Mechanical Unit Operations
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Lecture No 24
Flow through Fluidized Beds – 2

Welcome to the MOOCs course Mechanical Unit Operations. This is about fluidization. The title of this lecture is Fluidization part 2. It is a continuation part of the previous lecture Fluidization, what we have discussed. So, before going to the additional details about the fluidization, we will have just a recapitulation of what we have seen in our previous lecture on fluidization.

So types of fluidization we have seen, different types of fluidization pattern are possible depending on the type of fluidizing medium, that is, if the fluidizing medium is a liquid then different patterns are possible in general. And then if the fluidizing medium is a gas then different flow patterns are possible within the fluidized bed in general, ok.

That also depends on the density difference, and then velocity, etc and all those things. So, we have seen but the fluidization characteristics, or fluidization patterns in general may remain same up to certain extent, irrespective of whether the fluidizing medium is a kind of gas or liquid, ok. So that is what we have seen.

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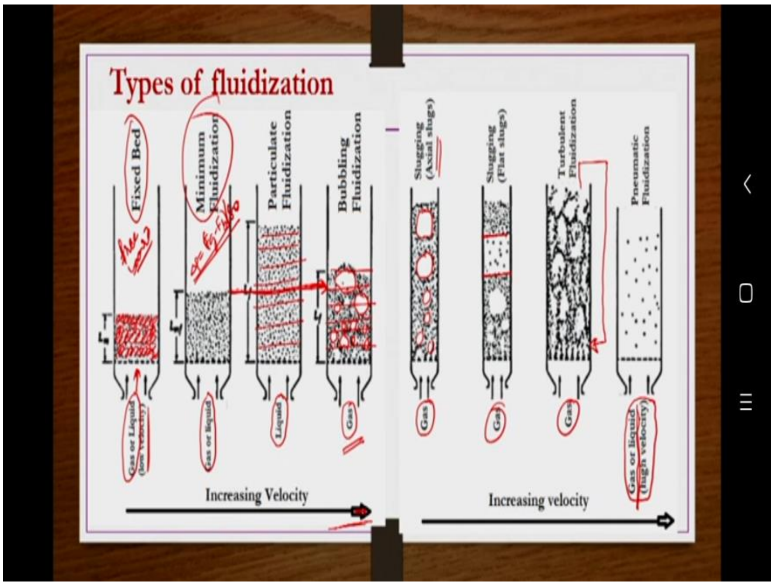

So have a kind of recapitulation. Let us say, you have a bed packed with a kind of particles as shown here. Right, so this is kind of bed which is packed with a kind of particle. We have a kind of column and then part of this column is packed with the few particles. At the bottom, there is a kind of perforation plate so that the particle should not fall down. And then from bottom, gas or liquid is flowing upward motion, like this. But, if the velocity is low, if the fluidizing medium velocity is low, whether the fluidizing medium is gas or liquid, that is not going to affect the packing structure of the bed.

So, if the velocity is as low as possible so that the packing structure of the bed is not being disturbed because of upcoming fluid, that is, upcoming fluidizing medium. Then we can say, such kind of situations are, you know, fixed bed conditions. Though at the top, it is not constrained. The bed is not constrained at the top but however, the velocity of the upcoming fluidizing medium is low.

Then, you know packing structure is not changing much. So, it is remaining kind of a similar situation as in a kind of fixed bed, right. So, under such low velocities, we can use Ergun's equations without any difficulty even if it is not packed at the top or even if the bed is not constrained at the top, ok.

So, this is anyway free board area that we have already seen. So free board area will be available anywhere for the particles to move on if it has some, if they are moving up. So, but subsequently if you gradually increase the velocity, right. So that means the arrow direction is increasing velocity we have shown here. So that means from one picture to the other picture as we are moving, the superficial velocity of the fluidizing medium is gradually increasing.

So, if we go to low to moderate velocity so, then what happens? The packing structure of the bed may be slightly, you know, being disturbed and then that becomes slightly looser and start expanding. And then this behaviour can obtain by using both gas or liquid type of a fluidizing medium, any fluidizing medium you can use it. So, this condition, you know, we call it minimum fluidization under which, you know, the total resistance offered by the particle is balanced by the buoyant weight of the bed, ok.

So, that is the balance under those condition or like you know, when the particles are moving, expanding or going away from each other slightly. So, there will be a point. The point corresponding to certain velocity. There will be a velocity corresponding to which the oral resistance offered by the particle for flow of this fluid is balanced by the buoyant weight of the bed. So, that condition we call it as incipient fluidization point or minimum fluidization point and then corresponding equations we can get by replacing $\Delta P = \frac{F_g - F_b}{S_0}$, ok. So, this is the balance.

So, after this point onwards, you know, once the velocity reaches certain point, certain velocity at which the minimum fluidization started, so from that point onwards even if you increase the superficial velocity of the fluidizing medium, the pressure drop is going to remain constant. It is not going to change. That is what also we have already seen.

So, that is the reason at minimum fluidization point, pressure drop we are equating by $\frac{F_g - F_b}{S_0}$. From there we get $\frac{\Delta P}{L} = g\Delta\rho(1-\varepsilon)$. And then ε should be replaced epsilon m that is the maximum fluidization, if the bed is under the fixed bed condition and then from that point, expansion of the bed takes places.

So, that is the minimum fluidization if the bed with respect to the fluidization condition if you take. So, after this point, further increasing the velocity, the expansion of the bed is going to take place and then voidage is going to increase and then bed will no longer be in fixed bed conditions, ok.

So, further if you increase the velocity then the effect of, you know, type of fluidizing medium is going to show in the patterns that are going to form. So, let us say if you have a, you know, the fluidizing medium is a liquid then further increasing the velocity superficial velocity of the liquid fluidizing medium, the bed uniformly expands. So, you can say, a kind of uniform expansion is there.

So, that you know at point within the bed if you take or at any cross-section within the bed if you take, the bed density is going to be same or it is going to be remain almost constant, that is the, (whatever's the) at any cross-section, at any position within the expanded width, the volume fraction of the particles and then the volume fraction of the, you know, fluidizing medium or void space is going to be remain almost constant. So, that we call as a kind of particulate fluidization or smooth fluidization.

However, if your fluidization medium is a gas, then as the velocity increases, there will be a kind of situation where you know the bubbles may be forming. Voids or bubbles may be forming which are almost you know free of particle. There may be 1 or 2 particles but in general, these voids are free of particles and then you can see, those bubbles are you know are moving up rather, you know, fast compared to the bed.

You can see, you can compare with the minimum fluidization conditions, the most of the particles are you know, you know, they are not raising up, much high much much larger height compared to the, you know, minimum fluidization condition. So, you can see, only the bubbles are rising up. Only there a few particles beyond this point also. What does it mean by bubbling fluidization condition which is obtained by the gas flow rate.

So, if you increase the velocity, most of the velocity, you know, most of the gas that is going to form a kind of voids or bubbles and they are moving quite rapidly or the expansion of the bed, whatever is taking place, that is primarily taking place because of the expansion or the rising of the bubbles or the voids. Or otherwise, the dense face or the [particle] particulate face is almost like you know, under the similar conditions as in the kind of a, you know, similar height as in kind of minimum fluidization conditions. That is what one can say from here, ok.

And then further the bed density at any location within the bed if you take, it is not going to be same. It is going to be different from one location to the other location.

Then further if you have kind of a, you know, increase in the velocity, fluidizing medium is a kind of gas. Now, you make the channel or the column in which you have done the packing of the material narrower, bit narrower. Then, this bubbles whatever are forming by the bubbling fluidization, they increase in the size and they the kind of slugs may be forming out which may be eventually, you know, occupying most of the cross-section of the bed.

This can happen at high velocity for a gas fluidizing medium, but the channel has to have a kind of a smaller dimension. The constraint or the narrow channel compared to the other cases like bubbling fluidized bed. Then, we can have the slugs. So, these slugs can be axial slugs as shown here or they can be almost like, you know kind of, flat slugs as shown in the subsequent figure.

Then, further if you increase the superficial velocity of the fluidizing medium gas then you know, the particulate phase and then the bubbles whatever are forming, they will under a kind of turbulent conditions. And then turbulent fluidization will take place.

Under such conditions, particles may even go out of the bed. However, those particles in general collected and then sent back to the column again, ok.

So, if you are recirculating like this then that bed is known as the circulating fluidized bed. But at large velocities, at very high velocities compared to the minimum superficial velocity, whether you are using a gas or liquid as the fluidizing medium, a kind of a pneumatic transport will take place where the particles are farly apart from each other and then they may be going out of the, you know, bed and then a kind of transport is occurring. So, these are the types of fluidization in general we have seen. And they are general observations. They may not necessarily be always be forming always.

Let us suppose, you know, whatever the pneumatic fluidization that you see in case of the liquids, you know, if the $\Delta\rho$, the density difference between the particle density and then fluid density is a kind of a very small then you kind of some kind of bubbles may form. Even when use a kind of a liquid as kind of fluidizing medium.

On the other hand, if your fluidizing medium even if it is gas, but the density difference is you know kind of very large, that is, $\rho P - \rho$. The density difference between particle and fluid density is kind of a very large. Then, even using this gas phase as a kind of fluidizing medium, you may expect some amount of particulate fluidization.

So, it depends on these properties. Other properties are also depends sometimes particle structure, nature of the particles. They are all going to have a, you know, effect in the flow patterns in general. But generally, we can experience this kind of a behaviour.

Depending on the particle size, type of the particle there are different types of particles sizes depending on size of the particle as well as the density difference between the particle and fluidizing medium. So, the particles are categorized as A, B, C and D type particles and you can have a kind of a mixed behaviour also possible in general, ok.

Further, what we have seen? In the previous class, we have a developed expressions for the minimum fluidization conditions or the equations we have developed for the minimum fluidization conditions and then from those equations we can evaluate what is the velocity required, minimum velocity, that so that to incur a kind of a fluidization within the bed. So, that velocity calculations we have done.

So, how we have done? So, we have taken a kind of a Ergun equation. In the Ergun equation, left hand side whatever $\frac{\Delta P}{L}$ by L is there, that is replaced by $g\Delta\rho(1-\varepsilon_m)$, where ε_m is the minimum voidage at minimum fluidization condition or maximum voidage possible for a bed to be behaving as a fixed bed condition.

Either way, we can say or it is a kind of a cut off between fluidized bed and then packed bed. So, at that cut off whatever the void is there, that is ε_m. And then corresponding velocity is known as the minimum fluidization velocity that we replaced with $\bar{v}_{om}$. That is, $\bar{v}_o$ is replaced by $\bar{v}_{om}$. So that, that is what we have done Ergun equation, right.

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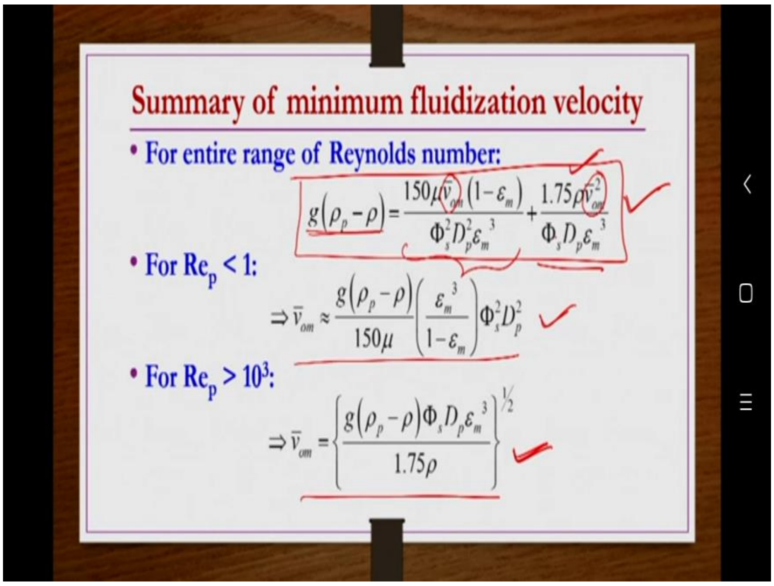

So, in the Ergun equation, left hand side whatever $\frac{\Delta P}{L}$ was there, that we replaced by $g\Delta\rho(1-\varepsilon_m)$ and then right hand side terms, wherever $\bar{v}_o$ was there, that was replaced by $\bar{v}_{om}$. So, there was here $(1-\varepsilon_m)^2$ and then here $(1-\varepsilon_m)$ was there so that $(1-\varepsilon)$ part can be cancelled out in the left hand side whatever it is possible. So, the final equation that we got this one. So, this equation can be used for the entire range of Reynolds number.

Let us say if you know the packed bed Reynolds number or the Reynolds number in the packed bed when a fluid is flowing through a bed. That Re_p if you know, then different equations we can use if Re_p is less than 1 and then Re_p is greater than 10^3. If Re_p is less than 1, only this term is going to be important and this term can be, second term can be neglected. Because that is because of the convection.

If Re_p is greater than 10^3, this first term can be considered in the right hand side term and then we can have only the second term in the right hand side because we need only convection part. So, we know that, the in the right hand side, first part first term is indicating about the viscous losses and then second part indicating about the kinetic losses.

So, viscous losses are predominant at small Reynolds number. Kinetic losses are renounced at larger Reynolds numbers. So, in order to have simplification and mathematical equations to develop for Re_p less than 1, we have taken left hand side is equals to first term of the right hand side and then we simplify. So then we got this equation.

Similarly, if Re_p is greater than 10^3, then left hand side we keep as it is, and the right hand side we have only taken this second term and then simplified the equation for minimum fluidization velocity that we got this equation. So, if you know the Reynolds number for packed bed condition then if it is less than 1 this equation you can use this for a minimum fluidization to occur or the to calculate the velocity for minimum fluidization to occur.

And then if you know that Reynolds number for packed, packed bed conditions, it is greater than 10^3 then you can use this third equation, you know, to get what is the minimum fluidization velocity, you know, so that you know fluidization start occurring. Or bed expansion start occurring.

So, if you do not know Reynolds number, this entire equation you have to use. You substitute the fluid properties and then geometry and then particle characteristics here. And then simplify this equation, you will get quadratic equation for $\bar{v}_{om}$. If you solve that equation, you get minimum fluidization velocity.

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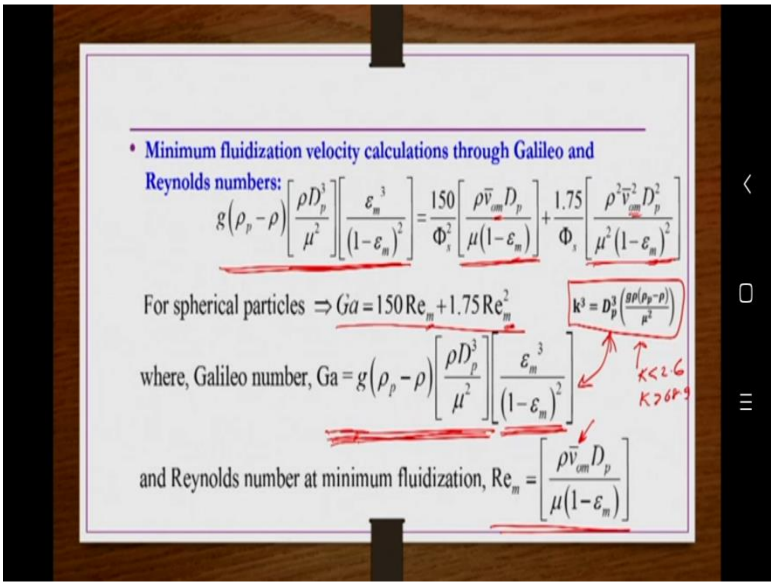

Minimum fluidization velocity can also be calculated in terms of Galileo and Reynolds number. For that, what we have done? Whatever the equation, that previous equation number 1 is there, that equation both sides we multiply it by $\frac{\rho D_p^3}{\mu^2}\left(\frac{\varepsilon_m^3}{(1-\varepsilon_m)^2}\right)$. Then we right hand's items, we simplified. Left hand's terms we keep, we kept it as it is. Right hand's terms, when we simplified, we got here $150 \times Re$ for packed bed $+1.75 \times Re^2$ for packed bed. But this Re is at minimum fluidization condition so $\bar{v}_{om}$ and then ε_m are used here, ok.

So, this equation for spherical particle if you write, Galileo number is equals to Ga = $150Re_m+1.75\ Re_m^2$. Re_m I am using is because this Reynolds number at minimum fluidization condition or the corresponding velocity that has been used to get this Reynolds number is the minimum fluidization velocity, where this left hand term of this equation is known as the Galileo number. And then Re_m as I already told, it is the Reynolds number for packed and fluidized bed conditions but at minimum fluidization conditions.

Further, we have seen Galileo number. This is quite similar to our k factor. k factor what we had in our single particle settling conditions. $k = D_p\sqrt[3]{\frac{g\rho\Delta\rho}{\mu^2}}$. That is what we have seen. So that $k^3 = D_p^3\left(\frac{g\rho(\rho_p-\rho)}{\mu^2}\right)$. So that you know, $\frac{g\rho\Delta\rho D_p^3}{\mu^2}$ term is also there. And then there is additional correction for a kind of a packed and fluidized bed conditions. So, is there any connection between this? That one has to find out, ok.

Anyway, but this, actually here k we have found, you know, in case of single particle if it is less than 2.6 then we say it is a Stokes regime and then corresponding free settling velocity we have used. If it is greater than 68.9 or between 68.9 and 2300, then we say that the settling velocity is under Newton's flow regime and corresponding velocity equation we have used. But in the case of packed and fluidized beds, there is no such kind of a sharp transition from one flow pattern to the other flow pattern. So we cannot have this less than this Reynolds number is a kind of a laminar flow.

So, some people say Re_p less than 1 is going to be a kind of laminar flow for packed bed but even some researchers argue that up to Re_p 5 to 10 also, the bed remains as a kind of you know, laminar flow conditions. So, those kind of ambiguities are there. So that is the reason corresponding limit numbers for Galileo number we cannot have for these conditions. Though by mathematical appearance, this k and g are appearing kind of similar way, ok.

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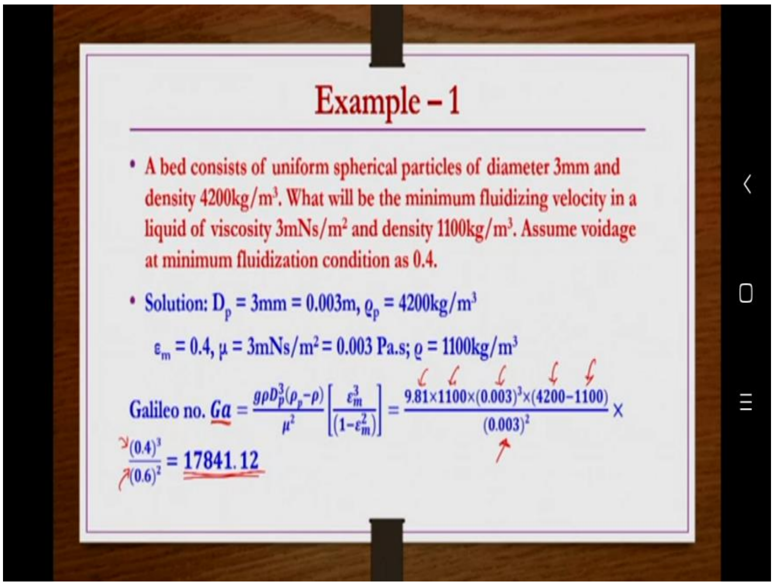

So, now we take a problem. This is what, you know, what we have studied in the previous class about the fluidization summary. Now we take a problem, example problem. A bed consists of uniform spherical particles of diameter 3 mm and density 4200 kg/m^3. What will be the minimum fluidizing velocity in a liquid of viscosity 3 milli Newton second/metre^2 that is you know 3 milli pascal seconds, so that is 3 x 10^3 pascal second. And density is 1100 kg/m^3. Assume voidage at minimum fluidization condition as 0.4.

Then, calculate what is the minimum fluidization velocity, that is the question. So, particle size is given, its density is given, ε_m is also given, then fluid viscosity, fluid density are also given. So, we can calculate Galileo number, correct. Galileo number $Ga = \frac{g\rho D_p^3(\rho_p-\rho)}{\mu^2}\left[\frac{\varepsilon_m^3}{(1-\varepsilon_m)^2}\right]$.

If you substitute all these terms here. You know, this is g, this is ρ, this D_p^3 and then this is ρ_p, this ρ, this is you know μ^2 and then this is ε_m^3 and then $(1-\varepsilon_m)^2$ this is. Now, $(1-\varepsilon_m)^2$, so then if you substitute all these values and then simplify. Ga Galileo number, you will be getting as 17841.12.

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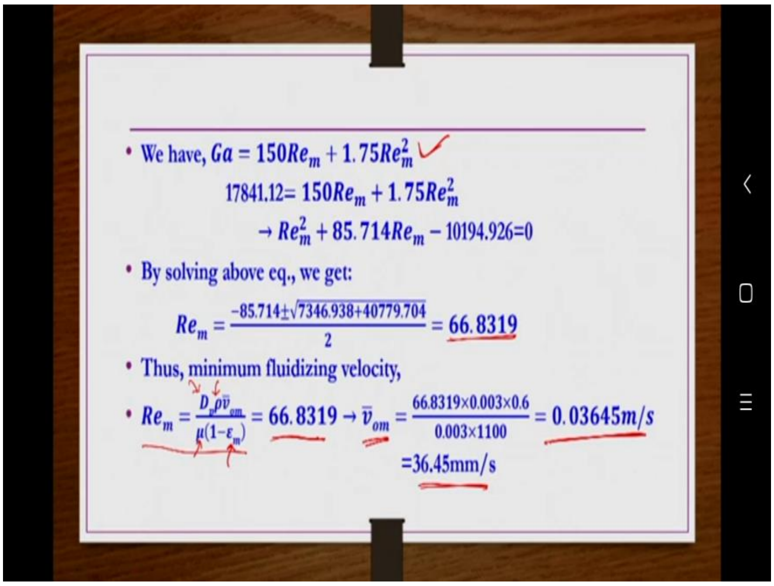

Now this one you substitute in the equation that we have derived. Galileo number is equals to 150 Re_m + 1.75 Re_m^2. And then you simplify this equation further. And then solve this equation to get the roots of this, you know, this Reynolds number so that you get Re_m. 1 positive 1 negative value you will get. Negative value is not having any meaning here. So, so we take positive value as the answers. So that comes out to be 66.8319.

So now, Re_m we know, as you know $\frac{D_p\rho\bar{v}_{0m}}{\mu(1-\varepsilon_m)}$, so that you, I know, use it here. So Re_m, we know $\frac{D_p\rho\bar{v}_{0m}}{\mu(1-\varepsilon_m)}$ for packed and fluidized beds. So, here this you equate to 66.8319. So, in this equation D_p is known, Re is known, μ is known, ε_m is known. So you can calculate $\bar{v}_{0m}$. It comes out to be 0.03645 metre per second or 36.45 millimetre per seconds. So, that is the minimum fluidization velocity.

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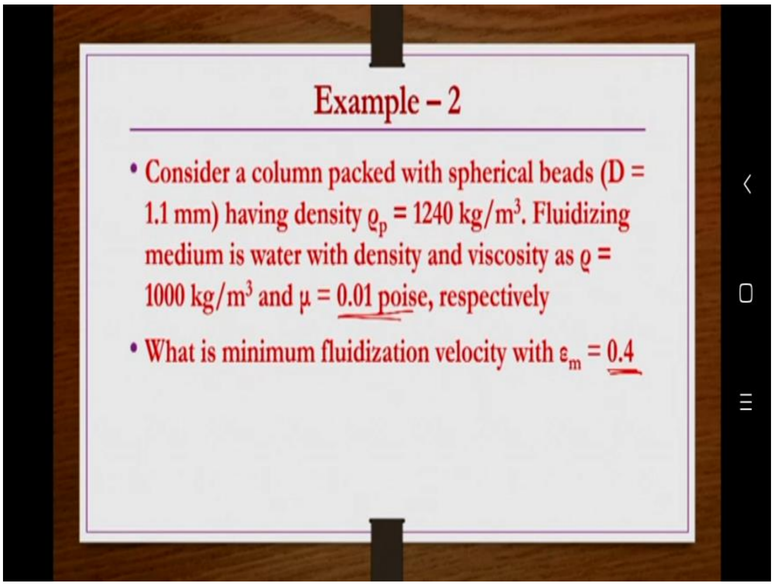

We will take another problem. Similar problem but now we directly use the equation to get the minimum fluidization velocity rather than the Galileo number calculations. So consider a column packed with spherical beads, diameter 1.1 mm, having density 1240 kg/m^3. Fluidizing medium is water with density and viscosity as given as 1000 kg/m^3 and 0.01 poise respectively. Viscosity is given in poise, ok. So, what is the minimum fluidization velocity with ε_m being 0.4?

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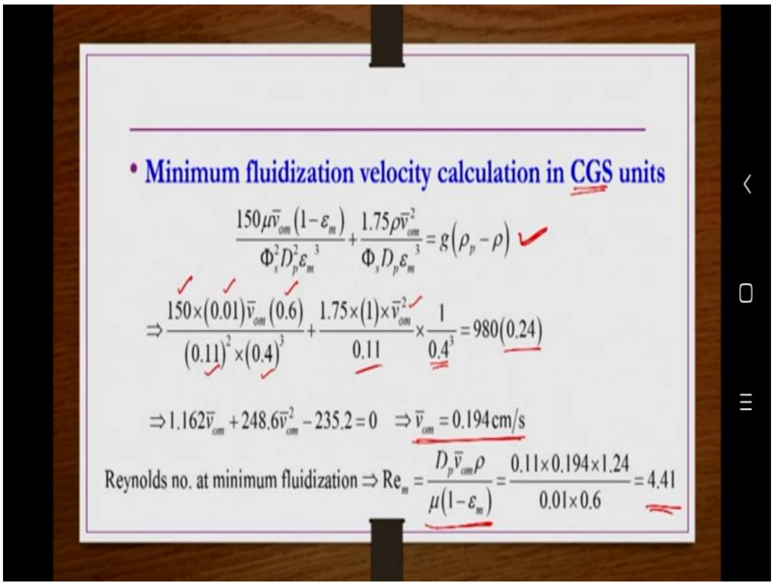

Exactly the similar problem what we have done previously, but we other way we calculate. So, this is the equations. Minimum at minimum fluidization condition this is the equation. $\frac{150\mu\bar{v}_{0m}(1-\varepsilon_m)}{\phi_s^2 D_p^2 \varepsilon_m^3} + \frac{1.75\rho\bar{v}_{0m}^2}{\phi_s D_p \varepsilon_m^3} = g(\rho_p - \rho)$. Here, except $\bar{v}_{0m}$ everything is known. We can substitute all these quantities. And then for a change we do this in CGS unit, ok. So, when you substitute all these numbers here, we can get the equation for minimum fluidization velocity. By solving that equation, we can get the minimum fluidization velocity.

So, 150 here. This is constant. Viscosity is 0.01 poise. $\bar{v}_{0m}$ is not known. 1 - 0.4 = 0.6. ϕ_s^2 is 1 because spherical beads. D_p is the 1.1 mm so 0.11 cm^2 and then ε_m is 0.4 + 1.75, ρ is 1000 kg/m^3 so 1 gram per cc. $\bar{v}_{0m}^2$, do not know, divided by ϕ_s 1. Dp is 1.11 mm. So, 0.11 centimetre and the m^3 is 0.4^3, g is 980 and then $(\rho_p - \rho)$ is (1.24 – 1). So that is 0.24, ok.

So, when you simplify you get this equation and then when you solve for $\bar{v}_{0m}$, you will get 0.194 cm/s because we have solved it in CGS units, ok. So, corresponding Reynolds number at minimum fluidization velocity if we calculate, it comes out to be 4.41 which is, you know, slight behind the, you know, laminar flow region.

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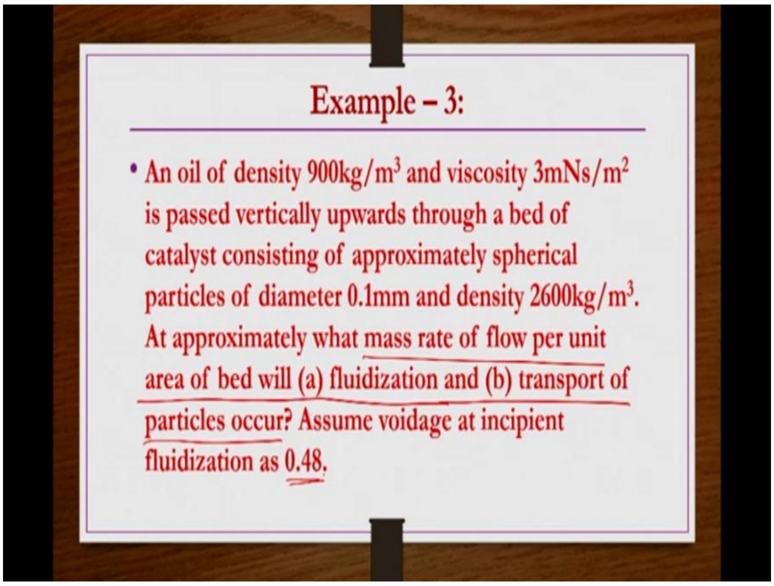

Another example. An oil of density 900 kg/m^3 and viscosity 3 milli newton second/m^2 is passed vertically upwards through a bed of catalyst consisting of approximately spherical particles of diameter 0.1 mm and density 2600 kg/m^3. At approximately what mass rate of flow per unit area of bed will fluidization and transport of particles occur? So it is not asking the minimum fluidization velocity. Corresponding mass rate it is asking per unit cross-section area, that is in kg/m^{2}s.

So, that is $\rho\bar{v}_{0m}$ we have to calculate. And then transport of particles in general that occurs at free settling velocity of the, you know, particle. So for those particles, free settling velocity we have to find out. Assume voidage at incipient fluidization as 0.48. ε_m is given 0.48

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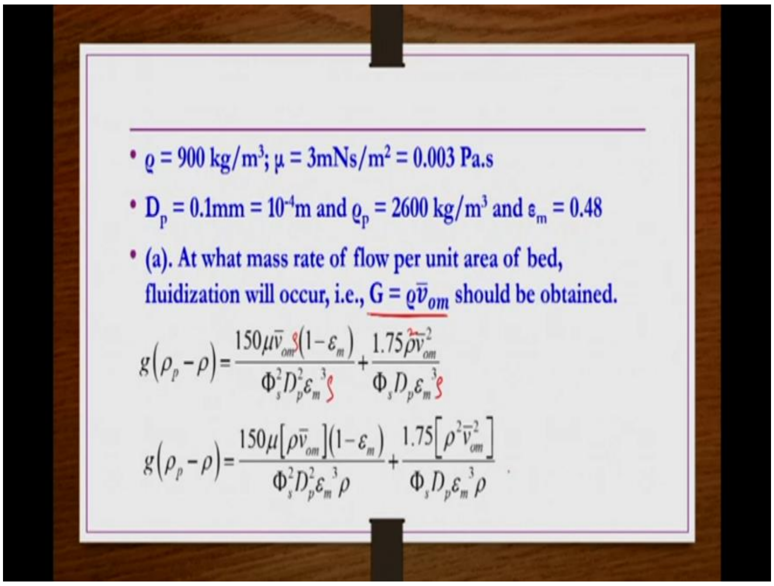

So fluid properties are given. Particle properties are given. Bed voidage is given. So, at what mass rate of flow per unit area of bed, fluidization will occur. So that is, density multiplied by minimum fluidization velocity, that we have to calculate. That we call it as a mass velocity or G. So, whatever this equation is there here. So, this equation you multiply by ρ and divide by ρ in the right hand side. So, $\rho\bar{v}_{0m}$ you can right it as G.

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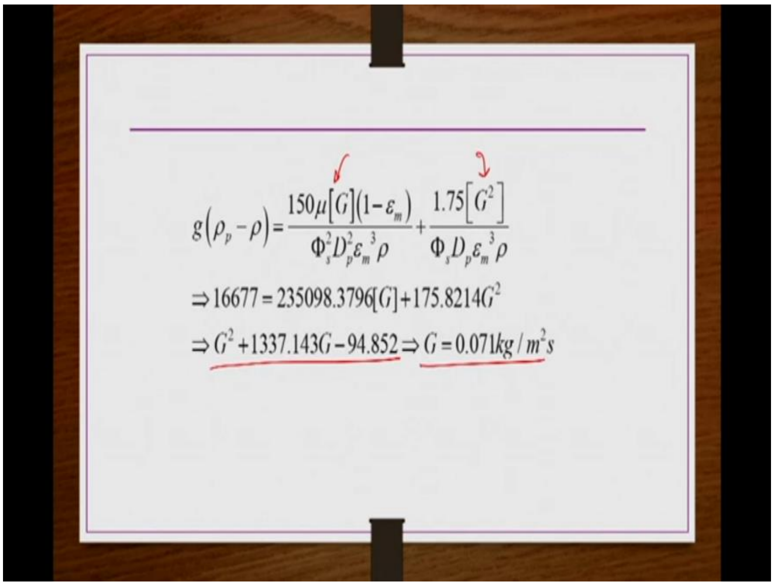

So this one. So, in terms of G, you can write this equation like this. So here also now except G everything is known so you substitute all the values. So then you get this equation. When you solve this equation, you get G is equals to 0.071 kg/m^{2}s.

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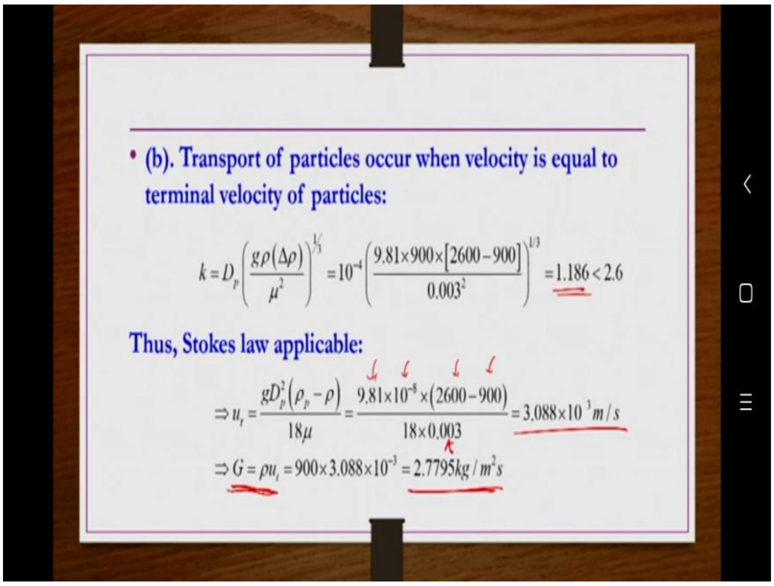

Now, similarly transport of particles occurs when velocity is equal to terminal velocity of particles. So we have to find out the terminal velocity of particles. But which equation should we use? We should we use the equation corresponding to Stokes regime or corresponding to Newton's regime, we do not know. So first we have to find out. That, how we find out?

We have to find out k value as we seen one of the previous chapters. So, $k = D_p\left(\frac{g\rho(\Delta\rho)}{\mu^2}\right)^{1/3}$. So you substitute all these values here. Then you will get k value 1.186 which is less than 2.6. That means it is settling in Stokes regime. So, Stokes law is applicable. Under the Stokes law, the settling velocity $u_t = \frac{gD_p^2(\rho_p-\rho)}{18\mu}$.

So there you substitute all the values. g, D_p^2, $(\rho_p - \rho)$ and then 18μ. Simplify, you will get settling velocity as 3 x 10^{-3} metre per second. So but this transport also, it was asked like you know, at what mass rate per unit cross-section area. So, that is G you have to find out. So whatever the u_t that you get from here, that you multiply by density of the fluid that is 900 kg/m^3. Then you will get g as 2.7795 kg/m^{2}s ok.

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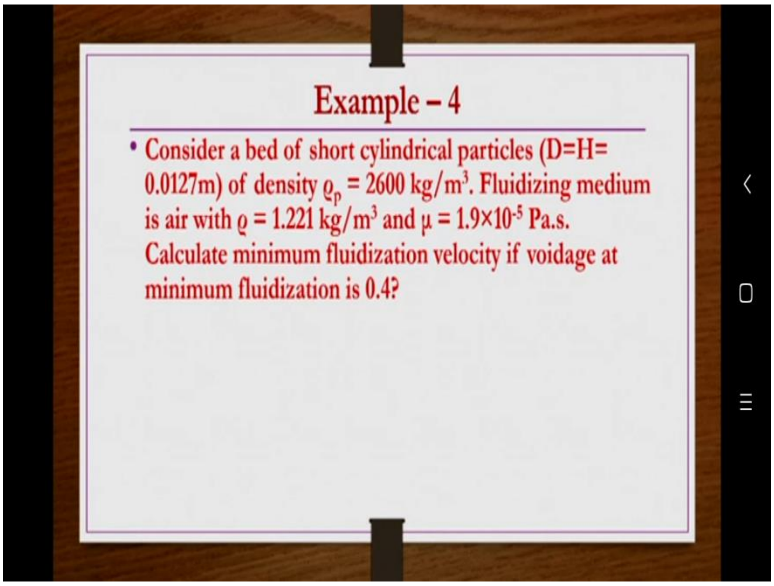

Now we take another example problem but you know the problem is quite similar but here we use short cylindrical particles where D and H of the particles, cylindrical particles, are equal to each other and then that value is 0.0127 metres. Density of the particle is given. Fluidizing medium is air. Density of air, viscosity of air are also given. The minimum fluidization velocity has to be found, if the minimum fluidization voidage or voidage at minimum fluidization condition is 0.4.

So, everything is known here but only the particle is not a spherical particle. So we have to find out sphere volume equivalent diameter D_p and then we have to find out ϕ_s. ϕ_s, we have already found in one of the previous chapter for the short cylinder as 0.847. That is, ϕ_s, the shape factor or sphericity does not depend on the size of the particle. It only depends on the shape of the particle for short cylinders. The sphericity is 0.847. That we have already done previously so we can use it.

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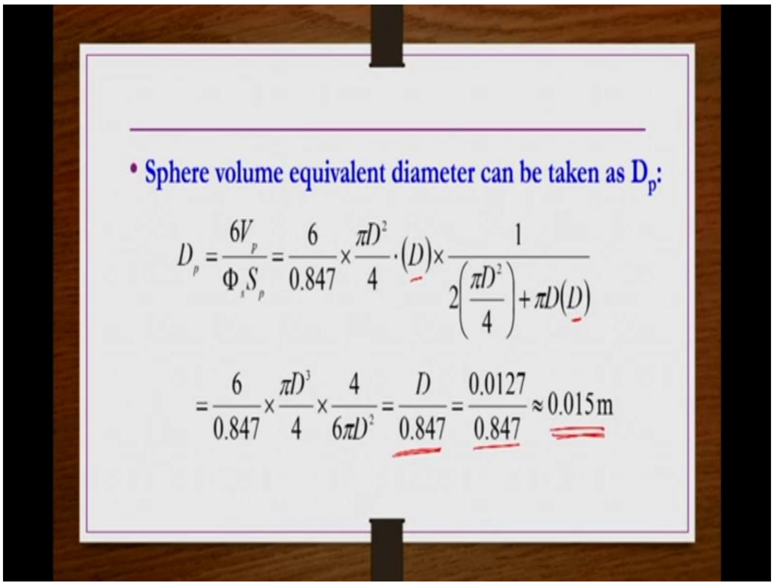

So now, sphere volume equivalent diameter D_p, you can find out by $\frac{6V_p}{\phi_s S_p}$. So, $6/\phi_s$ is 0.847. Vp is nothing but $\frac{\pi D^2}{4} H$. Now H is also equals to D. And then Sp is $2\pi r^2$, that is, $2\left(\frac{\pi D^2}{4}\right) + \pi D(D)$, H is now D anyway, so $\pi D(D)$. So H is equals to D. So that you substitute here. So when you do simplification because now everything is known here so this what you get here? From here, you get D_p is equals to D/0.847. So, that is 0.0127/0.847 that is approximately coming out as 0.015 metres. So this value you have to use in the corresponding equation for a minimum fluidization.

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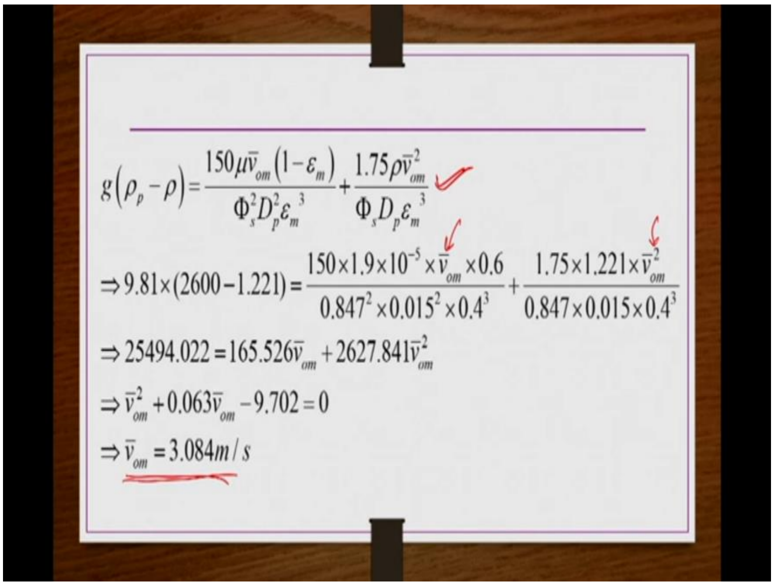

So, minimum fluidization equation is this one. That is, you know, we have been using the same equation. So here also we have to use the same equation. g, ρ_p,ρ , μ, ε_m, D_p, ϕ_s, everything is known here also in this equation also. So, you get minimum fluidization velocity as 3.084 metre per second. Quite high minimum fluidization velocity compared to the other previous example problems, ok.

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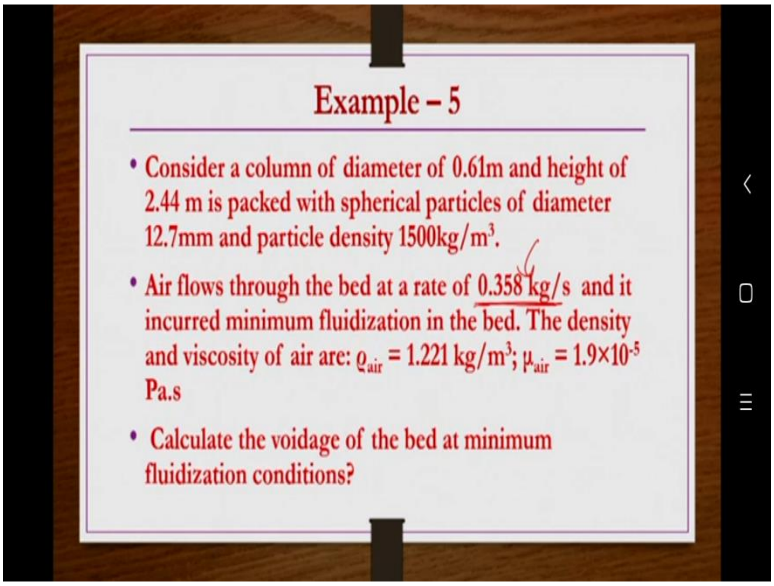

One more example problem. Here what we do? Minimum fluidization velocity if it is known we have to find out the voidage at minimum fluidization condition. So that is what we have to find, ok. So, consider a column of diameter of 0.61 metre and height 2.44 metre is packed with spherical particles of diameter 12.7 mm and particle density, 1500 kg/m^3. Air flows through the bed at a rate of 0.358 kg/s, ok.

So, the velocity flow rate is not given in a kind of, in terms of velocity, but it is given as mass rate in terms of kg/s. And it incurred minimum fluidization, when the air is flowing at this mass rate, so that that is providing, you know, minimum fluidization. So, this mass rate is corresponding to the minimum fluidization velocity, ok. The density and viscosity of the air are given. So calculate the voidage of the bed at minimum fluidization conditions, ok.

So, we need to find out what is G because here you know in the problem the $\bar{v}_{0m}$ is not given directly, right. So in the equation we cannot use directly. So, what we do? That equation, minimum fluidization velocity or equation corresponding to minimum fluidization. That we write in terms of G and then G we can find out from here, by dividing this mass rate by the cross section area of the column.

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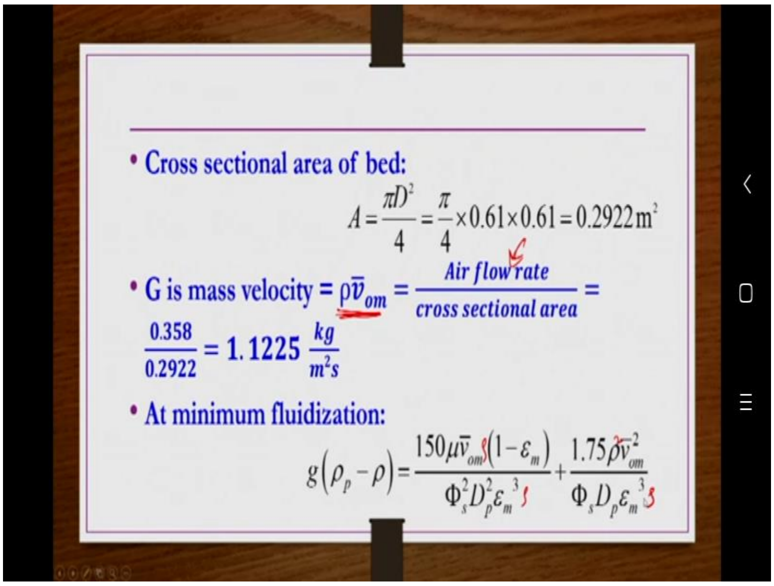

So, first we have to find out the cross-section area of the bed, that is, $\frac{\pi D^2}{4}$ that comes out to be 0.2922 m^2. Then mass velocity G that is corresponding mass velocity is corresponding to the minimum fluidization. It is given in statement. So, $\rho \bar{v}_0$ should be $\rho \bar{v}_{0m}$ because at this corresponding to at this mass flow rate, the minimum fluidization has occurred. So, we have to take $\rho \bar{v}_{0m}$. So, air flow rate divided by the cross-section area, so $\rho \bar{v}_{0m}$ or G we get as 1.1225 kg/m^2s.

So at minimum fluidization condition, we have this equation. Now again here we, right hand side, you know, we multiply and divide by the density so that we have a kind of a $\rho \bar{v}_{0m}$ terms. Those terms we can replace by G. So that this equation would be, you know, in terms of G or you know G is already we have calculated.

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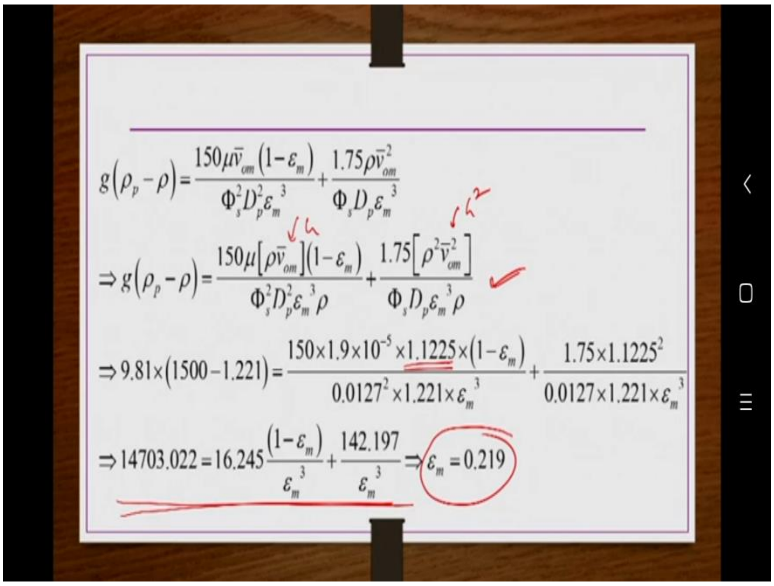

So, in this equation everything is known. This is nothing but G. This is nothing but G^2 So, g we have already calculated as 1.1225 and rest all the properties are also known here, ok, except the voidage at minimum fluidization condition. Except ε_m, everything is known. So, if you substitute all the numbers, you get equation for ε_m like this here. This equation if you solve, you will get ε_m closely 0.219, ok.

So voidage at minimum fluidization condition is 0.219 for this condition. So, the same equation, different problems we have taken. Different way, different quantities we can calculate using this equation. So, this is about the minimum fluidization and then corresponding problems on minimum fluidization.

Now, we go to the next part of the lecture, that is, other types of fluidization. What we can get? So expansion of fluidized beds. So you know the superficial velocity, when it is increasing, whether it is a in a gas or liquid fluidizing medium. The bed is expanding. The bed is, bed height is expanding. That is what we have seen, right. So now, how much is increasing, the bed height, or how much expansion is taking place. That we have to find out or when the minimum fluidization velocity, beyond the minimum fluidization velocity if you increase the superficial velocity, the due to the expansion, bed voidage is also increasing. How much, that is increasing,

So, that ε that expanded conditions or height of the expanded bed, that is L are ε for the expanded bed. For expanded bed velocity is much more higher than the minimum fluidization velocity, ok. So now, what is that here? The velocity is not a kind of a independent variable. It is input condition, dependent variable, right.

Independent variable is either L, bed height or you know voidage of the bed at expanded conditions. So those things we are going to find out for a different types of a beds, ok. Or different types of fluidization. Especially for expanded fluidized beds and then particulate fluidization and then bubbling fluidization. For three cases we are going to do this one.

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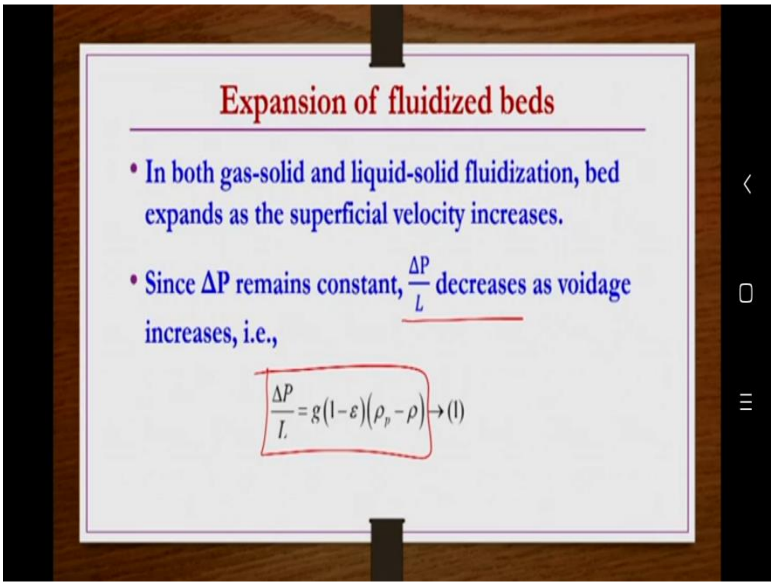

Expansion of fluidized beds. That we have seen irrespective of fluidizing medium, whether gas or liquid is the fluidizing medium. That is for both gas-solid system and then liquid-solid system, the bed expands as the superficial velocity increases. And then once the fluidization you know minimum fluidization has occurred, so as you increase the superficial velocity, pressure drop remains constant. It does not change with the superficial velocity, but the pressure gradient decreases as the ε_m or as the ε increases. So, so pressure gradient decreases as voidage increases because initially the voidage is low but now the velocity is increased, the superficial velocity is increased beyond the minimum fluidized conditions, right.

So that is the reason you know, the voidage is increasing. Bed expansion is taking place. Particle volume is remaining same so when the bed expansion is taking place, overall volume of the bed is increasing, but the particle volume is same. Corresponding, you know, the voidage volume will increase. So that means ε will increase.

So when ε increasing though the pressure drop is remaining constant but pressure gradient is decreasing according to this equation. That is what we have been using. Ok, so this is the equation one can solve and get the details of the, you know, voidage etc. That depends on the how much is the expansion, ok.

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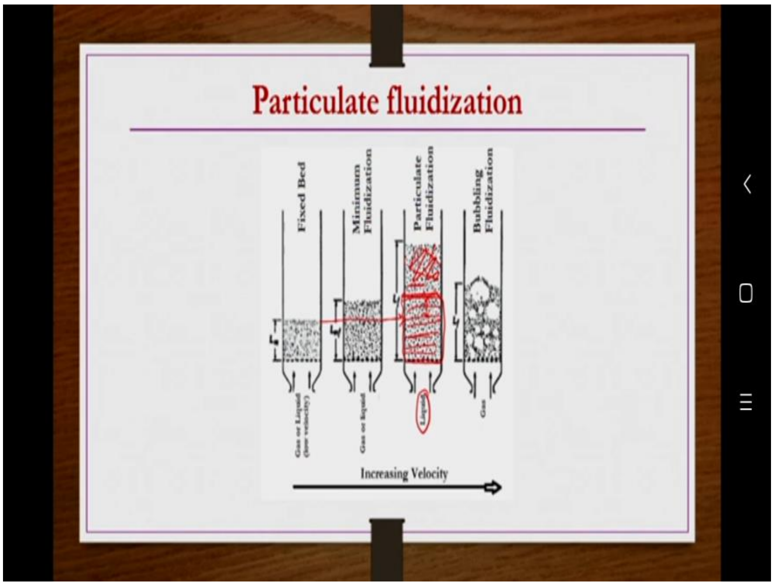

So, let us say now particulate fluidization. Particulate fluidization we just you know recapture. Here, you know, a kind of uniform expansion of the bed is taking place, right. So particulate fluidization first of all it occurs. Then the fluidizing medium is a kind of liquid. And then across the cross-sections, you know, the bed is, you know, so uniformly expanding that across the different cross-sections or at different locations in the bed if you try to measure the bed density, it is going to be almost constant, ok. That is what we have seen.

So now, what we see? So, if this expansion, pictorially it shows like you know it is almost like you know very, entire column has been, particles are reaching almost to the top of the column. But assume this, you know, let us say if you assume this corresponding to the initial packing height, right. The expansion is only small. It is uniform but it is expansion is only small. Initial bed height was this one, but now it is expanded. Slightly expansive, we are not taking that expansion is taking so much.

Why? Because we are making some kind of simplification so that we can use the existing equation to find out what is this expansion. How much it is expanded or how much, you know, the voidage has increased because of the expansion? That we can calculate, ok.

So the bed is so uniformly expanding that it is almost like is a kind of a packed bed structure only it is there. There is no abnormality across the different levels in the bed. As you move up from bottom to top or top to bottom, you know, at any location the uniformity of the bed is there almost. So, what we can say? Whatever the, whatever the Ergun's equation is there, that can also be used even for the particulate of fluidization provided the expansion is only slight expansion it is not large expansion. That is only slightly expanded beyond the minimum fluidization condition. Then, we can use the Ergun equation also here, ok. So, if you do that one what we get?

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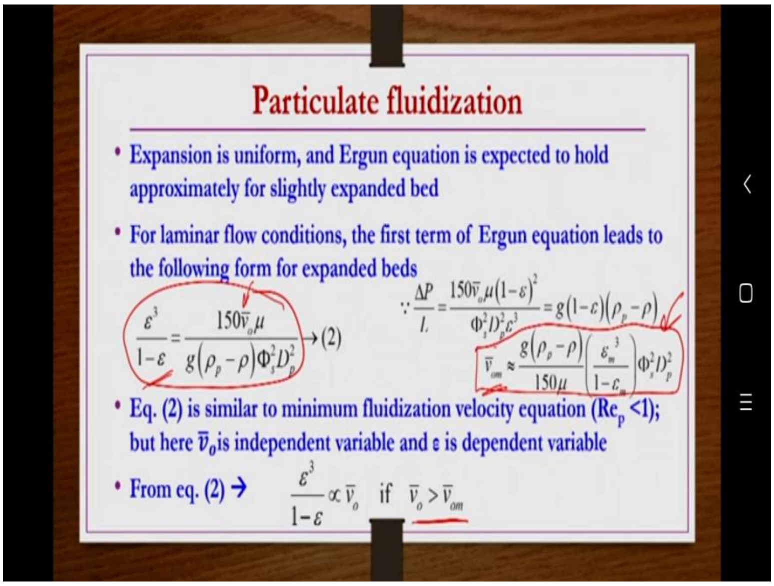

Here, what we can see, expansion is uniform so Ergun equation is expected to hold approximately well for slightly expanded bed. Not very largely expanded particulate fluidization but only slightly expansion we are considering. Under those conditions only these, these equation whatever Ergun equation we are going to use to get the things will be valid. So then for laminar flow conditions, the first term of Ergun equation leads to the following form for the expanded bed. So this is the Ergun equation, ok.

So this Ergun equation, this first, this is the Ergun equation here. Now in place of $\frac{\Delta P}{L}$, if you replace $g(1-\varepsilon)(\rho_p - \rho)$ then that will be corresponding to the, you know, expanded bed conditions, right. So, now if you equate this particular part, right. So, if you equate these two terms. What you will get? You will get an expression for ε. So you can write $\frac{\varepsilon^3}{1-\varepsilon} = \frac{150\bar{v}_0\mu}{g(\rho_p-\rho)\phi_s^2 D_p^2}$.

So, remember here, this equation here, you know, exactly same as a kind of equation for the minimum fluidization velocity. But when you use the same equation, this equality whatever we have done here, so when you use it for the calculation of minimum fluidization velocity. There ε_m or the voidage at minimum fluidization velocity is known. It was not independent variable. It was dependent variable and known variable, input variable. That is what we have seen. And then superficial velocity corresponding to minimum fluidization, that is the $\bar{v}_{0m}$ was independent variable so we equated, we used this equation to get the $\bar{v}_{0m}$ from here.

Now in this condition, expanded bed condition, the superficial velocity is beyond the minimum fluidization velocity but the expansion is very small. Expansion is very slight so then we are using the same equation. Here we are trying to find out what is the voidage, right. Because now the velocity is known. It is beyond the superficial velocity, some known velocity you are giving, ok. So corresponding ε we have to find out, that is the unknown now. Ok, L and ε are unknown, ok.

So, this is what we can see here. So, now you can see, this equation number 2 and then this equation for minimum fluidization velocity. They are, you know, same, similar to each other indeed, right, ok. So but only thing is that here in the minimum fluidization velocity is independent variable whereas in the present case you know, $\bar{v}_0$ is a kind of independent variable and then ε is a kind of dependent variable. Whereas in the minimum fluidization condition, you know, this $\bar{v}_{0m}$ is dependent variable and ε_m is known and independent variable. That is the different.

So, the same equation we are using here to calculate the minimum fluidization velocity. Same equation here, we are using to get the voidage for a expanded bed which is beyond the minimum fluidized conditions. So, superficial velocity is known. So, that means $\frac{\varepsilon^3}{1-\varepsilon}$ is proportional to the superficial velocity, provided the superficial velocity is greater than minimum fluidization velocity, ok. If $\bar{v}_0$ is equals to minimum fluidization velocity then ε is nothing but ε_m.

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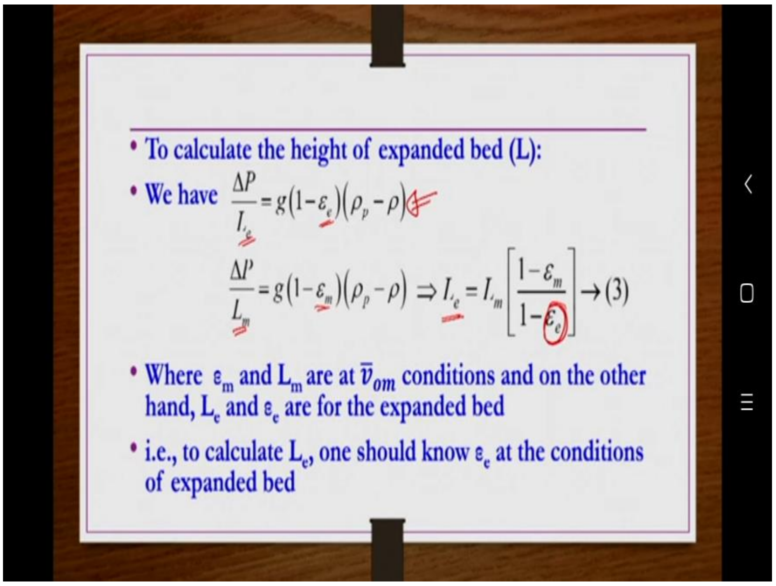

So now bed height if you wanted to calculate for the expanded bed then we have two equations. Indeed, one equations only. So, under the expanded bed conditions, $\frac{\Delta P}{L}$, we can write $g(1-\varepsilon)(\rho_p - \rho)$ but now L_e , ε_e I am writing just to indicate this is for the expanded conditions.

But the same equation is valid under the, for the minimum fluidization condition. Or from the minimum fluidization point onwards, whatever the velocity you increase, whatever the bed expansion takes place, the pressure drop is remain constant, ok. And then pressure gradient is equals to $g(1-\varepsilon)(\rho_p - \rho)$. So, under two, same equation, under two conditions we are using.

So, now this is here ε_m, L_m we are using because these equations are now under minimum fluidized bed conditions, right. The equation exactly same but one is at the expanded condition, another one is the minimum fluidized condition. Expanded conditions are beyond the minimum fluidization conditions so that the enough expansion is taking place but that expansion is not too high so that Ergun equation can be used.

So, from these two equations what we can write $L_e = L_m\left[\frac{1-\varepsilon_m}{1-\varepsilon_e}\right]$. So, that means from this equation if you wanted to find out the height of the bed expansion. How much expansion of the bed has been taken? If you wanted to know, you should know what is the voidage of the bed at that expanded conditions or the voidage of the expanded bed that you should know. Otherwise you cannot use, ok. Otherwise this equation is not the good enough to use. So, what we do here?

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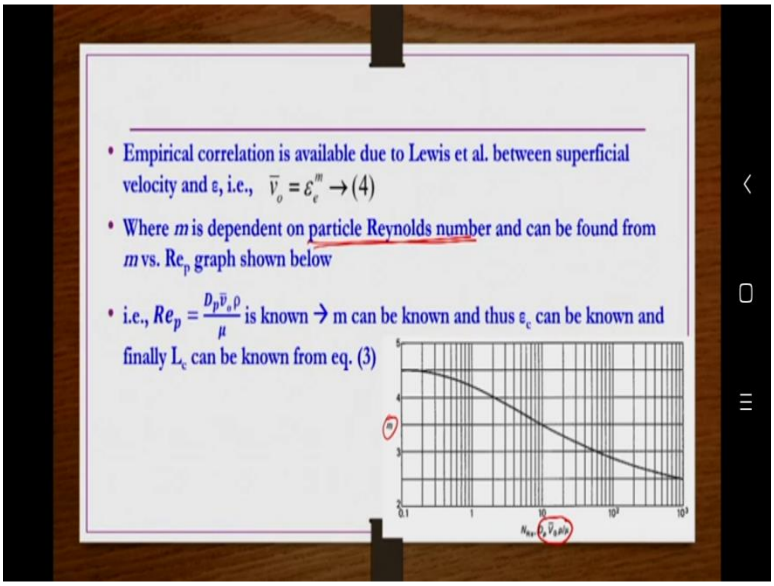

You know, there are some kind of empirical correlations based on the experimental studies are there, between superficial velocity and then, you know, voidage for expanded bed conditions. So, for expanded bed conditions, superficial velocity is equals to voidage of expanded bed power m, this m is a constant, which depends on the particle Reynolds number, right. It depends on the particle Reynolds number, not the Reynolds number for the packed bed. It depends only particle Reynolds number. It is similar like, you know, we have n vs Re_p for the hindered settling case, right. Similarly, m vs Re_p is available for the expanded beds also. Similar like that.

If you remember that hindered settling conditions, there also suspension velocity u_s is equals to u_t, that is, free settling velocity of single particle multiplied by ε^n and that n was depending on the particle Reynolds number. Similarly, here also, for packed bed or expanded fluidized beds, there is a relation between the superficial velocity and then voidage and then this power m is dependent on the particle Reynolds number.

So if you know the Reynolds number, $\frac{D_p \bar{v}_0 \rho}{\mu}$, this is for the particle because we are not dividing by $(1-\varepsilon)$, right. If you know this thing from this graph, you can know m. So once m is known then you can calculate the required things, ok.

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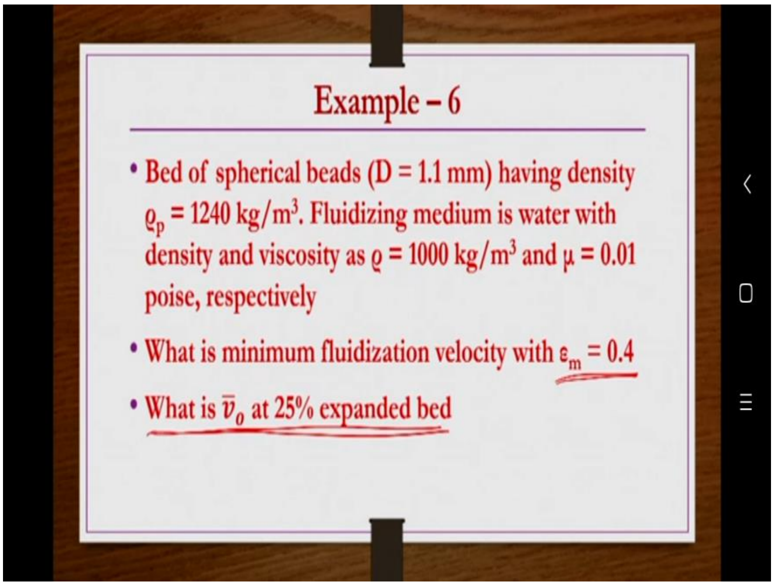

So, how to calculate, we see an example here. So, bed of spherical beads, diameter 1.1 mm, having density ρ_p is equals to 1240 kg/m^3. Fluidizing medium is water with density and viscosity of 1000 kg/m^3 and 0.01 poise respectively. So, what is the minimum fluidization velocity with ε_m equals to 0.4. Indeed, up to this part we have already calculated in one of the example problem. But here again we need to calculate because that is required for the next part of the problem.

Next part of the problem is what is the superficial velocity if you want to have 25 percent expanded bed. So, initial bed whatever there, let us say, you know, initial bed is let us say you know some like a 1 metre. So then if you wanted to have 1.25 meters of the bed height so how much superficial velocity should you provide and then corresponding voidage is what. That is what you have to find out.

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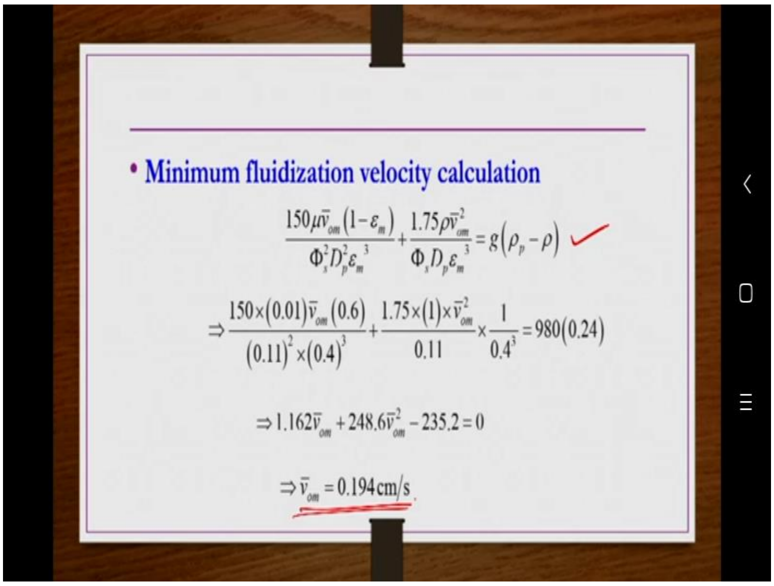

So minimum fluidization velocity. This is the equation. We have done several problems, right. Here everything is known except the minimum fluidization velocity $\bar{v}_{0m}$. Now here in this equation you substitute all the terms and then simplify, ok. I have done this one in CGS units, ok. So then you will get $\bar{v}_{0m}$ as 0.194 cm/s, ok.

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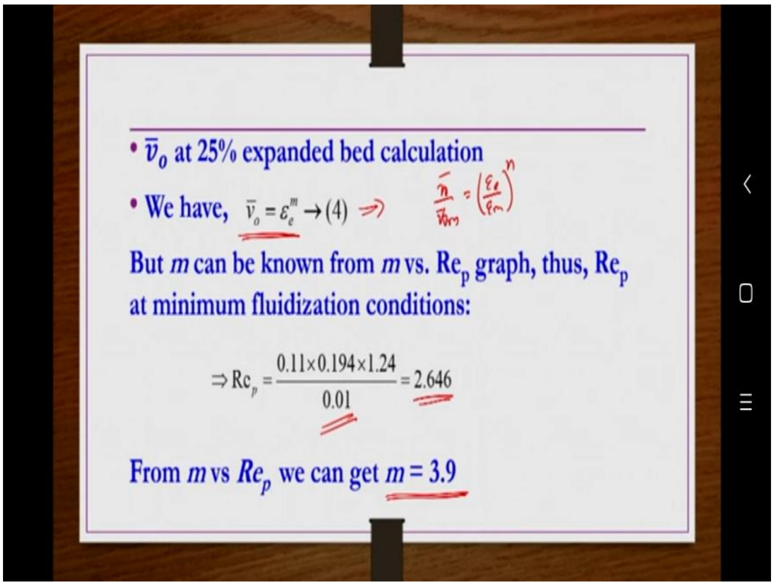

So, now second part. Superficial velocity at 25 percent expanded bed calculation. So, we have this equation for you know $\bar{v}_0$, for expanded beds. But we do now know what is ε_e. If we know that one, we can calculate it, ok. So, and then we do not know m also. So, we have to calculate both. m we can calculate from the graph, m versus Re_p. So first let us calculateRe_p. Re_p, if we calculate, substituting all the values again in CGS units, you will get Re_p as you know 2.646 and then from the graph for the corresponding, you know, Re_p is equals to 2.646. What is the m value? That is you read from the graph as 3.9. So this equation under 2 conditions if you apply for $\bar{v}_0$ and then $\bar{v}_{0m}$. So then what will have? You will be having $\left(\frac{\varepsilon_e}{\varepsilon_m}\right)^n$ because this condition is valid from the minimum fluidization condition onwards till the expanded bed.

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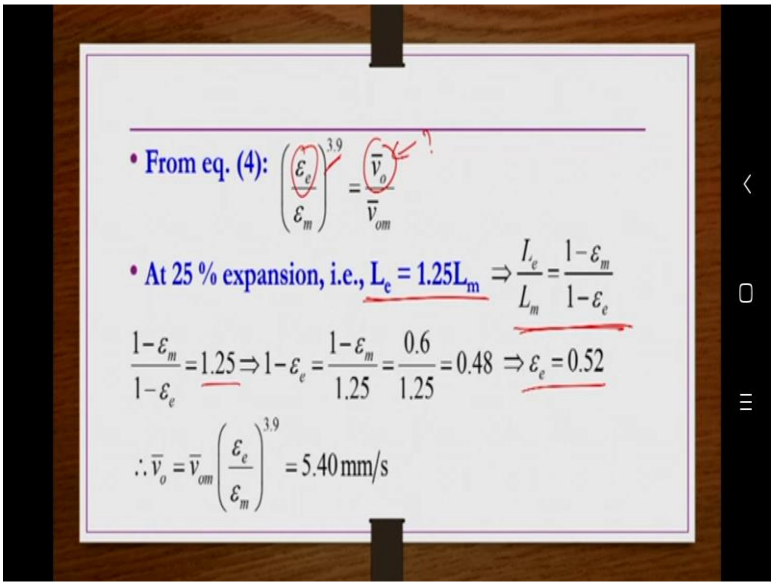

So, that is what this equation we can have. is equals to $\left(\frac{\varepsilon_e}{\varepsilon_m}\right)^{3.9} = \frac{\bar{v}_0}{\bar{v}_{0m}}$, m is 3.9 that is, we know it actually. Why we are writing it here? Because $\bar{v}_0$ is nothing but ε_e power 3.9 but how to find out ε_e? For that, we may be needing this expression, ok. So, at 25 percent expansion, that means, L_e is equals 1.25 L_m.

So, after minimum fluidization 1.25 percent it has expanded so 1.25 times the height at the minimum fluidization conditions. That is what you mean by the statement, ok. So then from here, $\frac{L_e}{L_m}$ you can write it as, I know, this equation already we have got it, you know, previously derived it. So here in this equation for $\frac{L_e}{L_m}$ you substitute 1.25. So, $\left[\frac{1-\varepsilon_m}{1-\varepsilon_e}\right]$ is 1.25. From here $(1-\varepsilon_e)$, you will get 0.48 means ε_e is 0.52. So, now this equation ε_e is also known. So, you can find out what is this one.

So, when you substitute this one you will get, you know, $\bar{v}_0$ as 5.40 mm per second. 5.40 mm per second you will get, you know, superficial velocity that is when you maintain superficial velocity of 5.4 mm per second then the bed expansion, 25% expansion of the bed will take place.

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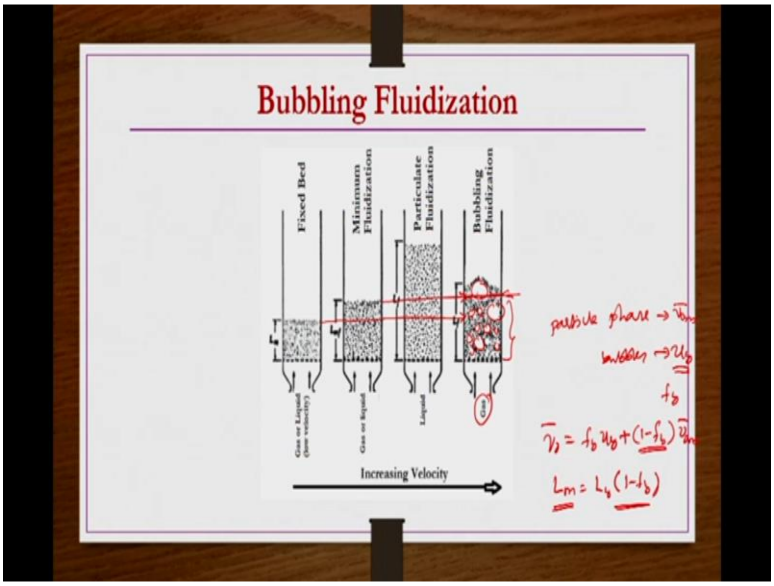

Now, bubbling fluidization. So, let us see the picture again. What happened? So, this is the initial fixed bed conditions, ok or initial height of the bed, it is like this, ok. So, now when the superficial velocity or the fluidizing medium when you use gas and then superficial velocity when you increase beyond the, you know, minimum fluidization velocity. So, bubbling fluidization is taking place here.

So now you can see most of the bed, you know, these voids are forming or bubbles are forming and they are rising up, ok. They are rising up. So most of the expansion of the bed here what we can see that is because of the expansion of the bubbles.

Whereas the particles are more or less remaining it is not exactly initial position, so they are more or less corresponding to the bed height at the minimum fluidization condition, whatever were there. Up to that one only they are, you know, mostly, you know, expanding. Beyond that is a more or less, you know, gas is only expanding. Of course there are a few particles but there are very few.

So what does we can say? Even under the bubbling fluidizing conditions, particle phase or dense phase is at minimum fluidization velocity conditions only. And then bubble phase or bubbles are at certain average bubble velocity u_b. So whatever the bubble fraction if you know or the volume fraction of the bubble if you know.

So what you can write, the total superficial velocity you can write it as, you know, bubble fraction multiplied by the average velocity of the bubble fraction. So, what is the remaining fraction other than the bubble is there? That would be the particle fraction or the dense fractions. So that would be $(1 - F_b)$, and that dense phase is at minimum fluidization condition, ok.

Similarly, L, whatever the bed is there so L_b, bed expansion. Now b I am using in order to distinguish in terms of you know you know the bubbling fluidization. So that would be $(1 - F_b)$ only. $(1 - F_b)$ is the particulate phase and that phase is rising almost to the height of the bed at minimum fluidization conditions, is not it. So that means $L_m = L_b(1 - F_b)$. So these are the approximations we are using. Approximation, but reliable approximation you can see from these pictures anyway.

So the bubbling fluidization where you know, most of the bed expansion is taking place because of the, you know, expansion of the voids or bubbles are forming. Whereas the dense phase is almost like you know expanding as a kind of minimum fluidization conditions only, ok. So under those assumptions we try to develop equations here.

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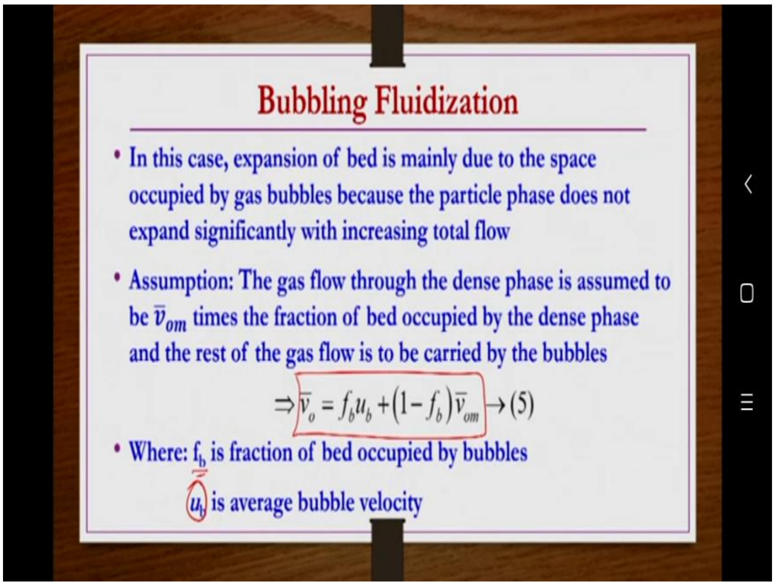

So this is the assumption that we are having. So then, we get this equation as I already described, ok. So here now, how to find out this minimum fluidization the bed height corresponding to the expansion. What is the expansion height and then what is the corresponding voidage? Those things we are going to calculate for this bubbling fluidization condition as well.

So here, as I mentioned, F_b is the fraction of bed occupied by the bubbles or bubble volume fraction. Then u_b is the average bubble volume velocity. So this equation if you wanted to know, if you wanted to know this equation, you should know u_b. What is the average bubble velocity? That is again a very difficult to find out experimentally for each and every experiment. So there are a few experimental studies have been done. And there are empirical correlations are available. That we will see anyway.

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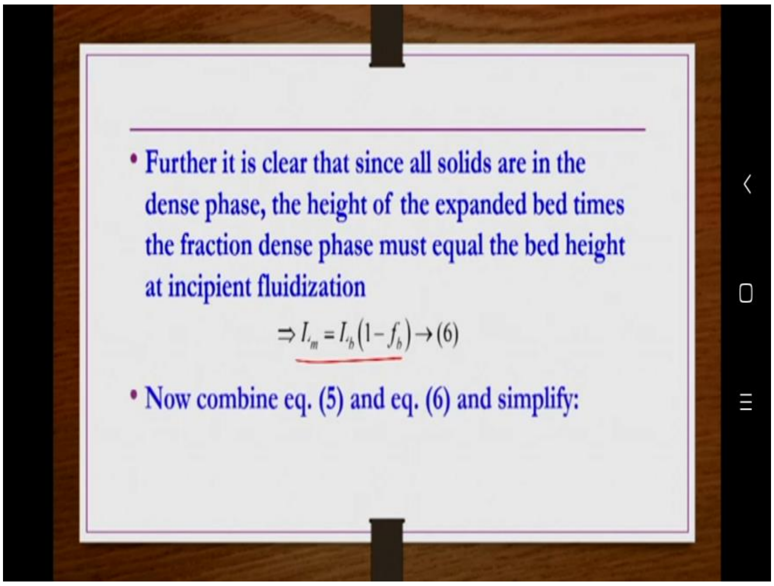

So, bed expansion also we can see. You know, the height of the expanded bed times the fraction of dense phase must equal to the bed height at incipient fluidization conditions as I explained using the figure just now, ok.

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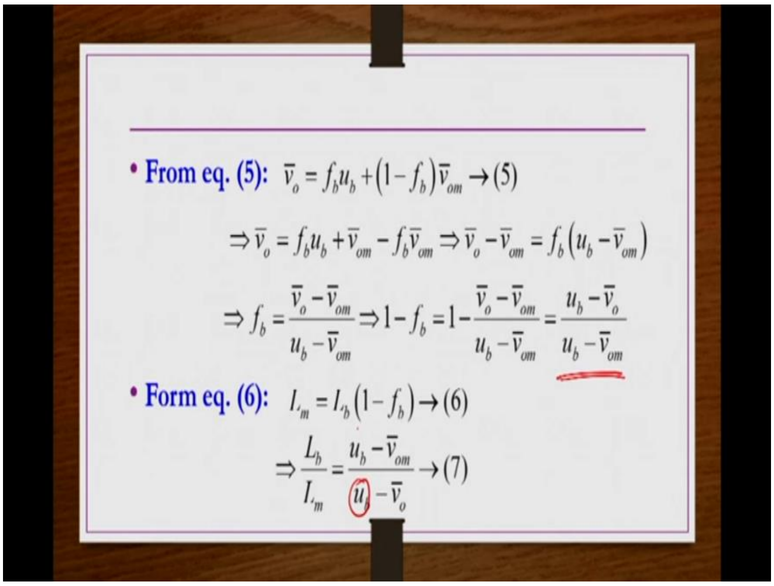

So, these equations when you combine together, from equation 5 we are having this equation, ok. So you expand this equation so that you can write $f_b = \frac{\bar{v}_0 - \bar{v}_{0m}}{u_b - \bar{v}_{0m}}$. From here, $(1 - f_b)$ you get this thing. Why are getting writing $(1 - f_b)$? Because if you wanted to know height of the expanded bed under the bubbling fluidization condition, that L_e if you wanted to know, you should know $(1 - f_b)$. Because $L_m = L_e(1 - f_b)$.

So, there that equation 6, $L_m = L_e(1 - f_b)$. Here, $\frac{L_b}{L_m}$, you can write it as $\frac{u_b - \bar{v}_{0m}}{u_b - \bar{v}_0}$. And then this Ub, you need to know. You need to know as I mentioned from experimental studies, each and every experimental studies finding them is very difficult.

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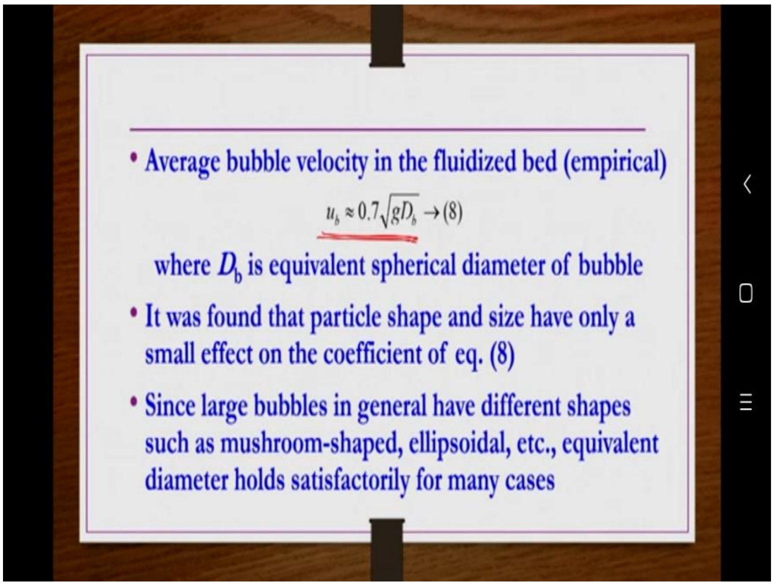

So already there are empirical correlations are available like this. u_b is approximately $0.7\sqrt{gD_b}$, D_b is nothing but equivalent bubble radius. Because in general, these voids are forming. They do not form the voids or bubbles which are rising through the impact area, you know. They are not in a kind of, you know, spherical shape. In general, they are in a something like, you know, mushroom-like shape or elliptical shape. Like that only they are there in general. So, that is the reason equivalent spherical diameter of bubble is considered here.

So, u_b is known so then you can use that equation number 7 previous equation to get Le, ok. So, this is about the bubbling fluidization. So, there are more other details are possible for each and every type of, you know, fluidized bed. So, fluidization is a kind of very complex and turbulent, very complex behaviour. Many times repeatability is also kind of very different task. So, most studies one can be seen as a kind of separate course on fluidization. So but at this course level, this information is sufficient about the different types of fluidization.

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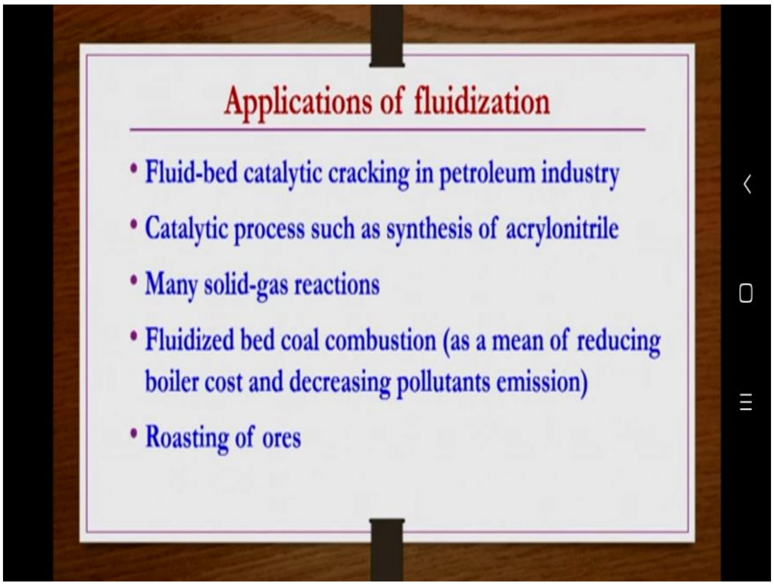

Now, before concluding the fluidization part we seen some applications in terms of advantages and disadvantages also we will see of this fluidizations. Coming to the applications, fluid-bed catalytic cracking in petroleum industry is the very famous one where the fluidization is used. Many catalytic processes such as synthesis of acrylonitrile etc or many polymeric reactions or many gas-phase reactions SO_2 conversion to SO_3 etc, there also we use fluidized beds.

Many solid-gas reactions, fluid-bed coal combustion. Coal combustion is in general done in a kind of fluidized bed reactor so that whatever the gases effluents carrying the energy are there, they are collected and then they will be used for the electricity production, right. Fluidized bed coal combustion is also used as a mean of reducing boiler cost and decreasing pollutants emission in general.

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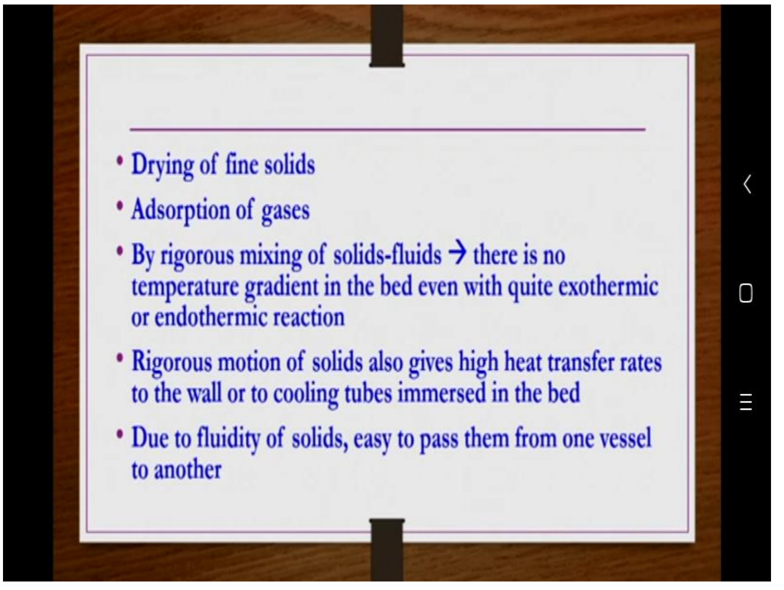

For roasting of many mineral ores before processing to the subsequent unit processes, drying of many fine solids, adsorption of several gases also done in fluidized beds. And then there is rigorous mixing of solids and fluids in general, so because of that there is no temperature gradient in the bed even with quite exothermic or endothermic reactions in general.

Rigorous motion of solids also gives high heat transfer rates to the wall or to the cooling tubes immersed in the bed in general. And then due to the fluidity of solids, these particles are easy to pass from one vessel to the other vessels.

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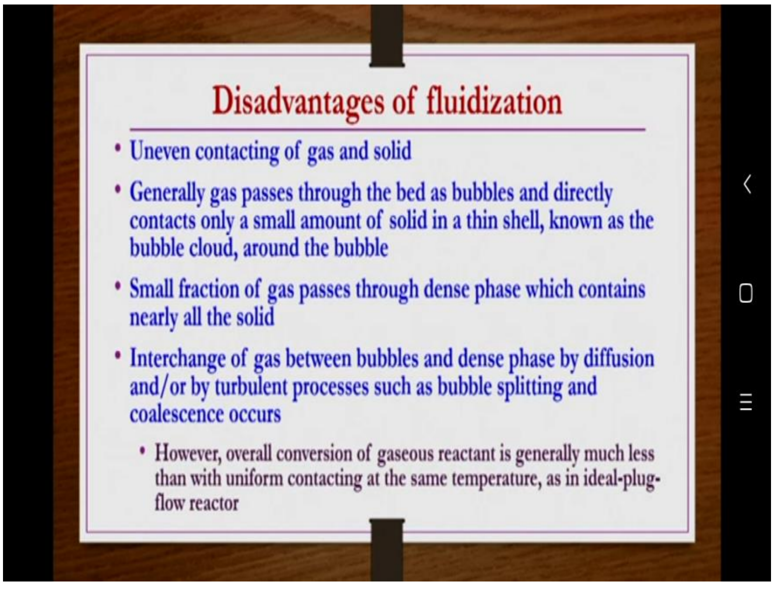

However, there are several disadvantages are also there with this fluidization. First of all, uneven contacting of gas and solid especially in bubbling fluidization as we have seen or (slug) slugging fluidization etc, those we have seen. There we have seen, there you know the contact is not even as even we have seen. The contact is very even in kind of particulate fluidization. So, for the gas-solid system, uneven contact is a kind of disadvantage. And then, that is generally because the gas passes through the bed as bubbles and directly contacts only small amount of solid in the thin shell, known as the bubble cloud etc.

As we have seen, these voids whatever the bubbles are forming, they are almost free of particles and sometimes, they are expanded so much, you know, they are having contact with the less number of particles. And then small fraction of gas passes through the dense phase which contains nearly all the solid, unevenity there is no evenity.

Interchange of gas between bubbles and dense phase by diffusion and or by turbulent processes such as bubble splitting and coalescence occurs. Those things was again a kind of disadvantage. However, overall conversion of gaseous reactant is generally much less than the same that obtain using a ideal plug flow reactor at a same temperature.

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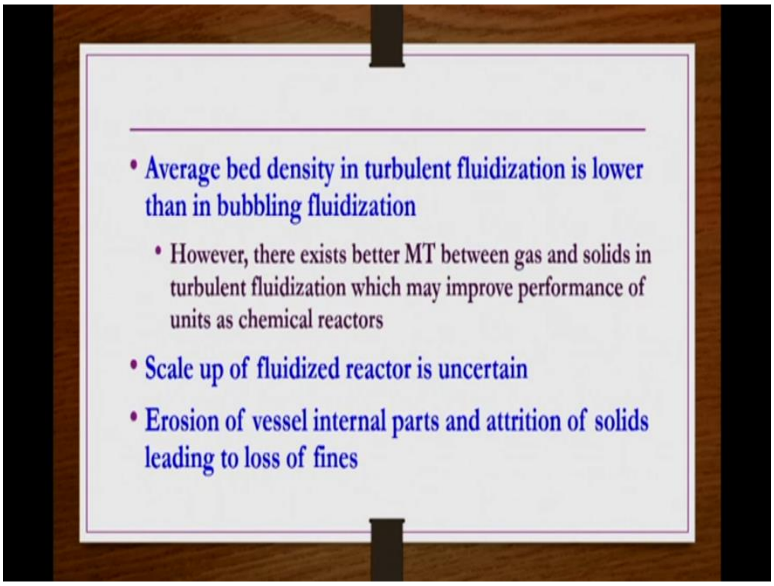

Average bed density in turbulent fluidization is lower than in bubbling fluidization in general. However, in turbulent fluidization mass-transfer between gas and solids is much better than compared to bubbling fluidization. Especially, that is going to be useful in improving performance of chemical reactors. The finally very important one, the scale up of fluidizing bed reactors is very uncertain, is very, very uncertain.

And then erosion of vessel internal parts and attrition of solids leading to loss of fines also occurs sometimes in this kind of a fluidized bed which may be taken as a kind of disadvantages in several cases.

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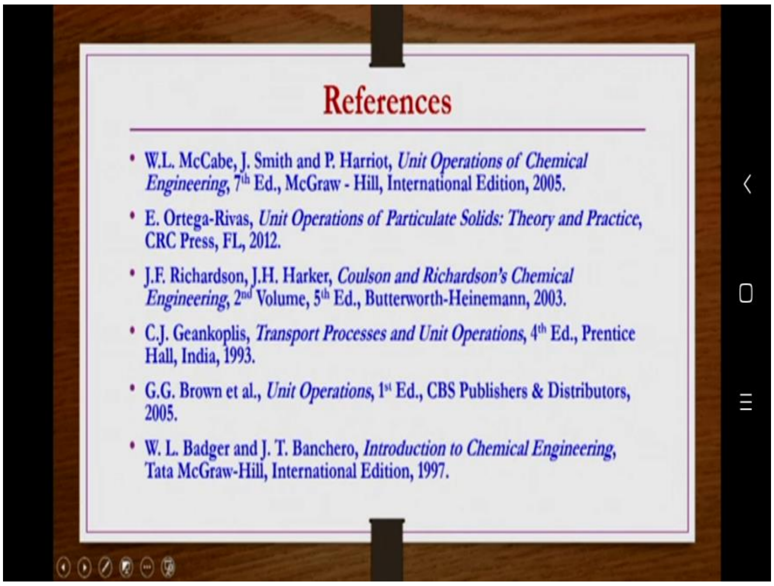

References for this lecture are, you know, McCabe, Smith and Harriot. Ortega-Rivas, Richardson and Harker. Geankoplis, Brown et al and then Badger and Banchero. But primarily, most of the lecture is prepared from McCabe and Smith and some of the problems are Richardson and Harker and then Geankoplis. Thank you.