Mechanical Unit Operations
Professor Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology Guwahati
Lecture No 26
Principles of Cake Filtration

Welcome to the MOOCS course Mechanical Unit Operations. The title of this lecture is principles of cake filtration.

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So, in cake filtration we have seen. Like you know there is a kind of filter medium onto which like you know normal to the surface of the filter medium. The slurry is allowed to pass through and then whatever the coarse particles, large amount of coarse particles are there. There will be forming a kind of a layer, which forms a kind of a layer of particles which is known as a kind of cake, cake formation.

So, under such conditions what are the working principles, etc those things we are going to develop now. Before going to develop this working principles we will see how this cake filtration process can be related to packed bed, flow-through packed bed that we have already seen one of the previous lectures.

Principles of cake filtration. The principles of this cake filtration are almost analogous to the case of, kind of a flow-through packed bed or fixed bed are kind of flow-through porous media. But, in the case of packed bed or porous media or fixed bed, what happens the resistance to the flow whatever is there, that is independent of the time. That does not depend on the time, okay.

So, but whereas in the case of cake filtration process, whatever the cake is say that, you know cake is actually behaving as a kind of a packed bed. Now, so that size of that cake is gradually increasing with the time. So more and more particles are coming in, and then accumulating on the surface. So cake technique is increasing. So, obviously the resistance you know gradually increases. So that means you know here in the case of cake filtration the resistance to the flow-through bed of particles that is form in cake is a kind of, you know function of time and then this resistance increases with increasing the filtration process time.

So just to recapitulate, what we have? We have this kind of filter medium in the, you know for a cake filtration kind of thing. So, here it is having some kind of a porous structure. This porous structures are shown almost kind of a channels kind of thing, you know straight or slightly straight channel kind of thing. But not necessarily there will be a straight channels, there can be a kind of a no interlinking between the channels etc.

The slurry comes through here, in the normal direction to the surface of the filter medium right. So the large amount of solid particles or whatever are there in the slurry, they will be deposited on the filter medium and then there will form a kind of cake. And then that cake size gradually increases as the filtration time increases.

Thus, the resistance to flow will also increase. So whatever the clear or almost clear filtrate is there, that will be collected from the bottom. So now, what we have to do? We have to see how the principles of packed bed that we have already develop. How we can make use of those principles for the case of a cake filtration. Basically the Ergun equation or Kozeny Carmann equation, whatever or Burke Plummer equation for different cases are there. So how we can make use?

Actually if the flow is small, low Renaults number flow, then Kozeny Carmann equation is valid. If the flow is high in the Newton's flow regime. Then you know, kind of Burke Plummer equation is valid. If you add the both the resistance together, then whatever the equations that you get that is known as the Ergun equation, right. So those equations or any of those equations can be make use here or not, that is what we are going to see. And then how to modify to those equation for a given cake filtration problem here, right.

So in cake filtration, resistance to flow increases as I mention with time as filter medium becomes clogged or filter cake builds up. So thus whatever the Ergun equation is there, that should be modified accordingly to bring in the effect of the filtration time or cake formation or you know clogging of filter medium or you know building of cake whatever the reasons are there. So, how to bring those concept here? So all of them can be, you know, combine together. If you can bring in the time factor into these equations. So that is what we are going to see.

Once you bring the time factor into this, you know principles, flow-through packed bed cases, whatever the equations are there. In those equations, if you bring in the time factor. So then accordingly the rest of the things will also automatically come into the picture. How do they come, that we are going to see now.

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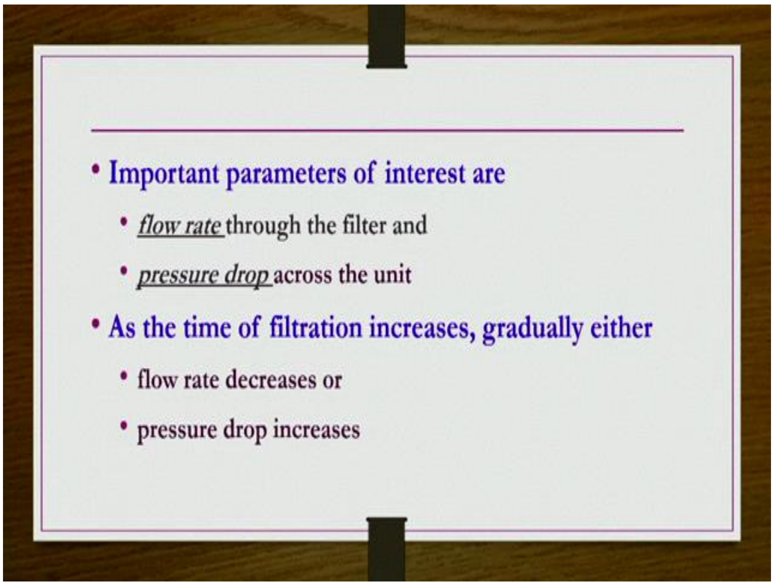

So in this cake filtration, what are the important parameters of interest? Obviously we have seen that the filtrate, the volumetric flowrate and then resistant to the flow. That is the pressure drop and that increases right. That pressure drop increases with time. So these are the kind of two important things. So these are flowrate through the filter and pressure drop across the unit okay.

And then as the time of filtration increases, what happens? As I mentioned, the more and more number of particles are deposited on the filter medium and then building a kind of cake or cake thickness increases. So accordingly, the filtrate volumetric flowrate of the filtrate that we collect from the other side of the filter medium that decreases, or the pressure increases in order to maintain the flowrate. The volumetric flowrate of a filtrate. So that is either of the two cases happens one is the flowrate decreases or pressure drop increases.

Now, since having this two important things are happening. That is either flowrate increasing or pressure drop is the increasing. So then it is possible that these filtration processes may be, you know, operated into such conditions. In one conditions where you can maintain the constant pressure.

And then let the volumetric flow rate to decrease with respect to the time or you know, you gradually increase the pressure drop in order to maintain the constant flowrate of the filtrate. So that the gradual increase in delta P with respect to the filtration time is a kind of a very rare case. So these two things are possible.

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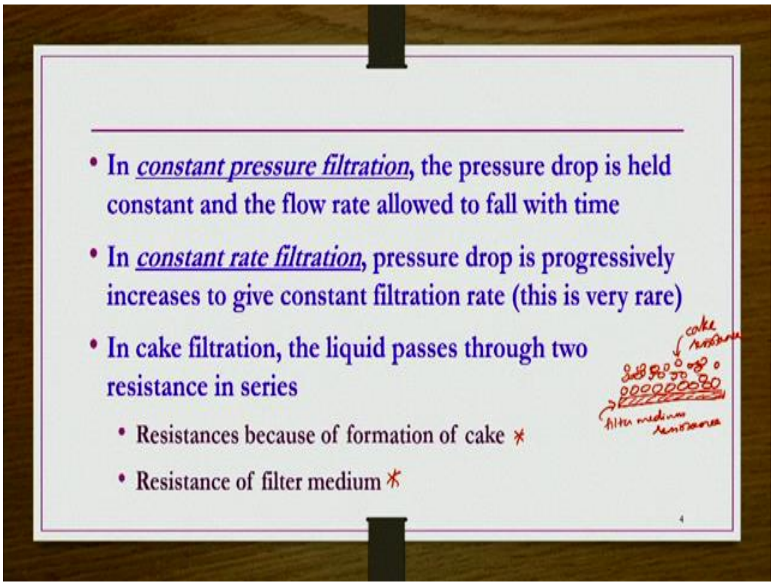

In constant pressure filtration, the pressure drop is held constant and the flowrate allowed to fall with time. In constant rate filtration, that is constant volumetric filtration rate. If you maintain constant volumetric filtration rate, the pressure drop is progressively increases to give constant filtration rate. But however, this is a very rare case.

So in cake filtration, the liquid passes through two resistance. One is the, whatever the filter medium is that resistance. The resistance offered by the filter medium as it is, and then another one is that as the time progresses these particles are depositing on the surface of the filter medium and then forming a kind of a cake.

So there are two resistances, the one resistance is the filter medium resistance. Another one is a kind of resistance due to the cake formation or you know cake resistance. These two resistance are possible in general, in a given cake filtration process. That is resistance because of formation of cake and then resistance of filter medium.

So one should have a kind of information about what is this resistance due to the formation of cake or cake resistance what is this of filter medium resistance. We need to have some kind of information or we should develop a kind of a equation. So that by using some experimental information we can calculate this resistance for a given situation. Where cake filtration is taking place.

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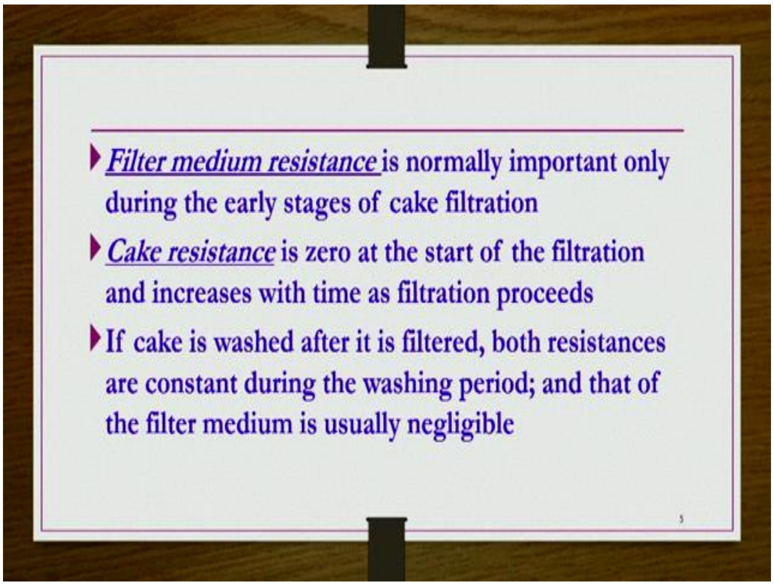

In general filter medium resistance is normally important only during the early stages of the cake filtration. Because you know what happens as the time of filtration gradual increases, more and more number of particles are depositing on the surface of the filter medium. And those particles are forming a kind of cake and that cake is offering much more resistance compared to the whatever the filter medium resistance is there.

So whatever the filter medium resistance is there, that is going to be having a kind of importance only at early stage of filter processes. Where the cake formation is not there, or it is almost kind of, minimum cake formation or negligible cake formation is there at early suggest. Under those early stages of filtration processes only this filter medium resistance is a kind of a important one.

Cake filtration is obviously zero at the start of filtration. Because at the start of filtration there is no kind of particle depositing on the surface of the filter medium. So there is no cake formation, so there will not be a kind of, any kind of a cake resistance. But as the filtration time progresses, more and more number of particles are forming and then gradually the cake thickness increases. Hence, the cake resistance also gradually increases with time as the filtration proceeds.

If cake is washed after it is filtered, both resistance are constant during the washing period; in general, and that of the filter medium is usually negligible compared to the resistance of a whatever the cake formation. Due to the cake formation whatever the resistances is there, that is compared to the cake resistance. The filter medium resistance is in general negligible, especially when cake is washed after the filtration.

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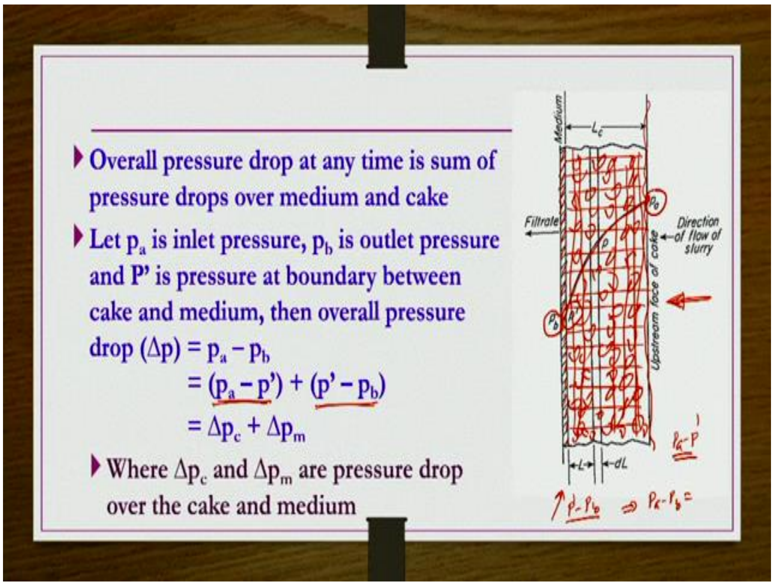

Now, what we understand from here? There are two resistance, so then obviously there will be a kind of two pressure drops. So the overall pressure drop at any time is going to be sum of pressure drops over the filter medium and across the cake formed. That cake formation or whatever the cake has form, across that cake form, whatever the pressure drop is there that one, plus the pressure drop across the filter medium.

Whatever the pressure drop is there. These two pressure drops when you add together, then you will get the overall pressure drop for that given cake filtration system at any given time. So because what we understand this overall pressure drop is also gradually increasing because of you know gradually cake formation or the thickness of cake is increasing or the more number of particles are being deposited on the surface of the filter medium. So this overall pressure drop is gradually is increasing with time, right.

So, but at particular specific time. If you take, if you have a kind of pictorial representation of this, you know pressure drops, then you can have a like this, right. So this is the kind of a filter medium that we have. And then slurry is coming down in this direction. Okay, slurry is coming in this direction right. And there is a kind of a cake formation and then whatever this cake is there that cake is a kind of a form like this kind of cake whatever this. Lc is represented here is a kind of thickness of the cake that has formed.

Usually you do not have a kind of sharp interface like this, you know. But in general, for a representation you can have a kind of a, you know sharp interface between the slurry and then first layer, top layer of the cake okay. So these are all a kind of particles that are being, you know, deposited on the surface of this, you know filter medium right. So now this particles are you know, almost a kind of accumulated and then or they are kind of, you know, kind of in a packed condition and then there will be behaving almost like a kind of a packed bed okay.

So now here, what we see? The pressure at the top surface of the cake whatever is there that we call it p_a. Let us take it as p_a and then pressure downstream in the filter side, whatever the pressure is there. Let us call it p_b and then there is one more pressure that is different from p_a and p_b. So that pressure is at the on the surface of a this filter medium and then it is in between this are at the interface between the filter medium and then cake form, whatever is there that pressure is known as the p', right.

So, what we can see here? So, the pressure drop is a kind of non-linear behaviour we can see here. Because the difference size of particles in general will be there and then when this particles in or you know, kind of packed as a kind of cake or a kind of bed of particles then whatever the fluid is coming because of that one. This packing structure may keep changing and then word is maybe came changing from one layer of the cake to the other layer. If you divide this cake into the separate layers, n number of layers like this. Each layer of the you know this, you know cake is going to have a different properties in general.

But however, we try to avoid such kind of complications that is one reason. Another is there, the on filter medium surface, whatever the particles are there, there will be having a kind of experience kind of largest compressive forces. And then compared to that one, the other sides the compressive forces are towards the slurry side, as we move away from the filter medium towards the slurry side. That compressive forces gradually decreases and there because of those reasons also sometimes it is possible that you know pressure drop that we are going to have a kind of non-linear pressure drop.

So the question maybe there, that why it cannot be a kind of linear pressure drop between this two. Because of this many complicated things like the size of the particles because the particles are not same size particles right. Shape of the particles are also not same and then the voidage of the each individual layer. If you break this cake into the different layers, so that voidage is also going to change from one layer to one layer in general.

So, because of those many reasons, you know the pressure drop is going to be a kind of non-linear. There are other reasons also like you know compressive force is a kind of a maximum at the filter medium surface and then it gradually decreases as we move towards the slurry from the filter surface through the cake. So now here, what we see? The pressure drop in the cake is what, is nothing but $(p_a - p')$. And then pressure drop across the filter medium is nothing but $(p' - p_b)$.

So let p_a is the inlet pressure, p_a is the inlet pressure that is the upstream side of the cake, the top layer of the cake that is, whatever is there. So at that layer, you know pressure is p_a. And then p_b is the outlet pressure that is at the filtrate size, whatever the pressure is there, that is p_b. And then p' is pressure at bottom boundary between cake and then medium, whatever at the interface between the filter medium and cake is there, at that interface, the pressure is p', right.

So, p_a you can measure, p_b you can measure in general for experimentally. If you wanted to measure by some means. But p' is not possible to measure in general so easily. So, that is the reason you know what we can write overall the pressure $\Delta p = (p_a - p_b)$ that is nothing but $(p_a - p') + (p' - p_b)$. So that you know you can avoid, you know calculating, this p', and then without that you can get the overall pressure drop.

However, this $(p_a - p')$ is nothing but the pressure drop across the filter and then $(p' - p_b)$ is nothing but the pressure drop across the filter medium. So in that we, if we note it Δp_c and Δp_m respectively. So $\Delta p = \Delta p_c + \Delta p_m$ respectively. So then where Δp_c , Δp_m are nothing but pressure drop across the cake and then across the filter medium.

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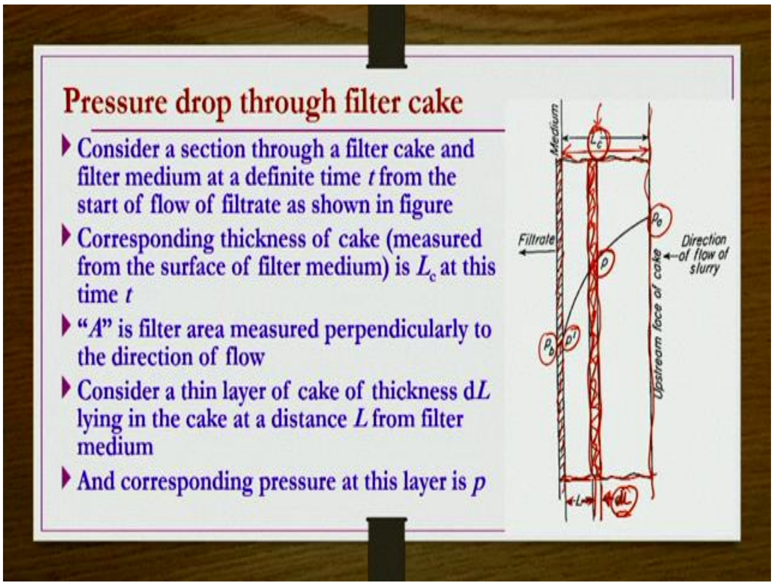

Now, you wanted to calculate this pressure drop of through the filter cake. Because these pressure drop equation for. Now this cake is behaving as a kind of, you know, almost kind of a packed bed kind of thing. And then for the packed bed pressure drop equations, you are having. So one way or another way you can make use of that flow-through packed bed cases for which whatever the pressure drop equation is there.

That equation, you can use your and then modify for this kind of problem for the current situation of the problem. Where cake formation is there and then cake thickness gradually increases because of which pressure drop gradually increases. For that situation you can make use of this equation. So that what we are going to do now here.

So let us take the same picture once again here. But now what we do, within this cake thickness is L_c right. So we have this filter medium right and then there is a kind of cake formation. The cake thickness at certain instant of time, it is going to change with time. But we are taking at one particular instant of time, the thickness of this cake is L_c. The pressures as already discussed p_a and p_b at either extremes of cake of surface and filtrate side respectively. p' is the pressure at the interface between filter medium and then cake.

So now, within this L_c cake what you do? You take a particular location, you know at distance L from the filter medium. The distance L from the filter medium you take. Then further, please be noted this L_c is measured from the filter medium side, from the filter medium side. So L_c is initially is 0 at the filter medium and gradually as time increases that L_c gradually increases from filter medium side towards the slurry side that particles being deposited, right.

So now at location L within this cake you take a kind of a thin layer of, you know, whatever the cake that has formed. So one small layer of that cake you take whose thickness is kind of a dL right. So now, the corresponding pressure for this layer of cake that we have taken is p, right. So, pressure drop for this layer of cake that has been sliced out of the overall cake of thickness L_c is there. So, overall cake thickness L_c from out of which that, out of which we take a kind of a slice of cake at distance L from the filter surface medium, filter medium surface.

And then the thickness of this slice of a layer of this cake is dL. So the pressure is for this layer is p. So for across this layer. Now this layer is a kind of maybe thin-layer of thickness dL. But across dL there is a pressure drop alright. That pressure drop you can calculate using the Kozeny Carmann equation okay.

So consider a section through a filter cake and filter medium at a definite time t from the start of flow of filtrate as shown in the figure. Corresponding thickness of cake measured from the surface of filter medium is L_c at this time t. Okay, this is at particular time t whatever the picture we have shown.

A is the filter area measured perpendicular to the direction of flow normal to the direction of flow, whatever the surface area or the area of exposed the filter medium area that is exposed for the separation or the for the filtration process is A. Consider a thin layer of cake of thickness dL lying in the cake at a distance L from filter medium. And then corresponding pressure at this layer is p, right.

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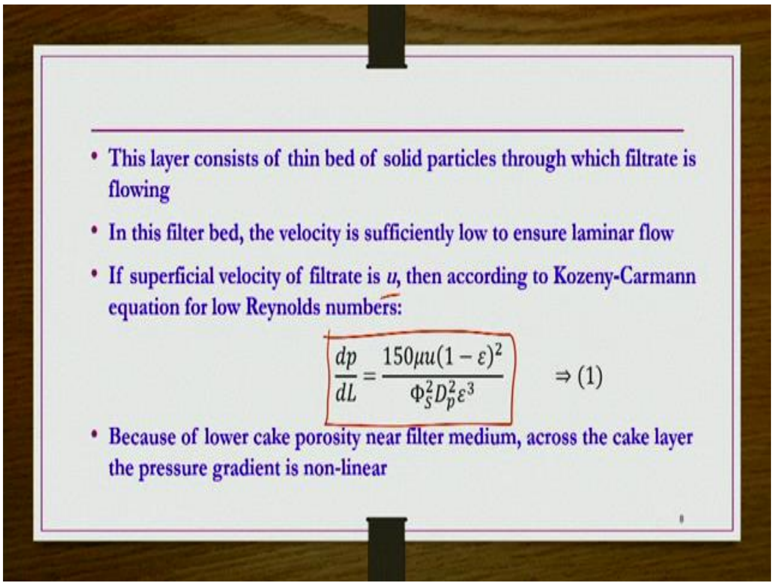

Then, this layer consists of thin bed of solid particles through which filtrate is flowing. So in this filter bed, the velocity is sufficiently low, usually in filtration process the volumetric flowrate of filtrate is in generally small enough to consider the flow is in a kind of a laminar flow region right. So in the same we can apply here for this layer of cake for which, you know the velocity is sufficiently low. So that we can make sure, so that we can ensure that laminar flow is, you know. The laminar flow conditions are existing for this layer of the particles. So in this filter bed, the velocity is sufficiently low to ensure the laminar flow.

And then if superficial velocity of filtrate is u, then using the Kozeny Carmann equation for low Reynolds number flows, for low Reynolds number flows we have this, $\frac{dp}{dL} = \frac{150\mu u(1-\varepsilon)^2}{\varphi_s^2 D_p^2 \varepsilon^3}$. μ is the viscosity of the filtrate, u is the superficial velocity of filtrate, ε is the voidage of the cake, φ_s is the shape factor of the particles that are forming the cake and D_p is the nominal diameter of the particles which are forming the cake, ε is anyway the voidage of the cake that we have already seen. This equation, we can modify it.

So because of lower cake porosity near filter medium, across the cake layer the pressure gradient is in general non-linear. So because of lower cake porosity near filter medium, across the cake layer the pressure gradient is non-linear.

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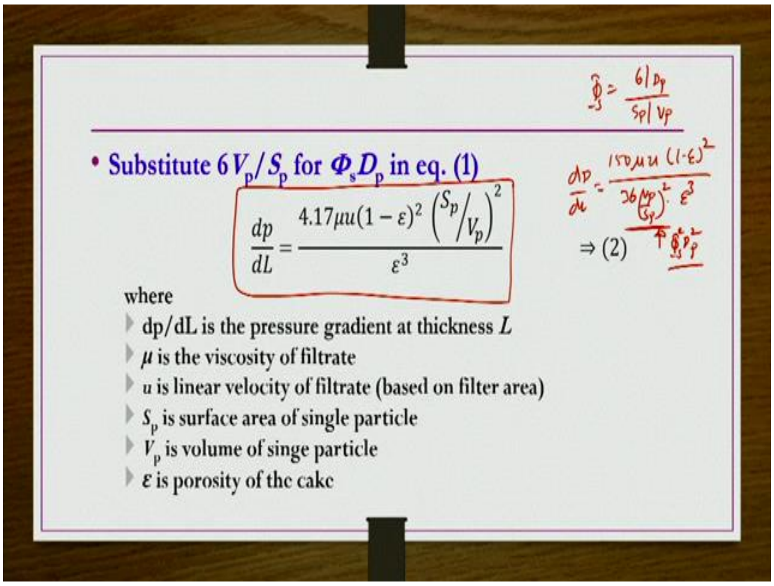

Now in this equation, wherever $\varphi_s D_p$ is there you can you can substitute $\frac{6V_p}{S_p}$ because we have already seen this φ_s is nothing but $\frac{6/D_p}{S_p/V_p}$. So $\frac{6V_p}{S_p}$ if you write in place of $\varphi_s D_p$ in that above equation. The same equation will be, you know written as a kind of $4.17\mu u(1-\varepsilon)^2 \left(S_p/_{V_p}\right)^2$ and then $\varphi_s D_p$ is nothing but $\frac{6V_p}{S_p}$. So that we can write, you know $\left(S_p/_{V_p}\right)^2$ on the numerator side. Okay and then ε^3 is anyway.

So here what we had? Actually this equation what we had $\frac{150\mu u(1-\varepsilon)^2}{\varphi_s^2 D_p^2}$.. So that we have now here, $\left(\frac{36V_p}{S_p}\right)^2$ and then ε^3. So this is nothing but $\varphi_s^2 D_p^2$. So in place of $\varphi_s^2 D_p^2$, we are writing$\left(\frac{36V_p}{S_p}\right)^2$. So that we rearrange so you are having this equation right. $\frac{150}{36}$ is kind of 4.17 right.

Here, $\frac{dp}{dL}$ is the pressure gradient at thickness L, μ is the viscosity of filtrate, u will is the linear velocity of filtrate based on the filter area. And then S_P is the surface area of single particle. V_P is volume of single particle right. So, but this equations in general, and then $\frac{dp}{dL}$ is the porosity or the voidage of the cake.

But this equation here, whatever you have taken. So that is taken for a kind of single particle basis on single particle, surface area of single particle, surface area of a volume of a single particle, etc right. So, but in general the suspension is made up of, you know, kind of a several different types of particles, different size, different shapes are there right.

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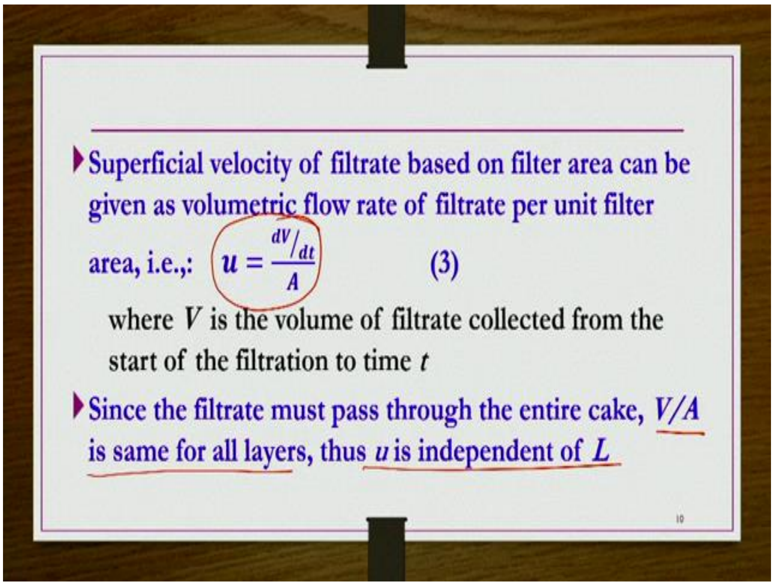

Superficial velocity of filtrate based on filter area can be given as volumetric flow rate of filtrate per unit filter area. So now, the so-called time factor is coming into the picture in this pressure drop equation now. So that is $u = \frac{dV/_{dt}}{A}$. $dV/_{dt}$ is nothing but volumetric flowrate of filtrate. And then you divide it by filter medium area which is exposed for the filtration then you will get a kind of a whatever the superficial velocity of the filtrate okay.

So now, this if you substitute that equation 2, you will get a different equation. So here in this equation, V is the volume of filtrate collected from the start of the filtration to time T. Since the filtrate must pass through the entire cake, $V/_A$ is same for all layers, thus u is independent of, this is a kind of a required analysis, assumption is required. Filtrate must pass through the entire cake. So then, we have taken only one layer of the cake for this analysis. So whatever $V/_A$ is there, that should be same for all layers. So that the superficial velocity u can be taken as independent of L. That is the thickness of filter, whatever the cake, thickness of cake or cake thickness.

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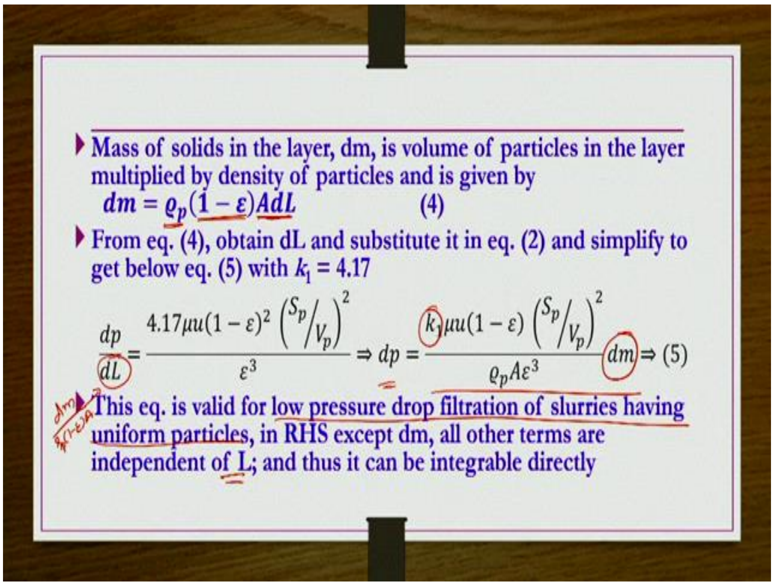

So now, as we know that as the time progresses, more and more particles are depositing on the filter medium. So the mass of solids in the layer, that is also going to change with time alright. So how much that mass is being deposited or what is the mass of solid in that layer of cake of thickness dL that we have taken. So that we should calculate, that let us say dm. So then, let us say the volume of the layer of cake that we have taken is area of the, area is now same as the filter area, filtration area.

Because entire filtration area particles are being deposited, right. So that area multiplied by the thickness of cake layer that we have taken, that will give you a kind of volume of that particular cake layer and then if you multiply that one by $(1-\varepsilon)$ that will give the volume fraction of the, you know that 1 minus epsilon is nothing but volume fraction of the particles in that layer.

So that will give a kind of volume of particles. $(1-\varepsilon)AdL$ is nothing but volume of particles. And then, if you multiply that one by the density of the particles, you know you will get the mass of the particles that are present in that layer of a cake of having thickness dL, okay.

So now this equation, we can substitute in equation number 2 and simplify to get this equation. So this is equation number 2. In this equation what we are going to do only in place of dL, we are going to write $\frac{dm}{\rho_p(1-\varepsilon)A}$. This is what we are going to write in case of a dL and then rearrange it. So that whatever this ρ_p is there, and then $(1-\varepsilon)$ would be there. A is already there. So this actually here $(1-\varepsilon)^2$ is there. So the square and then this $(1-\varepsilon)$ is cancelled out. So that finally we get this equation, dp is equal to k_1 constant, whatever 4.17 is there, that we can take it as k_1.

Because whatever 150 is there in the flow-through packed bed constant 150 is there, that is also based on some kind of experimental observation, which best suit into the factors etc. So those constants may slightly change. So not necessary to stick to those constant value as it is. Now henceforth we can take this constant whatever is there, as a kind of k_1 okay k_1 is nothing but 4.17, if you compare equation number 2, so $\frac{k_1\mu u(1-\varepsilon)\left(S_p/V_p\right)^2}{\rho_p A\varepsilon^3}dm$ is nothing but pressure drop across that layer of a cake of thickness dL that we have taken, right.

This equation is valid for low pressure drop filtration of slurries having uniform particles, alright, in RHS of this equation 5, except this dm, rest everything, rest all other parameters are independent of L, rest all were all other parameters are independent of L. So that what we can do? We can directly integrate this equation 5. When integrate this equation 5 then you will get a kind of a, you know required equation between pressure drop and then the properties of, you know particles and then void fraction of a cake etc, surface to volume ratio, particles, etc in those kinds of terms. We get a equation for pressure drop right.

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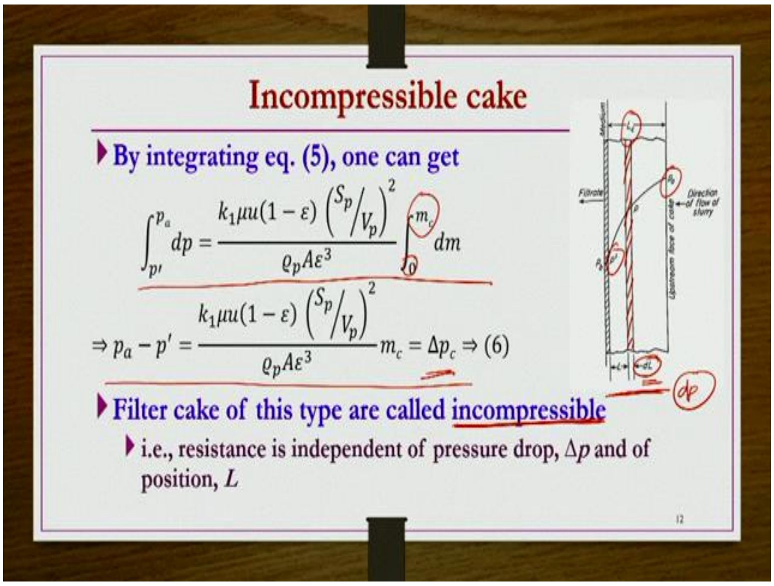

So now, incompressible cake. So these cakes are in general can be compressible or incompressible cakes, whatever the cake resistances is there, that is independent of the pressure drop and then that is not going to change with change in the L. Then we can call such cakes are kind of incompressible cake. But in general, most of the cakes are in compressible only. So we see both what is incompressible cake and then compressible cake right now.

So let us start with the incompressible cake, same picture we take whatever we have taken previously here. So if equation 5, if you integrate we can get this equation, integrating. Now this whatever the layer that we have taken is within the cake only. This layer is there, that is within the cake only. So what are the limits of the pressure that.

Because now initially this equation, you have developed for dL, for dL what is dp you calculated only for this layer right, only for whatever this layer of cake that you have taken within the total cake of thickness L_c at a given time t. At a given time t the cake thickness is L_c within that case, we have taken one layer of cake, for this layer of cake just now we develop a kind of equation for pressure drop. That equation if you integrated so you will get this equation.

Here the limits for this integration, especially for pressure should be like, you know p' to p_a , p' to p_a because this is, this layer is there within the cake and then this integration we are doing only for cake. So the pressure limit should be p' to p_a. And then at p' that is when there is no filtration or there are no particles deposited on the surface of filtrate initially right. So the mass of particles at the interface is almost 0.

And then at p_a., when the cake builds up and then forms a kind of thickness of Lc. Let us say whatever the solids are there, the mass of the solids in the cake is mc. So there should be two limits of dm. So 0 to mc, so when you do this one you get this equation and then $(p_a - p')$. $(p_a - p')$ is nothing but pressure drop across the cakes. So that is Δp_c.

Now, filter cake of this type are called incompressible cakes, where we have taken that, you know the pressure drop, the cake resistance and the cake that has formed is independent of kind of a L value. That is resistance is independent of pressure drop, and of position L. So such case, whatever the cakes are there. So they are called as a kind of incompressible cakes.

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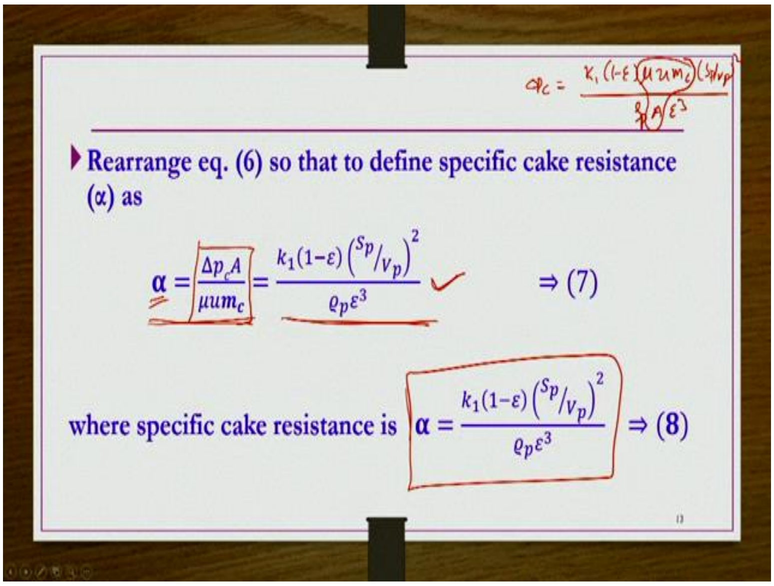

Now, if you rearrange equation number 6, so that to define specific cake resistance. So actually, we now coming to the position where we can describe or develop equation for two different resistance that we have. We have two resistance for the cake filtration cases, one is the cake resistance, another one is the filter medium. So we have arrived to a kind of point, where we can measure the cake resistance.

How, we can measure? That equation number 6, if you rearrange whatever this equation number 6 is this one only $\Delta p_c = \frac{k_1(1-\varepsilon)\left({}^{S_p}/_{V_p}\right)^2}{\rho_p \varepsilon^3} \mu u m_c$. So what have done? So and then divided by A is there. So whatever these $\frac{A}{\mu u m_c}$ or whatever here in equation number 6. If I rewrite this equation number 6, so we will have $\Delta p_c = \frac{k_1(1-\varepsilon)\mu u m_c\left({}^{S_p}/_{V_p}\right)^2}{\rho_p A \varepsilon^3}$.

So out of this one $\frac{\mu u m_c}{A}$, whatever is there, that I am taking into the left inside. So that I can join with Δp_c as $\frac{\Delta p_c A}{\mu u m_c}$ and then keep all other terms in the right-hand side itself. So whatever this $\Delta p_c A$, whatever this $\frac{\Delta p_c A}{\mu u m_c}$ is there, that is nothing but the cake resistance in terms of the particle properties and voidage if you wanted to write. This is nothing but $\frac{k_1(1-\varepsilon)\left({S_p}/{V_p}\right)^2}{\rho_p \varepsilon^3}$, okay.

This α, if it is independent of pressure drop D_p and then independent of and the position L, then we call it as a kind of incompressible cake. So where here specific cake resistance is nothing but this one. Remember in experiment cases we do not know what is ${S_p}/{V_p}$ for unknown particles size and shape in general. Okay, so we are trying to further simplify this equation. So that without having this information way can calculate this α, so that is what we are going to do now anyway.

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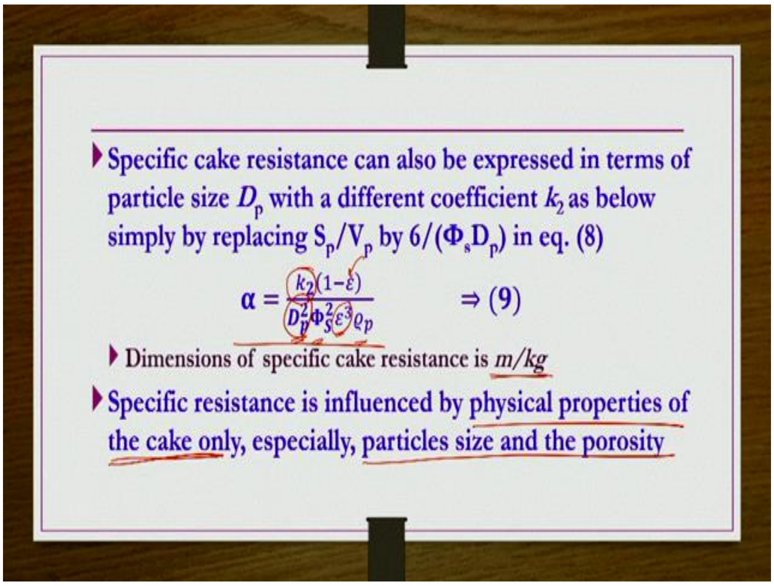

But in general, specific cake resistance can also be expressed in terms of particle size Dp with the different coefficient k_2 as below by simply replacing ${S_p}/{V_p}$ by ${6}/{(\varphi_s D_p)}$ right. So then we get α is equal to that constant will be modified that, you know whatever 4.17 is there, now here that 6^2 is also there right. So that whatever the new thing is coming, constant is coming that we can call it as k_2 okay.

So we do not need to worry about what is this exact value of k_2. So let us take that revenue on that k_1 is combined with this 6^2. Then whatever the new constant is there, that we call it k_2. So all files $\frac{k_2(1-\varepsilon)}{D_p^2\varphi_s^2\varepsilon^3\rho_p}$.

Dimensions of specific cake resistance is metre per kg. Specific cake resistance is influenced by the physical properties of the cake only. That is what we can say, especially particle size and the porosity. What we can see here from the alpha definition, this alpha is going to be affected by the particle size and then this porosity ε along with the density and then sphericity, etc. But this particle size and then voidage is going to have a kind of a strong influence on this one, on the cake resistance.

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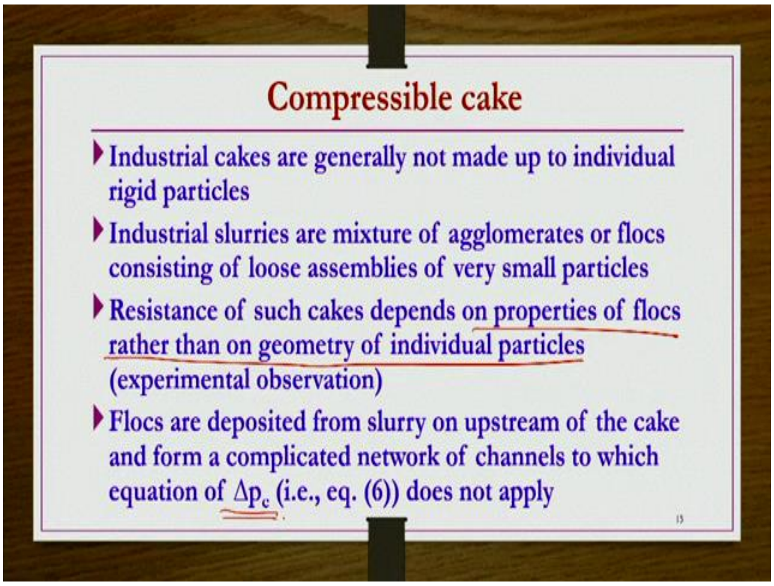

Okay, now we take the case of compressible cakes. But in general, we know that the cake is, whatever the cake resistance is there, that is function of Δp because as gradually filtration time increases the cake thickness increases and then the pressure drop increases right. So industrial cakes are generally not made up of individual rigid particles, as we have taken the previous case. We assume that all particles are of same size as we did assumption in the kind of packed or fixed bed cases.

Further industrial slurries are mixture of agglomerates or flocs consisting of loose assemblies of very small particles. And the resistance of such cakes depends on properties of flocs rather than geometry of individual particles, as you know been observed by the many experimental results okay. So in the previous case incompressible cake what we have seen α is going to be a kind of function of a particle properties. Like you know size of the particle and then voidage of the bed etc right.

But in general, in reality the industrial cake whatever the cake resistance is there, that depends on the properties of flocs rather than geometry of individual particles. And then flocs are deposited from slurry on upstream of the cake and form a complicated network of channels to which equation of Δp_c does not apply.

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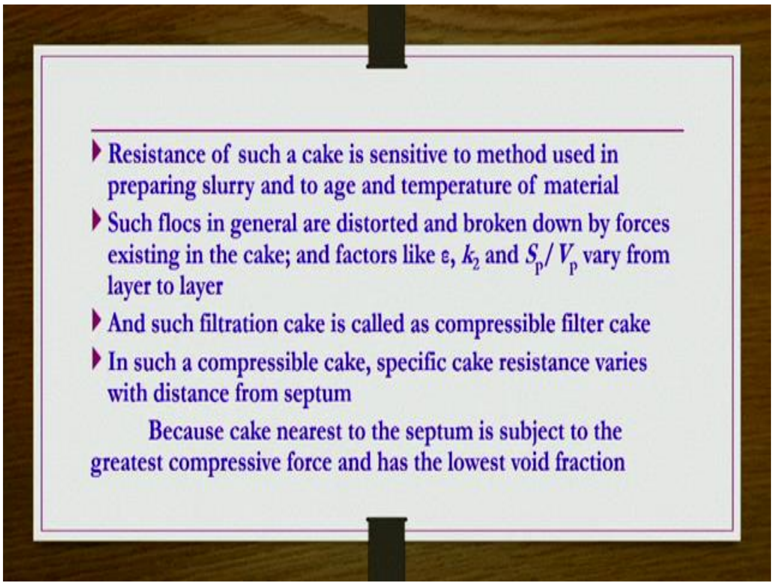

However, resistance of such a cake is sensitive to method used in preparing slurry and the age of slurry and temperature of material, which has been you know applied for the separation or required of filtration process. Such flocs in general are distorted and broken down by forces existing in the cake; and factors like ε, k_2 and $\frac{S_p}{V_p}$ in general vary from layer to layer. Because of that one pressure drop is also going to be non-linear.

And as such filtration cake is called a compressible filter cake, where the cake resistance is a function of a pressure drop. In such compressible cake, specific cake resistance varies with distance from septum that is L, position L. Because cake nearest to the septum is subjected to the greatest compressive force and has a lower void fractions.

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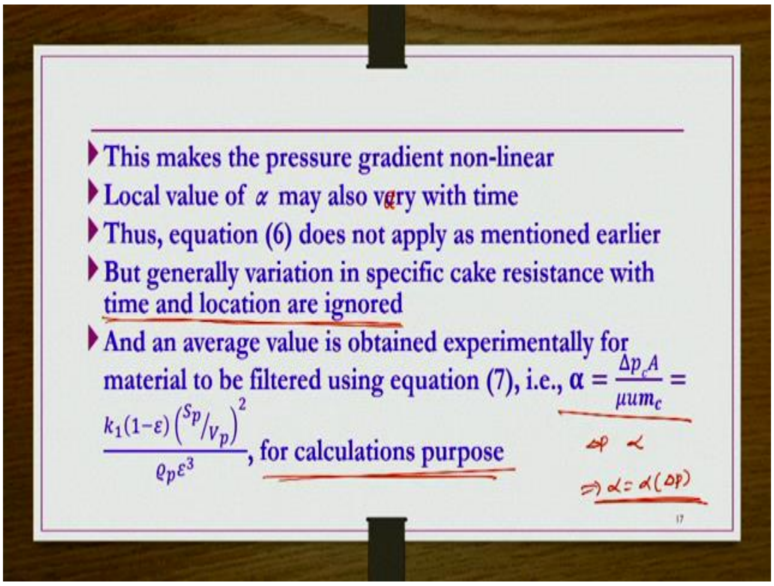

And because of that one, the pressure drop or pressure gradient becomes a kind of non-linear. And then local value of α may also very with time in general. Thus, whatever the equation number 6 that have been developed for cake resistance does not apply as mentioned earlier for a case of a compressible cakes that can be applied for incompressible cakes. But for compressible cake that is not applied.

But generally variation on specific cake resistance with time and locations are ignored. And an average value is obtained experimentally for material to be filtered using the same equation $\alpha = \frac{\Delta p_c A}{\mu u m_c}$ for cancellations purposes. Okay, so here only the variations, variation with respect to the time and location are ignored. But not the variation with respect to the pressure.

In order to know the variations with respect to the pressure what you do, for different pressure drops you do the experiments, you calculate the α and then you can have a kind of relation between α as function of a pressure drop or you can have relation between α and then pressure drop. So that will give the compressibility information of a this compressible cakes. Though we ignore the influence of a duration of filtration and then a location.

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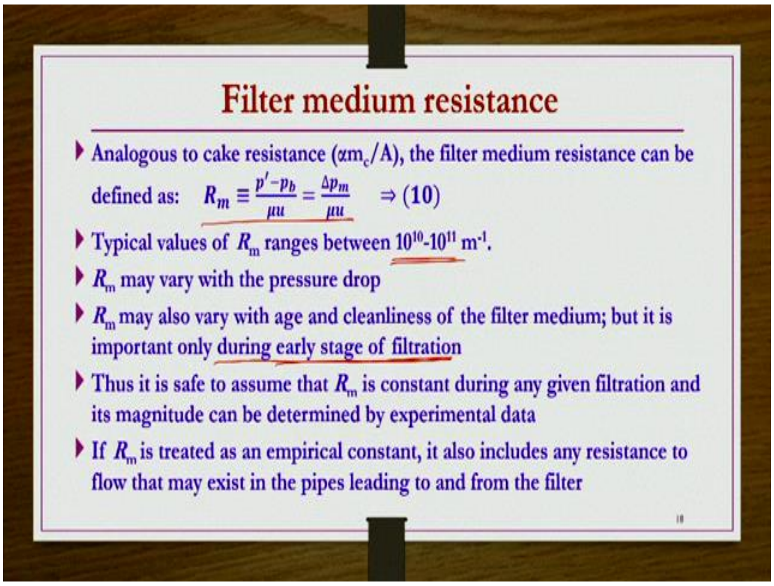

Now filter medium resistance, analogous to cake resistance $\alpha m_c/A$, the filter medium resistance can also be defined as $R_m = \frac{\Delta p_m}{\mu u}$, where Δp_m is nothing but $(p' - p_b)$. Here typical values of Rm in general ranges 10^{10} to 10^{11}. So now we have a expressions in terms of a maybe pressure drop, we have expression for the cake resistance as well as the filter medium resistance, okay.

Rm may vary with the pressure drop. Rm may also vary with age and cleanliness of the filter medium; but it is important only during early stage of filtration as already mentioned. Because as the filtration process progresses more amount of particles deposited. And then cake resistance would be dominating over the filter medium resistance. So that you know this may be a kind of a important only at the early stage or the filter medium resistance is going to be important only at the early stage of the filtration.

Thus, it is safe to assume that Rm is constant during any given filtration and its magnitude can be determined by experimental data. How to determine this magnitude that we are going to see, we are going to take some examples to see how to obtain this cake resistance as well as the filter medium resistance. If Rm is treated as an empirical constant, it also includes any resistance to flow that may exist in pipes leading to or from the filter.

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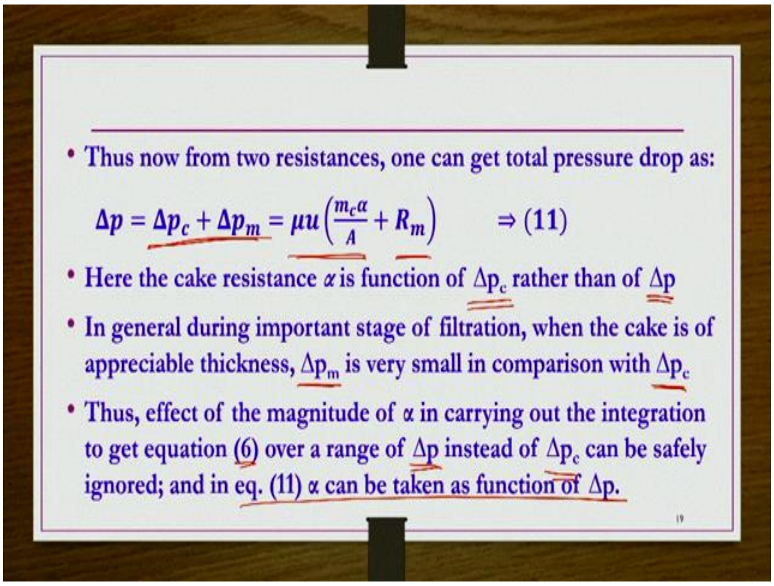

Now from two resistance, one can get the total pressure drop as Δp is nothing but $\Delta p_c + \Delta p_m$. So from α definition Δp_c can be written as $\frac{\mu u m_c \alpha}{A}$ and then Rm definition Δp_m can be written as $\mu u R_m$ or Rm is equals to $\frac{\Delta p_m}{\mu u}$. So Δp_m you can write $\mu u R_m$. So when you add these two Δp_c and Δp_m then you will get the total pressure drop for the system. Here the cake resistance α is function of Δp_c rather than Δp.

But however, in general during important stage of filtration, when the cake is of appreciable thickness, Δp_m is very small in comparison to Δp_c that is already we know Δp_m is going to be important only at the early stage of the process. But later on, it may not have a kind of very significant magnitude compare to the Δp_c.

Thus, effect of the magnitude of α in carrying out the integration of equation number 6, whatever that α definition is there, over a range of Δp. Instead of Δp_c can be safely ignored. Okay and then in equation 11 α can be taken as a function of Δp, okay.

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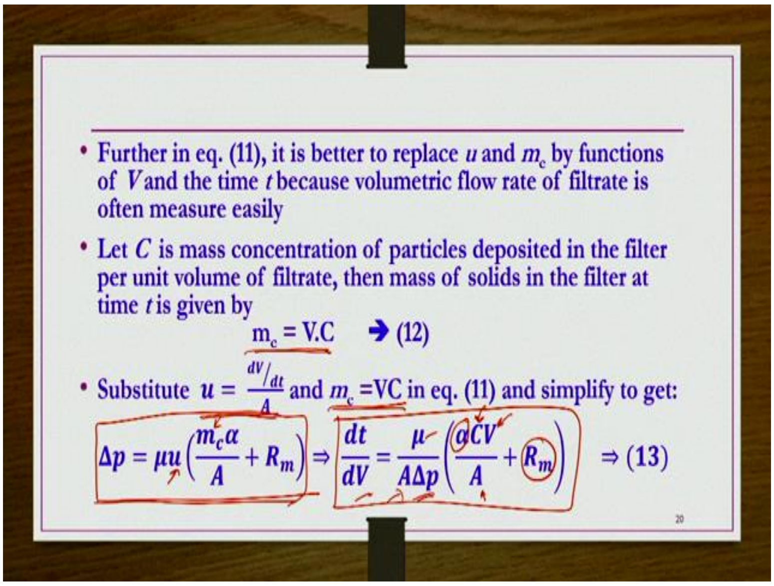

So further in equation 11, whatever this total pressure drop equation is there, that is equation number 11, it is better to replace u and μ by functions of V and the time t because volumetric flow rate of filtrate is often measured easily, experimentally. Rather than u or m_c okay. Let C be the mass concentration of particles deposited on the filter medium per unit volume of the filtrate, then mass of solids in the filter at time t is given as V.C, where V is nothing but the volume of filtrate connected.

And then further u we already know it as $\frac{dV/dt}{A}$. So in that equation number 11, wherever u is there, you substitute $\frac{dV/dt}{A}$ and then wherever mc is there, you substitute VC. Then we have this Δp, this is the equation 11, this part is nothing but your equation number 11. In this equation in place of m_c, you write VC, in place of u you write $\frac{dV/dt}{A}$ and then rearrange this equation. So that you can have $\frac{dt}{dV} = \frac{\mu}{A\Delta p}\left(\frac{\alpha CV}{A} + R_m\right)$right.

In this equation experimentally the volumetric flow rate or volume of filtrate collected versus time information you can get for a given Δp value. Let us say you take a filter medium and then you take a slurry and then you allow the slurry to pass through this filter medium at certain pressure, alright. So with respect to the time you can collect the, you know volume of filtrate and then you can tabulate that one. So that information, you know actually.

And then viscosity of the filtrate is in general, you know you can calculate, area of the filter medium is also known right. In this equation. So V is known, A is known alright. So this equation, if you wanted to calculate alpha and Rm everything is known, except this C. What is this C? How to calculate? Experimentally you cannot measure how much mass is deposited on the filter medium per unit volume of the filtrate, especially during the process of the filtration okay.

So however, there is a process we can do a kind of mass balance and then we can develop a kind of relation for C. In general, for problems they given straightforward. If there are not given there should be obtained as like this.

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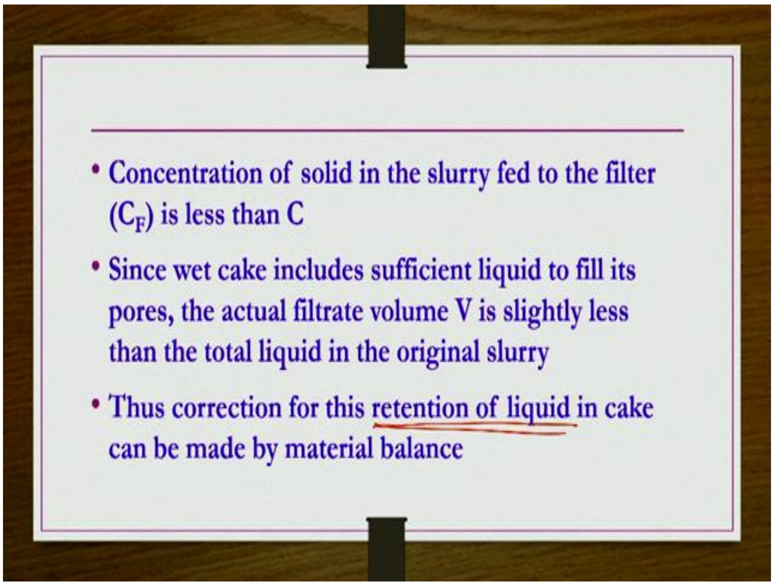

Let us assume, concentration of solid in the slurry fed to the filter is C_F and that is less than C okay. Because wet cake includes sufficient liquid, to fill its pores, then actual filtrate volume V is slightly less than the total liquid in the original slurry. Because the entire liquid is not coming as a kind of filtrate, some amount of the liquid is retained interstitial space between the particles whatever is there within the cake.

That space is occupied with the some amount of the liquid. That is the reason the actual filtrate volume V is less than the total liquid that is present in the original slurry okay. Thus correction for this retention of liquid in cake has to be made, and that can be made by the material balance.

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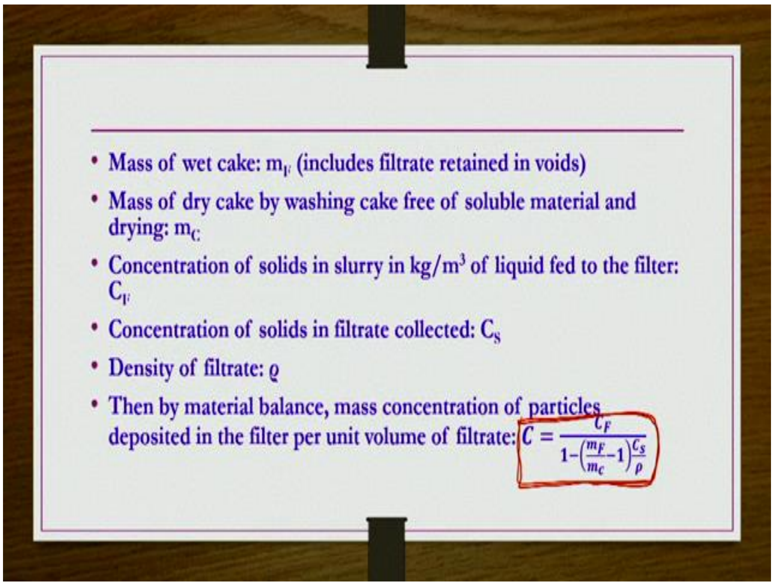

How it can be made? Let mass of wet cake is mf, which includes the filtrate retain the voids or the liquid that is retained inside the or between interstitial spaces of particles within the cake. And then mass of dry cake that is after washing the cake, free of soluble materials and drying it, whatever the mass is there that if you take mc.

Than concentration of solids in slurry in kg per metre cube of liquid fed to the filter medium, if you take C_F. And concentration of solids in filtered collected, if it is C_S. Density of filtrate is ρ. Then by material balance, if you do a kind of material balance and then simplification you do, then you get this C is nothing but $C = \frac{C_F}{1-\left(\frac{m_F}{m_C}-1\right)\frac{C_S}{\rho}}$ is always going to be less than 1. So this denominator value whatever is there, that is going to be less than 1. So C is going to be more than the C_F, C is going to be more than the C_F in general.

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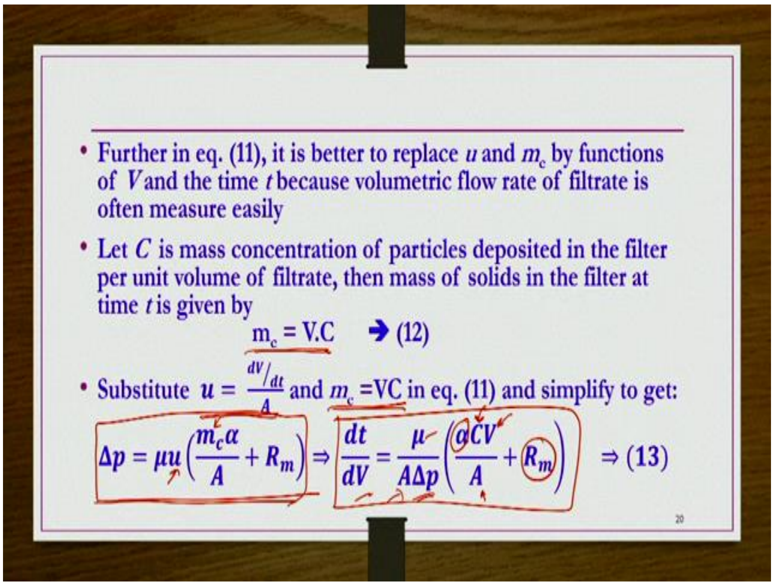

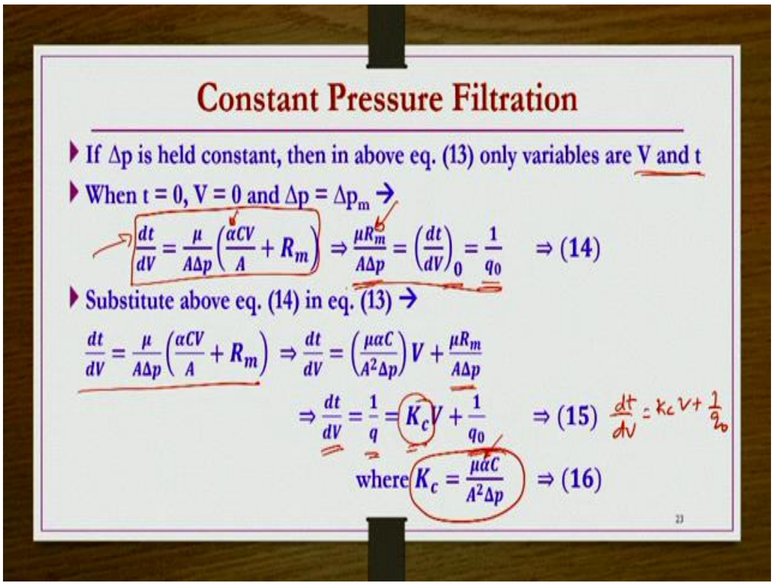

Okay, so now, so that equation, we have this basic equation, whatever have $\frac{dt}{dV} = \frac{\mu}{A\Delta p}\left(\frac{\alpha CV}{A} + R_m\right)$. If you operate, or if you use this equation for constant pressure filtration. Then you have a kind of a methodology to calculate this α in Rm.

Constant pressure filtration, if Δp is held constant, then above equation 13 only variables are V and t right. So when t is equals to 0, V is equals to 0 and Δp is equals to Δp_m. Because when there is no cake formation whatever the pressure drop is there, is that is only across the filter medium.

Then this $\frac{dt}{dV}$ equation whatever this is the equation number 13, right. This equation, now if you do a kind of a, you know integration or you take a time t is equals to 0, at time t is equals to 0, whatever $\frac{dt}{dV}$ is there, let it $\frac{1}{q_0}$. So that you know why are we doing? Because we need to have a kind of information for Rm and then alpha also.

At t is equal to 0, there is no cake formation. So then this α is 0, so then, what we can write? $\frac{dt}{dV}$ at t is equal to 0 is equals to $\frac{\mu R_m}{A\Delta p}$, that is nothing but $\frac{1}{q_0}$. So this is nothing but from this information if you know $\frac{1}{q_0}$ by some experimental information, you know, you can get the Rm value.

Substitute this equation 14, this equation back in this equation number 13. So wherever $\frac{\mu R_m}{A\Delta p}$ is there, you write $\frac{1}{q_0}$, when you write it $\frac{dt}{dV}$ is equals to. This is equation number 13, so $\frac{dt}{dV}$. Now this taking $\frac{\mu R_m}{A\Delta p}$ inside, so $\left(\frac{\mu uC}{A^2\Delta p}\right)V + \frac{\mu R_m}{A\Delta p}$. So this I am writing $\frac{1}{q_0}$, $\frac{\mu R_m}{A\Delta p}$ I am writing $\frac{1}{q_0}$. And then whatever the $\left(\frac{\mu uC}{A^2\Delta p}\right)$ is there, that am writing as a K_c.

If $\frac{dt}{dV}$ at t is equals to 0 is $\frac{1}{q_0}$. So at some other instant of time $\frac{dt}{dV}$, we can write it as a kind of $\frac{1}{q}$ or simply you can have this equation as $\frac{dt}{dV} = K_c V + \frac{1}{q_0}$. So where this K_c, if you know this K_c information in by somehow by experimentally. So you can get this alpha value, if you know this $\frac{1}{q_0}$ value somehow by experimentally. Then you know this Rm value. So that is the purpose we are doing now.

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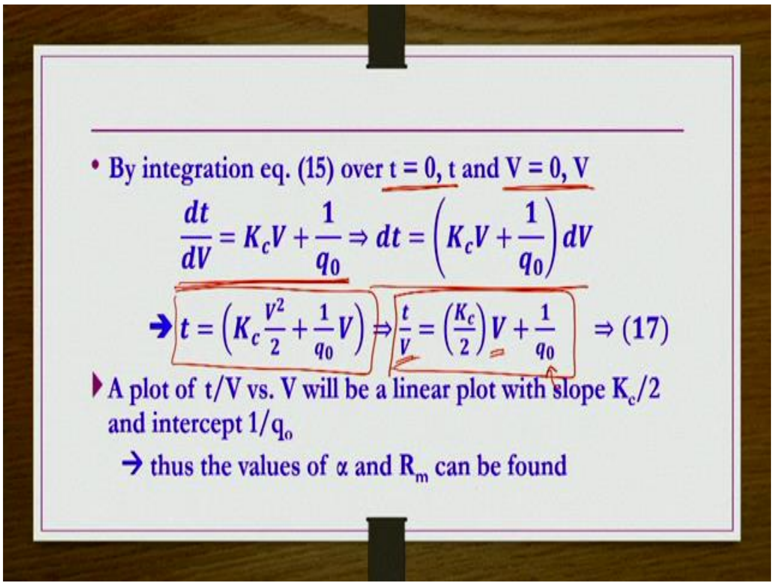

Okay, so by integration equation 15 over the limits t is equal to 0 to some instant t and then volumetric flow rate initially it is 0 anyway. At time t if volumetric flow rate or volume of filtrate collected is V, then if you use them as a kind of limits, then this equation number 15 is this one, you integrate this one. So integration for that purpose dv you take to the right inside.

Then if you integrate you have $t = \left(K_c \frac{V^2}{2} + \frac{1}{q_0} V\right)$ and then integration constant that will be anyway 0. Because anyway, we are taking the definite limits, so then there will not be any kind of integration constant anyway. So then here this equation, if you simply rearrange $\frac{t}{V} = \left(\frac{K_c}{2}\right) V + \frac{1}{q_0}$. Then t, V information you get the experimentally right.

So experimentally for given a pressure drop whatever the t, V information is there, you rearrange $\frac{t}{V}$ versus V and then plot $\frac{t}{V}$ versus V. Then you will get a kind of straight-line with intercept $\frac{1}{q_0}$ and then slope $\frac{K_c}{2}$. So from slope you get K_c and from K_c you get the α value, K resistance and then from intercept you get $\frac{1}{q_0}$ and then from $\frac{1}{q_0}$ you can get the Rm filter medium resistance.

Okay, so this is of now, at constant pressure, whatever the experiment that you do, you collect volume of filtrate with respect to time that information you can make use to calculate the specific cake resistance and filter medium resistance like this.

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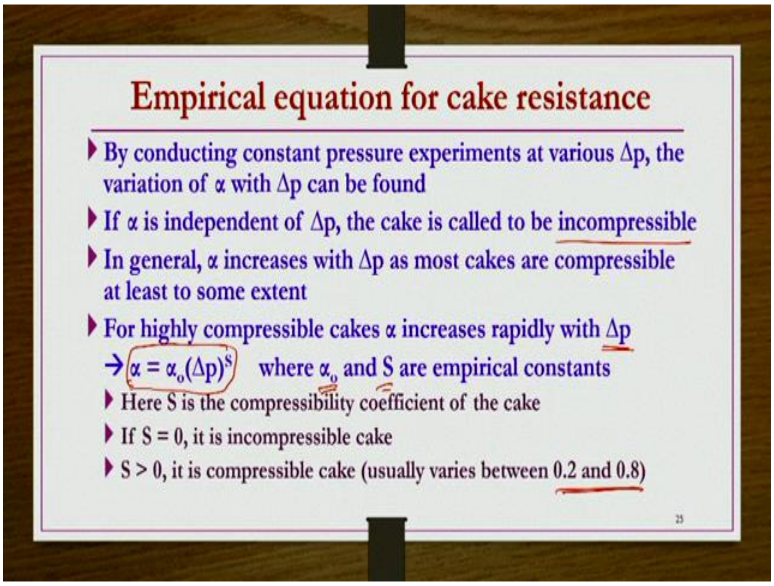

Now empirical equation for cake resistance, by conducting constant pressure experiments at various Δp, the variations of α with Δp can be found right, as I already mentioned. If α is independent of Δp, then cake is said to be incompressible cake. In general, α increases with Δp as most cakes are compressible at least to some extent. For highly compressible cakes α increases rapidly with Δp.

Then under such conditions, if you do experiments at several Δp values and can calculate corresponding alpha values that if you correlate, you may get a kind of equation like this $\alpha = \alpha_0(\Delta p)^S$. Where α_0 and then S are kind of an empirical constants. S is the compressibility coefficient of the cake, if it is 0, then cake is incompressible. If it is greater than 0. It is compressible cake and then gives value for compressible cakes it varies between 0.2 to 0.8.

So whatever the equations till now that we have for compressible and incompressible cake that is for constant pressure filtration but batch process. Now this same equations we can modified or rearrange in a different way. So that the same equations we can use for the continuous filtration, but constant pressure right, constant pressure filtration but continuous process.

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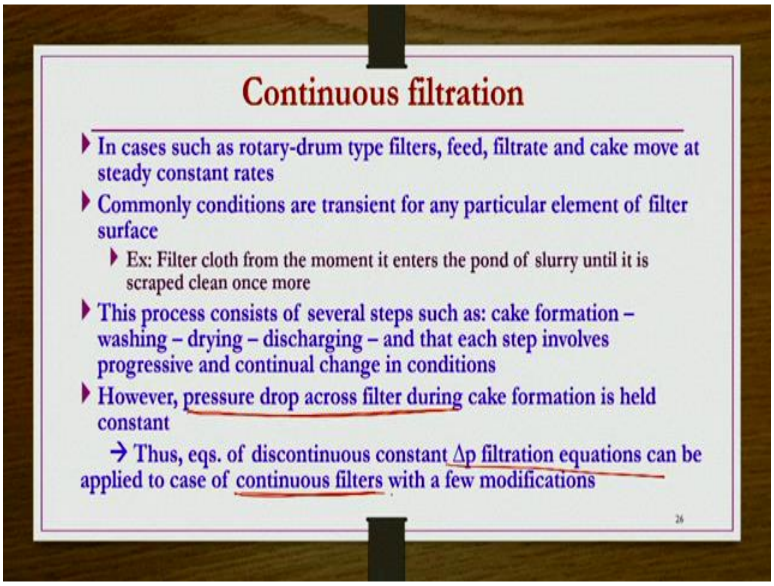

So continuous filtration, here in cases such as like in a rotary drum type filters, there are different types of industrial filter equipment are there. So some of them we are going to see anyway in the next lecture right. So one of the type is the rotary drum type filters, where the feed, filtrate and cake move at steady constant rates. All of them are moving at different constant rates right. So the cake is also continuously removed. So that the processes is not stopped in between just to discharge the cakes etc right.

So such in continuous filtration processes, we know all this, you know feed coming in, filtrate continue collecting and then cake removal or discharging of cake is continuously going on as long as the cycle is, you know progress. So for discharging of cake we are not going to stop the experiment. So under such conditions, you know we can call as a kind of continuous filtration and then rotary drum type filters are kind of a continuous filters.

Commonly conditions are transient for any particular element of filter surface as we know. Example filter cloth from the moment it enters the pond of slurry until it is scrapped clean once more. It is going to be a kind of function of time. So this process consist of several steps such as: cake formation, washing, drying, discharging and that each steps involves progressive and continual change in the conditions with respect to the time okay.

So however, the pressure drop across the filter during the cake formation is held constant. Though the process is continuous, the pressure drop is a kind of a held constant. In the previous case, the whatever the $\frac{t}{V} = \left(\frac{K_c}{2}\right)V + \frac{1}{q_0}$ equation that we developed, there the processes is a kind of batch process okay, but the pressure is constant.

Now here, the processes continuous, the content is filtration process, but the pressure drops again constant. So both of the, the previously developed the equation and this equation, whatever we are going to develop for this case are valid for a kind of a constant pressure filtration processes. But one thing the previous one is for the batch process and that this is whatever we are going to do is for the continuous process.

So since the pressure drop across the filter medium is maintained constant during the cake formation, whatever the discontinuous or batch constant pressure drop or constant pressure drop filtration equations are there, that can be used or applied to the case of continuous filters as well with a few modifications.

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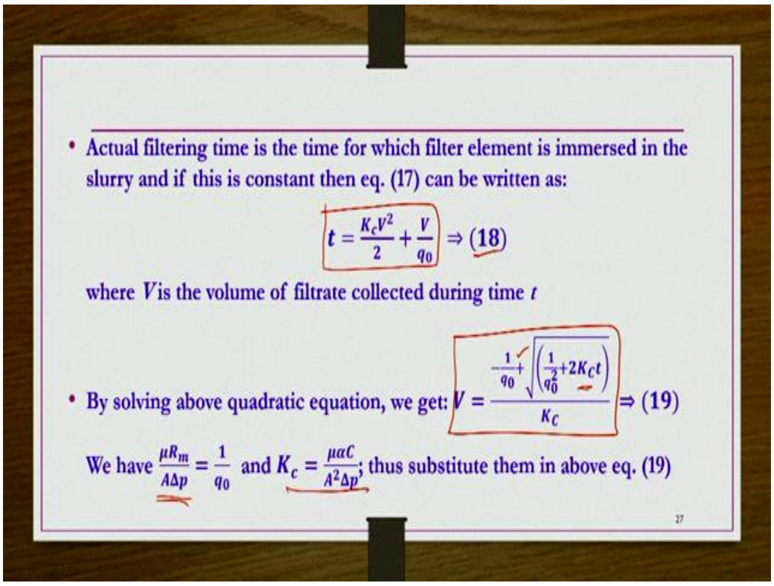

What are those modification? Actual filtering time is the time for which filter element is immersed in the slurry and if this is constant then equation 17 can be rewritten as like this that whatever the $\frac{t}{V}$ was there, what V was there in the left-hand side that has been taken to the right-hand side and then given equation number 18. So that $t = \left(K_c \frac{V^2}{2} + \frac{1}{q_0} V\right)$.

And the continuous process what is the volume of, volume of filtrate that is being collected it with respect to the time that is what we are going to check for a constant pressure right. So this equation now should be solved to get a kind of a expression for V as a function of time okay.

So here V is the volume of filtrate collected during time t and then by solving this equation, this quadratic equation, you will get $V = \frac{-\frac{1}{q_0}+\sqrt{\left(\frac{1}{q_0^2}+2K_c t\right)}}{K_c}$. This should be plus or minus as per the, you know solution method. But you know we cannot have a kind of a negative volume of filtrate.

So it has to be plus now, if both are minus then it is going to be negative that is not possible in real situation that is the reason it is. We have taken, you know, though it is minus B plus or minus square root of B square minus 4 AC/2A from you have taken only plus here, okay. So here we know what is K_c? K_c is nothing but $\frac{\mu u C}{A^2 \Delta p^3}$ and then $\frac{1}{q_0}$ also we know it as $\frac{\mu R_m}{A \Delta p}$.

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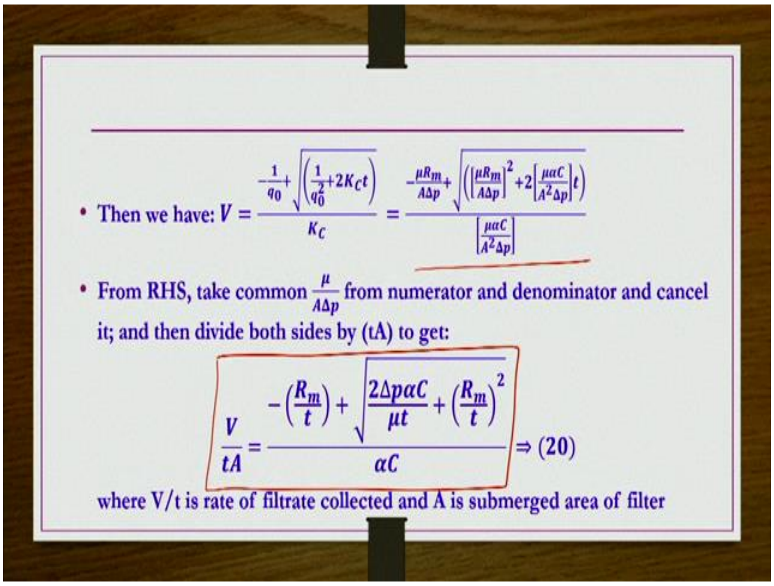

So, if you substitute them here in this equation, equation number 19 then we have this equation. From this equation, what you do? You take $\frac{\mu}{A\Delta p}$ common from the both numerator and denominator of the right-hand side and strikeout. And then after that step you divide both sides of this equation by At. Then you will get this equation $\frac{V}{tA} = \frac{-\left(\frac{Rm}{t}\right)+\sqrt{\frac{2\Delta p \alpha C}{\mu t}+\left(\frac{Rm}{t}\right)^2}}{\alpha C}$, okay.

So basically this difference between equation number 19 and 20 that, you know equation number 19 we have in terms of K_c and $\frac{1}{q_0}$, here directly we have this equation in terms of alpha and Rm. V by t is rate of filtrate collected and A is submerged area of the filter. It is not the total filter area, in continuous process, continuous filtration process only fraction of a filter medium is submerged in the slurry.

That we are going to see in the equipment part, only how much area is submerged in the filter area in the slurry that is known as the A here in this case. In the case of continuous filtration. Whereas in the case of batch filtration the entire surface that is exposed for separation for slurries into liquid and solids that entire area of the filter medium is taken as the area. But in the case of continuous filtration only the fraction of area of the filter medium that is submerged in the slurry is A.

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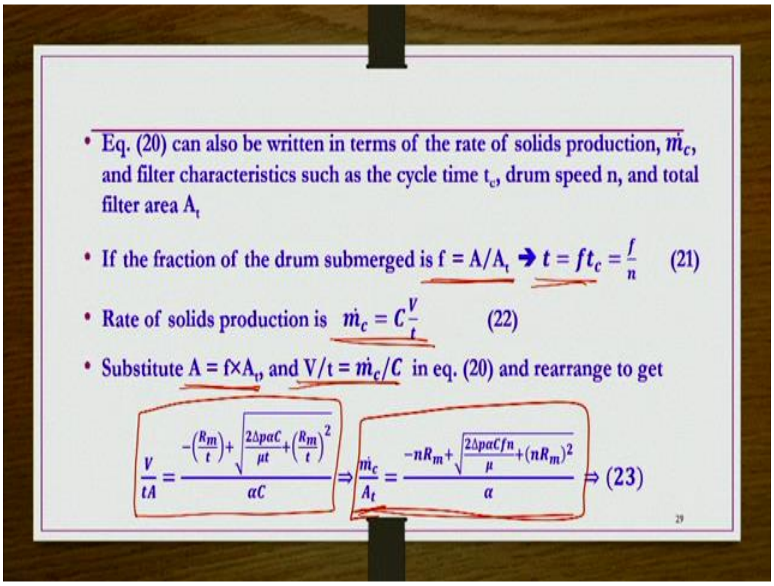

So this equation 20 can also be written in terms of the rate of solids production, mc and then filter characteristics such as the cycle time t_c, drum speed n, and total filter area A_t etc. So, if the fraction of the drum submerged is f then A/A_t is nothing but f. Then cycle time, we can write ft_c is nothing but the t, which we can also write as f/n. That is the n is nothing but drum speed.

And then m dot c for this kind of continuous process, we already know that it is $C\frac{V}{t}$. If you substitute this equation number 21 and 22 in equation 20 or $A = f \times A_t$, and then $\frac{V}{t} = \frac{\dot{m}_c}{C}$ in equation number 20. Then we get this equation by rearranging simply, this is your equation number 20. Here now in place of $\frac{V}{t}$ you substitute $\frac{\dot{m}_c}{C}$ and then in place of A you substitute fA_t.

So, and then further you simplify, here you will get left-hand side $\frac{\dot{m}_c}{A_t}$ is equals to this equation. This is much more convenient form of the equation to be used for continuous filtration process. Because in general, in continuous filtration process this $\dot{m}_c$ or n, f etc are available, rather than V and t.

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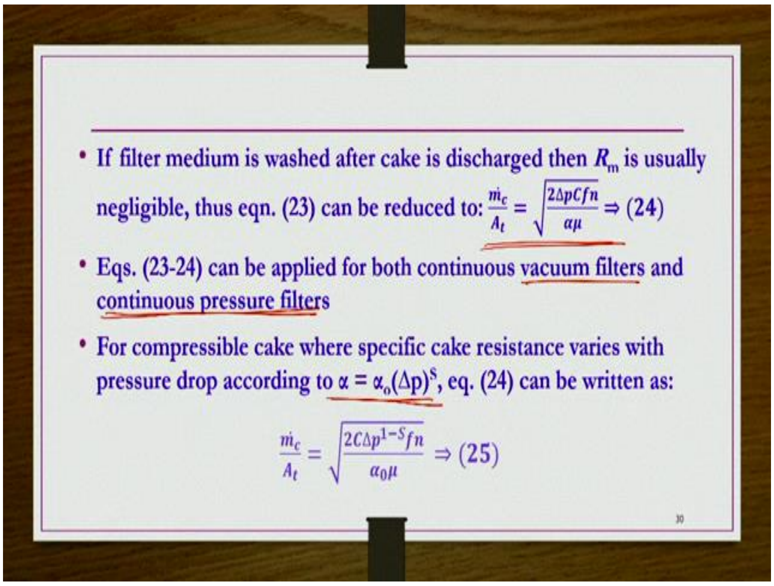

If the filter medium is washed after cake is discharge, then whatever the filter medium resistance Rm is there that is usually negligible. Then that equation number 23, from that equation number 23, if you strike of the Rm terms, then you will have $\frac{\dot{m}_c}{A_t} = \sqrt{\frac{2\Delta pCfn}{\alpha\mu}}$.

So these equations 23 and 24, this equation without Rm and then previous equation with Rm term can be applied for both continuous vacuum filters, as well as the pressure filters. Though we are waiting for the pressure filters kind of things, so they are also kind of valid for a kind of vacuum filters.

For compressible cake where specific cake resistance varies with pressure drop, according to $\alpha = \alpha_0(\Delta p)^S$. Then this equation 24 can further be modified like this.

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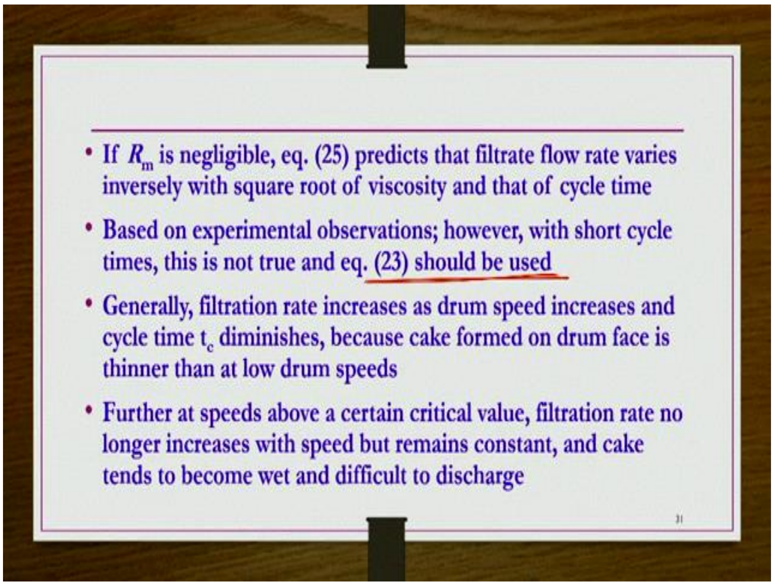

So if Rm is negligible, equation 25 predicts that filtrate flow rate varies inversely with square root of viscosity and that of cycle. But in general, that is not true based on the experimental observations; especially, for short cycle times, this is not true and then equation 23 should be used, okay.

Generally, filtration rate increases as drum speed increases and then cycle time t_c diminishes, because cake formed on drum face is thinner than at low drum speeds. Further at speeds above certain critical value, filtration rate no longer increases with speed, but remains constant, and cake tends to become wet and difficult to discharge.

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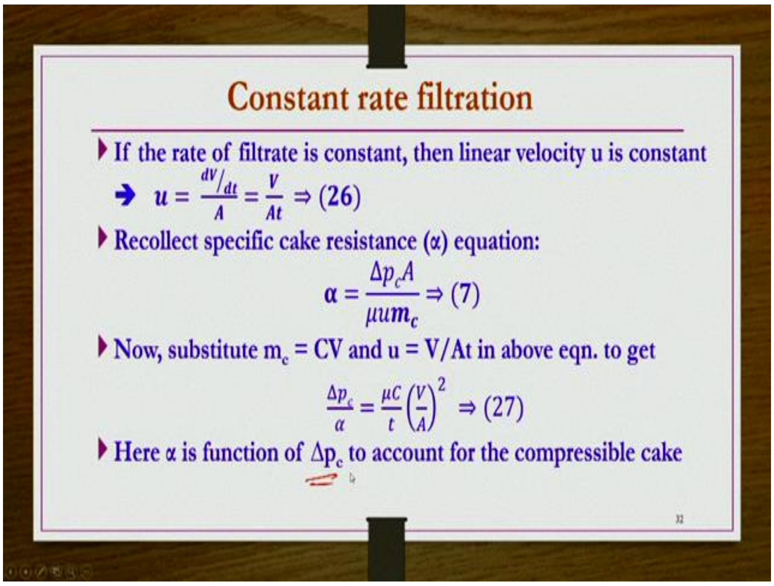

So the finally constant rate filtration. The previous two cases, whatever we have the constant pressure filtration for batch and then continuous processes. Now we are going to have equations or principles of a constant rate filtration under the category of cake filtration, okay. So though it is a very rare, we need to have a kind of equations. If the rate of filtrate is constant, then linear velocity u is constant. So u we can write rather than $\frac{dV/dt}{A}$, we can write it as $\frac{V}{At}$. Because now here the rate, filtration rate is constant, okay.

Then alpha equation, we have this equation $\alpha = \frac{\Delta p_c A}{\mu u m_c}$. In this equation, m_c you can substitute as CV and then u you substitute as $\frac{V}{At}$. Then we get $\frac{\Delta p_c}{\alpha} = \frac{\mu C}{t}\left(\frac{V}{A}\right)^2$. Here α is function of Δp_c to account for the compressible cake.

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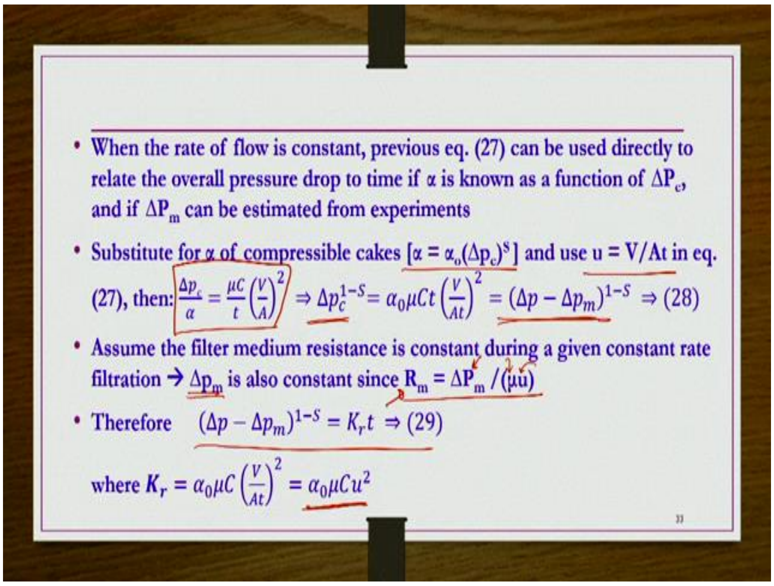

When the rate of flow is constant, the previous equation number 27 can be used directly to relate the overall pressure drop to time if alpha is known as a function of Δp_c, and if Δp_m can be estimated from experiments. So in that above equation substitute $\alpha = \alpha_0(\Delta p_c)^S$and use u = V/At in equation number 27. So this is your equation number 27.

So here α, in place of α if you write $\alpha_0(\Delta p_c)^S$ and then in place of V/At you write u. So then you get this equation. So, and then rearrange all Δp_c terms, you write one side. So $(\Delta p_c)^{1-S} = \alpha_0 \mu C t \left(\frac{V}{At}\right)^2$ and Δp_c is nothing but $(\Delta p - \Delta p_m)^{1-S}$ as it is.

Assume the filter medium resistance is constant during a given constant rate filtration. So Δp_m should also be constant. Because Rm is nothing but $\frac{\Delta P_m}{(\mu u)}$, u is constant for a constant filtration rate, viscosity of a filtrate is in general constant. So, if Rm is constant then ΔP_m should also be constant.

So this equation, we can write it as $(\Delta p - \Delta p_m)^{1-S} = K_r t$ and then K_r is nothing but $\alpha_0 \mu C \left(\frac{V}{At}\right)^2$. And then this $\frac{V}{At}$ if you write as a kind of u, then Kr can also be written as $\alpha_0 \mu C u^2$. That is about constant rate filtration.

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The references for this lecture, the entire lecture is prepared from this reference book McCabe, Smith and Harriot, Unit Operations of Chemical Engineering. There are other reference books, where you may find some information as well. Thank you.