $2.32\epsilon_m^{3/2}$. Now, let us say, spherical particles with ϵ_m is equals to .45 as we have taken previously. So, if you substitute here then you can see this ratio $\frac{u_t}{\bar{v}_{om}}$ is just 7.7 it is not 50 or 52 as previously we have seen, okay. Just you know, you know you can see that the reason and I am saying I was mentioning depending on the flow condition whether the laminar flow conditions you want to have are the turbulent flow conditions you want to have accordingly the minimum fluidization velocity you have to find out and then accordingly those number of times you how to multiply. So, that to have a proper fluidization, okay.

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• If the flow regime is not known, then eq. (2) should be solved for \bar{v}_{om}

$$\Rightarrow g(\rho_p - \rho) = \frac{150\mu\bar{v}_{om}(1-\epsilon_m)}{\Phi_3^2 D_p^2 \epsilon_m^3} + \frac{1.75\rho\bar{v}_{om}^2}{\Phi_3 D_p \epsilon_m^3} \rightarrow (2)$$

• Alternatively it can be done as below: Multiply eq. (2) both sides by $\frac{\rho D_p^3}{\mu^2} \frac{\epsilon_m^3}{(1-\epsilon_m)^2}$ and simply

$$\Rightarrow g(\rho_p - \rho) \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1-\epsilon_m)^2} \right] = \frac{150\mu\bar{v}_{om}(1-\epsilon_m)}{\Phi_3^2 D_p^2 \epsilon_m^3} \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1-\epsilon_m)^2} \right]$$

$$+ \frac{1.75\rho\bar{v}_{om}^2}{\Phi_3 D_p \epsilon_m^3} \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1-\epsilon_m)^2} \right]$$

So, these previous two expressions for low Reynolds number and the high Reynolds number we have seen. So, if you do not know flow regime, if you wanted to know flow regime you should know Re_p that is $D_p \rho$ you know $\frac{\bar{v}_{om}}{\mu(1-\epsilon_m)}$ that you know. So, that is velocity you should know then only you know the Reynolds number then only you can use those minimum fluidization velocity conditions. Indeed those simplification we have done in order to get a minimum fluidization conditions, minimum fluidization velocity.

So, so, that means, you cannot know the Reynolds number. So, you cannot use the equation number three and four which we just derived. So, in order to overcome such kind of problems, you know what we do we use equation number two, equation number two, whatever is there you know, when you substitute the particle characteristics and then fluid physical properties rho, mu, etc. voidage etc. if you substitute you will get a kind of a quadratic equation for minimum fluidization velocity that you can solve to get the minimum fluidization velocity.

But alternatively there is another method, you know, where you know this is the equation number two we are having this equation both sides what you do, you multiply by $\frac{\rho D_p^3}{\mu^2} \frac{\epsilon_m^3}{(1-\epsilon_m)^2}$.

This particular term you multiply both sides of this equation number two. When you do these things, you get this one. What we have done here nothing but you just multiplied by this term here and then in the right hand side also both the terms.

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$$\Rightarrow g(\rho_p - \rho) \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right] = \frac{150 \bar{\mu} \bar{v}_{om} (1 - \epsilon_m)}{\Phi_s^2 D_p^2 \epsilon_m} \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right] + \frac{1.75 \rho \bar{v}_{om}^2}{\Phi_s \bar{\mu} \epsilon_m} \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right]$$

$$\Rightarrow g(\rho_p - \rho) \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right] = \frac{150}{\Phi_s^2} \left[\frac{\rho \bar{v}_{om} D_p}{\mu (1 - \epsilon_m)} \right] + \frac{1.75}{\Phi_s} \left[\frac{\rho^2 \bar{v}_{om}^2 D_p^2}{\mu^2 (1 - \epsilon_m)^2} \right]$$

Then what happens that again, I have written the same equation here. So, now, you can see in the left hand side I am not going to do anything. Right hand side what we can do here. So, the square of the viscosity and then this viscosity is gone cancelled out. We can cancel out and then, this $(1 - \epsilon_m)$ and then the $(1 - \epsilon_m)^2$ is cancelled out ϵ_m^3 and then ϵ_m^3 is cancelled out and then this D_p^2 is cancelled out with the D_p^3 . So, there is one D_p , right.

So, then what we have here, $\frac{\bar{v}_{om} \rho D_p}{\mu (1 - \epsilon_m)}$. So, that is nothing but Reynolds number. Forget about this ϕ_s^2 . So, we take for the spherical cases as of now. Similarly, here also, this D_p here and then D_p^3 if you take. So, you have the D_p^2 here and then \bar{v}_{om}^2 is already there. This ϵ_m^3 this ϵ and give you can cancel out, μ^2 is there, $(1 - \epsilon_m)^2$ is as it is.

This room multiplied by rho what we can have rho square. So, here $\frac{\bar{v}_{om} \rho D_p}{\mu (1 - \epsilon_m)^2}$. you can write it up. So, that means nothing but that is the Reynolds number. So, left hand side is as it is, right hand side $\frac{150}{\phi_s^2}$ multiplied by this term is remaining plus $\frac{1.75}{\phi_s}$ multiplied by this term is remaining and this is nothing but our Re_m . So, this is nothing but Reynolds number definition. But Reynolds number packed bed we have defined it as you know $\frac{\bar{v}_{om} \rho D_p}{\mu (1 - \epsilon_m)}$.

This is how we defined the Reynolds number for packed bed. So, since we are doing this is at the minimum fluidization condition this ϵ should be replaced by ϵ_m and then \bar{v}_0 should be replaced with minimum fluidization velocity. So, that is the reason Re_m I am calling it are the

Reynolds number at minimum fluidization conditions, okay. So, left hand side is equal to $150Re_m + 1.75Re_m^2$ for spherical particles that is what we get here.

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The slide contains the following equations and notes:

$$\Rightarrow \left[g(\rho_p - \rho) \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right] \right] = \frac{150}{\Phi_s^2} \left[\frac{\rho \bar{v}_{om} D_p}{\mu(1 - \epsilon_m)} \right] + \frac{1.75}{\Phi_s} \left[\frac{\rho^2 \bar{v}_{om}^2 D_p^2}{\mu^2 (1 - \epsilon_m)^2} \right]$$

For spherical particles $\Rightarrow Ga = 150Re_m + 1.75Re_m^2 \rightarrow (7)$ $k^3 = D_p^3 \left(\frac{g\rho(\rho_p - \rho)}{\mu^2} \right)$

where, Galileo number, $Ga = \left[g(\rho_p - \rho) \left[\frac{\rho D_p^3}{\mu^2} \right] \left[\frac{\epsilon_m^3}{(1 - \epsilon_m)^2} \right] \right] \rightarrow (8)$ Stokes regime
 $k < 2.6$
 $k > 68.9$

and Reynolds number at minimum fluidization, $Re_m = \left[\frac{\rho \bar{v}_{om} D_p}{\mu(1 - \epsilon_m)} \right] \rightarrow (9)$

So, left hand side whatever is there this term in general is known as the Galileo number for spherical particles, right. So, this phi s we have taken because now, we wanted to do it for the spherical particle conditions. So, then left hand side whatever this term is there that is known as the Galileo number. So, this is a just a kind of in order to avoid the calculation of \bar{v}_{om} and then finding out the Reynolds number and from the Reynolds number we can find out the minimum fluidization velocity. This is just an alternative approach.

But this approach is having no advantage it is similar to whatever the k factor that we have developed for the case of single particle. For the case of single particles you know, what we have we have this k, k is nothing but $D_p^3 \left(\frac{g\rho(\rho_p - \rho)}{\mu^2} \right)$. This is what we have a k for single particle. If $k < 2.6$ we know this is a Stokes regime, if $k > 68.9$ we know it as a kind of Newton's flow regime, in between it is intermediate reason so, then calculation has to be done.

So, that advantage was there, here finding out this k factor right in this case of single particle case. But here in the case of packed bed you know the fluidized bed that advantage is not there because in the case of packed or fluidized bed we do not know exact transition from laminar to turbulent conditions, right. Like, you know, Re_p if less than one then we say that it is Stokes flow regime for a single particle condition. If $Re_p > 10^3$, then we can say it is a kind of you know, Newton's flow regime so, accordingly we do calculation.

So, this is the advantage of having this key factor or k parameter which is independent of velocity, it was useful in calculating the free settling velocity. Similar analysis we have done here, people say this is a kind of Galileo number, you know, we can see it is a kind of k^3 only. It is just sees this term in the Galileo number or you can see here also in the Galileo number whatever this term is there. The first two this term, this exactly is nothing but your k^3 . So, probably, if you multiply this one, so, then it may be k^3 or cube root of this one is going to be k for a kind of packed or fluidized bed conditions.

But there is no advantage of doing such kind of analysis here because packed bed or fluidized bed, we do not know there is no clear transition between a laminar to turbulent flow condition. So, there is no point of using it. So, even if you have written this equation in terms of Galileo and Reynolds number like this, the calculation is again going to be trial and error kind of thing or solving this equation for Reynolds number and from that Reynolds number you are going to find out the minimum fluidization velocity or rather doing that one directly you can calculate the equation number two you can use and then applying quadratic equation which you can solve to get the minimum fluidization velocity.

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The references for this lecture are: Unit Operations of Chemical Engineering by McCabe, Smith and Harriot. Unit Operations of Particulate Solid: Theory and Practice by Ortega-Rivas. And then Coulson and Richardson's Chemical Engineering Series, Second Volume, by Richardson and Harker. Transport Processes and Unit Operations by C.J. Geankoplis and then Unit Operations by Brohn et Ashok Leyland, and then Introduction to Chemical

Engineering by Badger and Bancho. But however, most of this lecture is taken, you know, obtained from these two difference that is McCabe, Smith and Harriot and then Richardson and Harker. These two books are referred to prepare today's lecture slides. Thank you.