Mechanical Unit Operations
Professor Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology Guwahati
Lecture 30 - Cross Flow Filtration - 2

Welcome to the MOOC's course Mechanical Unit Operations. The title of this lecture is Cross Flow Filtration Lecture Number 2, on this cross flow filtration. We have already seen a basic, a few basic things about the cross flow filtration and different types of membrane, membrane properties, those kind of things we have seen in our previous lecture. Now we are going to see a few more details about the cross flow filtration, especially with respect to ultra-filtration. So we will be developing some equations for the flux, permeate flux, concentration, polarization etc for the case of ultra-filtration membranes.

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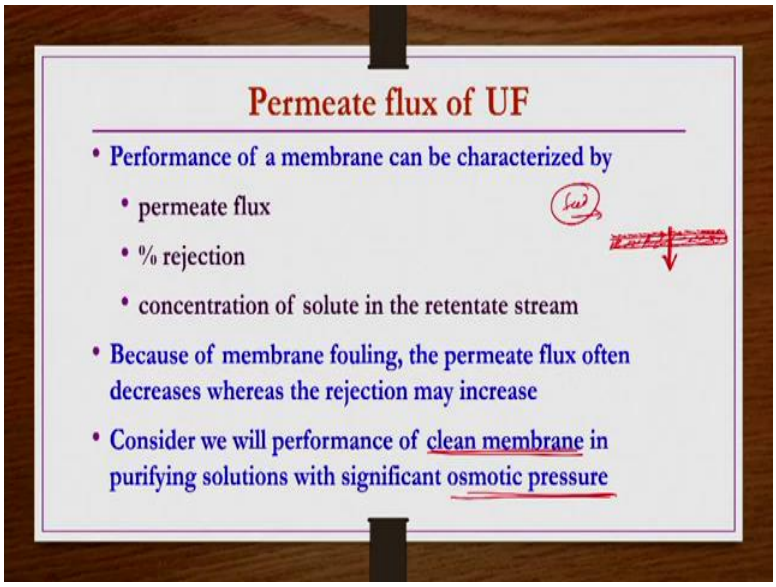

So permeate flux of ultra-filtration membranes, so the performance of a membrane in general can be characterized by three important points. First is the permeate flux. Second one is the percentage rejection of the solutes by using the membranes and third one is the concentration of the solute in the retentate stream, because here in the membranes we, one of the phase you know retentate phase becomes more concentrated compared to the feed.

So what is the concentration of the solute in the retentate stream is also kind of important factor as we are going to discuss. We are going to find out each and every detail of these things now in this particular lecture. So, because of membrane falling the permeate flux often decreases

whereas the rejection may increase. What happens you know when there is a kind of membrane surface here, along with, you know, when a kind of feed which is having small particle, a low concentration solution which is feed is here nothing but a solution having low concentration of particles and particle sizes are also in general very small of order of microns or even less; then only we use this cross flow filtration using membranes.

So when this operation is taking place then what happens, some of the particles will be deposited on the layer and then they form a kind of a very thin gel kind of layer here, very thin gel kind of layer would be there, right. Because of that layer, additional resistance to the flow of permeate to pass through this membrane would be there. Membrane resistance is already there, and then because of that one there is a kind of some permeate flux but and in addition to this membrane resistance the gel layer whatever the particles they are forming on the surface during the process of, you know membrane separation, so this gel layer will also be offering additional resistance.

Because of that one, permeate flux would be reduced but however more resistance is this and more particles would be retained towards the retentate site, so then the rejection may also increase. Now what we consider? We consider performance of a clean membrane used for purifying a solution, a solution having certain amount of osmotic pressure, okay? So this osmotic pressure, you know is function of concentration as we understand.

So as the concentration of the solute increases the osmotic pressure of the feed solution whatever is there, that increases. Okay so then you know accordingly the correction has to be made in the performance. So all those things we are going to see now here.

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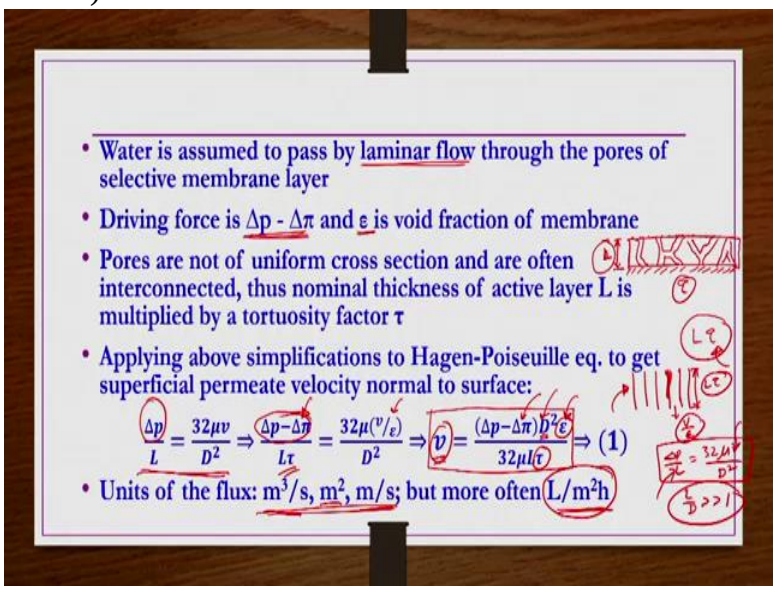

Now let us assume the water is flowing through porous structure of the selective membrane layer within the laminar flow, not necessarily assuming because in general whatever the flow is there through the membrane that permeate flow rate is in general very, very small. So the Reynolds number would be in general very small.

So under such conditions, laminar flow conditions or equations related to the laminar flow can be happily considered. And the driving force here for the performance whatever we are trying to develop, performance equation we are trying to develop for the membrane, for that case driving force is not just the pressure drop but the pressure drop minus osmotic pressure because we are taking a case where the solution is having a significant amount of osmotic pressure and epsilon is the void fraction of the membrane. So membrane in porous structure is the, whatever is the porous structure void fraction is there, so that is ε.

In general pores are not uniform across section and then often they are interconnected as we have seen several cases. Because of that one, whatever the nominal thickness of the active layer, so let us say if this is the active layer, you know, these active layers are in general in a few micron sizes, but just to explain I am writing, I am drawing a kind of thick layer so that we can see the detail.

So this here, this, this is the active layer, membrane layer. This is mounted on a kind of support, it is a supported membrane kind of thing. And the support is a kind of a macro porous material, okay. Now here, you know these pores, whatever the membrane surface on which the pores

would be there, pores would not necessarily be a kind of, you know uniform pore structure something like this. It is not possible.

So they may have a kind of, you know different degrees of interconnection and then different levels of, you know, porous structure across the cross-section, then this is the L of the membrane, you know across the cross-section of this active layer as we pass on, the path that the molecule, permeate molecule is passing through the membrane that is more than L because the porous structure is not uniform like this. So because of that one, we have to multiply by some tortuosity factor τ. It is similar as kind of what we have done in the case of flow through packed bed, okay.

So here also the pores are interconnected and they are not even, they are not uniform and then their length, the molecule may be traveling the length more than the active layer thickness. So L has to be multiplied by some factor, tortuosity factor that is τ. Now for this case whatever the Hagen-Poiseuille equation is there, what we assume that you know we take this now L if you multiply by τ, that τ is taking care of the tortuosity factor.

So now we can say that these pores are, you know kind of a uniform cross-section $L\tau$, that τ is taking care of the non-uniformity factor so then, through this pore, through individual pores now each pore is like this, of length $L\tau$ and then permeate through which is coming, you know $\frac{v}{\varepsilon}$ because there are n number of such kind of you know pores are there, right, so similarly exactly the same way that we have done for the packed bed. So these factors we have to bring into the picture in Hagen-Poiseuille equation, whatever the $\frac{\Delta p}{L} = \frac{32\mu v}{D^2}$ is the, this is the, you know Hagen-Poiseuille equation for flow through a pipe, uniform cross-section pipe.

So then this is what we have Hagen-Poiseuille equation which is valid for a infinitely long cylinder or for the cylinder where the case L/D is very, very much higher than the, you know 1. So that is L is very much larger than the D so then it is valid. So under such assumptions, those assumptions we can also apply for the simplified case here as we have given here, right, okay but with a kind of modification this L has to be multiplied by V and then this v has to be divided by the ε in order to bring the effect of the voidage through which only the material is flowing, through which only the permeate is flowing.

When we do this one, we get this thing. So this is $\frac{32\mu v}{D^2}$ is nothing but $\frac{\Delta p}{L}$ and also this pressure, now whatever the resistance is driving force for the steady state, it is not a simple pressure drop but also there is a kind of effect of osmotic pressure. So that, that should also be taken care into this equation. So $\frac{\Delta p-\Delta \pi}{L\tau}$has to be now $32\mu L\tau$ L , v has to be $\frac{v}{\varepsilon}$ and D is as it is, we are keeping it as it is though it is not the same as the kind of this thing, equivalent diameter kind of thing we have to write but let us not go into all those details because we are not going to simplify this equation further anyway.

So what we get here, superficial permeate velocity normal to the surface v is nothing but $\frac{(\Delta p-\Delta \pi)D^2\varepsilon}{32\mu L\tau}$, right. So here the, from this equation though v is having the units, you know meter per second so this is the permeate flux but in general this permeate flux is also represented in terms of m^3/s, m^2, not only just meter per second, they are just you know multiplied by some kind of factor so that you know reported in this kind of forms but more often the permeate flux is reported in terms of units L/ m^2h.

So now, but if you wanted to use this equation, let us assume, you know the concentration, solution concentration so then you can find out the osmotic pressure, that is not the problem. But what is D? The pores, this membrane is having several pores on the surface, right and then these pores are having different cross-section. So which diameter should you take? That you do not know and then in general voidage is also very difficult to find out though you have the facilities now you can find out for a given membrane, alright.

So, but it is not readily available, in a sense it is not readily available though it can be measured. And then what is the tortuosity factor? We do not know actually how much it is. In packed bed we found for some cases it is 2.1 something like that, it is appropriate one, appropriate for the laminar conditions but other turbulent conditions it may be very much higher, very much different from the laminar case tortuosity factor. So having these many ambiguities getting a reliable information about permeate flux is going to be very difficult.

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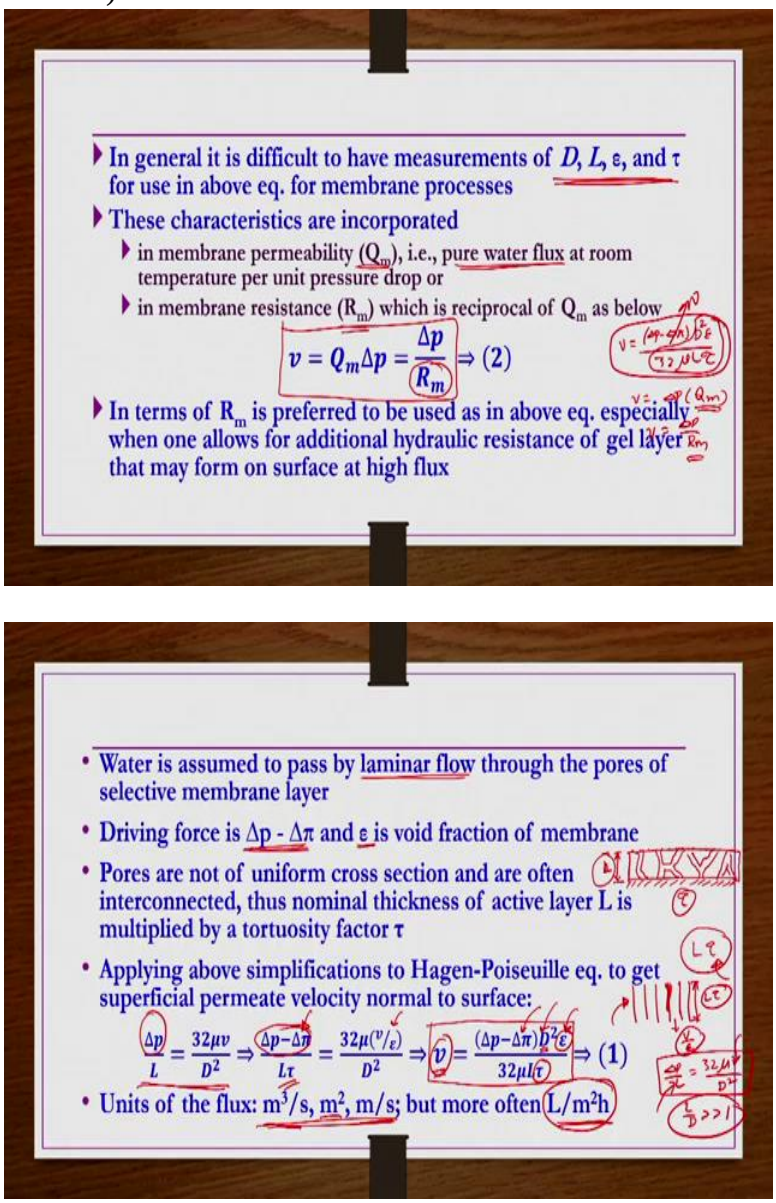

So what we do, we try to overcome this, you know measurements of the L, ε and τ et cetera and then rearrange this equation in a different kind of form. So how we can write a different form? These parameters we can, or these characteristics D, L, ε and τ et cetera we can incorporate in kind of, in some kind of permeability or in some kind of resistance. So, in two ways the permeate flux can be reported. So now when you report...we can report in terms of the permeability that is pure water flux at room temperature per unit pressure drop or membrane resistance which is reciprocal of Q_m as below here.

So the same equation we, whatever we have, so v is equals to delta p minus delta pi and then something is there, right, so and then something is there that you would like to know, if you specifically the previous equation 1, we write $v = \frac{(\Delta p - \Delta \pi) D^2 \varepsilon}{32 \mu L \tau}$. This equation now we are

writing for a pure water flux. Q_m is kind of for a pure water flux kind of condition. So for pure water if there is no solute, then there will not be any kind of osmotic pressure. So then we can write this equation, Δp multiplied by some constant.

So all these things are, you know, we measure, we incorporate or include in something like that Q_m that we can call it as a kind of permeability, okay. So, or if you take the reciprocal of this thing so then we can take this one as $\frac{\Delta p}{R_m}$. R_m is the resistance which is the reciprocal of the permeability. So the either of the way it can be written. Either of the way it can be written, okay.

But in general, you know if, if you wanted to write the same equation for the case where you are using for purifying a solution, if you have a solution there would be a kind of solute and when there is a kind of solute particles in a solvent then that, you know that overall final solution would be having some amount of osmotic pressure. So you have to incorporate that osmotic pressure also in a generalized equation and then when we are, you know doing the separation by using membrane for those solutions then you know particles will be forming a kind of gel layer, not as a kind of cake as we have seen in cake filtration but you know very thin gel layer kind of thing would be forming on the surface. So that surface, that gel layer is also offering some additional resistance in addition to the resistance due to the membrane.

So then whatever the R_m is here is, you know, is resistance due to the membrane only. There is no resistance due to the gel in this equation because this equation we have written for the pure water case. Pure water case there are no particles so there is no gel formation kind of thing. So but when you take the solution then this R_m should be added with the R_{gel} or the resistance due to the gel and then this Δp has to be subtracted with the osmotic pressure that is minus $\Delta \pi$ should be added. That we are going to see.

So this equation, though we can write $v = Q_m \Delta p$ but it is writing $\frac{\Delta p}{R_m}$ kind of form is going to be more useful because you know wherever we have this kind of gel kind of formation things are there, so those things you know can be easily incorporated, you know. Their resistance can be easily incorporated if you are writing in the form of $v = \frac{\Delta p}{R_m}$ form.

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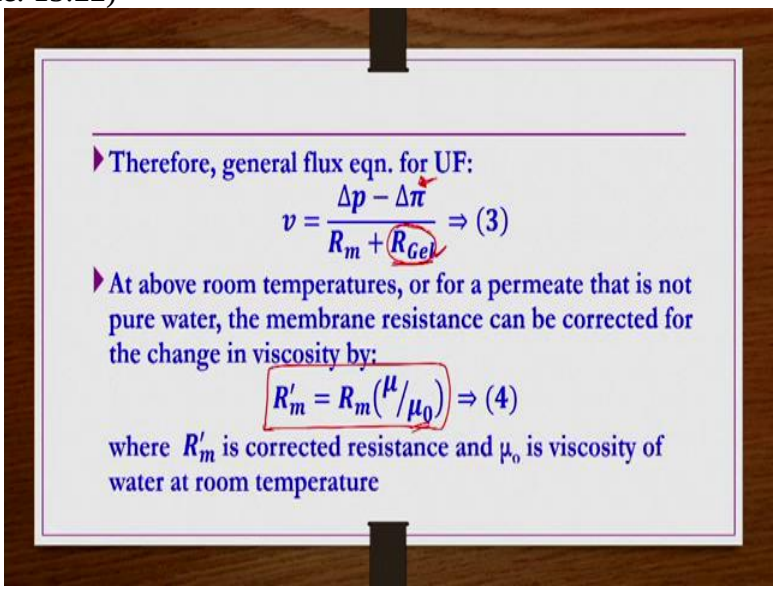

So now generalized flux equation for ultra filtration membrane should be what? It should be $v = \frac{\Delta p - \Delta \pi}{R_m + R_{gel}}$, right, okay. So if you use the pure water then $\Delta\pi$ is 0. For pure water osmotic pressure is 0 for pure substance and then there will not be any kind of gel, so R_{gel} would also be 0. That would be the case of previous case of equation number 2. But if you are using this one for purifying the solution using this ultra-filtration membrane so then because of the particles in the feed, feed solution you know, there will be a kind of osmotic pressure that should be brought into the picture, so $\Delta\pi$ has come into the picture.

And then there will be layer of particles, fine particles forming on the surface as a kind of layer, that is known as the gel, so that would also be offering a kind of additional resistance when we are using this membrane for purifying the solution. So that resistance due to the, that whatever additional gel has formed that should also be brought into the picture, so that is here.

So if you take the pure water case, $\Delta\pi$ and R_{gel} would be 0 and then equation 3 would be same as equation number 2 as we have just seen. But further what happens, you know the, at room temperature it is, this equation is fine, okay. But above room temperature or for a permeate that is not pure water then the membrane resistance can be corrected for the change in the viscosity. The change in the viscosity of the permeate with respect to the temperature can be brought into the picture like this.

So $R'_m = R_m \left(\frac{\mu}{\mu_0}\right)$ where as μ_0 is a kind of viscosity of the permeate at room temperature and mu is the viscosity of the permeate at a higher temperature at which we are operating. Remember the same equation number 3 in some books is also written in terms of, you know μ terms will also be there. So then accordingly the unit should be changing in general. So here R'_m is corrected resistance and μ_0 is viscosity of water at room temperature or whatever the solution that we have taken.

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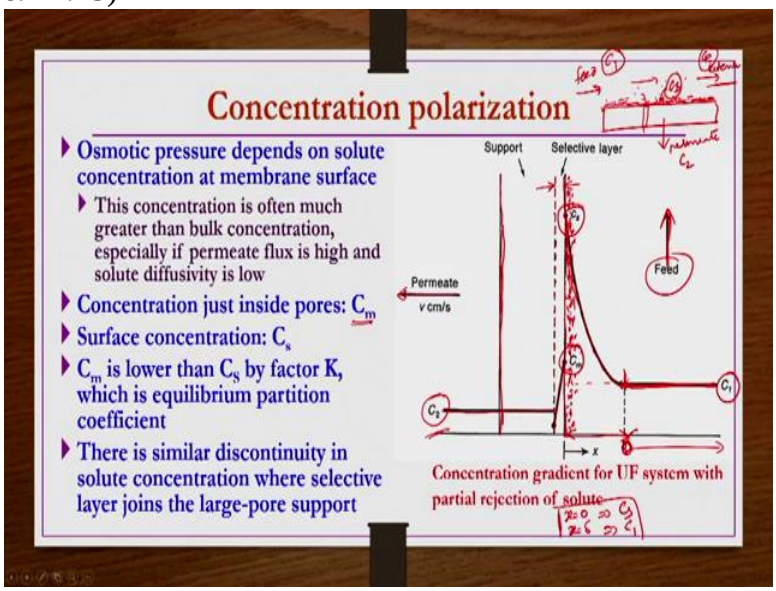

So now concentration polarization, concentration polarization is nothing but whatever the gel formation is there, you know, let us say this is the membrane surface active layer, okay when there is a cross flow of feed solution having a low concentration of fine particle, so this is flowing across like this. So then retentate are taken like this and then permeate is collected through bottom like this, okay. Now this is containing fine particles, so the particles are you know retained over on the surface. They are separated, right and then almost clear or almost clear permeate is collected at the bottom. This will also be having some kind of solute because 100% rejection in general is not possible.

So permeate will also be having some solute particles but very few particles, very less fraction will be there, right and then retentate, you know it is a kind of concentrated solution. So retentate concentration would be much more higher than the feed concentration.

So while this process is going on the layer of the particles would be forming as a kind of a gel here and then that will be offering a kind of a concentration which is very much higher

compared to the bulk feed concentration. Let us say bulk feed concentration is c_1, on the surface if you take concentration is c_s, the c_s is going to be very much higher than the c_1 because of this concentration polarization or because these particle are forming.

So the particles are forming as a kind of gel, so that gel, you know increasing the solute concentration on the surface of the membrane, that is known as the concentration polarization. So this c_s at the surface is much more higher than the c_1 and let us say permeate concentrate or the solute concentrate in permeate is c_2. So c_2 is not 0. It is 0 only when the 100 percent rejection is there. But 100 percent rejection, it is in general not possible for any case. So then c_2 will also have some values, okay. And then retentate concentration, let us say C or something like that, you know, that must be very much different.

Now because of this concentration polarization whatever the flux is there, the flux gradually decreases or the pressure drop increases, okay. And then when the pressure drop increases the c_s also increases current value, okay. So this kind of phenomena occur for ultra-filtration membrane, so this concentration polarization concept we are going to see now, and then we are going to develop equation for this case, okay. So now osmotic pressure depends on solute concentration at membrane surface.

So now what happens, if you obtain the osmotic pressure based on the c_1, so that is not going to give the correct results because the solute concentration is very much different on the membrane surface or whatever the osmotic pressure is playing role in separation, that is playing role only at the surface because membrane surface, because there only from that point only separation is taking place. So when you calculate the osmotic pressure for the solution you have to use the c_s concentration rather than the c_1, right.

So that is the reason you know, the osmotic pressure though is very small for a kind of dilute solution, dilute feed solution you take, let us say very, very small dilution like 0.01% of solids are only there in the kind of feed solution, despite of that one, so in general when you say that 0.01, you know gram per liter of solutes in a feed solution, it is kind of very small concentration. So then what you assume that osmotic pressure is going to be very small. That can be neglected.

But because of this concentration polarization what will happen? Now, the c s is going to be much more higher. It is going to be much more higher than 0.01. How much higher, that is what we are going to develop equation to obtain this expression. So even for the dilute solutions you cannot avoid, you know considering the osmotic pressure because the osmotic pressure is based on the c_s, surface concentration which is going to be much higher than the feed solute concentration c_1, okay.

So this concentration is often much greater than the bulk concentration c_1 especially permeate flux is high and then solute diffusivity is low. And then concentration just inside the pores, let us assume it is c_m, pictorially we are representing, going to represent. c_s is the surface concentration here. Just let us say the pore you know is starting here, so right at this point, right at this point at the just inside the pore the concentration of the solute is again going to be different from c_s and c_1, okay. What is that? We do not know.

Let us assume it is c_m. And then surface concentration is c_s which is much more higher than the feed concentration c_1. c_m is definitely different from the c_s but it is lower than c_s by a factor K which is known as the equilibrium partition coefficient, right. Such kind of similar discontinuity would be there for the solute concentration where selective layer joins the large pore support in general. So pictorially if you see this concentration variation or concentration gradient for ultra-filtration membrane, so this is the kind of membrane surface towards the support and this is, this line is a kind of, you know membrane surface exposed to the feed solution which is going to be separated, right.

So now this is the selective layer thickness. We do not know what it is, okay; we do not want to know as of now. It is L. And the support is, you know, thickness of the support is much more higher than the membrane thickness in general. So this is the support.

On this support there is a selective membrane layer is there. That is this one. And then the feed solution is flowing around the surface like this because it is a cross flow filtration. It is not dead end filtration. So the solute concentration in the feed that is in the bulk solution is c_1, right. But you know as the process moves on so then particles will be deposited on the surface, on the surface of membrane. So because of that one, you know the surface concentration or the

concentration of the solute on the surface is going to be much higher, much higher compared to the bulk concentration of the feed, because till this point there is no separation.

There is no separation or concentration change are supposed to be. But you know the solute concentration near the selective layer, on the selective layer surface is going to be very much higher because of this gel formation or because of the concentration polarization, right. That concentration we call it as c_s at the membrane surface. And let us say this is the membrane surface, right. So here on this membrane surface, on this membrane surface we have taken one particular pore.

So on the surface the concentration is c_s, okay but just inside the, just inside the membrane pore, whatever the pore, one selected pore we have taken here, the concentration is c_m which is going to be less than the c_s, which is going to be less than the c_s that is c_m. But it is definitely going to be more than the c_1, more than the c_1. So this the level, c_1 is at lower level, c_s is at the higher level, c_m is the, you know between these two but towards the c_1, okay. And then towards the permeate you know once this whatever the solution passes through here, so some solutes will also be passing through.

So then from here the permeate is collected. So permeate is in general, you know, pure, almost pure but not completely pure. So there will be some solute particles. So then solute concentration in the permeate side is c_2. So pictorially if you represent concentration polarization this is what you get. So now here what we can say, the concentration from the bulk, you know it is almost stable to certain distance, almost constant to certain distance but after that it is gradually increasing and then increasing to the such high concentration c s at the surface, right.

So now we can separate the distance in two parts towards the membrane. So towards the bulk flow, bulk fluid side, whatever the distance is there, you just leave it as it is. Let us say this point, from here the concentration gradually increases and then it increases to such large concentration as c_s at the membrane surface. So this distance, this distance you take it as a kind of delta, okay. Looks like 8 here but it is a kind of a delta. So this distance what we do, you take as a kind of delta. So at x is equals to 0, the membrane surface, at x is equals to 0 what is the concentration?

Here is c_s, at x is equals to delta onwards and towards the bulk side, what is the concentration? It is nothing but c_1. These are the boundary conditions. So now some changes, concentration change with respect to distance is happening only from x is equals to 0 to x is equals to delta. So what is that function we have to find out if we wanted to find out what is c_s, okay? So that is what we are going to do in here.

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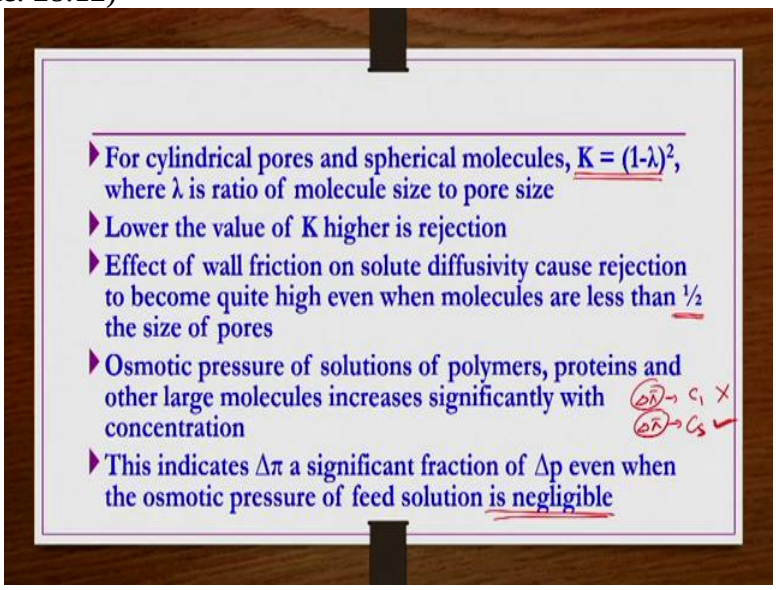

Now in general for cylindrical pores and then spherical molecules whatever the partition coefficient K is there, that is given as you know $(1-\lambda)^2$where the λ is the ratio of molecule size to the pore size. Lower the value of K higher is the rejection in general. But however effect of wall friction on solute diffusivity also cause the rejection to become quite high even when the molecules are less than the half the pore size, that is even when λ is less than half, you know what happens there is a kind of wall friction on solute diffusivity.

Effect of wall friction on solute diffusivity is there. That makes this rejection very high even when this λ value is less than 1/2. Further osmotic pressure of solutions of polymers, proteins and other large molecules increases significantly with the concentration. So osmotic pressure if you calculate based on the c_1 that is feed concentration, that is not going to give correct thing because this $\Delta\pi$ in general increases for any of these polymer solutions, protein solutions etc with increase in the concentration.

So at the surface, the concentration c_s is much higher than the c_1 so whatever the osmotic pressure using the c_s concentration, that is going to be the correct one, okay. This indicates, what we indicate that you know, $\Delta\pi$, a significant fraction of Δp should be considered even when the osmotic pressure of feed solution is negligible, even if osmotic pressure of feed solution is negligible because c_1 is very, very small in general.

Despite of that one, $\Delta\pi$ has to be incorporated in this equation because what happened, in the ultrafiltration membrane because of the concentration polarization, surface concentration gradually increases, and then it reaches certain maximum concentration for a given thickness of the feed layer, okay. At that maximum concentration c_s, this osmotic pressure is going to be much higher compared to the osmotic pressure calculated based on the feed concentration. So because of that reason even if the feed solution osmotic pressure is negligible, the has to be incorporated in the equation.

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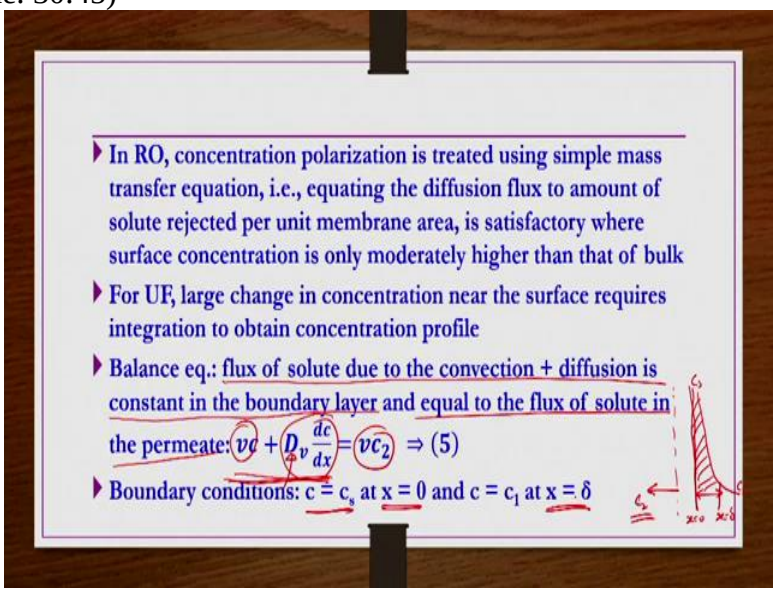

But on the other hand in general in reverse osmosis concentration polarization is treated using simple mass transfer equation that is equating the diffusion flux to amount of solute rejected per unit membrane area. That is itself is satisfactory where surface concentration is only moderately higher than that of bulk, like in reverse osmosis also such kind of concentration polarization would be there.

But there we do not need to worry about you know variation of concentration with respect to the x distance, from x is equals to 0 to δ because the c_s and c_1 variation is in general very, very

small in the case of a reverse osmosis. Like $c_s - c_1$ is very small, that can be in general neglected in the case of reverse osmosis so then you do not need to find out how this c_s is changing from c_s to c_1 as x is increasing from x is equals to 0 to x is equals to delta, that you do not need to do in case of reverse osmosis because that difference is very small.

But in ultra-filtration that is going to be very significant one. So we cannot avoid that, you know variation, whatever the concentration variations with x direction near the surface we cannot avoid in ultra-filtration. So that is what we are going to do now. So for ultra-filtration, large change in concentration near the surface requires integration to obtain concentration profile. So if you want to integrate you have to develop a kind of differential equation. That you can get by doing the basic balance equation.

So what kind of balance that we can have near the surface, membrane surface? Flux of solute due to convection plus diffusion is constant in the boundary layer and it is equals to the flux of the solute in the permeate. So let us say if I redraw, this is the kind of membrane surface, this is a kind of support and then here we have a kind of permeate of concentration c_2. So on the surface here the c_s concentration is there. And then at the bulk c_1 is there, so in this distance x is equals to 0 to x is equals to delta the concentration is changing from c_s to c_1.

So this variation whatever is there, that you know we have to take care and then where the convection and diffusion both are taking place, so whatever the near the surface that is in the boundary layer, this boundary layer δ so within this boundary layer whatever the flux, solute flux is there because of the convection and diffusion that should be equal to the flux of the same solute in the permeate, okay. That will give you the basic flux balance at the membrane surface.

So in the boundary layer the solute flux due to the convection is vc and then in the boundary layer the solute flux due to the diffusion is $D_v \frac{dc}{dx}$. Let us take D_v as a kind of diffusivity and then solute flux in the permeate side is vc_2 . And then what are the boundary conditions? At x is equals to 0 that is on the membrane surface c is equal to c_s and x is equals to δ that is the outermost layer of the, boundary layer whatever the concentration polarization because of the concentration polarization whatever the boundary layer is formed that thickness is δ.

So outer layer of that, I know the boundary layer is exposing towards the bulk fluid, so the concentration should be c_1. So from this point onwards you know concentration is everywhere. Here it is c_1, okay. So now this equation you solve and then apply these boundary conditions so then you will be having expression to get c_s as function of c_2. That is what we are going to do now.

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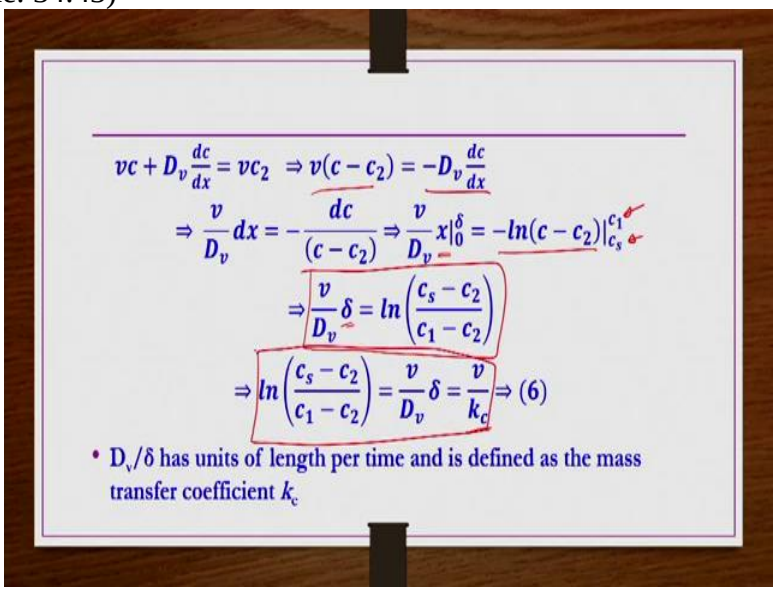

So $vc + D_v \frac{dc}{dx} = vc_2$ that we have done just now flux balance equation at the membrane surface. So then now v terms we keep one side, other terms we take to the the remaining right side, right.

Now we can rearrange this equation $\frac{v}{D_v} dx = -\frac{dc}{(c-c_2)}$. When you integrate either side so d x is, integration of dx is x and then x is varying from 0 to δ, that is within the boundary layer, okay, due to the concentration polarization whatever the boundary layer is there, that the thickness is δ and then that is varying between 0 to δ from membrane surface towards the feed side, ok bulk solution side. And then $-\frac{dc}{(c-c_2)}$ is nothing, when you integrate you will get $-ln(c - c_2)$ and then surface concentration at x is equals to 0, concentration is c_s.

At x is equal to δ concentration is nothing but the bulk concentration c_1. So when you apply these boundary conditions, limiting conditions we have $\frac{v}{D_v}\delta = ln\left(\frac{c_s-c_2}{c_1-c_2}\right)$. So this is the equation. Now here if you know the diffusivity and then thickness, boundary layer thickness

then you can know the c_s because c_1, c_2 are known. The feed solution concentration you know and then permeate solute concentration also you know while doing the experiment. So c_s you can know from here.

The same equation if you rearrange this what $\frac{D_v}{\delta}$ is having a kind of meter per second units, so that is we can call it as kind of a mass transfer coefficient kc. So $ln\left(\frac{c_s-c_2}{c_1-c_2}\right)$ we can write by, we can write it as $\frac{v}{k_c}$. $\frac{D_v}{\delta}$ has units of length per unit and is defined as the mass transfer coefficient k_c, okay. This is the case when the rejection is partial. Indeed, in reality most of the cases, the partial rejection only takes place. Complete rejection never takes place; very rarely it takes place, right. So in case if you have a kind of a complete rejection then c_2 should be 0 in the same equation. Then $ln\frac{c_s}{c_1}$ should be equal to $\frac{v}{k_c}$.

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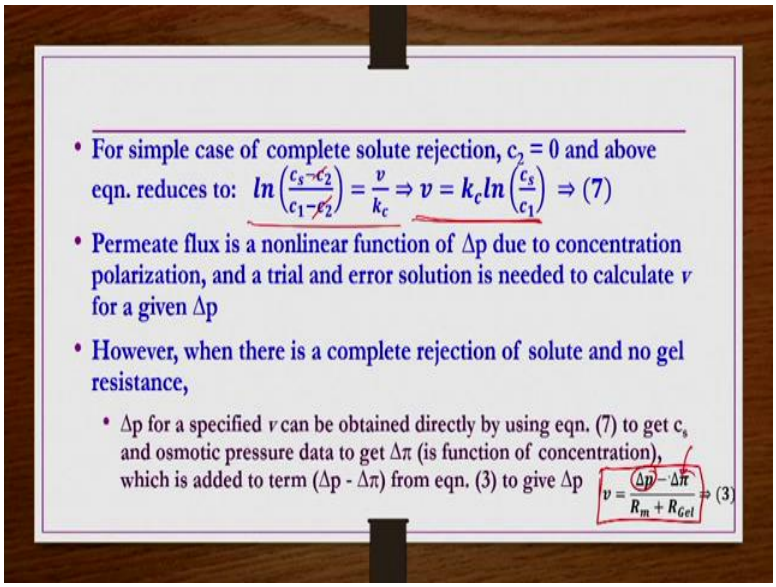

For simple case of complete solute rejection which is not possible in general, however if you have a case where complete rejection is taking place then c_2 is equals to 0 and then in above equation $ln\left(\frac{c_s-c_2}{c_1-c_2}\right) = \frac{v}{k_c}$. If you substitute c_2 is equals to 0 then we have $v = k_c ln\frac{c_s}{c_1}$, okay.

So now what we understand further, permeate flux is non-linear function of Δp due to this concentration polarization and trial and error solution is needed to calculate v for a given Δp. Actually if it is pure water then you know water flux you know linearly increases with the Δp.

If you have a dilute solution which you wanted to purify by some membranes, by using membranes in this ultrafiltration then that flux of the solution, you know it is non-linear. It is non-linear, okay.

We are going to see that one also. So however when there is complete rejection of solute and no gel resistance is there, Δp for specified v can be obtained directly by using equation 7. From here you can get the c_s and then once you know the c_s you can know the $\Delta\pi$ and once you know the $\Delta\pi$, so then we can use this equation to get Δp for a required v.

Basically what this statement says that, let us say if you wanted to have 0.0001 L/m^{2}h of permeate flux how much Δp should you apply? That is what you wanted to calculate, okay. So then for that you need to know delta pi also. How much Δp is required, if you want to calculate how much Δp is required for a specified permeate flux, let us say 0.001 L/m^{2}h of permeate flux, if you want to obtain by using ultra-filtration membrane then how much Δp should you apply. That you can calculate by these equations.

For that you need $\Delta\pi$. So this $\Delta\pi$ can be obtained by the c_s concentration. Andc_s you can obtain from the equation number 6 or 7 depending on the rejection. If the complete rejection then equation number 7 can be used, if partial rejection then equation number 6 can be used; that is the point of using this equation. So this developed equation is used in 2 ways. You can directly, you know obtain what is the surface concentration or the solute concentration at the surface c_s or you can obtain you know Δp required to maintain the specified permeate flux. Either way you can use these equations.

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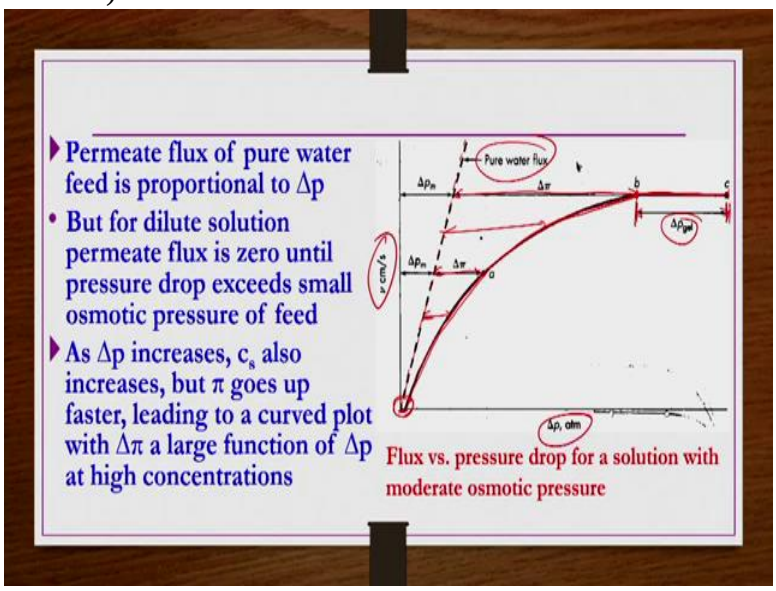

Now let us say, as I mentioned already you know, whatever the flux is there if it is for the pure water it is going to be linear in general. So then, but if you have a kind of a solution, if you have a kind of dilute solution and then using the membrane for separating that solution so that you can have a kind of clear permeate and then concentrated retentate as a kind of separate layer.

Then this flux is not going to be linear with Δp, why? Because there is a kind of osmotic pressure because of these solute particles, right. Because of that osmotic pressure even if you apply certain pressure initially so for initial certain delta p there will not be any kind of flux. Now here you can see at the origin this pure water flux is passing through the origin. But the other line, the solid line is a kind of, you know permeate flux for a kind of a solution.

We need to take a dilute solution then the flux would be having like this and then this is not passing through origin. That means the solute, you know, retention will be there. And then permeate flux will only start after certain Δp has to be applied and that Δp is nothing but you know, that is osmotic pressure has to be overcome then only flux, permeate flux will start coming out, okay the permeate will start coming through the membrane.

Until that pressure, until and unless that pressure is crossing that osmotic pressure the separation will not take place. So this, this is the region, here it is not passing through the origin. So permeate flux of pure water feed is proportional to Δp and then you can see this

proportionality, kind of linear plot you can see here, okay. So this is increasing with increase in the Δp linearly.

But for dilute solution, permeate flux is 0 until pressure drop exceeds small osmotic pressure, whatever small osmotic pressure is there because of the presence of solutes in the solution, you know there is a small osmotic pressure. Until and unless the applied pressure exceeds that osmotic pressure the permeate flux will not be there, it will be 0, okay. Then after that, you know after crossing this osmotic pressure, further if you increase this pressure, when you start applying the pressure, gradually you increase the pressure, right.

So there will be a kind of pressure drop which is equal to the osmotic pressure and after that permeate is started coming. But after that osmotic pressure if you further increase the applied pressure then what happens? Let us say 1.1 is a kind of, 1.1 atmosphere is a kind of osmotic pressure for a given solution. Then up to 1.1 pressure drop there will not be any permeate and permeate will be 0. And after 1.1 atmosphere, you further gradually increase the pressure drop, the flux is going to increase gradually but this increase is a kind of non-linear like this and then after certain point let us say, from this b to c point it is going to be constant. That is the point where the gel formation is there and gel is offering a lot resistance.

That resistance is known as the Δp_{gel} which is not allowing further flux increase. With increase in Δp, c_s increases, right. c_s, that is the concentration, solute concentration on surface increases because more and more particle would be you know forming or contributing towards the gel formation. So c_s concentration would increase. But c_s concentration may be slightly increasing but the corresponding osmotic pressure you can see, it is largely increasing. How largely it is increasing from one level to the other level gradually you can see.

So this is called $\Delta\pi$. So as the pressure drop increases, the osmotic pressure is increasing to the larger extent; faster, faster rate increases. This is the permeate flux versus pressure drop plot for a kind of ultrafiltration membrane. In general this is what you find, okay.

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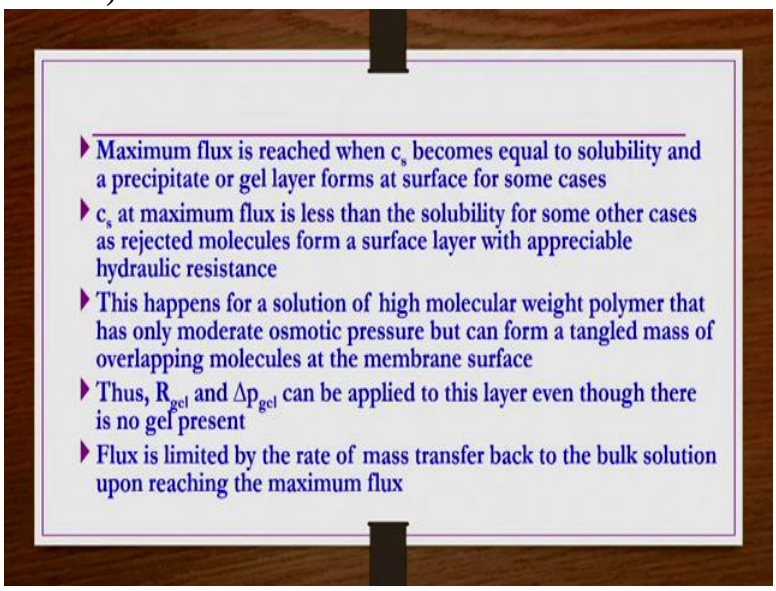

This maximum flux whatever is there, that is reached when c_s becomes equal to the solubility and precipitate or gel layer forms at surface for some cases. But however in some cases c_s, if maximum flux is less than the solubility, as the rejected molecules form the surface layer with appropriate hydraulic resistance which is, because you know, whatever the particles are there, tangled mass of overlapping molecules at the membrane surface, they cause a kind of increased rejections for these kind of cases where the maximum flux is attained and then c_s is less than the solubility. Despite of that one also rejection increases.

So now what we understand here? So whether the gel formation is there or not, R_{gel} and Δp_{gel} can be applied to this layer even when there is no gel present because the gel formation is going to take place gradually with increase in the Δp. So flux is limited by the rate of mass transfer back to the bulk diffusion upon reaching the maximum flux. After this point whatever the flux is there, here it is controlled by the mass transfer, okay.

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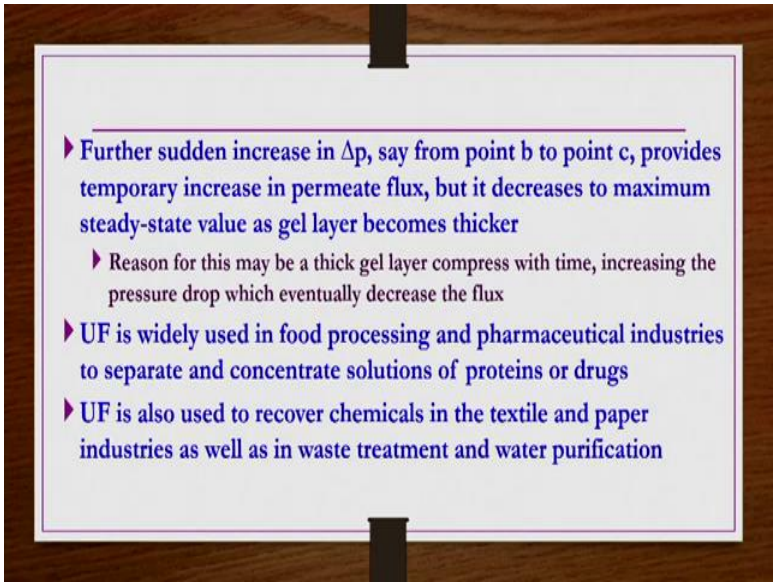

Further sudden increase in Δp, say from that point b to c in the picture we have seen, in the picture we have seen, that provides a temporary increase in permeate flux but it decreases to maximum steady state value as gel layer becomes thicker because you know thick gel layer compresses with time and increasing the pressure drop would eventually decrease the flux only. So this is about the, you know concentration polarization and permeate flux for the ultra-filtration membrane and then how the permeate flux is changing with the pressure drop et cetera, those kind of things we have seen about the ultra-filtration membrane.

So in general these ultra-filtration membranes are used widely in food processing and pharmaceutical industries to separate and concentrate solutions of proteins or drugs. UF is also used to recover chemicals in the textile and paper industries as well as in waste treatment and water purifications. We have seen a few specific applications in the previous lecture and then some generalized applications are given here for ultra-filtration membranes.

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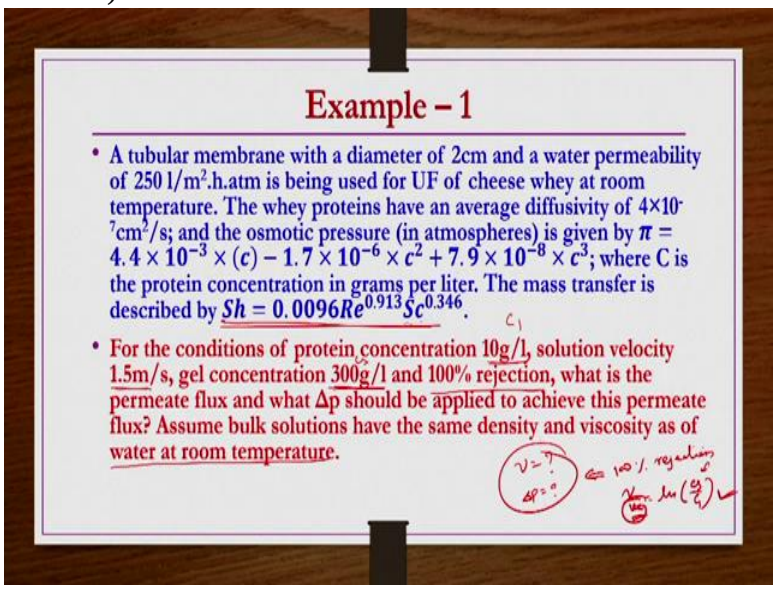

Now what we do? Based on this you know permeate flux and then concentration polarization concept for ultra-filtration membrane whatever we have seen, we try to solve couple of problems. A tubular membrane with a diameter of 2 centimeter and water permeability of 250L/m^{2}h.atm. is being used for ultra filtration of cheese whey at room temperature. The whey proteins have an average diffusivity of 4 x 10^{-7} cm^2/s and osmotic pressure is given by this equation where C is the protein concentration in g/L.

The mass transfer is described by the Sherwood number and Sherwood number as function of Reynolds number and Schmidt number is given here. So now for the conditions of protein concentration, protein concentration 10 g/L that is the c_1 is given, and then solution velocity 1.5 meter per second and gel concentration that is c_s is given, 300 gram per liter. c_1 is 10 g/L so now you can see c_s is 300 g/L, 30 times higher, okay. And 100% rejection is there.

So under such condition what is the permeate flux and what Δp should be applied to achieve this permeate flux? So we need some kind of properties such as density and viscosity. So for that bulk solutions have the same density and viscosity as of water at room temperature, that we can assume as per the problem statement, okay. So we need to find out what is permeate flux and then what is pressure drop to maintain this permeate flux under 100 percent rejection conditions, right?

So c_s, we have this $\frac{v}{k_c} = ln\left(\frac{c_s}{c_2}\right)$. From here we can find out the, you know this v. But kc we have to find out. How to find out the k_c? k_c we can find out from this Sherwood number expression. So Re you can calculate because the solution velocity is given. Density and viscosity of the fluid is given, same as the water. Schmidt number also you can calculate, so then Sherwood number you can calculate, from Sherwood number you can calculate the k_c value. Once k_c is known, from this equation you can calculate the v because c_s is also given. And then c_2 is 0. This is $ln\left(\frac{c_s}{c_2}\right)$.

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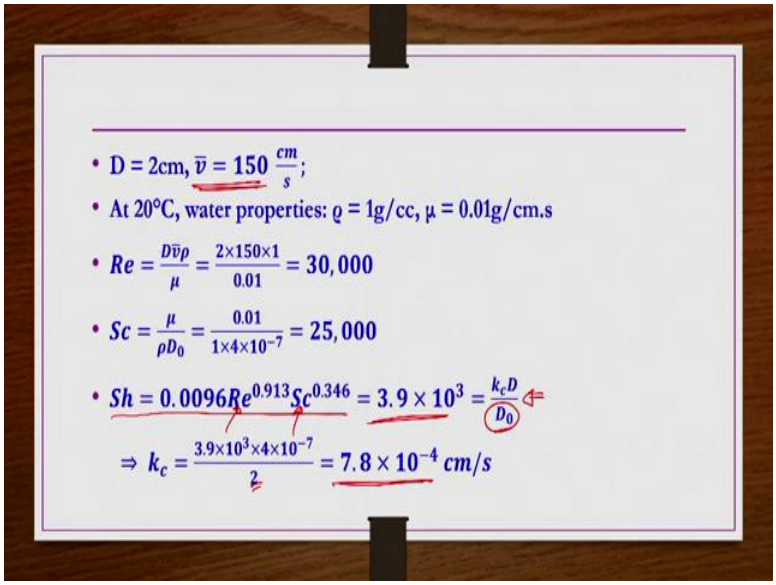

So D is 2 cm. $\bar{v}$ is 150 cm/s. This $\bar{v}$ is not the permeate flux. It is the velocity. It is the velocity of the fluid that is given for the condition of protein concentration 10 g/L; solution velocity is 1.5 m/s at that whatever the across the membrane surface, feed is coming at high velocity rate. That high velocity is nothing but 1.5 m/s. It is not the permeate flux, okay. So 1.5 meter per second, I will write 150 cm/s because we will be solving this problem in CGS units.

At 20 degree Centigrade water has the 1 g/cc, μ and 0.01 g/cm.s viscosity. So $Re = \frac{D\bar{v}\rho}{\mu}$, D is 2, $\bar{v}$ is 150, ρ is 1, μ is 0.01 so Re is coming out as 30000. $Sc = \frac{\mu}{\rho D_0}$ so which is given as a kind of, you know, μ is given as 0.01, D_0 is 4 x 10^{-7} and then ρ is 1. All are in CGS units.

So it is coming 25000. So the Sherwood number, it is given as like this in the problem, so in the Sherwood number expression, you substitute Schmidt number and Reynolds number, so you will get Sherwood number is 3.9 x 10^3 which is, definition of Sherwood number is nothing but $\frac{k_c D}{D_0}$ is the diffusivity. So now from here you can find out the k_c. k_c is equals to 3.9 x 10^3 into diffusivity is $\frac{4 \times 10^2}{2}$is the diameter. Then you will get k_c you will get it as 7.8 x 10^{-4} cm/s.

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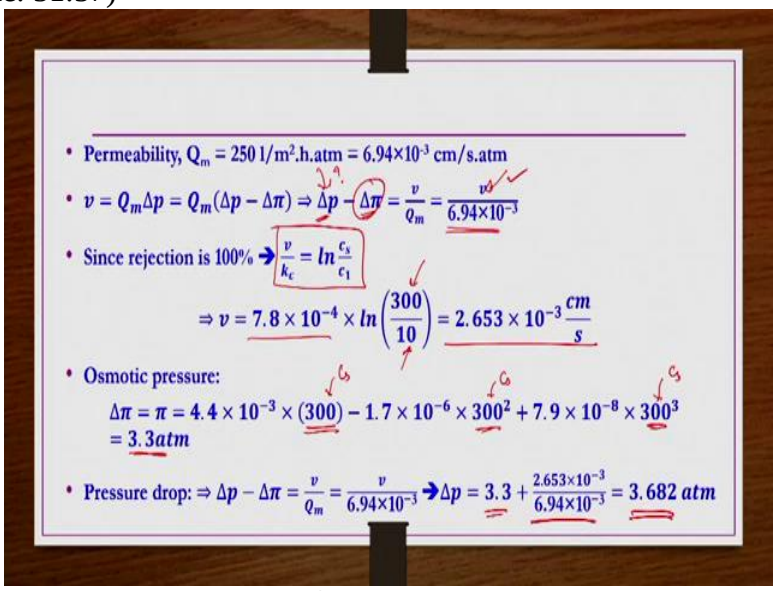

So permeability, Q_m is given as 250 L/m^2.h.atm, so that we have to convert into the cm/s, so then you will get this one. And then $v = Q_m\Delta p = Q_m(\Delta p - \Delta\pi)$ we have to have because now it is a dilute solution, dilute cheese whey solution so $(\Delta p - \Delta\pi)$ should be $\frac{v}{Q_m}$.

So v is the, you know whatever the permeate is there, that we have to find out and then Q_m is the permeability that is 6.9 x 10^{-3}, that is again given, okay. So now in this equation if you wanted to find out v you need to find out $\Delta\pi$. You need to find out Δp also, okay. But for 100% rejection case we know that $v = k_c ln\left(\frac{c_s}{c_1}\right)$. So we do not need to find out v from here, we can find out from here because it is given in the problem statement for 100 percent rejection conditions.

So from here v you will get, k_c that is 7.8 x 10^{-4} cm/s, lnc_s is 300 gram per liter, c_1 is 100 gram per liter given in the problem. So then you will have this 2.653 x 10^{-3} cm/s as a kind of permeate

flux. So permeate flux is known. So now v is also known from here. So now you need to find out what is Δp. For that you just need to find out $\Delta \pi$.

$\Delta \pi$ osmotic pressure expression is also given. In place of c you have to substitute c_s values because we wanted to know the osmotic pressure, you know at the membrane surface. At the membrane surface the concentration is c_s. So 300 when you substitute here in this osmotic pressure equation which is given then you will get osmotic pressure as 3.3 atmosphere. So $\Delta p - \Delta \pi = \frac{v}{Q_m}$. Then v is already 2.65 x 10^{-3} that we have calculated.

Permeability Q_m is given as 6.94 x 10^{-3}. Osmotic pressure you just calculated as 3.3, so 3.683 atmosphere of pressure drop you should apply if you wanted to have a kind of 100% rejection with permeate flux, 2.65 x 10^{-3} cm/s, okay.

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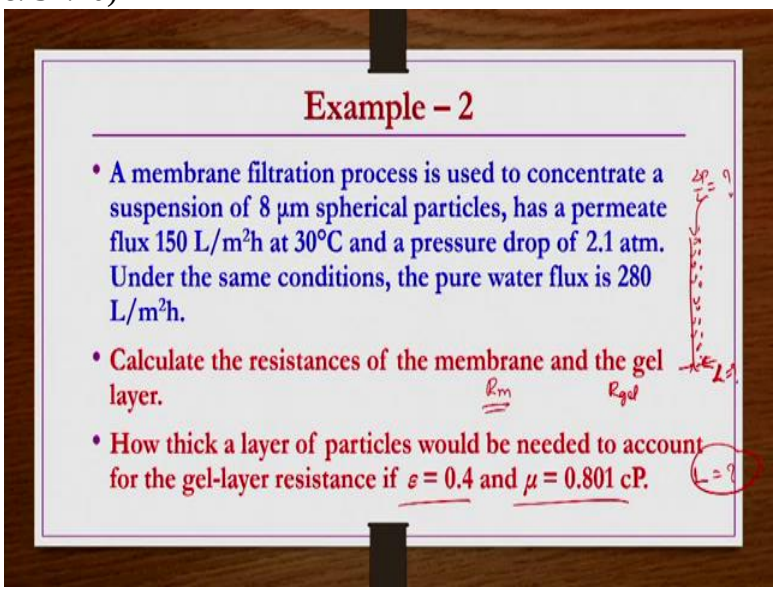

We take another example problem. A membrane filtration process is used to concentrate a suspension of 8 μm spherical particles, has a permeate flux of 150 L/m^2h at 30 degree Centigrade and the pressure drop of 2.1 atmosphere. Under the same condition the water flux is 280 L/m^2h. Calculate the resistance of the membrane and the gel layer, that is R_m and then R_{gel} we have to find out. In addition we have to find out how thick a layer of particles would be needed to account for the gel layer resistance if ε is 0.4 and then μ is 0.801 cP that is L we have to find out.

That is L is like you know, on the membrane you know this is the membrane, okay. On this membrane the particles, gel layers are formed. The particles are forming a kind of gel layers, right. What is thickness of this gel layer L? This we have to find out. So for this we can use the Kozeny Carman equation that kind of form that $\frac{\Delta p}{L}$ equals to, from the Kozeny Carman equation we have that $\frac{\Delta p}{L} = \frac{150\mu v(1-\varepsilon)^2}{\varphi_s^2 D_p^2 \varepsilon^3}$.

That expression we can use, right. In that expression Δp is known, only L that we can know. L is nothing but the length of the packing. Now here that length of the or the thickness of the gel whatever is there, that is L here in this case. Okay, so before that we have to find R_m and R_{gel}. Let us see that one.

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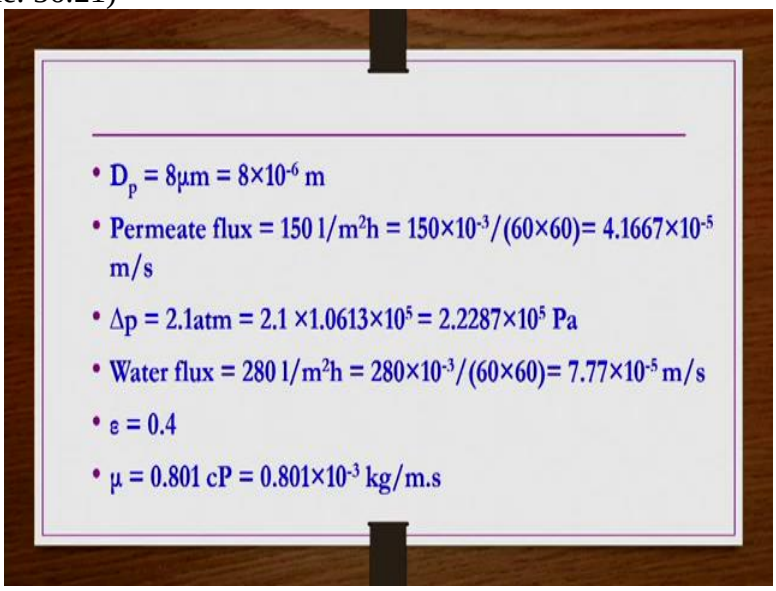

D_p is given, permeate flux is given so we converted in meter per second, Δp is given, it is converted into the Pascal. Water flux is also given in L/m^{2}h. If you convert into the m/s units it comes like this. ε is given, μ is given, okay. So in kg/m.s it is 0.801 x 10^{-3}.

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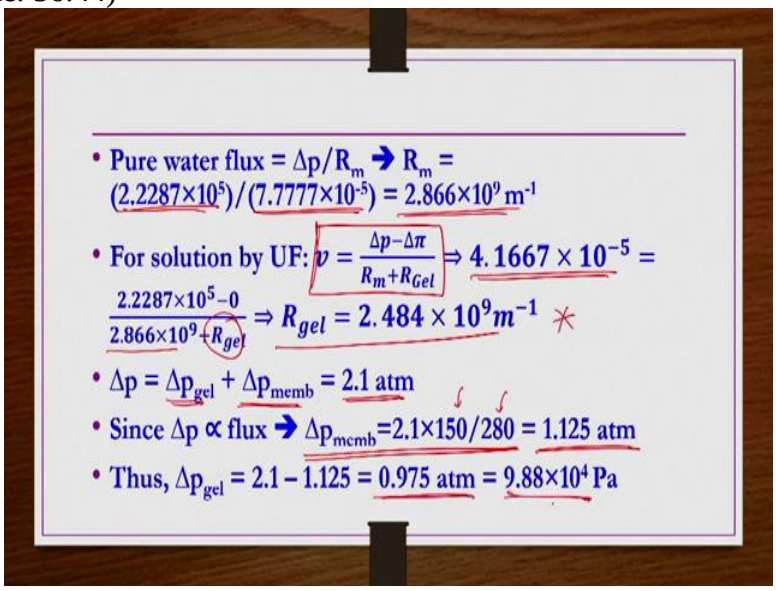

So the pure water flux is nothing but $\frac{\Delta p}{R_m}$. Right, so from here Δp is given so R_m should be Δp by pure water flux. Pure water flux is also given. So Δp is nothing but, we converted in Pascal so it is 2.1 atmosphere but in Pascal it is 2.2287 x 10^5 Pascals divided by the permeate flux is given in pure water, it is pure water so pure water flux is given in L/m^2h. In meter per second if you convert it comes out to be 7.77 x 10^{-5}.

So R_m comes out to be 2.866 x 10^9 m^{-1}, straight forward. So similarly for solution this equation we know permeate flux is nothing but $\frac{\Delta p - \Delta \pi}{R_m + R_{gel}}$. So permeate flux is also given 150 L/m^2h, so that we converted into m/s like this. Now Δp it is given 2.1 atmosphere, in Pascals it is 2.2287 x 10^5 Pascals minus $\Delta \pi$ we can take it as 0 because it is first of all the solute concentration is not given, so we assume that the solute concentration is very, very small.

So then we can take it as a kind of 0. So then from here, R_m is also known. So from here R_{gel} we can find it out. So R_{gel} is equals to 2.484 x 10^9 m^{-1}. What we can see here, the gel resistance is not small compared to the membrane resistance. Indeed both of them are almost equal. So whatever the membrane resistance the same amount of resistance is, almost same amount of resistance is also offered by the gel. So gel layer one cannot avoid easily otherwise results may be going wrong.

So gel layer resistance is also very significant in ultra-filtration. So Δp overall is 2.1 atmosphere but that should be $\Delta p_{gel} + \Delta p_{membrane}$, whatever the resistance offered by the membrane and

whatever the resistance offered by the gel are there, so across these 2 layers whatever the pressure drops are there they would be added together to get the total pressure drop as same as in cake filtration we have done, right.

So now, but you know Δp is proportional to the flux, is not it? So then delta p membrane you can find it out, 2.1 x 150/280, 150 is the permeate flux, 280 is the pure water flux so from here $\Delta p_{membrane}$ you will get 1.125 atm. So then Δp_{gel} you can get it as (2.1 - 1.125) so that is 0.975 atm which is 9.88 x 10^4 Pascals, right.

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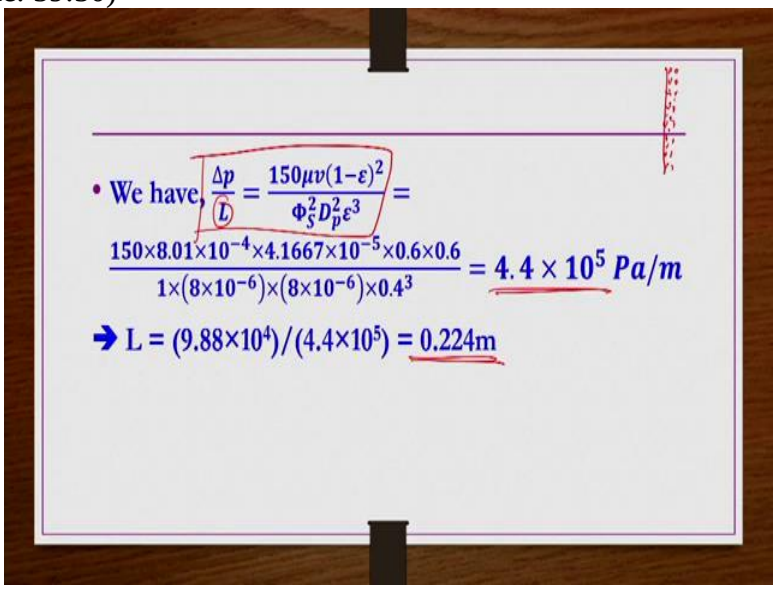

Then we have to find out this layer of particles, whatever the gel layer has formed so that is having some thickness that we have to find out. So as I mentioned so if this is membrane, on the surface the particles are forming. So this layer, though it is very thin, so we can apply the principles of flow through packed bed and then we can use this Kozeny Carman equation that is $\frac{\Delta p}{L} = \frac{150\mu v(1-\varepsilon)^2}{\varphi_s^2 D_p^2 \varepsilon^3}$. Here except L everything is known. So what we do? $\frac{\Delta p}{L}$, right hand side when you calculate you will, after substituting all these values you will get 4.4 x 10^5 Pascal per meter, right. Now Δp is also known.

So L should be Δp divided by whatever this 4.4 x 10^5 is there, that when you do you will get L as 0.224 meters. So 0.224 meters thick particles are formed as a kind of gel layer. So gel layer thickness is nothing but 0.224 meters. It is very significant one.

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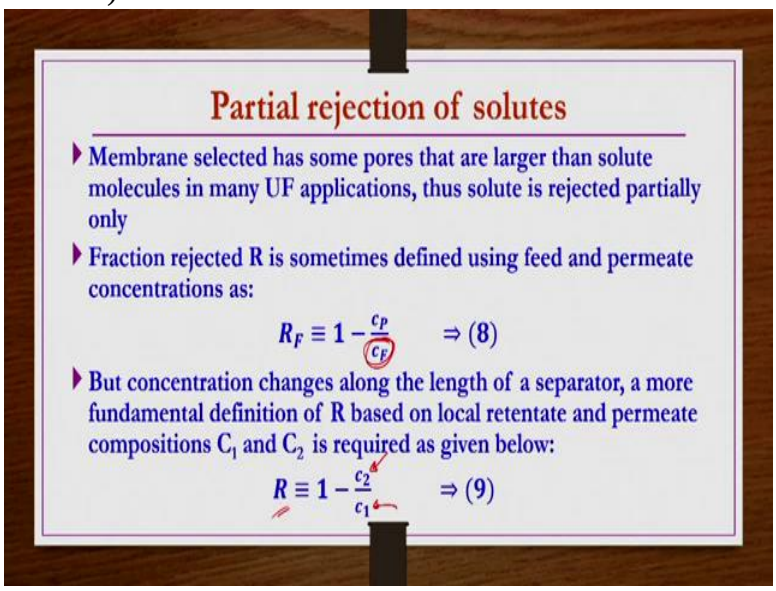

So finally we see the partial rejection of solute. So we have seen there are 3 cases, permeate flux and then you know percentage of rejection and then solute concentrate and retentate are the kind of three important things which describe the performance of the ultrafiltration membrane. So permeate flux we have seen. Now we are going to see the partial rejection of solutes. In general membranes whatever we select they have the pores which are larger than the solute molecules in many ultra-filtration applications.

So because of that one only partial rejection takes place. It is not possible to get a kind of complete rejection in general. So fraction rejected R is sometimes defined using the feed and permeate concentrations as $R_F = 1 - \frac{c_p}{c_f}$. But the c_f we have seen especially in ultrafiltration membrane it is very much different. It is very much different from one location to the other location, especially the boundary layer that is forming because of the concentration polarization.

The concentration varies from $\frac{c_1}{c_s}$. So feed concentration c_1 we cannot use, c_s in general we do not know. So what happens? People in general use, you know retentate concentration. So that is $1 - \frac{c_2}{c_1}$ is as a kind of R, rejection. c_2 is nothing but the permeate concentration. c_1 is nothing but the retentate concentration. Terminology is different here, okay.

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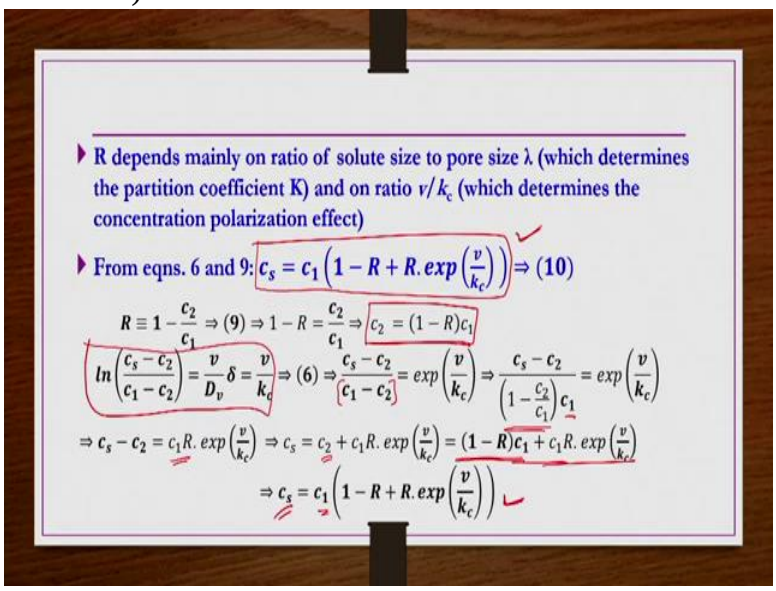

So c_2 is permeate concentration and c_1 is the retentate concentration here, not the feed concentration, okay. And then R depends mainly on ratio of solute size to pore size which determines the partition coefficient and ratio of $\frac{v}{k_c}$ which determines the concentration polarization effect. Both are very much important factor in ultrafiltration. So we have to bring in those factors here.

So whatever the equation number 6 to 9 we have, so if we combine then we can get c s concentration like this. How we get? We do this one. So equation 9 we have just now seen, $R = 1 - \frac{c_2}{c_1}$. So from here $1 - R = \frac{c_2}{c_1}$ or $c_2 = (1 - R)c_1$. And then equation number 6 we have already derived, so this $ln\left(\frac{c_s - c_2}{c_1 - c_2}\right) = \frac{v}{k_c}$, this we have already derived. This is equation number 6.

So now what we do? We take exponential either side, so then we get this one. And now from here, from this part $c_1 - c_2$ what we do? We take c_1 common so that we have $1 - \frac{c_2}{c_1}$, rest everything keep same. So now this $1 - \frac{c_2}{c_1}$ what we can write? We can write it as R from equation number 9, so whatever the c_1, R, that you can take it to the right hand side. So $c_s - c_2 = c_1R.exp\left(\frac{v}{k_c}\right)$, that is $c_s = c_2R.exp\left(\frac{v}{k_c}\right)$ we can write and then from equation number 9 we can also write $c_2 = (1 - R)c_1$. So in place of c_2 you substitute $(1 - R)c_1$ and then plus

$c_1 R.exp\left(\frac{v}{k_c}\right)$ as it is. So now from these 2 terms if you take c_1 common then $c_s = c_1\left(1 - R + R.exp\left(\frac{v}{k_c}\right)\right)$. So this is what you get.

So that means if you know the retentate concentration or if you know the feed concentration you can know the surface concentration or solute concentration at the membrane surface through this expression once you have the $\left(\frac{v}{k_c}\right)$ and c_1 information along with the rejection.

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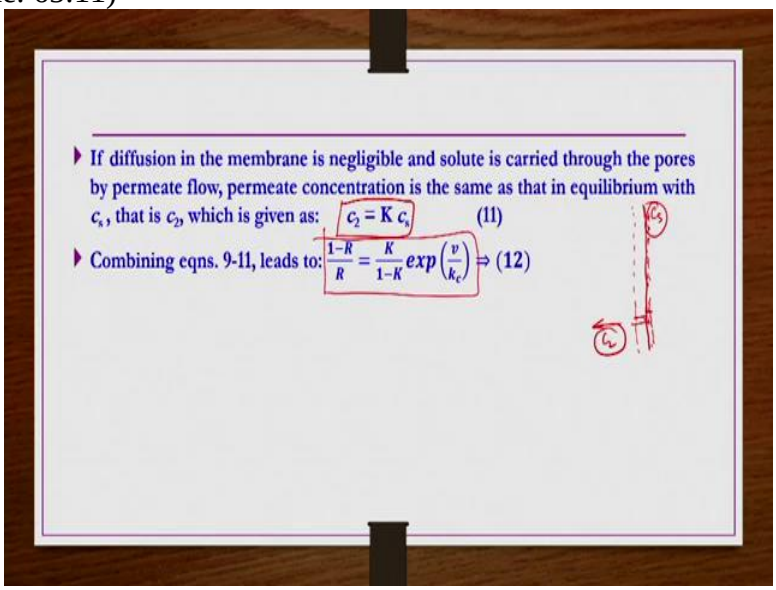

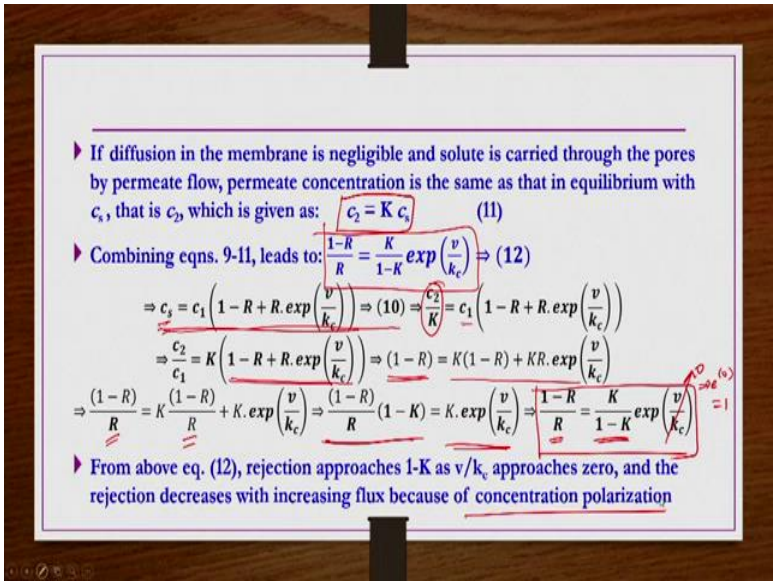

So if the diffusion in the membrane is negligible and solute is carried through the pores by permeate flow, permeate concentration is the same as that in equilibrium with the c_s, that is c_2.

That picture we have, right. So what we have, like you know this membrane surface we are having, right. Now here the particles are being deposited as a kind of gel layer on the membrane surface, right.

So this is c_1 concentration. But other side you know c_2 concentration is there, okay. So within the c_s just inside you know c_m is there, okay just outside of the pore let us say in this pore here it is c_m, here is the and then c_2 is there. So this c_2 and then c_m, c_s are in a kind of equilibrium, right? Because they are just either side of the membrane.

One is just one side of the, upstream side of the membrane. Other one is the, just you know downstream of the membrane. And then within that membrane thickness layer the concentration variations are, you know, some variations are there. So that variations can be represented by the equilibrium partition coefficient and then how we can represent that? $c_2 = Kc_s$. Now if you use this one in equation number 11 then we get this expression for the rejection, okay.

How we get? So that also we do it here. Just now equation number 10 we derived, okay. This equation what you can do, c_2, left hand side we have c s actually. In place of c_s let us write you know c_2 by K from equation number 11. That is the first step you do, right; in place of c_s you write c_2 by K. Then next step what you do? This k you take to the right hand side and this c_1 you bring it to the left hand side. So left hand side you have $\frac{c_2}{c_1}$ is equals to K multiplied by whatever the remaining step, terms, remaining terms are as it is.

Now $\frac{c_2}{c_1}$ is what? From the equation number 9 that is rejection definition is nothing but (1 – R), okay. In the right hand side you can just, you know write it as 2 terms, $K(1-R)$ as one term, and another term $KR.exp\left(\frac{v}{k_c}\right)$, both the terms are added here, just expanded it. Now this equation what you do, both the sides you divide by R. Both the sides you divide by R. So left hand side you have $\frac{(1-R)}{R} = K\frac{(1-R)}{R} + Kexp\left(\frac{v}{k_c}\right)$ because R and R are from the second term would be canceled out when you are dividing it.

So now what you do, this $\frac{(1-R)}{R}$ terms you take one side, so then you have $\frac{(1-R)}{R}(1-K)$ in the left hand side and then right hand side $Kexp\left(\frac{v}{k_c}\right)$ on the right hand side. So we can write $\frac{(1-R)}{R} = \frac{K}{1-K} exp\left(\frac{v}{k_c}\right)$. This is what we have, okay. So now from this equation when this $\left(\frac{v}{k_c}\right)$ approaches 0, $\left(\frac{v}{k_c}\right)$ approaches 0 then what we have? The exponential of 0 is 1, so that means rejection approaches $1-K$. Rejection approaches $1-K$ and then rejection decreases with increase in flux because of the concentration polarization.

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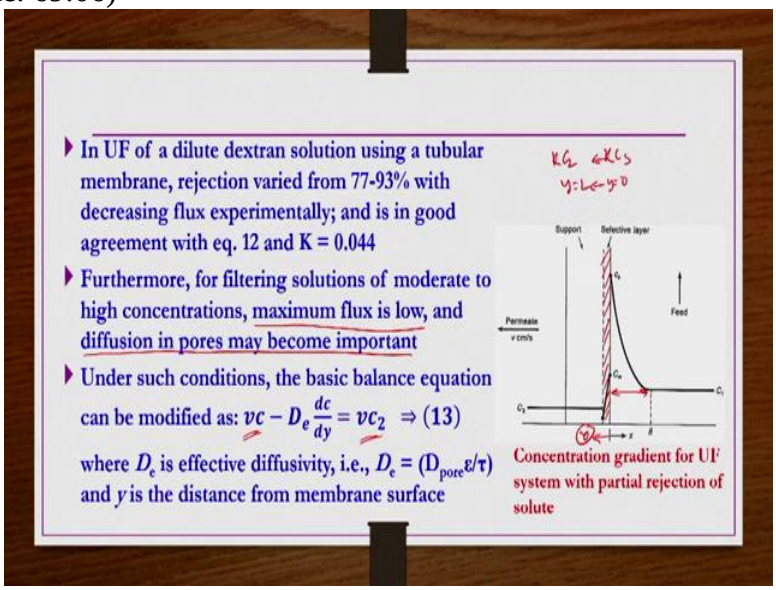

Further in this kind of expression whatever we have found, they are found to be kind of reliable in ultra-filtration of a dilute dextran solution using a tubular membrane, rejection varied from 77 to 93% with decreasing flux experimentally. And it is found in good agreement with the equation number 12 just we derived by substituting K is equals to 0.044. But in general what happens in the layer, filtering solutions of moderate to the high concentration, maximum flux is in general low and then diffusion in pores may become important. In some cases it is possible that the diffusion in pores is much more dominant than the other cases, right. Under such conditions how these equations would change?

So now in the previously whatever the changes were there, the concentration changes those things we have seen in this side, right. So but now this membrane layer, within this membrane layer, selective layer it is having certain thickness maybe in some microns but it is having certain thickness. In this thickness there are several pores are there. In the pores the diffusion

is dominating, right. That part we have not taken care while deriving the concentration polarization.

So under such conditions, you know what could be the equation for this c s and v et cetera? So again we have to see the basic equation here, basic balance equation. Now what happens here, the diffusion is inside the pore. So then, at the membrane surface whatever the flux is there, that is the flux is only because of the convection if you take, so that convection is vc and other side the flux is the vc_2 and that should be added with the flux due to the diffusion inside the pore, so that is $D_e \frac{dc}{dy}$. So $D_e \frac{dc}{dy}$ should be added here so that if you take into the left hand side you will get this one, right.

And then D_e is nothing but effective diffusivity that $D_{pore}\varepsilon/\tau$, and then y is the distance from the membrane surface towards the permeate, from the membrane surface towards the permeate that is in the y, reverse x direction, so that is y here. So the membrane thickness is L. So y limits are, limits of y is nothing but y is equals to 0 here and then here y is equals to L. At y is equals to 0 C is nothing but, you know c_s. At y is equals to L, you know c should be c_2 but they are having some partition coefficient equilibrium, they are in equilibrium with each other. So then the corresponding concentration should be multiplied by the partition coefficient scale.

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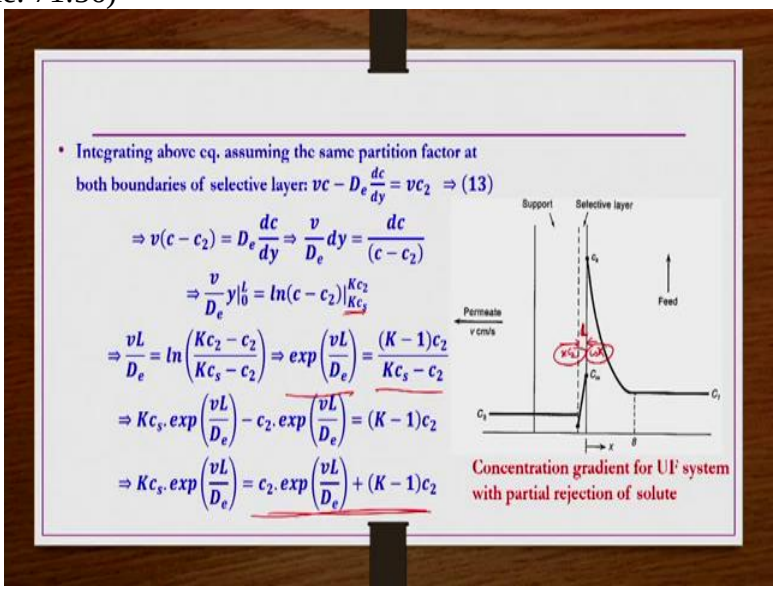

So now this equation we will be integrating and then applying the boundary condition exactly the same way that we have done for the concentration polarization then we have, v terms if you

take one side and D terms if you take the other side so then $vc_2 = D_e \frac{dc}{dy}$. Then $\frac{v}{D_e} dy = \frac{dc}{(c-c_2)}$, when integrate it $\frac{v}{D_e}$, integration of dy is y and then limits are 0 to L, 0 is the membrane surface, L is the membrane thickness towards the permeate side, okay.

So right hand side $\ln(c - c_2)$ should be the, this thing you know, after integration of $\frac{dc}{(c-c_2)}$ you will get $\ln(c - c_2)$. That limits at y is equal to 0 is the membrane surface towards the feed side, so that is at the membrane surface or the top surface, the concentration is c s but now we are assuming the partition factor for both the boundary layers or selective layer are there.

So then k has to be multiplied, so kc s and then towards the, inside the other side of the pore so that is here in this pore, this side; now this is layer actually. This is your thickness of the membrane, actually, L, okay. This side the surface concentration is c_s. This side moment it comes, this side other side it is c_2. But you know it has same partition coefficient, that is the assumption because we do not know exactly what is c_s, we do not know exactly c_2 inside the pore, okay. So they are in equilibrium with a kind of partition coefficient K. So that has to be multiplied here.

So when we take these limits then $\frac{vL}{D_e} = ln\left(\frac{Kc_2 - c_2}{Kc_s - c_2}\right)$. Then you take exponential so here $\frac{(K-1)c_2}{Kc_s - c_2}$, when you rearrange this equation then do the simplification you will get $Kc_s . exp\left(\frac{vL}{D_e}\right) - c_2 . exp\left(\frac{vL}{D_e}\right) = (K-1)c_2$. Further you take all c_2 terms one side, $Kc_s . exp\left(\frac{vL}{D_e}\right) = c_2 . exp\left(\frac{vL}{D_e}\right) + (K-1)c_2$.

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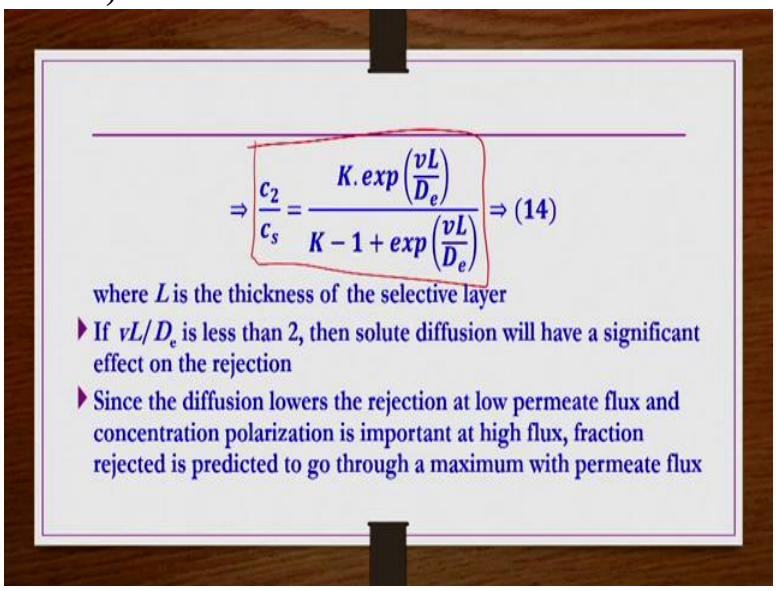

This equation, if you write $\frac{c_2}{c_1} = \frac{K.exp\left(\frac{vL}{D_e}\right)}{K-1+exp\left(\frac{vL}{D_e}\right)}$. This is what we have this equation for a surface concentration if the pore diffusion is predominating. If the pore diffusion is predominating, under such condition the surface concentration on the membrane surface can be obtained by this expression. Or the relation between the surface concentration and permeate flux can be written as this equation number 14. So L is the thickness of the selective layer, that membrane thickness.

If $\frac{vL}{D_e}$ is less than 2 then the solute diffusion will have a significant effect on the rejection and then $\frac{vL}{D_e}$ value goes further beyond 2, it is going to have a pore diffusion is going to take the lead. Since diffusion lowers the rejection at low permeate flux and concentration polarization is important at high flux, fraction rejected is predicted to go through a kind of maximum with the permeate flux.

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References for this lecture are given here but most of the notes is prepared from this first reference book McCabe, Smith and Herriot, Unit Operations of Chemical Engineering. Thank you.