Mechanical Unit Operations
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Lecture 03
Particle Shape and Density

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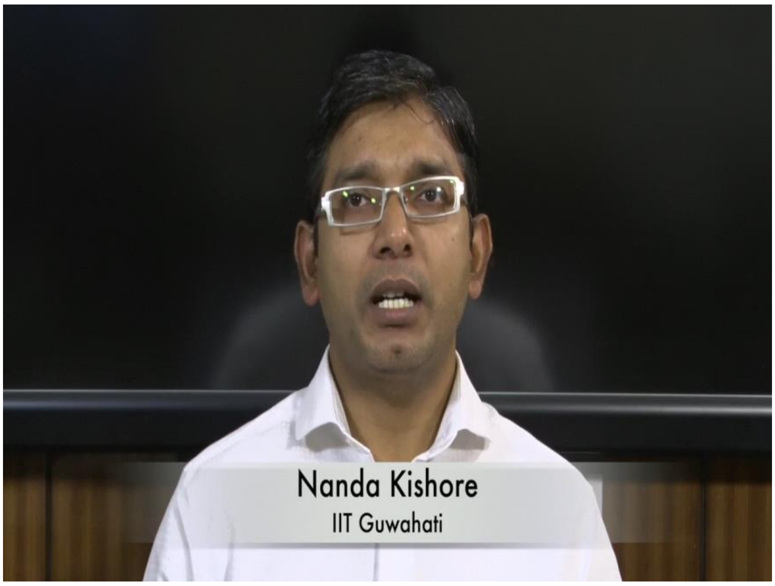

Welcome to the mock course Mechanical unit operations, the title of this particular lecture is Particle shape and density, we have been studying the different distinctive properties of particulate solids which can have a kind of influence if properly not monitored, they made have the effect, strong effects and a kind of product distribution, sometimes they also be needed to be controlled in order to have a kind of a proper requirement at the reactant level itself.

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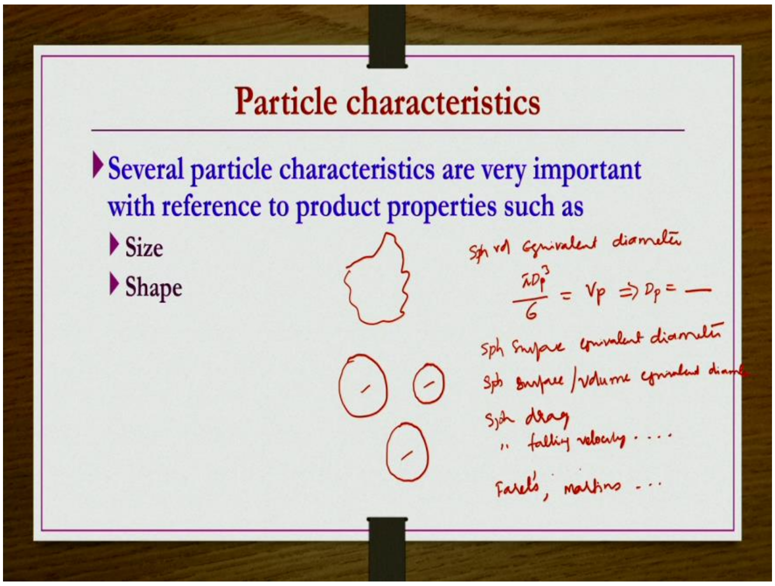

What are those properties that we are saying so far? That we have listed, enlisted these kinds of properties, these properties are the size, shape, density, hardness, adsorption, etc this kind of things. So out of which we have also seen that the size, shape and density are the most important one, under this size of the particles we have seen different types of representations of the irregular particles because when you take a bulk material and then crush it as per requirement of the process, then what will happen, you will get regular particles and then irregular particles may not be having a kind of a similar degree of irregularity as well.

So how to define the size of those particle is a kind of big question, when this number of particles are you know some kind of millions of particles are involved in a kind of process, then this becomes even a difficult task, so then what we have to do, we have to make a kind of equivalent size representation, so for irregular particles what we had? We had kind of equivalent property of sphere and then accordingly equivalent diameter for those irregular particles we have seen.

Like you know sphere, volume equivalent diameter that we have seen, one of the methods of representing the size of irregular particles is that you know, sphere, volume equivalent diameter, what we have done here, we have measured volume of the particle, irregular shape whatever, irregular shape particle is there we measure its volume and then we equated that volume to a kind of a spherical particle of size D_P, whose volume is same as particle diameter $V_p = \frac{\pi D_p^3}{6}$, that is whatever the spirit of particle that is having a volume equal to the volume of the particle right.

So the size of that spherical particle is D_P, then from here, this whatever the V_P that is the volume of the particle you calculate that you equate to the volume of a spherical particle of size D_P, D_P is the nominal size or equivalent diameter of the particle, so then from here, whatever the D_P that you get that is the nominal diameter, this diameter is nothing but sphere volume equivalent diameter. Likewise we also had a kind of a sphere surface equivalent diameter, where we measure the surface area of the particle and then equated that one to a spherical particle whose surface area is same as the kind of surface area of the irregular particle.

From there, whatever the D_P that we get that is again the equivalent diameter or nominal diameter, but that is sphere surface equivalent diameter, likewise we can have sphere surface to volume ratio equivalent diameter that also we have seen, so here what we can do? We can take a irregular particle, measure the surface of that irregular particle, measure the volume of that irregular particle, then you take the ratio, the surface area of the particle divided by the volume of that particle $\frac{surface\ area\ of\ the\ particle}{volume\ of\ the\ particle}$, irregular particles.

This ratio equal to the surface to the volume ratio of a spherical particle, whose surface to volume ratio is same as the surface to volume ratio of the irregular particle, from their whatever the D_P that you get, you get spheres surface volumes or sphere surface to volume ratio equivalent diameter, likewise we also had sphere drag equivalent diameter, similarly sphere, you know falling velocity equivalent diameter, like this, several type of or equivalent diameters that we have seen in order to represent the size of the particle right.

So we have also seen the statistical measurements of the regular particles, like you know Faret's dimensions and then Martin's dimensions etc, those statistical measurements have also been seen, the statistical measurements in general obtained by the microscopic that is not a kind of practically applicable method in industrial applications as process specially, why because we have relatively infinite number, that is thousands of, lakhs of particles in general are there in the mixtures, so each and every particles you cannot take and then measure in the microscopic in order to have the analysis, so that is the reason, though it is a reliable kind of one, it is not going to be helpful.

Further we have also seen kind of, if you have a kind of irregular particles like this, so it is sphere volume equivalent diameter, if you measure and then pictorially represent it, it may be of size like this, similarly for the same size particle, if you measure the sphere surface equivalent diameter then it may be of this size, then similarly, if you measure the surface to volume ratio equivalent diameter for this particle, it may be having like this, so what we see here, see for the same particle different equivalent representations having a kind of a different sizes, this size is different from this size, this size is different from this size.

So all three different equivalent the representations of the size of particles are different from each other, so which one should be taken, that is again a big question, which equivalent diameter should be taken, that is again questionable thing, important question one to answer, so however this can be answered by the experience of the process engineer that has been involved in a kind of process or it can be based on a kind of applications kinds of thing one can do.

Let us say if you have a kind of cyclones operators, then sphere close equivalent diameter would be sufficiently reliable one, if you have a kind of you know regular particles like you know, something like you know, flow through packed bed etc, then sphere surface to volume ratio equivalent diameter would be more reliable, so based on the applications also one has to have a kind of intuitive feeling, which kind of equivalent diameter one has to use and then this equivalent diameter, in the previous class we have seen only for a kind of single particles only, we have not seen for a kind of mixture, those thing we are going to see in a kind of subsequent lectures, where we do the screen analysis.

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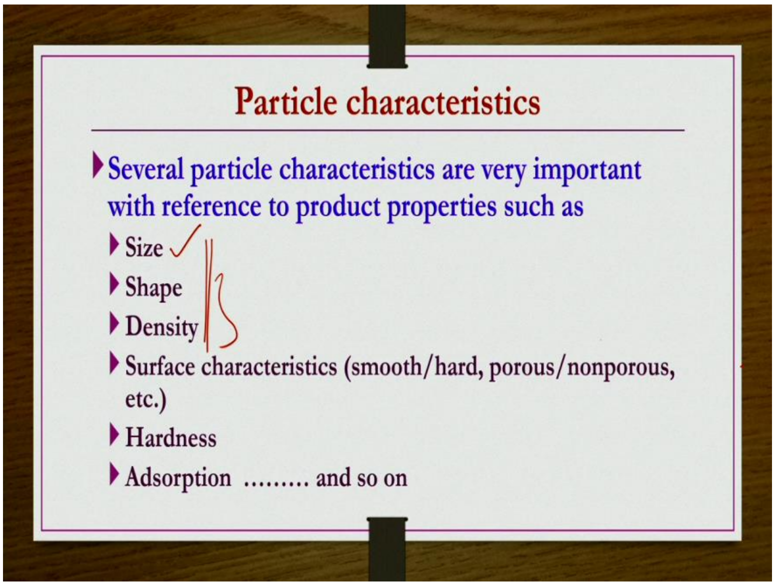

So in addition to this size of the particles what are the other properties that we are going to see? We are going to see the shape of the particle, density of the particle, so these two things we are going to take up in this particular lecture in detail now so, but there are additional properties that can also be important depending on the applications surface characteristics, hardness, adsorption etc, so we are going to see only this three things, so this we have already seen in the particular lecture today, we are going to discuss shape and density of a given irregular particle.

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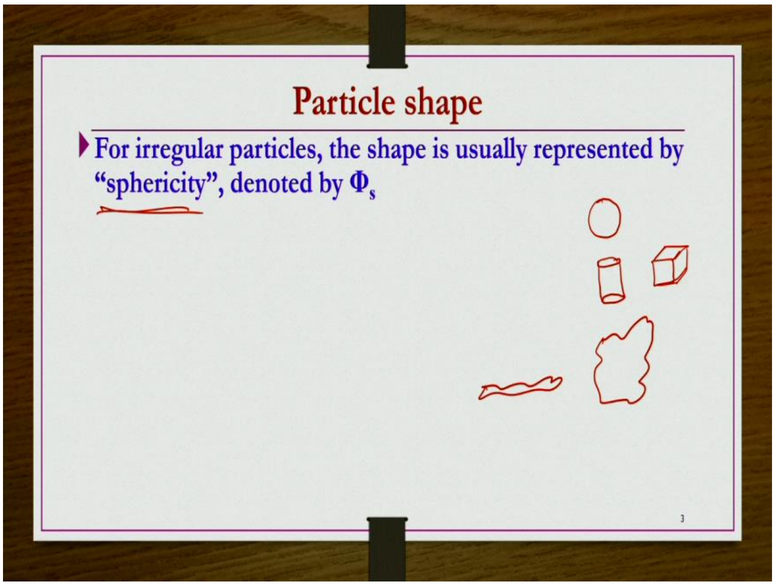

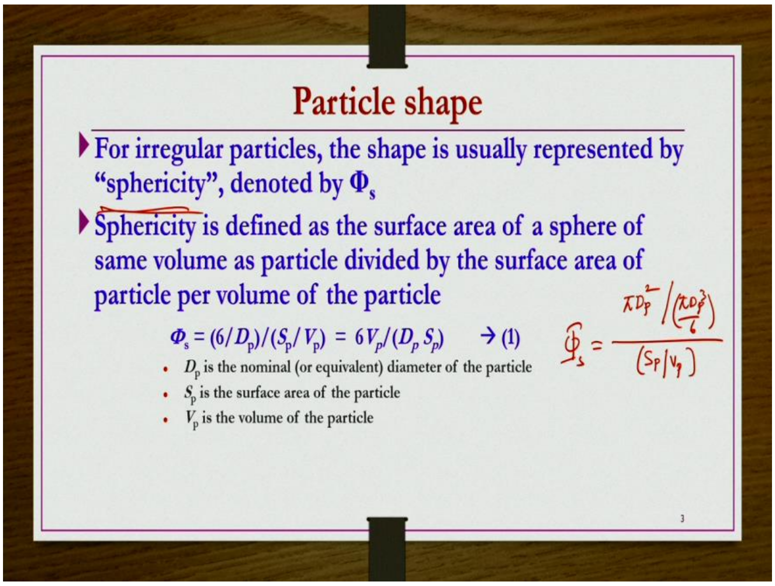

So particle shape, for a given irregular particle that is when you take a bulk material and you crush it into the smaller fraction, so you get a kind of small particles size, but they are irregular shape, they are not in a kind of regular shape, they are not in a kind of spherical shape, they are not in a kind of cylindrical shape, they are not in a kind of completely needle like shape, even if they are mostly like needle like shape, they are you know not perfectly needle like shape, even if they are kind of rounded shape, they are not perfectly spherical shape.

So how to represent this irregular particle, so that is what we are going to see for a kind of individual particles, in this lecture what we are going to see? We are going to see the particle shape, how to represent the shape of the particle for a given individual particle, later on, we are going to do in the subsequent lecture for a mixture of particles also we can define a kind of what is this sphericity kind of thing, especially for kind of non-uniform mixtures right.

So the particle shape with a reference can be defined, what we have for irregular particles, we can define the shape by a kind of sphericity $\emptyset_{S},$ the sphericity as we are by the word it relates how much is the kind of spherical nature of that particle, how closely can we represent in terms of this sphericity, spherical nature of that particle, let us say you have a kind of a spherical particle, so then you can say that is sphericity is a kind of one that we are going to see, if you have a kind of small cylindrical particle like this, so then what is the spherical, equivalent spherical shape of this particle that is also we are going to see.

If you have a kind of small cubicle particle like this, then what is its equivalent, you know kind of sphericity, those things we are going to say, now in this particular lecture, so it basically represents like you know how much closely we can say, how much close it to the spherical shape, reference materials spherical shape that it is right.

So let us say like you have a irregular particle like this, if you have a kind of irregular needle particle like this, so like that if you have a kind of different shapes, so then how much it is spherical, that we can calculate by this sphericity and then that is what we are going to do here particularly in this lecture, so the sphericity is a kind of you know representation how much spherical it is, so how closely related to the spherical shape, that is what with reference to the spherical shape that we are going to see here.

So it is defined as the surface area of a sphere of same volume as particle divided by the surface area of particle per volume of the particle, so what that is what sphericity, like you know surface area of a spherical particles of sizes D_p square and then it is volume is $\frac{\pi D_p^3}{6}$ right and then divided by the surface area of the particle, irregular particle, let us say Sp and then volume of the irregular particle, let us say, V_p , so this, you do ratio, then you will get the sphericity that is sphericity is defined as the surface area of a sphere of same volume as particle divided by the surface area of the particle per volume of the particle.

So that is when you put in a kind of equation, so now this, you are going to get $6/D_P$ divided by S_p/V_p,

$$\frac{\pi D_p^2 \Big/ \frac{\pi D_p^3}{6}}{S_p \big/ V_p}$$

$\emptyset_s = \left(\frac{6/D_p}{S_p/V_p}\right) = \frac{6V_p}{D_p S_p} \rightarrow (1)$, that means for a given irregular particle, if you know what is the volume of the particle? What is the surface area of the particle? And then what is the nominal diameter of the particle? Especially sphere volume equivalent diameter that DP, then you can know the sphericity for that given particle easily right.

So here what we have D_p, D_p is the nominal or equivalent diameter of the particle and then S_P is the surface area of the particle and then V_p is the volume of the particle right, so now you may be wondering if this sphericity is it going to be depended on the size of the particle that is the question.

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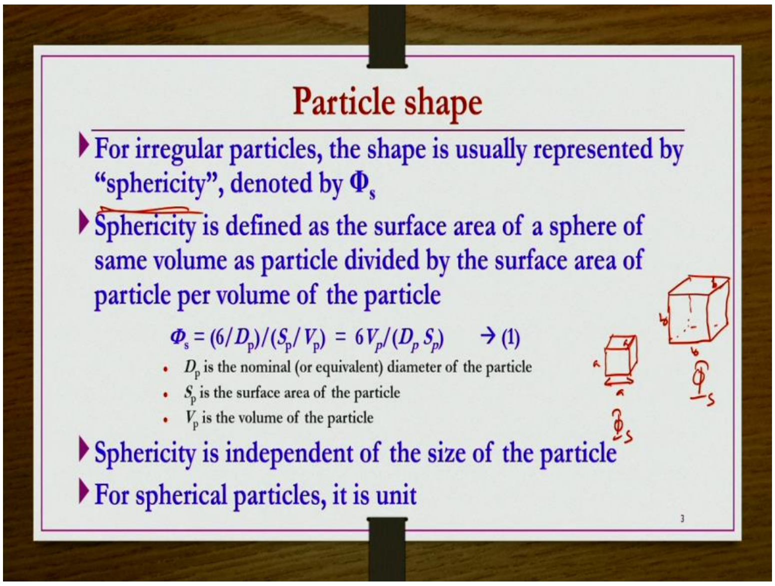

Let us say, you have a kind of small cube size let us say all the sizes are same, this size is a, same is this also and then same is this also, now you take another cubicle particle right, but of the different size, so b, b, b, and then where a is not equals to b and they are not in a kind of any proportional to each other and a is not equals to b and the let us say b is much higher compared to the a.

So then is this sphericity for the smaller cubicle particle and then sphericity for these larger cubicle particle are not going to be different, they are going to be same that we can prove, we can take the different cubes and then we can calculate the sphericity and then you can find like this and then same is to for any kind of shape of the particle, if you take short cylinders, if you take hemispheres, if you take raschig rings or if you take any irregular particles, you maintain the shape of the irregular particle same but the size you change, you change the size of those irregular particles and then you calculate sphericity, you are going to get the same sphericity there.

So the sphericity is dependent only on the shape of the particle, it is not going to depend on the size of the particle right, whether it is regular, non-spherical particle or irregular arbitrary shape particle, whatever it is, if the shape is same, but size is changing the sphericity is not going to change, the sphericity is only depended on the shape of the particle but it does not depend on the size of the particle, it is not going to change with the size of the particle. And obviously the sphericity we have defined with reference to the surface to volume ratio of a spherical particle of size D_P, so then for sphericity, for a spherical particle is always going to be one, it is always going to be one for spherical particles right.

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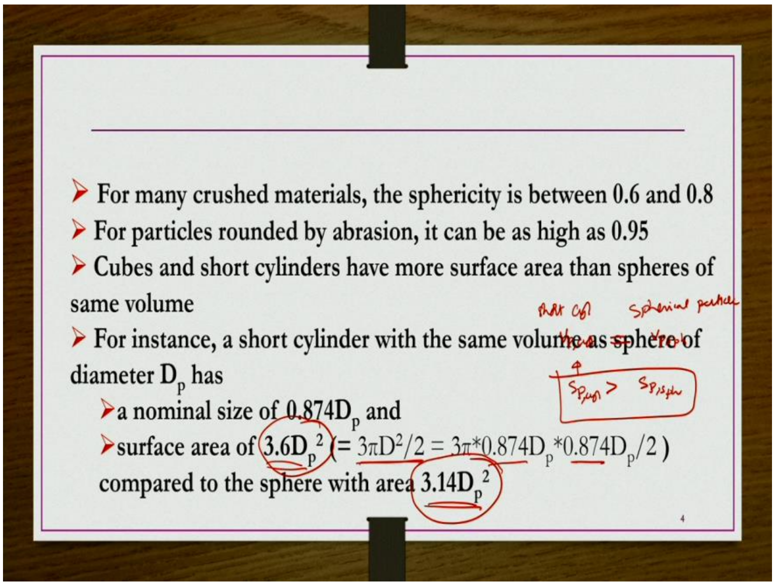

Now we have in general, we do not have a kind of cubical shapes or you know small cylinders kind of things or raschig rings always, we may be have a mixture of particles or when you take a bulk material and you crush it, then you may be having a different types of shapes of the particles, depending on the bulk material is homogeneous or not, if it is heterogeneous or composite kind of material or it is consisting of different materials, A, B, C, so depending on the molecular structure of A component, B component and C component that is consisting in the bulk material.

The shapes of those crush material, after crushing the shape of the particles may also be varying because A maybe in general crushed into kind of a cubical shape or B may be always giving more or less like a plate-like kind of things, let us say B is, mica, so it is always going to crush into kind of plate-like shape, if it is PBS Galina it is more or less going to give a kind of cubical shape.

So when you crush if your bulk material is consisting this different types of material, it is not type of pure one component, then obviously when a crush material is also going to have a kind of different shapes right, so then it is going to be very difficult to maintain, to measure this sphericity but there are few methods that theoretically also we can find out, also we can find out from the experimental background also, experimental methodology also that we can see.

In general, many crushed materials they have the sphericity between 0.6 to 0.8, but in the crushing there is a kind of abrasion while they are transporting from one process to the other process there is a kind of sufficient of degree of abrasion is taking place, in general because of the abrasion particles may become more or less kind of rounded shape, so for those kind of particles it can be, the sphericity can be as high as 0.95 and then some cubes and short cylinders have more surface area than sphere of same volume, that is we are going to see, we are going to prove it again in subsequent slides.

So that is let us say if you have a short cylinder with L is equals to D or you we have a cubes with a custody is equals to D, if the surface area, then the volume of the particle, whether the short cylinder you take is same as the volume of a spherical particle of size D_P, then surface area of a cylindrical particle is going to be higher, that is, let us say you have a short cylindrical particle and then spherical particle right.

Now in these two cases, if let us say V_P cylindrical particle and then we have V_P spherical particle, if these volumes are same $V_{short\ cylindrical\ particle} = V_{spherical\ particle}$, then whatever this for the short cylinders are there, S_p is going to be more than the S_P cylindrical particles is going to be more than the S_P spherical particle $S_{short\ cylindrical\ particle} > S_{spherical\ particle}$, why? Because of the nominal diameter, if you calculate nominal diameter, if you calculate for this cylindrical particle that is going to be more than the its original diameter of the cylindrical particle right.

So that nominal diameter that we have already calculated in the previous lecture, we are going to do once again, so because of this reason if you take a short cylinder, if you take a spherical particle, if the volume of these two particles are equal to each other, then the surface area of the cylindrical particle is going to be the more compared to the surface area of the spherical particle of same volume as a kind of cylindrical particle.

Same is true for the cubes also but it is not necessary for all cases, let us say if you have a hemisphere case, if you have a hemisphere particle, if you have a spherical particle and then volume of these two particles are equal to each other, then the surface area of the hemisphere is going to be less than the surface area of the spherical particle right, that depends on the equivalent diameter, how much it is. Okay, that is what we are going to see.

So let us say, if you have a short cylinder with same volume as a sphere diameter of D_P, then nominal size is going to be 0.874 D_P, that is D is going to be $\frac{1}{0.874D_p}$, so that is the reason 1/0.874 is going to be more than 1 that is the reason it is going to be having higher value right, let us say the surface area of short cylinder is $3\pi D^2/2$, now D is nothing but 0.874 D_P that if you substitute here it is going to be $3.6D_P^2$ $\left(= 3\pi D^2/2 = 3\pi * 0.874D_p * 0.874D_p/2\right)$, whereas the surface area of the spherical particle is going to be πD_p^2, so that is $(3.14D_p^2)$, so that is the reason now here the surface area of a spherical particle is smaller compared to the surface area of the shorter cylinder provided the volume of these two particles are equal to each other. Okay, the similar way we can check for the cubes also.

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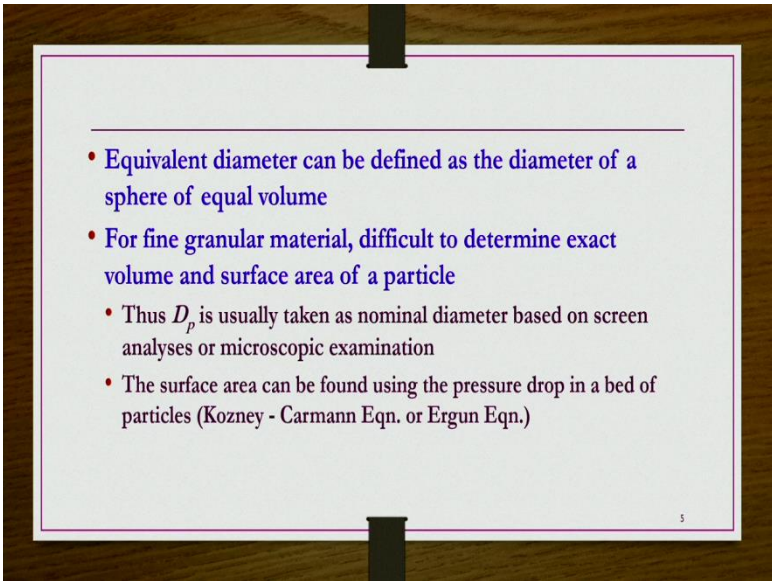

So equivalent diameter, the Dp that we are going to use here in this sphericity calculation, in general, we can use the diameter of a sphere of equal volume or a sphere volume equivalent diameter that is what we are going to use here in D_P calculation and then for fine granular material, it is difficult to determine exactly what is the volume? What is the surface area of the particle right?

For granular material of very small size, it is going to be very difficult, if you have a kind of particles irregular shape but size is in a kind of, you know or volume is some mm cube or some centimetres cube, it is quite possible that you can measure S_P, V_P and then you can get the sphericity but particles are in some microns size like you know size of the particle is something, like you know 50 microns or 20 microns, so then it is going to be very difficult for you to measure the V_p and S_p that is the surface of the particle and volume of the particle is going to be very difficult measurement, that is not possible even for a you know slightly regular particle, if the particle is irregular in larger dimensions, larger degree then it is going to be very difficult.

For that how to measure this volume and surface area of particles, so there we have a kind of screen analysis or we can do by some kind of a microscopic examination right and then there are some kind of experimental methods also are available from where we can measure the surface areas, surface area like you know, you take a packed bed and then you pack with the kind of irregular material for which you wanted to measure the surface area and do the experiments, you measure the pressure drop and then these pressure drop values along with the size of the particle etc, then fluid properties etc you substitute in a kind of Kozeny–

Carman equation and then from there you can find out the surface area of that particular unstructured material packing right.

This is also be are going to do in a subsequent lecture probably in module 6, this is we are going to do this one, but this particular lecture we are going to take only for the individual particle, so let us not worry about them. For nonporous particles, from adsorption measurements, one can find out the sphericity as well.

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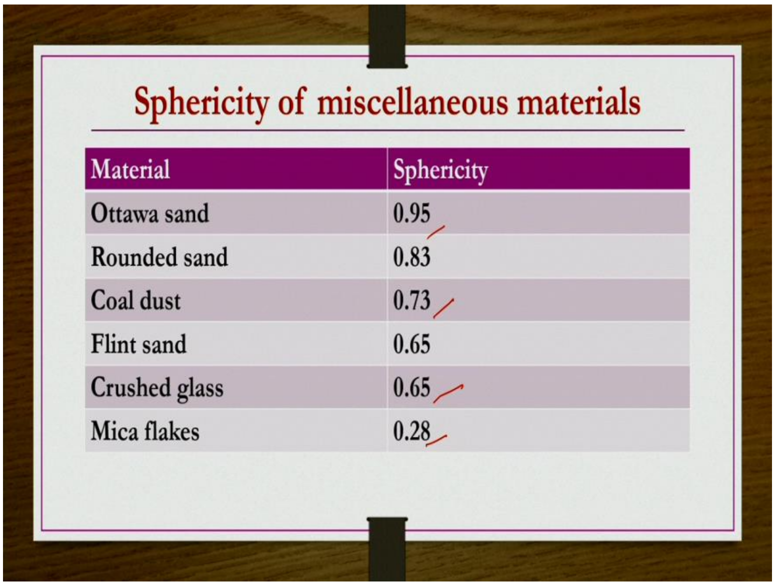

So what we do? We see some example materials like you know and then what is the sphericity of those materials? In general, let us say you have the Ottawa sand, so the sphericity can be as much as 0.95, so more or less a kind of they are having a spherical shape and then there are some kind of rounded sand, the sphericity is almost 0.83, whereas the coal dust in general, coal dust is very fine size, very small size kind of particles, finer particles that is the finest sizes that is possible, not a crust entire thing, the coal dust whatever we have, for the coal dust the sphericity is as much as 0.73.

Flint sand it is 0.65, crushed glasses in general use kind of packed material, some applications, some chemical applications where there is no reactions are there, so those crushed classes have in general 0.65 and then mica flakes have a 0.28. So mica flakes are in general, you know mica material when you crush them into the smaller strings smaller sizes, you may get approximately a kind of almost you know this plate like materials, so since the shape of the plate-like material is not very much spherical, not at or close to the spherical shape, so that is the reason the sphericity for this mica flakes kind of material, it is very small.

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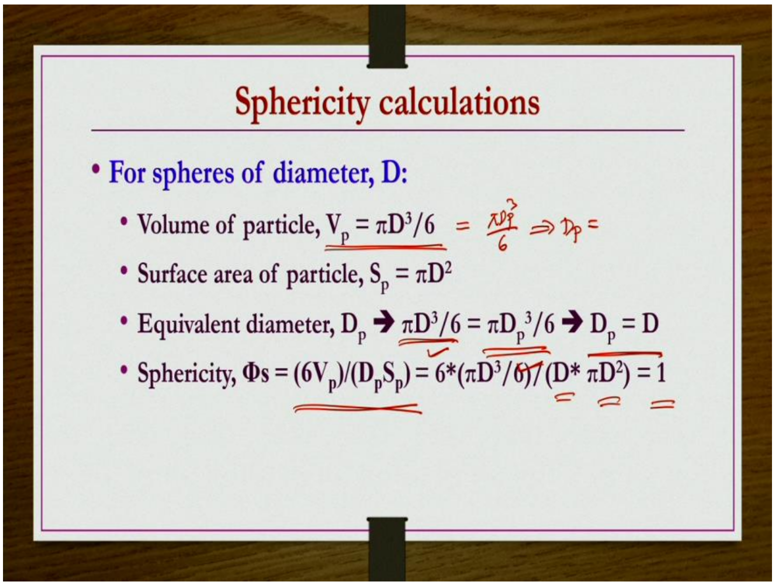

Now we are going to do sphericity calculations, so how to measure a sphericity of a given particle that is what we are going to do now, so sphericity is going to be one for a kind of spherical particle that is what we have seen because this sphericity is defined with respect to the surface to volume of a spherical particle basis, so that is the reason it is going to be 1, it is going to be 1, so sphericity is always going to be less than or equal to 1 only always, it is not going to be greater than 1 any for any case, so it is going to be 1 for a perfectly spherical shape particle and then as the deviation from the spherical shape increases the sphericity decreases, so since spherical shape we know that it sphericity is equals to 1, so we start with that one and then we go to the different shapes of the particles.

Let us say spherical particle of diameter D we have, so what is the volume of the particle? It is, $V_p = {\pi D^3}/_6$ and then volume of this particle and then surface area this spherical particle is $S_p = \pi D^2$, then equivalent or nominal diameter D_P you have to find out, so how you do? You equate the volume of this particle to the volume of a spherical particle whose volume is same as the volume of the particle right. Let us say spherical particle is having the size D_p right, so ${\pi D_p^3}/_6$ is the volume of that particle and then this volume of the spherical particle is equal to the volume of the particle $D_p \rightarrow {\pi D^3}/_6 = {\pi D_p^3}/_6 \rightarrow D_p = D$.

So from here, whatever this D_p you are going to get that is nothing but the nominal or sphere volume actual diameter, so equivalent diameter D_P calculation, so volume of the particle should be equated to the volume of a spherical particle whose volume is same as a kind of volume of the whatever the particle that we have considered, here since both of them are you know spherical particles, so ${\pi D^3}/{6}$ is equals to ${\pi D_p^3}/{6}$, this is the material that for which we are calculating, this is a kind of that nominal diameter.

So now the sphericity we have the $\left(\frac{6V_p}{D_p S_p}\right)$, so here in this V_P is ${\pi D^3}/{6}$, D_P is now nothing, but D and then S_P is nothing but πD^2 when you substitute all this things you are going to get 1 $\left(\emptyset_s = \left(\frac{6V_p}{D_p S_p}\right) = 6 * \frac{({\pi D^3}/{6})}{(D * \pi D^2)} = 1\right)$, so this is how we have to do, so this reference this is we have done reference to cross check whether are we doing correctly or not.

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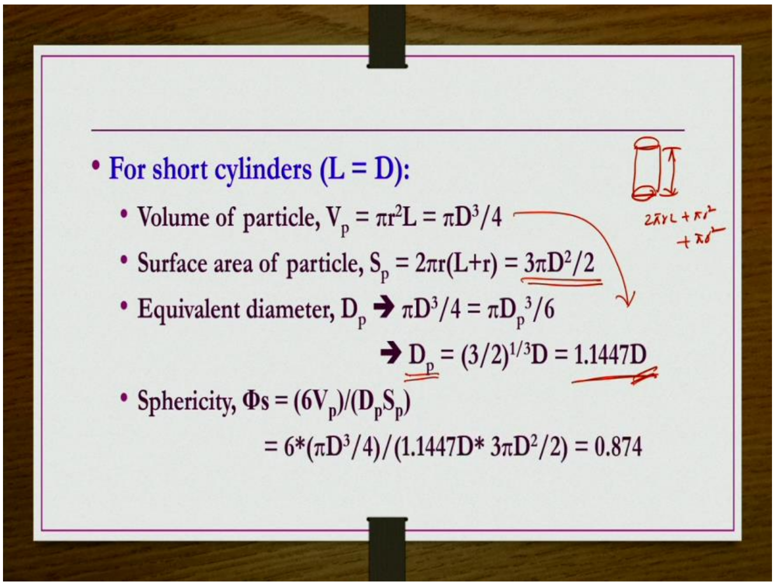

Now what we do we take a short cylinder whose size L is equal D (L=D), can say the cylinder a shorter one if its diameter and then length are equal to each other in general or diameter or length closely equal to each other, then we can say that kind of a short cylinder, let us take a short cylinder case L is equals to D, then volume of this particle is going to be this $\pi r^2 L$, $\left[V_p = \pi r^2 L = {\pi D^3}/{4}\right]$. So if you write in terms of D you get ${\pi D^3}/{4}$ and then surface of area of the particle is nothing but the surface area the $2\pi r(L+r)^2$, $2\pi rL + \pi r^2$ one circle and then plus πr^2 the other circle that is $2\pi r^2 + 2\pi rL$ that is $2\pi rL + r$ then you convert in terms of D by using r = D/2 and L = D, you get ${3\pi D^2}/{2}$; $\left[S_p = 2\pi r(L+r) = {3\pi D^2}/{2}\right]$.

Now the nominal diameter of the particle we have to find out, so the volume of the particle. Now we have to equate it to a volume of a spherical particle who sizes D_P and then that volume of spherical particle is equals to the volume of the particle, then we get equivalent diameter D_P is equals to 1.14470 that is $\pi D^3/4 = \pi D_p^3/6$, so from here, D_P you get 1.1447 D right $D_p = (3/2)^{1/3} = 1.1447D$. So this is the reason now what we have here, we have you know for a short cylinder the equivalent diameter is more than the diameter of the cylinder itself, that is the reason if the surface, if the volume of short cylinder and then volume of a spherical particle are equal to each other, the surface area are not going to equal to each other, surface area of the short cylinder is going to be more than this, surface area of the spherical particle because the equivalent diameter or nominal diameter of the short cylinder is more than its original diameter so that is the reason the surface area of a short cylinder is going to be more than a surface area of the spherical particle provided their volumes are equal, that is the volume of particle, spherical particle and volume of short cylinders are equal to each other.

Then sphericity now we have $\frac{6V_p}{D_p S_p}$, you substitute V_P, D_P, S_P etc here, then you get this as 0.874, so short cylinders with L = D sphericity 0.874.

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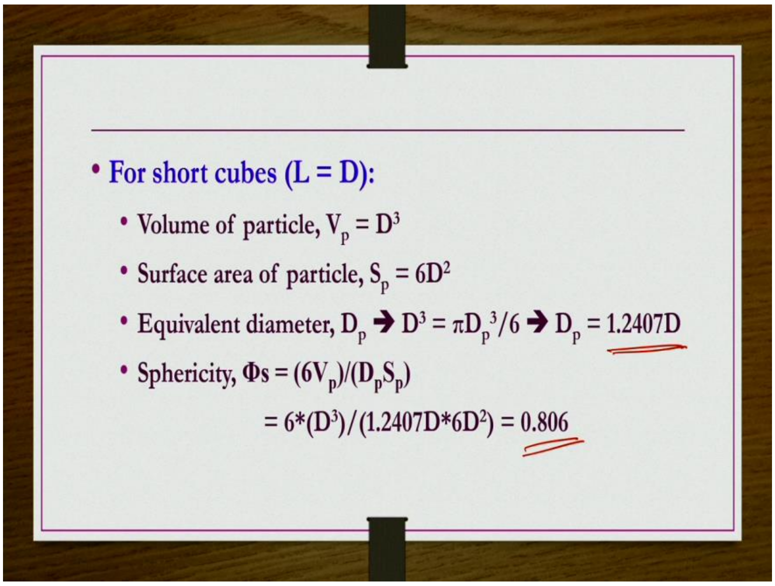

Similarly, if you take a short cubes with L = D are the size of the cube is equals to D, then volume of the cube is D cube $V_p = D^3$, surface area of the cube $S_p = 6D^2$ because 6 phases are there, each phase is having D^2 area, so $6D^2$ is the surface area of the particle and then the nominal diameter, the D^3 should be equated to the $\pi D_p^3/_6$, $D^3 = \pi D_p^3/_6$, so from here, what you get Dp is equal to 1.2407 D,

$D_p \rightarrow D^3 = \pi D_p^3/_6 \rightarrow D_p = 1.2407D$, this is again the reason as we have mentioned in one of the previous slides.

If you take two material, one is sphere another one is a cube, but of the same volume, there volume is same, despite the volume is being same the surface area of the cube is going to be more compared to the surface area of the sphere because of this reason, that the equivalent diameter of the cube is more than its original size, more than D is its original size, but now equivalent diameter is 1.2407, 0.24 times more than its original size right, so that is the reason the surface area of the cube is going to be higher compared to the surface area of the sphere of equal volume. Then sphericity $\emptyset_s = \left(\frac{6V_p}{D_p S_p}\right)$, when you substitute here Vp is D^3, Sp is $6D^2$, Dp is 1.2407 D and then do calculation sphericity is 0.806 for short cubes it is 0.806

$\left(\emptyset_s = \left(\frac{6V_p}{D_p S_p}\right) = 6 * \frac{(D^3)}{(1.2407D*6D^2)} = 0.806\right)$.

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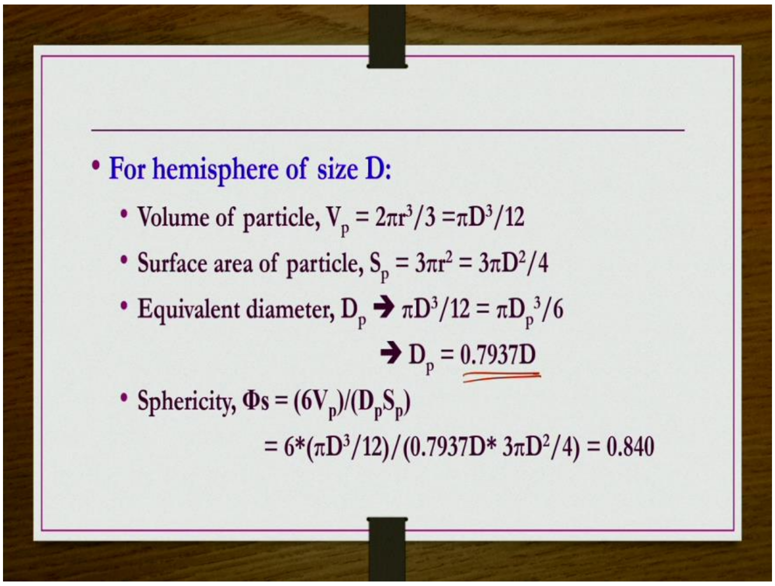

Now likewise, if you take hemisphere of size D, volume of the hemispherical particle $V_p = 2\pi r^3/_3$, in terms of D if you write it is $\pi D^3/_{12}$

$$V_p = 2\pi r^3/_3 = \pi D^3/_{12}$$

, then surface area of particle hemispherical particle is $3\pi r^2$ in terms of D, it is ${3\pi D^2}/_{4}$,

$$S_p = 3\pi r^2 = {3\pi D^2}/_{4}$$

so its equivalent diameter how to find out, the volume of this hemispherical particle, if you equate to the volume of a spherical particle of size D_P that is ${\pi D_p^3}/_{6}$, then you get the equivalent diameter, that is ${\pi D^3}/_{12}$ should be equated to the ${\pi D_p^3}/_{6}$ and then from here equivalent diameter D_P you get it as 0.7937 D

$$D_p \rightarrow {\pi D^3}/_{12} = {\pi D_p^3}/_{6} \rightarrow D_p = 0.7937D$$

Now here the equivalent diameter of the hemisphere is the smaller than its origin size D, it is 0.7937 D, that is the reason if you take a spherical particle, if you take a hemispherical particle of equal volume each other, that is, if you take a spherical particle same as the volume of a hemispherical particle, then the surface area of the hemispherical particle is going to be smaller than the surface area of the spherical particle because the equivalent diameter of the hemispherical particle is less than its origin size.

So now this sphericity, likewise if you get $\emptyset_s = \left(\frac{6V_p}{D_p S_p}\right)$, when you substitute V_p, D_p, S_p here and then simplify, you get 0.840 as the sphericity for hemispherical particles

$$\left(\emptyset_s = \left(\frac{6V_p}{D_p S_p}\right) = 6 * \frac{\left({\pi D^3}/_{12}\right)}{\left(0.7937D * {3\pi D^2}/_{4}\right)} = 0.840\right)$$

, likewise for any kind of particle you can find out the volume of the particle, surface area of the particle, you can find out the nominal equivalent diameter of the particle, so then you can find out the sphericity.

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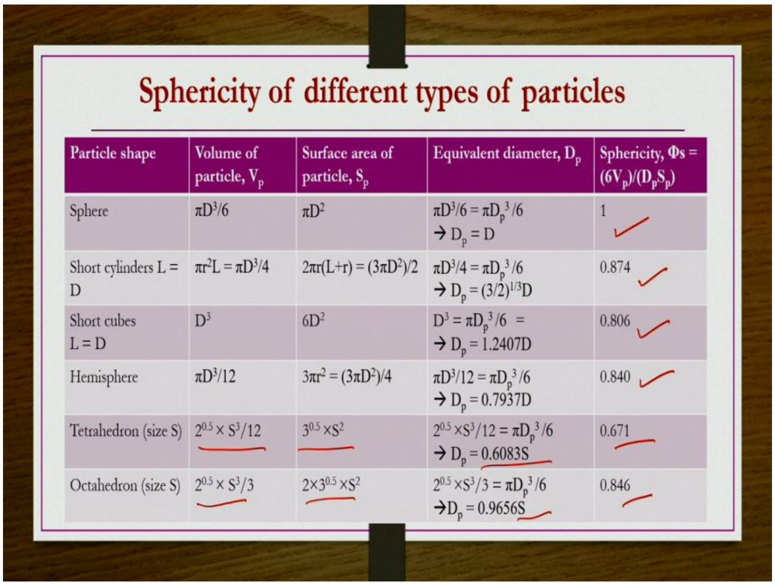

Particle shape	Volume of particle, V_p	Surface area of particle, S_p	Equivalent diameter, D_p	Sphericity, $\Phi s = (6V_p)/(D_pS_p)$
Sphere	$\pi D^3/6$	πD^2	$\pi D^3/6 = \pi D_p^3/6 \rightarrow D_p = D$	1
Short cylinders L = D	$\pi r^2 L = \pi D^3/4$	$2\pi r(L+r) = (3\pi D^2)/2$	$\pi D^3/4 = \pi D_p^3/6 \rightarrow D_p = (3/2)^{1/3}D$	0.874
Short cubes L = D	D^3	$6D^2$	$D^3 = \pi D_p^3/6 = \rightarrow D_p = 1.2407D$	0.806
Hemisphere	$\pi D^3/12$	$3\pi r^2 = (3\pi D^2)/4$	$\pi D^3/12 = \pi D_p^3/6 \rightarrow D_p = 0.7937D$	0.840
Tetrahedron (size S)	$2^{0.5} \times S^3/12$	$3^{0.5} \times S^2$	$2^{0.5} \times S^3/12 = \pi D_p^3/6 \rightarrow D_p = 0.6083S$	0.671
Octahedron (size S)	$2^{0.5} \times S^3/3$	$2 \times 3^{0.5} \times S^2$	$2^{0.5} \times S^3/3 = \pi D_p^3/6 \rightarrow D_p = 0.9656S$	0.846

So some materials that we have done, spheres we have done here, short cylinders we have done, short cubes we have done, hemisphere we have done, tetrahedron also, volume of the particle is given, surface area of the particle is given, D_P you can calculate it as 0.6083 S, so sphericity is going to be 0.671, similarly octahedron also of this particular size is S, so V_P S_P are given, one can calculate the D_P, one can calculate the sphericity as 0.846, likewise you can do for any other kind of material also.

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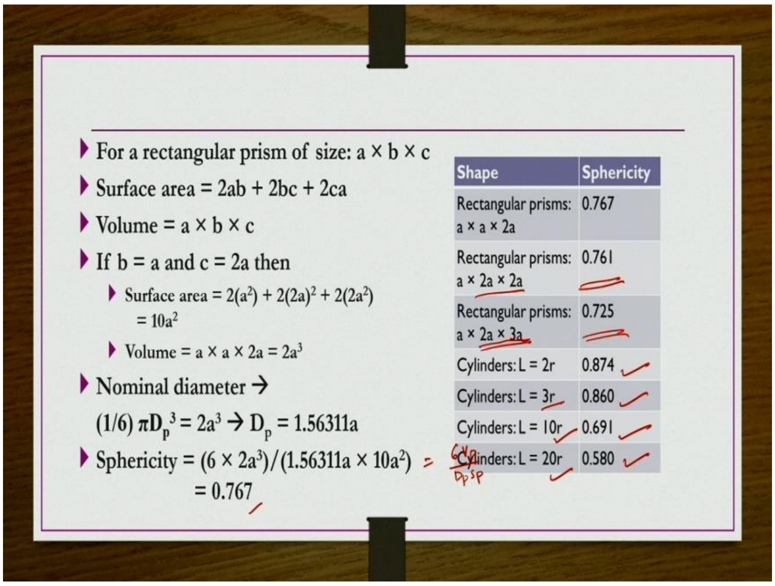

Let us say, if you have a rectangular prism of size $a \times b \times c$, then what is the surface area of that particle, rectangular prism particle, it is going to be $2ab + 2bc + 2ca$

$$surface\ area = 2ab + 2bc + 2ca$$

, what is the volume of that particle, it is $a \times b \times c$ right, now if you take this $b = a$ and $c = 2a \; if \; b = a \; and \; c = 2a$, then what should be the surface area? It is going to be $10a^2$

$$surface\ area = 2(a)^2 + 2(2a)^2 + 2(2a)^2$$

$$surface\ area = 10a^2$$

and then what should be the volume question market is going to be $2a^3$ right

$$volume = a \times a \times 2a = 2a^3,$$

so under this condition, if you equate this $2a^3$ to $\left(1/6\right)\pi D_p^3$ you are going to get a nominal equivalent diameter of this particle as 1.56311a

$$\left(1/6\right)\pi D_p^3 = 2a^3 \rightarrow D_p = 1.56311a$$

and then sphericity is going to be V_p is $2a^3$, Dp is 1.56311a and S_p is $10a^2$, when you substitute these things you get sphericity is 0.767

$$sphericity = {(6 \times 2a^3)}/{(1.56311a * 10a^2)} = 0.767.$$

Let us say, if b=2a, and then c = 2a, then sphericity is not going to change much, it is going to be 0.761 only, now if you have a b=2a and then c = 3a, then again, it is slightly decreased 0.725, similarly, if the short cylinders, this, we have done L is equal D then it is 0.874, if L is equals to 1.5 D, then it is going to be 0.86, you can calculate and get it this one also, if L is going to be 5 times the D, then it is going to be 0.691, the sphericity is going to be 0.691, similarly, if you take L is equal to 10 times the D then sphericity is going to be 0.581, so as the size of the cylinder increasing the length of the cylinder is increasing getting the diameter same, the sphericity is decreasing, that is what we can see here from this table.

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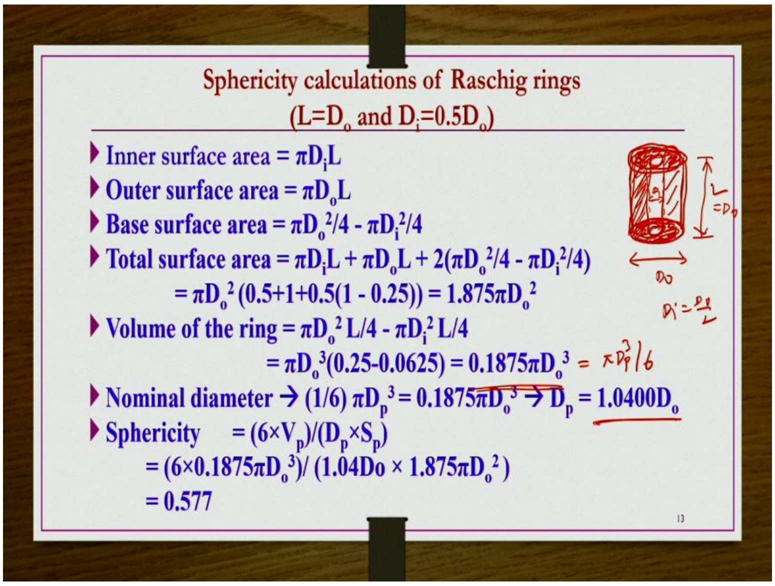

So let us say we take a Raschig rings, raschig rings are kind of, you know hemispherical cylindrical particles, you can take something like this, so diameter outer diameter is D_0, the inner diameter is D_i and then this, let us say height is L, so this is a kind of hollow kind of cylinder, so this is solid structure, this is of and then in between a kind of hollow structure is then, there used as a kind of very good packing material in most of the chemical engineering applications, so for these what is going to be the sphericity? That also we can calculate.

So here, what is the inner surface area of that is because now it is hollow cylinder

$$inner\ surface\ area = \pi D_i L$$

, hollow short cylinder kind of thing we can take, we can take $L = D_0$, L is equal to D_0 we can take and then D_i we can take $D_0/2$ right

$$L = D_0\ and\ D_i = {}^{D_0}/_{2}$$

, so the inner surface area or the surface area of the inner portion, the hollow portion that is $\pi D_i L$ and then outer surface area is $\pi D_0 L$

$$inner\ surface\ area = \pi D_i L$$

$$outer\ surface\ area = \pi D_0 L$$

Similarly base surface area, that is going to be the ${}^{\pi D_0^2}/_4 - {}^{\pi D_i^2}/_4$

$$base\ surface\ area = {}^{\pi D_0^2}/_4 - {}^{\pi D_i^2}/_4$$

, whatever this, the base surface area is this one only, only this fraction, so the outer circle area whatever is there, that minus the inner circle area if you do that you are going to get the base surface area and that it is two times, two side, here also the same thing is there, here also it is going to be same thing right, so it should be multiplied by 2.

So the total surface area is going to be the inner surface area $\pi D_i L$ plus outer surface area $\pi D_0 L$ plus 2 base surface areas, the bottom one and top one, so $2\left({}^{\pi D_0^2}/_4 - {}^{\pi D_i^2}/_4\right)$

$$total\ surface\ area = \pi D_i L + \pi D_0 L + 2\left({}^{\pi D_0^2}/_4 - {}^{\pi D_i^2}/_4\right)$$

, if you substitute L = D_0 and D_i = D_0/2, you are going to get this one as $1.875\pi D_0^2$

$$total\ surface\ area = \pi D_0^2\big(0.5 + 1 + 0.5(1 - 0.25)\big) = 1.875\pi D_0^2$$

Similarly volume of the particle, this raschig ring is the volume of the outer cylindrical object and minus inner cylindrical surface, whatever the volumes are there, that if you subtract, so you will get the volume of the entire ring, that is $\left({}^{\pi D_0^2 L}/_4 - {}^{\pi D_i^2 L}/_4\right)$

$$volume\ of\ the\ ring = \left({}^{\pi D_0^2 L}/_4 - {}^{\pi D_i^2 L}/_4\right)$$

when you do this L = D_0 and then D_i = D_0/2, you are going to get this one as $0.1875\pi D_0^3$,

$$volume\ of\ the\ ring = \pi D_0^3(0.25 - 0.0625) = 0.1875\pi D_0^3$$

this volume of the particle if you equate to the one of spherical particle $\left(\frac{\pi D_p^3}{6}\right)$ or equate to the spherical particle volume, whose size is D_P, so then you will get, D_P that nominal diameter, so $\left(\frac{1}{6}\right)\pi D_p^3 = 0.1875\pi D_0^3$, so D_p nominal size is almost close to the D_0, that is 1.0400 D_0.

$$nominal\ diameter = \left(\frac{1}{6}\right)\pi D_p^3 = 0.1875\pi D_0^3 \rightarrow D_p = 1.0400 D_0$$

Then sphericity we have $\frac{6V_p}{D_p S_p}$, when you substitute this, D_P, V_P, S_P here and then simplify, you get sphericity of this raschig rings is close to 0.577

$$sphericity = \frac{6 \times 0.1875\pi D_0^3}{1.04\mathrm{D}_0 \times 1.875\pi D_0^2} = 0.577$$

, so this is how we can take any shape of the particle and then calculate the nominal diameter or equivalent diameter, in addition to that one we can also calculate the sphericity, that is sphericity represented the shape of the particle.

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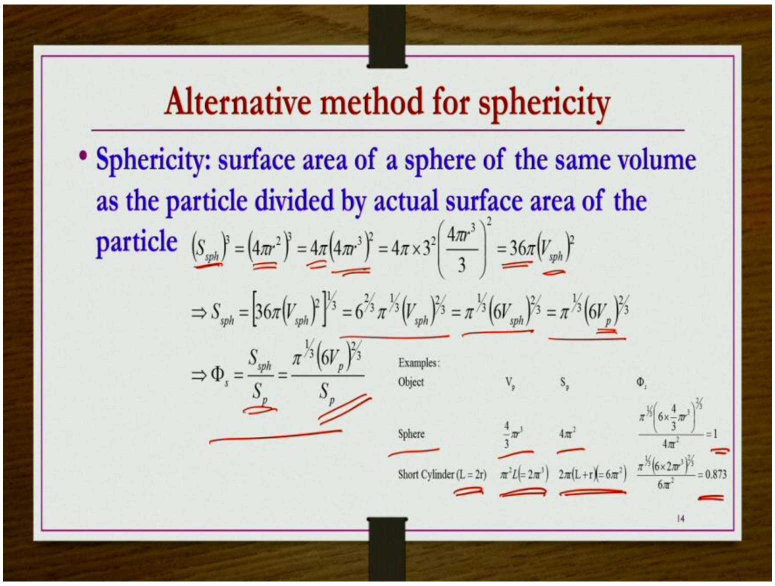

So there are some alternative to methods for sphericity also, that we can see from the definition, sphericity we can say as a surface area of a sphere of the same volume as the particle divided by the actual surface area of the particle, that way also we can see the sphericity when we do this one we can have a kind of slightly different approach to get this one, let us say you have the surface area of sphere as $4\pi r^2$.

Now you do the cube both sides, so surface S spherical, surface area of the $(S_{sph})^3 = (4\pi r^2)^3$, now what you do here? You take 4π common, so then you have $4\pi(4\pi r^3)^2$, this is how you can write the same thing. Now here what you do, you multiply and divide by 9, so then you have $4\pi * 3^2 \left(\frac{4\pi r^3}{3}\right)^2$, so these 9 x 36 π and then $\frac{4\pi r^3}{3}$ is nothing but the volume of a spherical particle of size R, that is what we get $36\pi(V_{sph})^2$ right.

So you do the cube root both sides, then you have the S spherical is equal to $[36\pi(V_{sph})^2]^{1/3}$, if you expand this one you can write it as $6^{2/3}\pi^{1/3}(V_{sph})^{2/3}$, now the 6 and then 6, this V spherical you can combine because powers are same, so you have $\pi^{1/3}(6 * V_p)^{2/3}$. Now what we have this definitions, the nominal diameter, the definitions the volume of the particle should be same as a kind of a spherical particle of size D_P or you should select a kind of spherical particle whole size is D_P, but its volume is same as a kind of particle volume.

$$(S_{sph})^3 = (4\pi r^2)^3 = 4\pi(4\pi r^3)^2 = 4\pi * 3^2\left(\frac{4\pi r^3}{3}\right)^2 = 36\pi(V_{sph})^2$$

$$\Rightarrow (S_{sph}) = [36\pi(V_{sph})^2]^{1/3} = 6^{2/3}\pi^{1/3}(V_{sph})^{2/3} = \pi^{1/3}(6 * V_{sph})^{2/3} = \pi^{1/3}(6 * V_p)^{2/3}$$

$$\Rightarrow \emptyset_s = \frac{S_{sph}}{S_p} = \frac{\pi^{1/3}(6 * V_{sph})^{2/3}}{S_p}$$

So this spherical volume V_S spherical should be equals to V, V_p that is the particle volume that means from here $\emptyset_s = \frac{S_{sph}}{S_p}$, that is the surface area of the spherical particle divided by the surface area of the particle whose volumes are same, whose volumes of the spherical particle and then particle, regular particle whatever, for which we are measuring sphericity, their volumes are same, so then their surface area ratio is going to give us the sphericity. So S spherical particle we obtain it as $\frac{\pi^{1/3}(6*V_{sph})^{2/3}}{S_p}$, now V_P, S_P you calculate for a given particle, you substitute here, you get the $\emptyset_s$. Only thing that you know we just bypass the calculation of the nominal size of the particle by doing this alternative method, otherwise it is the same thing as the previously.

So example like spherical particles, sphere if you take V_P is going to be $\frac{4\pi r^3}{3}$ and then S_p is going to be $4\pi r^2$, this V_P, S_P if you substitute here, so you get sphericity as 1, this same as previous results, if you take short cylinder L = 2r, then V_P is $2\pi r^3$ or $\pi r^2 L$ and then S_P is $6\pi r^2$ or $2\pi rL + r$, so these things if you substitute here in $\emptyset_s$ definition, you get 0.873, that is the same as whatever the value we got previously, so this is how we can get for any material, the sphericity of any regular or irregular material.

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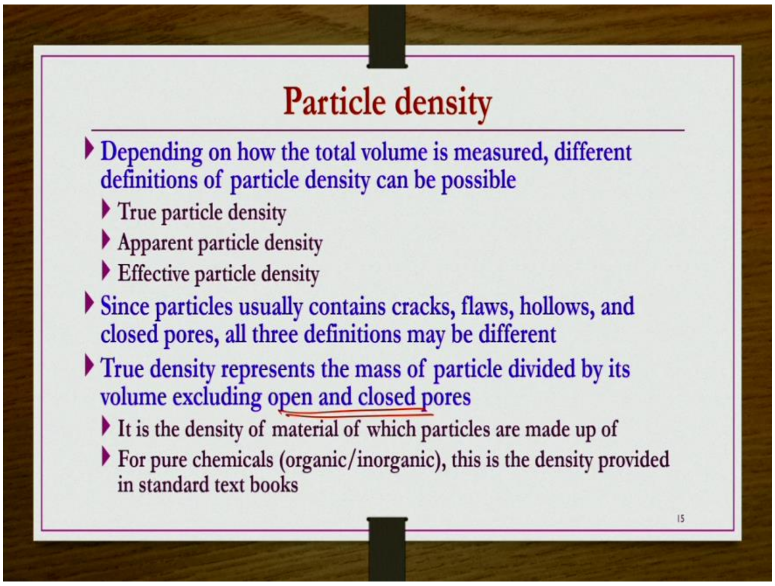

Then the particle density right, depending on the how the total volume is measured, different definitions of particle density can be possible right, because in general, the particles may be having the flaws, cracks, hollow structures, etc those kinds of things, maybe there, so how you measure the total volume based on that one whether you are including in your volume circulation, when you calculate the volume of the particle, if you are including these flaws, cracks or porous structures, etc are not those kind of things is going to make a kind of different volume measurement.

So different, since you have different volume measurements for the same mass of the particle, you are going to get the different density, so depending on how the total volume is measured, different definitions of particle density can be possible, they are like true particle density, apparent particle density and then effective particle density. This is why we have because particles in general contains cracks, flaws, hollows, closed pores, open pores, etc possible that is the reason all three types of definitions are possible.

True density represents the mass of particle divided by its volume excluding the open and closed pores, that is the only you take the effective volume, you do not take the cracks, flaws etc, so those volumes you subtract, so that is the reason the density whatever you get that is the true density and it is nothing but the same as the bulk material density, that is the density of the material from which these particles are made up of, this particle densities we are measuring for the particles not the bulk material, after crushing now you have the regular shape of particles.

So size of the particles measurements that we have already seen, shape of the particle how to represent that we have seen, so how to represent, how to get the true density of those particles, it is not necessary that it is going to be same as a kind of bulk density or the density of the bulk material from which we have got the particulate material, crushed material right, so if you are not going to consider, if you are excluding those cracks, pores etc within the particles whatever are there.

If you are excluding them from their volume, then whatever the density that you are going to get that is the true density and it is same as the bulk density or the material from which the particulate material we have got and then this is the same as the standard density that you get for pure chemicals in general, in standard textbooks.

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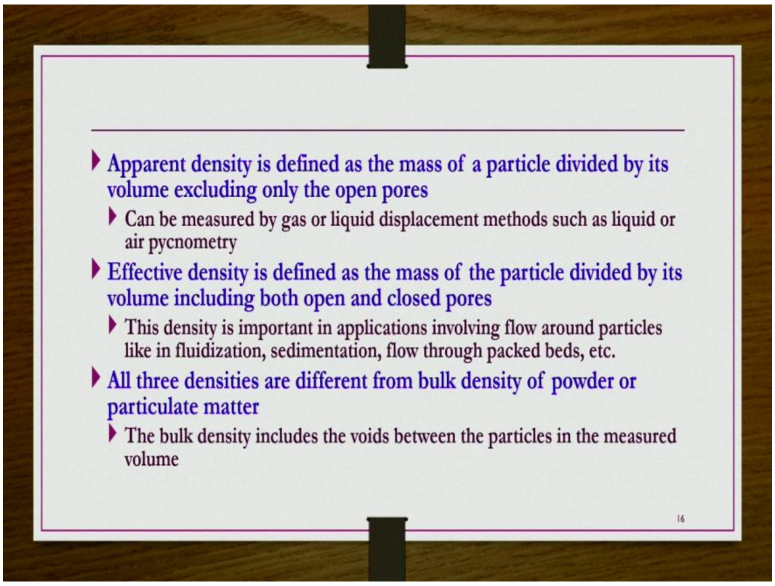

But if you exclude only open pores right, if you from the total volume of the particle, if you subtract the volume of the open pores alone not the other closed pores etc, then whatever the density that you get you get the apparent density right, but if you include the both density, both volumes that is open and closed pores also you include in the, that is volume of the open pores as well as the close pores of the particles also if you include in the volume of the particles, then whatever the density mass by volume that you get, that is going to be effective density.

Apparent density, one can measure by gas liquid displacement method, such as liquid or air pycnometry okay, so effective density is important in applications involving flow around particles like in fluidization, sedimentation, flow-through packed bed etc, and then all three densities are going to be different in general from the bulk density of powder or particulate material.

The bulk density includes the voids between the particles is the measured volume also, whatever these true density, apparent density, effective density that we have seen, therefore, the individual single particle, but if you take the particulate powder material of such small, small particles, so there is a interstitial spaces also there, so that volumes is also there, that is the reason this is going to be different also from the application point of view, the bulk density includes the voids between the particles in the measured volume of the sample that we have taken.

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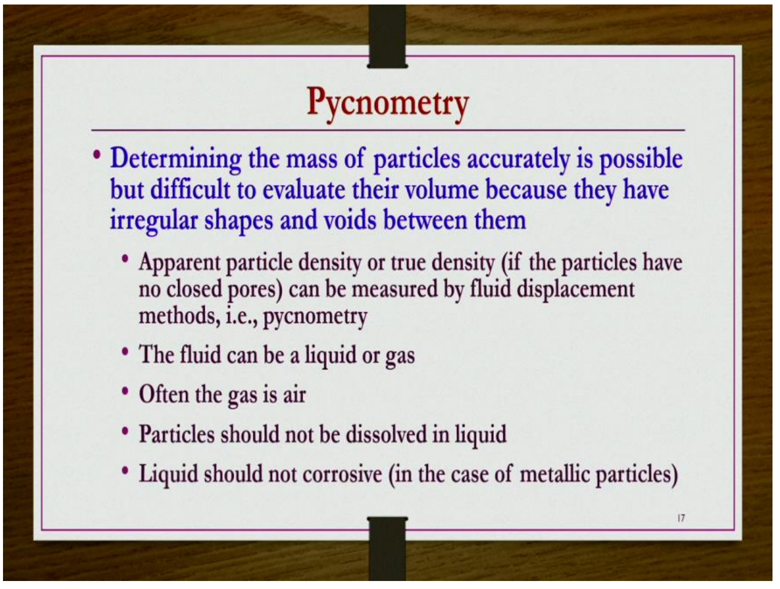

And there are kind of pycnometries are available, which can be used for measuring the density because this are going to be important one because measuring the mass or the particles accurately is possible, but such kind of irregular particles and then where there is a kind of pores etc are there, measuring the volume is going to be very tough, is going to be very difficult, especially for the irregular shape particles and then if there are voids between them right, for such cases pycnometry can be used.

So apparent particle density or true density that is, if the particles have no closed pores, then the true density or apparent density are the same thing, so they can be measured by the fluid dispersion method by pycnometry right, so in the pycnometry in general liquid or fluid liquid or gases can be used and then gas in general air and then particles should not be dissolved in the liquid, the selection for this pycnometry whatever the liquid that we are going to select that should be such a way that, in that liquid this particle should not be dissolving, should not be reacting, should not be corrosive kind of thing, liquid should not be corrosive in the case of metallic particles.

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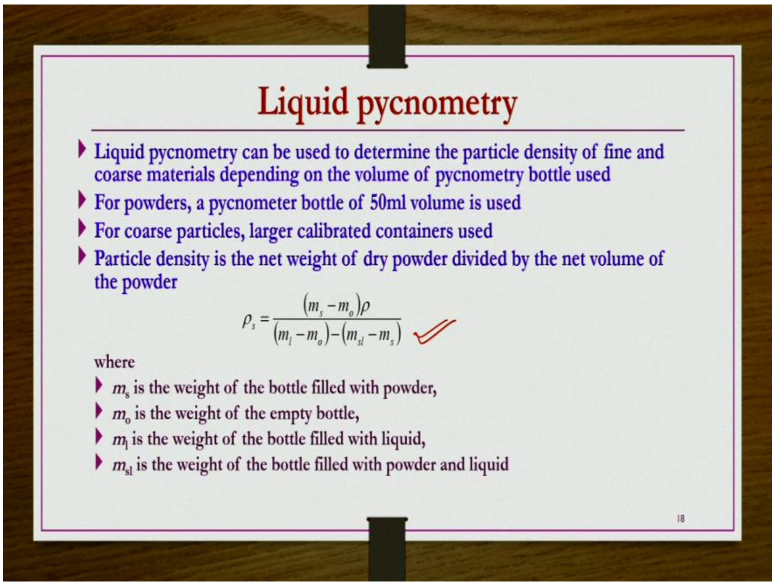

So in the liquid pycnometry what we do, liquid pycnometry can be used to determine the particle density of fine and coarse material is depending on the volume of pycnometry bottles used in general, for powders a pycnometer bottle of 50 ML volume is sufficient enough but for coarse particles larger calibrated containers are needed to be used. Particle density is the net weight of dry powder divided by the net volume of the powder and then when you use the liquid pycnometry this you can get ρ as $\rho_s = \frac{(m_s - m_0)\rho}{(m_l - m_0) - (m_{sl} - m_s)}$, where m_s is the weight of the bottle filled with the powder, then 'm' not is the weight of the empty bottle that you use for this measurements, and then m_l is the weight of the bottle filled with only liquid and then m_{sl} is the weight of the bottle filled with both powder and liquid.

$$\rho_s = \frac{(m_s - m_0)\rho}{(m_l - m_0) - (m_{sl} - m_s)}$$

So using this principles we can find out the density of the particles okay or powder material, so there are different other methods and then facilities, experimental methods available for measuring this particles density, but we are not going to consider any way them.

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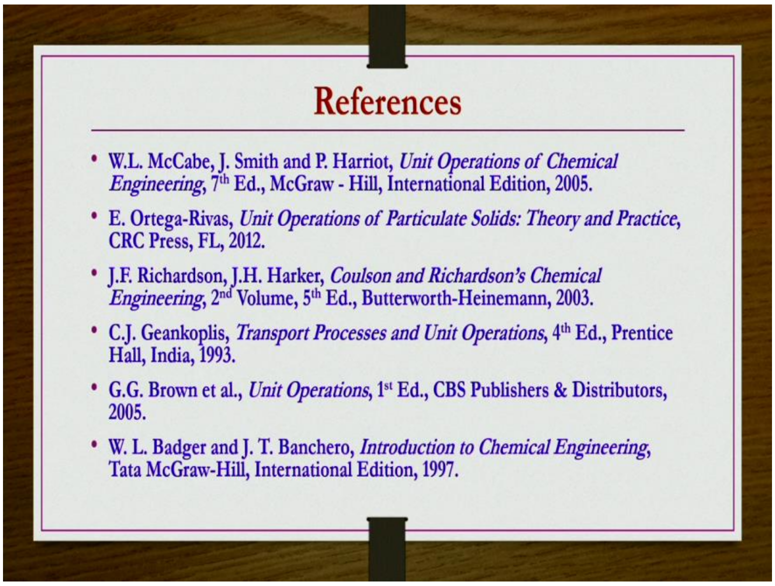

So the material presented in this particular lecture, I have been taken from this reference books primarily, McCabe and Smith, Ortega-Rivas, Coulson and Richardsons Second volume, Geankoplis, Brown et al and then Badger and Banchero. Thank you