Mechanical Unit Operations
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Lecture 31 - Gravity Sedimentation - Classifiers

Welcome to the MOOCs course Mechanical Unit Operations. The title of this lecture is Gravity Sedimentation, under which we will be discussing classifiers. In the flow past solid objects model, we have already seen the sedimentation or the settling of the particle, single particle under the gravity and then settling of large number of particles under gravity. And then what are the corresponding equations for those settling velocities, those things we have already seen. Now, why we have seen? Because we know that, there are a kind of few mechanical unit operations where you know solid, fluid, both the phases are interacting. Under such conditions if you wanted to separate solids from the mixture of solid liquid, then some of these principles may be useful. That is the reason we have seen.

So now we are going to discuss the gravity sedimentation process in which we will be using the working principles or the equation that we developed for a kind of settling of particles in the gravity field. Those kind of things whatever we have did for single particle and then multiple particles probably, so those things we may take here as a kind of input. And then those equations we may be using for the designing and then operating of this kind of equipment which fall under this gravity sedimentation category.

So gravity sedimentation primarily can be used for separation of particles, especially by two different ways. So one is by the classifying action and other one is by the clarifying action. In the clarifying action or the clarifying processes what happens, all the particles of the slurry, they will be settled down in a kind of tank called as a kind of sedimentation tank so that at the bottom, we will have a kind of solids. And then at the top almost clear liquid is there. In the classifiers what happens, we have a kind of mixture of solids. So then we try to apply the principles of sedimentation or apply the principles of settling of particles under the gravity and then try to separate those mixture of particles into pure fractions as much pure as possible.

So those kind of, two kind of mechanical unit operations we can consider in this chapter where we will be using the concept of gravity settling or settling of particles under gravity.

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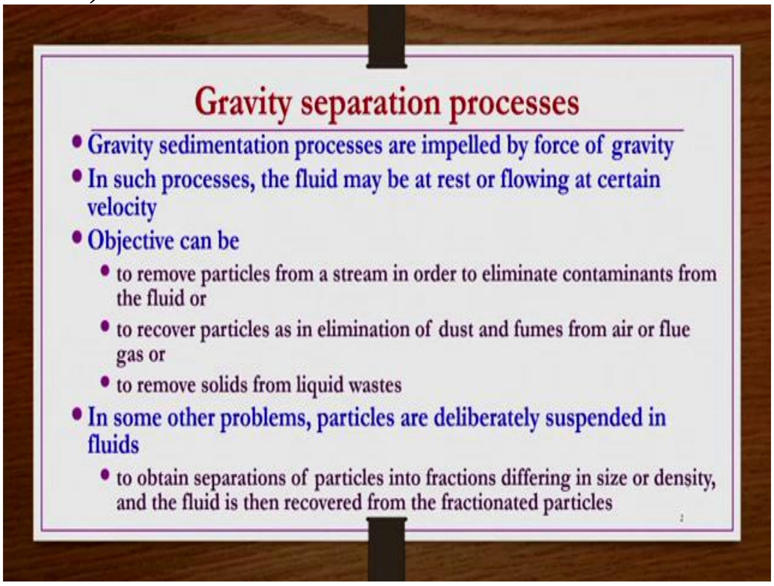

Gravity separation processes: Gravity sedimentation processes are impelled by force of gravity, because of the gravity the sedimentation is taking place. In such processes, the fluid may be at rest or flowing at certain velocity. Under this gravity sedimentation or gravity separation processes, objective can be different as per the applications. One of the objective can be to remove particles from a stream in order to eliminate contaminants from the fluid or recover particles as in elimination of dust and fumes from air or flue gas or to remove solid from liquid waste. This can be a kind of objective, whenever we have this kind of requirements then we can try going for a kind of gravity separation processes.

In some other problems particles are deliberately suspended in fluids to obtain separations of particles into fractions differing in size or density, and the fluid is then recovered from fractionated particles. So what we do, deliberately you know particles whatever the particles are there they will be suspended in fluid deliberately to obtain separation of particles into fractions differing in size or density and the fluid is then recovered from the fractionated particles.

As I mentioned sometimes it is possible that you know, you have a mixture of particles. You need to separate those particles into pure fractions or almost pure fractions kind of thing by using a kind of fluid medium. So for that case also we can use this gravity sedimentation or settling under the gravity principles.

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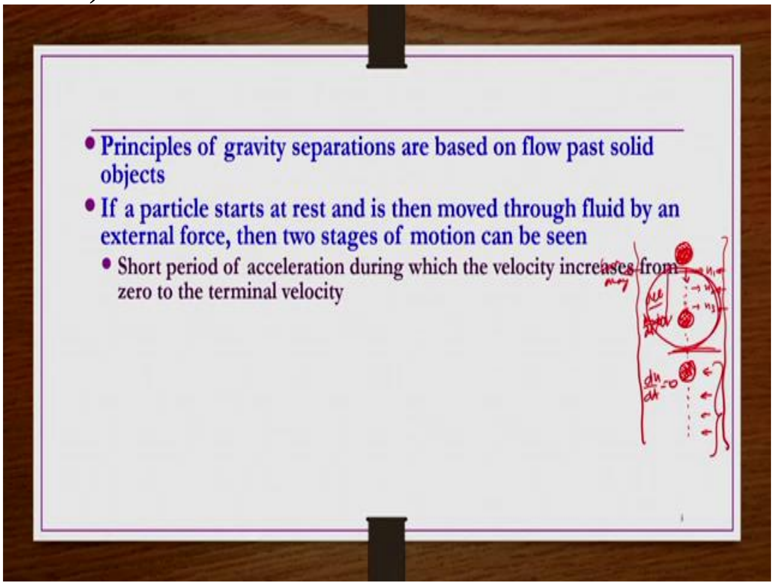

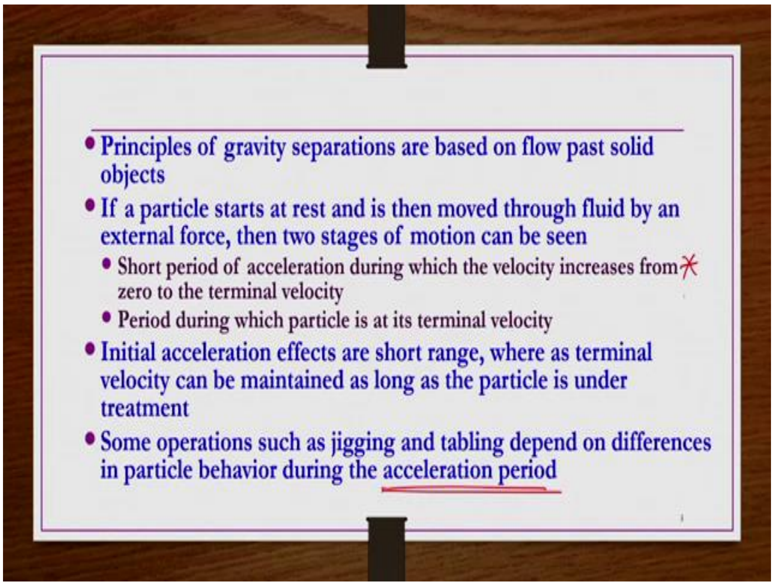

Principles of gravity separations are based on the flow past solid objects that already we have seen, we have seen a kind of single particle settling in a Newtonian fluid which is unbounded. So under such conditions when the particle is settling what are the settling velocity conditions, those kind of things we have listed out and then under those free settling velocity conditions whatever the settling velocity expression is there, that we have developed for a generalized case of a particle settling in a kind of Newtonian fluid without assuming any shape of the particle. Then we have taken spherical particles and for spherical particles we have developed the settling velocities or free settling velocities or terminal velocities for the those particles settling in gravity.

And then for different flow regimes what are those settling velocity, those kind of things we have seen and then several problems also we have solved. So those concepts may be useful here in these gravity separation processes.

If a particle starts at rest and is then moved through a fluid by an external force, then two stages of motion can be seen actually. What happens, let us say there is a kind of unbounded cover as we have already seen. If a particle is settling here, let us say a spherical particle is settling here, so I have already mentioned, so when it settles assuming these walls are far away, so that unbounded flow conditions are there. And then there is only single particle, there are no other neighbouring particles that they may be disturbing this settling velocity of this particle. So nothing is there in the surrounding, only infinitely bounded stagnant fluid is there. In that fluid particle is settling.

So when it settles initially there will be a kind of conditions where this velocity, let us say velocity at this location may be u_1, velocity at location u_2, velocity at this location u_3. Like these different locations if you get this velocity, that is you have to note down the time, how much time this particle took to reach up to this point. So then that distance by time if you give you will get the u_1. Like that you know what is the time the particle has taken to fall down up to this distance if you note down, so then that distance by time, that time u_2 you will get. Like that different locations if you measure and then you can see, what you can see initial stages there will be kind of acceleration of the particle. That is you know this $\frac{du}{dt}$whatever is there, that increases and then that does not become zero kind of thing.

So but after certain more distance when it settles, there will be a kind of stage like you know where these particles settling with a kind of zero acceleration. That is further with distance it is settling, its settling speed does not change with time. So that is what you can see. That is these conditions we call it as a kind of free settling conditions where acceleration is zero. So but before attaining these free settling conditions whatever the conditions are there, there will be a kind of you know a kind of acceleration would be there.
So this region whatever the settling process is there, when the particle is settling because of the external force, of course here external force we have taken gravity. It can be any kind of external force. Then we can have this kind of two stages. Short period of acceleration during which the velocity increases from 0 to the terminal velocity and then period during which particle is at its terminal velocity. That is, it is at $\frac{du}{dt} = 0$ conditions. There are two stages,

one stage $\frac{du}{dt} = 0$, velocity increases from 0 to some increasing velocity and then it increases upto a maximum velocity which is called as a terminal velocity.

Once this terminal velocity is reached, the particle settles as a kind of without any acceleration and it settles with a constant velocity without changing with respect to the time. So that is the second stage period during which the particle is at its terminal velocity. And then at the terminal velocity, that means the free settling conditions and if the terminal velocity is established that means $\frac{du}{dt} = 0$. So mathematically there are two regions, in one region the in the first region, $\frac{du}{dt} \neq 0$and subsequently, second region when the terminal velocity conditions are established, $\frac{du}{dt} = 0$.So initial acceleration effects are short range where is terminal velocity can be maintained as long as the particle is under treatment. Some operations such as jigging and tabling depend on differences in particle behaviour during the acceleration period or during the initial period where the acceleration is important.

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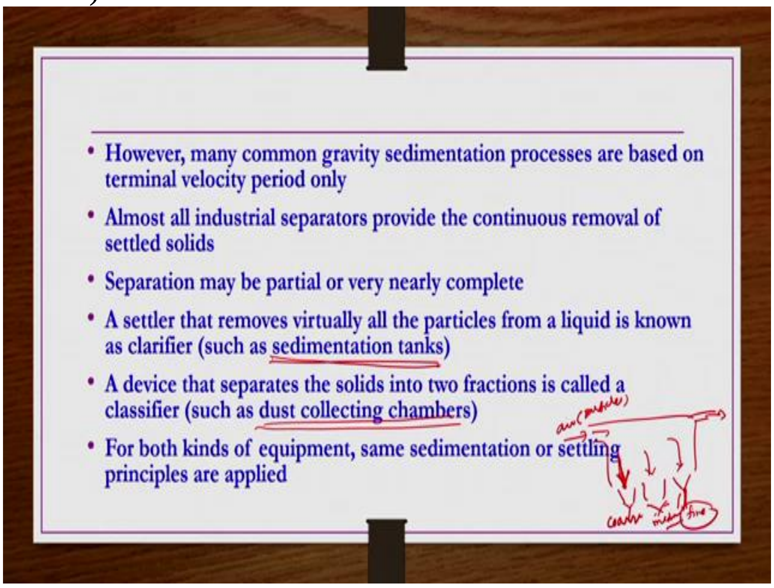

But majority of gravity sedimentation processes are based on the terminal velocity period only. That is in this section where $\frac{du}{dt} = 0$. Almost all industrial separators provide the continuous removal of settled solids in general. Separation may be partial or very nearly complete, it is not necessary that you know it is going to be complete separation is going to take place. That is not possible in any kind of separation processes. But however we can achieve a kind of stage where almost all particles are being separated or nearly complete separation taking place.

So now under the settling velocity or the settling by gravity whatever the principles are there, based on those principles we can design two kinds of equipment. So in which I know different types of separation of particles are taking place. So one type is that a settler tank that removes virtually all the particles from a liquid is known as the clarifier such as the sedimentation tanks.

Another one is a device that separates the solids into two fractions is called as a classifier such as dust collecting chambers etcetera. So equipment we are going to see for this one anyway. So the gravity settling principles we can apply for two different kind of separation processes, as I mentioned sedimentation tank where you can take a slurry and then allow the particle to settle for sometime. You have to give sufficient time so that all the particles will be settled at the bottom and then clear liquid would be at the top.

So those equipments are known as the kind of a clarifier or sedimentation tanks. They can be batch or continuous and there are kind of other sections where you know you need to separate particles into fractions in different fractions by making use of kind of fluid. You take the mixture of solids and mix in a fluid so that and then allow them to settle. So there will be a possibility that a pure layer of one particular type of particles may be settling at the bottom and then pure layer of other type of particles maybe settling on top of it. Like that it is possible. Or something like you know dust laden particles are there, if you allow to pass through this dust laden gas or the particle laden gas or air whatever is there, you allow to pass through a kind of dust collecting chambers. So then what happens?

The particles having the highest settling velocity would be collected at the moment the at the entry inside internal entry of the particle. That we are going to see anyway.

Let us say this is a kind of dust collecting chamber is there. So whatever the air carrying the particles, air with particles whatever is there so moment it enters the particles of higher settling velocity will immediately collected here. And then particles having the lower settling velocity will be collected at the end and then the particles which are having a kind of intermediate velocity will be collected at the center. And then almost all clear air without any particles will be taken out from here outside through the outlet.

We are going to see details of this equipment and all that anyway. So whatever the larger particles, heavier particles, larger size, larger they will be having higher settling velocity. So the moment they enter here, so their settling velocity is high. So then they will be immediately settling down here. So here a kind of coarse particles and then here the fine particles may be having the smaller or the smallest settling velocity amongst the all other particles. So they will be settling at the last, so they will be collected as a kind of fines at the last. And then medium fractions are collected at the in the middle. So for both kinds of equipments same sedimentation or settling particles are applied.

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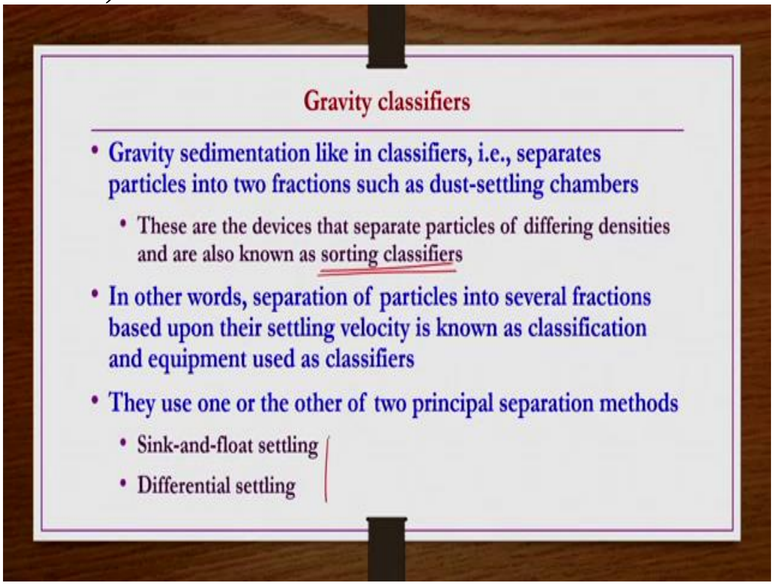

So let us start with the gravity classifiers. Gravity sedimentation like in classifiers, that it separates particles into two fractions such as dust settling chambers are known as the gravity classifiers. These are the devices that separate particles of differing densities and are also known as sorting classifiers. So the separation whatever occurs here is of course because of gravity. But the difference in density whatever the particles different types of particles are there, they maybe having different densities.

Because of the density differences the particle separation from the mixture of particles will take place in these classifiers. Of course the external force that is applied for the separation to occur is the gravity. So these classifiers are also known as sorting classifiers in some books. In other words, separation of particles into several fractions based upon their settling velocity is known as the classification and equipment used is known as classifiers. They use one or the other of two principal separation methods shown here. The first one is the sink-and-float settling method. The second one is the differential settling method. So we see details of these two processes now.

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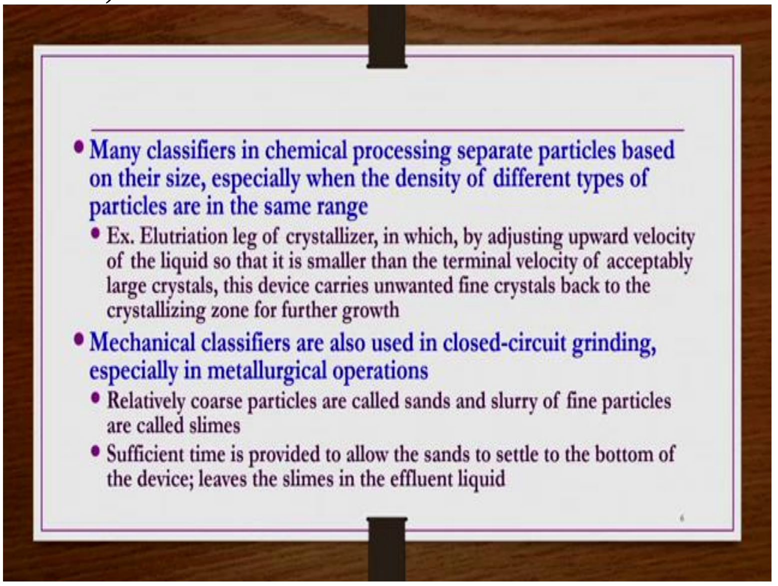

But before going to the details of these processes we see some examples. Many classifiers in chemical processing units separate particles based on their size, especially when the density of different types of particles are in the same range. Something like elutriation leg of crystallizer in which by adjusting the upward velocity of the liquid so that it is smaller than the terminal velocity of the acceptably large crystals, this device carries unwanted fine crystals back to the crystallizing zone for a further growth.

Let us say you have a kind of you know crystals like which you are making a kind of enlargement of those crystals so that you can have a kind of bigger one. So when you allow to pass through a fluid, so these crystals initially are having you know different size fractions, size ranges like small particles and then bigger particles. So let us say whatever the bigger particle size range is there out of 10^{-3} m or something like 1 mm or 2 mm that you want, so you find out what is the terminal velocity of that largest crystal within the sample.

Then you allow this material to flow through elutriation lake. And then by using a kind of fluid, by using a kind of external fluid you allow this material to flow through elutriation lake. And then the velocity of the fluid should be such a way that the velocity of the fluid should be slightly lower than the settling velocity of the largest particle or largest crystal whatever is there. So because of this one you know, the largest (particles) whatever the finer particles are there, they will be you know deviated by the elutriation lake and then collected back and sent back to the crystallization unit for further kind of growth of the crystals.

And then mechanical classifiers are also used in closed circuit grinding, especially in metallurgical operations where relatively coarse particles are called sands and slurry of fine particles are called as slimes. Sufficient time is provided to allow the sands to settle to the bottom of the device leaving the slimes in the effluent liquid itself. So these are the kinds of you know real life applications in general we can see by using these you know classifiers.

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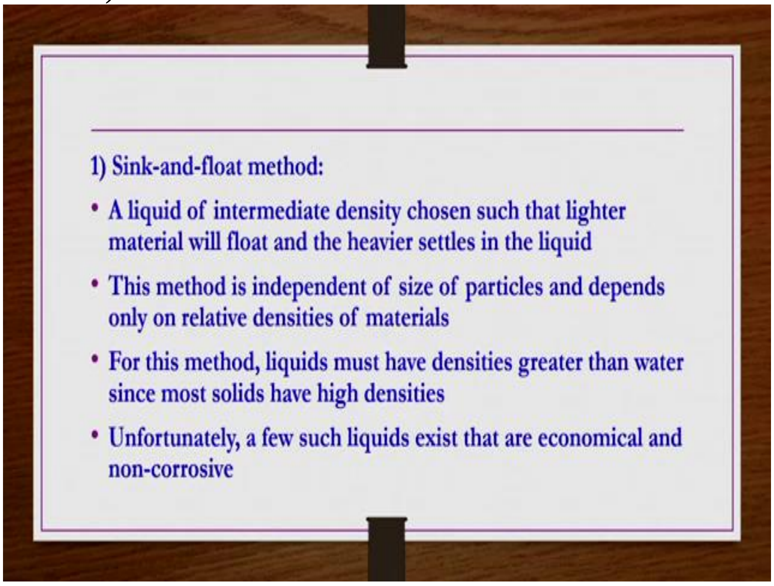

In sink-and-float method a liquid of intermediate density chosen such that lighter material will float and the heavier settles in the liquid. You take a mixture of a particle, let us say you have taken a mixture of solid particles of nature A and B. Material A, material B you have taken, their densities are very different. One is let us say heavier material something like 2,500 kg/m^3 having the density. Another one is the lighter material let us say that is having a kind of density approximately 800 kg/m^3.

Then you find out a kind of fluid which is having a density in between of these two, let us say something like a material, something let us say a kind of liquid which is having a density 1200 kg/m^3. Then you mix these particles in that fluid. Then what happens, whatever the heavier particles are there, they will be their density is higher than the liquid density because heavier particles are having the density approximately 2500 kg/m^3 and then liquid density is approximately equal to 1200 kg/m^3. So those heavier particles will settle at the bottom whereas the lighter particles which are having the density approximately 800 kg/m^3, their density is lower than the liquid density, which is having the these densities you know approximately 1200 kg/m^3.

So that lighter particles having the density lower than the liquid density will be floating on the surface of the liquid. So almost a kind of clear separation will take place. So that is what sink and float method. That is basically you take a mixture of material having different densities and then you select a kind of liquid which is having density in between of these two, and then mix them together so that whatever the lighter material lighter solid material is there that will be floated. And then heavier solid material is there, that will be sunk at the bottom. So that is the reason this method is known as sink and float method.

This method is independent of the size of the particles and it depends only on relative densities of the materials. For this method, liquids must have densities greater than water, since most solids have high densities. Unfortunately, a few such liquids exist, that are economical and non-corrosive. That is a problem. So in this material so though the operation is based on the density, what happens the particles if they have different sizes? Let us say if you have a kind of lighter particle but very big size. And then if you have a kind of heavier material but smaller size, then it is possible that their settling velocities maybe same in general.

So then in order to avoid such kind of complications before separating these materials of different densities what you have to do? You have to crush them to same as size. Like you crush them to almost equal size, that is let us say 1 mm or 2 mm. But all the materials should be crushed to the one uniform size. So if the particle size are uniform, then what happens? The difference in settling velocity would be there because of the difference in the densities of these materials. So lighter material will be floating and then heavier material will be settling. Because all the particles whether the lighter medium or the heavier medium that are present in the mixture, all of them are having same size now. You crushed in such a way that they are having almost kind of a uniform size, same close to each other size.

So whatever the separation is there that will take place only based on the density. So if you have a kind of density difference but particles are of are not of uniform size then separation may not take place. Because the particle size is also playing role in a settling velocity that we know already.

So basically if you wanted to make a kind of separation based on the density difference, you have to make sure that the particles having different densities are crushed to one same size. And then, then mixed them with a kind of liquid here which is having the density in between of these two densities heavier and then lighter, medium densities. So liquids must have densities greater than water since most solids have high densities. But unfortunately a few such liquids exist that are economical and non-corrosive, that is a big problem.

And then also in general it is not possible to find out a kind of a liquid which is having a kind of density in between these two solid medium densities. Then again it becomes a problem.

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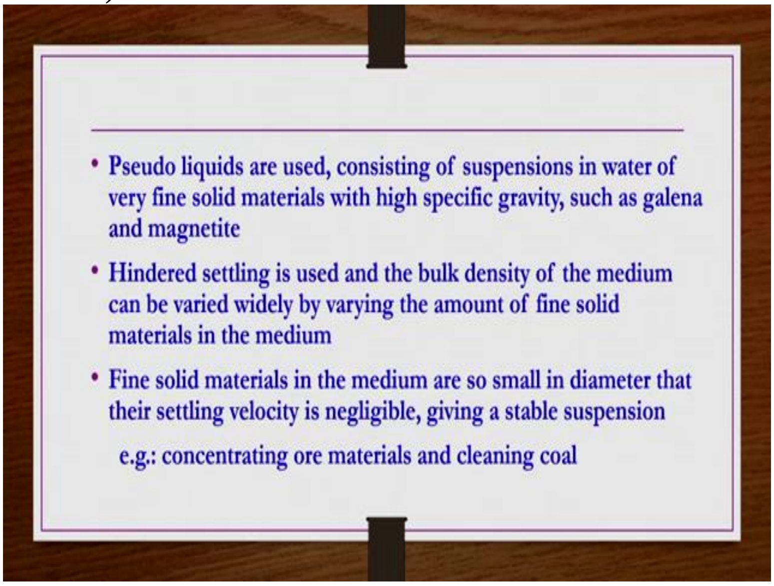

So then in order to avoid such kind of problems people have used the pseudo liquids, where these pseudo liquids are consisting of suspension in water of very fine solid materials with high specific gravity such as galena and magnetite. So when such fine particles of these materials which are having high specific gravity when you mix with a kind of liquid or water something like water, then they will be forming a kind of suspension. Suspension and then stable suspension they may form. Then their density would be higher than the water densities in general because the specific gravity of these materials is very very large compared to the specific gravity of water.

Then probably it is possible that you can have a kind of suspension which is having the density in between of the in between the density of these two materials, heavier and then lighter material. Hindered settling is used and then bulk density of the medium can be varied widely by varying the amount of fine solid materials in the medium. Find solid materials in the medium are so small in diameter that their settling velocity is negligible, giving a stable suspension. Example, concentrating ore materials or cleaning coal etcetera.

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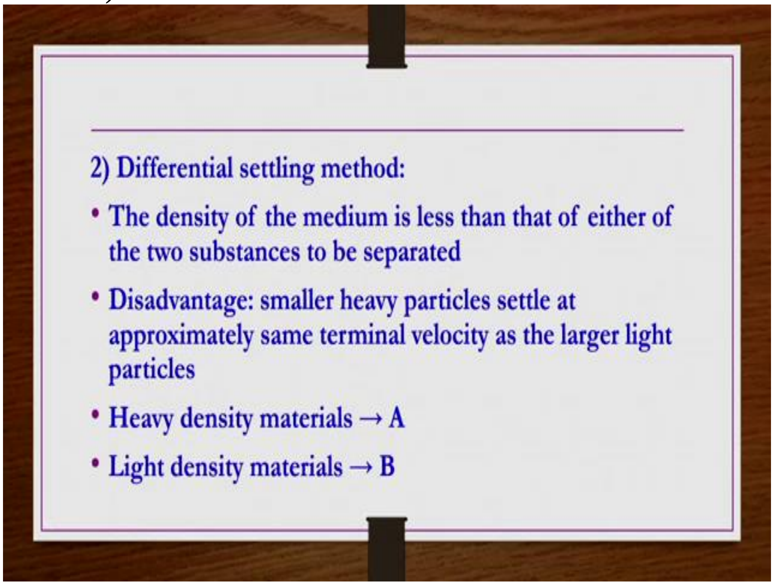

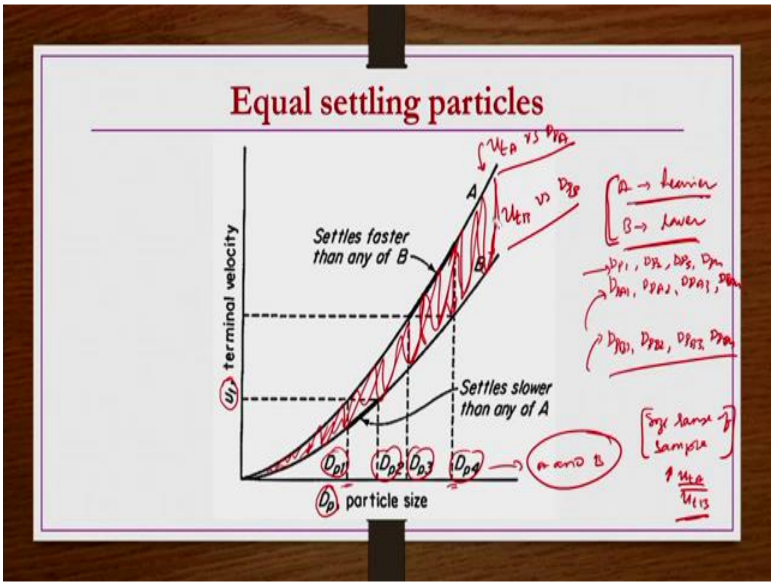

The next one is the differential settling method. So here the density of the medium is less than that of either of the two substances to be separated. And then disadvantage of this method is that smaller heavy particles settle at approximately same terminal velocity as the larger light particles. Let us say assume the heavy density material designated as A. Lighter density material is designated as B. Then if you measure their settling velocity for different size of these materials, then equal settling particles what we can have. So now these settling velocities u_t with respect to the particle size. Whatever the A heavier material or the higher density material and then B lower density material is there, so these materials now having different size ranges.

Let us say both of them are having D_{pA1}, D_{pA2}, D_{pA3}, D_{pA4}. Actually this material here in the picture D_{p1}, D_{p2}, D_{p3}, D_{p4} are designated here. So what does it mean by like you know, they are valid for this A and B also. B is also having you know this wide range of particle. Both A and B type particles are having wide range of distribution from D_{p1} to $D_{p4.}$ It is not like that they are valid only for A or B, this is valid for both A and B. Both A and B are having wide range of particle size distribution varying from D_{p1} to D_{p4}. Now these particles are in a kind of mixture, you wanted to separate them, you wanted to separate them in a kind of a two fractions as a kind of pure fractions, pure fractions as much pure as possible.

So what you do, first you take different size of these particles individually. And then their measure terminal velocity within this range of D_{p1} to D_{p4} because D_{p1} to D_{p4} are the size range of the sample. So within this size range of the sample for both A and B type particles, you find out u_{tA} and u_{tB}. So that you plot. So the this is u_{tA} versus D_{pA} and this is u_{tB} versus D_{pB}. Why this A is above B curve? Because A particles are heavier, so their settling velocity is obviously going to be larger than the settling velocity of the lighter particles. So that is the reason like this.

And then this gap is you know, you can see the gap is increasing gradually, so that between these two curves why it is increasing? Because you know the not only density particle size is also making a difference. When let us say particle size range is small for the given sample, then the difference between the settling velocity of these two types of particles maybe small. But when you have bigger particles, the particles are big and then density difference is also big. So because of that one, the difference between this terminal velocity of these two type of particles is going to be large. So that is the reason the space between these two curves is a kind of increasing with increasing the size of the particle.

If the particles are very small sized, fine microns or something like that then their setting velocities are going to be almost same because almost close to each other, even if one of the material is lighter and another one is a kind of heavier kind of thing. Because you know usually under stocks region these settling velocities are going to be very very small. That is the one.

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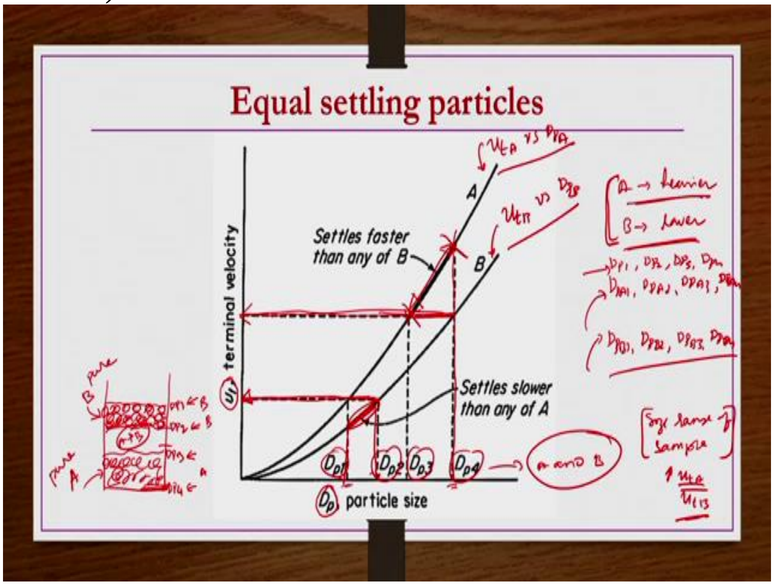

The second one is that now, how do you make a kind of separation of this particle when you are mixing in a kind of liquid? And then liquid is having the density lower than both the material. Because we want this material to be settled in a kind of classifier but settle such a way that you know the particles which are having the largest settling (particles) settling velocity should be at the bottom and then particles having the lower setting velocity should be at the forming a kind of top layers. And then liquid is above that one. We do not want like in a sink and float kind of thing.

In the sink and float you have to find out the liquid of intermediate density so that one material is floating, the lighter material is floating the heavier material is settling. But here we want a kind of liquid which is having the lower density compared to the density of these two different types of material $\rho_A$and ρ_B. ρ should be less than ρ_A and less than ρ_B as well so that both the particles settle. But the particles you know heavier particles larger size should be forming as a kind of bottom layer. And then finer and then lighter particles should be forming as a kind of top layer.

So now you select two different terminal velocities, equal terminal velocity. So here at this case when you have the one terminal velocity, whatever this range of the size range of this particle is there, this size range of the particle is going to have a kind of the largest terminal velocity for A type particles compared to the any of the B type particles, compared to the any of the B type particles. So that you know these particles will settle as a kind of bottom like this, like this. So this would be A material. And then what happens? The other lower end of the terminal velocity you to take another terminal velocity.

So here you can see this range of the material whatever is there D_{p1} to (D_{pB}) D_{p2}, the B type material is having the lowest settling velocity compared to any other particle of A type. So they may be forming a kind of pure layer because their settling velocity lowest compared to the all other material of A or B, whatever you take. So then this is B. This is almost pure A. This is almost pure B. Almost complete pure fraction is not possible, pure fractions making is not possible for any of these methods.

So in between what happened? We can have a kind of mixture A and B, we can have mixtures can be there. So let us say this is D_{p4} particles having the largest settling velocity. So they will be settling at the bottom and then D_{p3} particles having something like this. And D_{p2} and then D_{p1} are these smallest size particles which is having the lowest settling velocities. So they are being at the top. And then these two should be for B, because the lighter ones. These two should be for A, because the heavier ones.

And then in the mixtures both the particles solid and whatever these ranges are there, these two ranges between D_{p2} to D_{p3} or D_{p1} to D_{p3} or D_{p2} to D_{p4} are possible some fractions. This is what happens in equal settling particles. So by these principles we can make a kind of separation based on the equal settling velocities.

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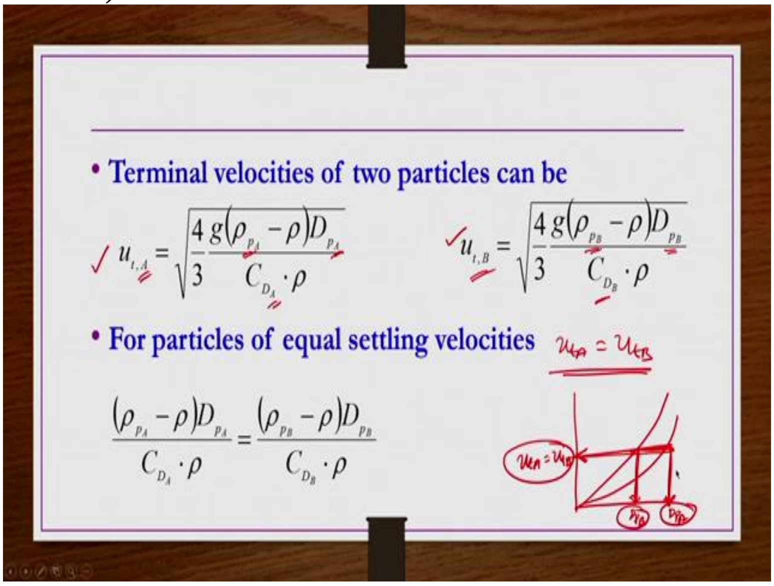

So terminal velocity of two particles, let us say we have already derived it as $u_{tA} = \sqrt{\frac{4}{3}\frac{g(\rho_p-\rho)D_{p_A}}{C_{D_A}\rho}}$ also. This is generalized one, we are not taking any Newtonian or Stokes flow regime that we are not taking. Similarly for particle B, the terminal velocity generalized equation if you write, so you can have this suffix B in this case.

So this is the terminal velocity of A particle, this is the terminal velocity of B particle, generalized one without knowing the flow regimes. So now for particles of equal settling velocities that means when u_{tA} is equals to u_{tB}, the previous one differential setting whatever the graph we have seen that line you know, so that means this line, so this line now here corresponds to u_{tA} is equals to u_{tA}. So these let us say this terminal velocity when it is same equals for both the particles A and B particles then corresponding size ranges are whatever there. So this is D_{pB}, D_{p_A}. Because A are the heavier size, so they will have a kind of the largest size particles you know, will have a kind of largest settling velocity.

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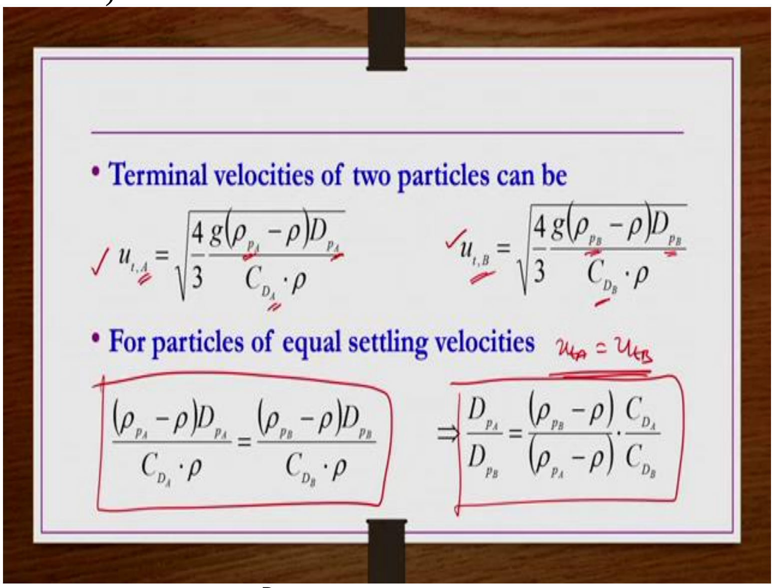

So then from here what we can get? $\frac{D_{pA}}{D_{pB}}$ from here when you take u_{tA} is equals to u_{tB} from these two from these two expressions, you will get $\frac{(\rho_{pA}-\rho)D_{pA}}{C_{DA}\rho}$. From here $\frac{D_{pA}}{D_{pB}}$ if you do, you will get this expression that is $\frac{(\rho_{pB}-\rho)C_{DA}}{(\rho_{pA}-\rho)C_{DB}}$. So this is generalized one without taking any assumptions of Stokes or Newton's flow regime. So that is the reason C_{DA}s are written say C_{DB} are written as it is.

C_{DA} is nothing but drag coefficient for the A particle, C_{DB} is nothing but drag coefficient for the B particle. u_{tA} is nothing but the terminal velocity of A particle and then u_{tB} is nothing but terminal velocity of the B particle. Let us say in the mixture, we have a kind of some particles whose terminal velocities are equal. That is there would be some range of particles though their density differences are different, the different sizes maybe there. So there D_{pA} or

D_{pB} their ranges are different. So there will be some range of particles, some size of particles for A type, for B type, for which the terminal velocities are going to same. u_{tA} is equals to u_{tB} that is what going to become.

So what is that size of A, size of B or what is the ratio of the size of A and B type where their terminal velocity are equal? So that is what this is. So that we can find out these things.

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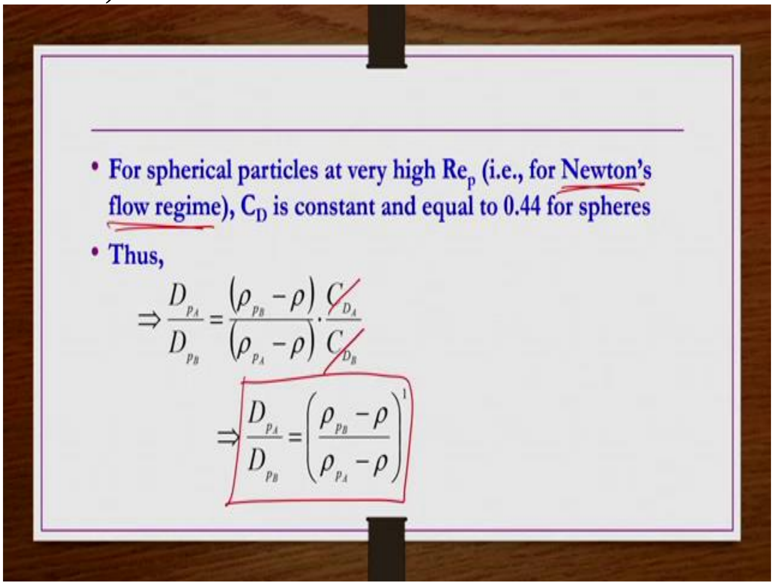

Now let us say for spherical particles if you take Newton's flow regime that Re_p greater than 10^3, then C_D is constant and equals to 0.44 for spheres. So then from this equation if you substitute C_{DA}, C_{DB}, both are equal because 0.44, 0.44 are the values for both of them under the Newton's flow regime. So then what does mean by, what does it mean by $\frac{D_{pA}}{D_{pB}} = \frac{(\rho_{pB}-\rho)}{(\rho_{pA}-\rho)}$.

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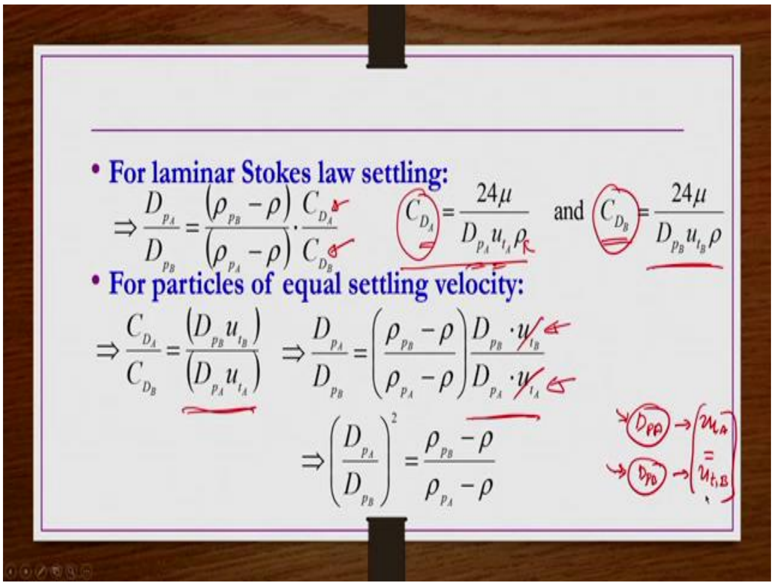

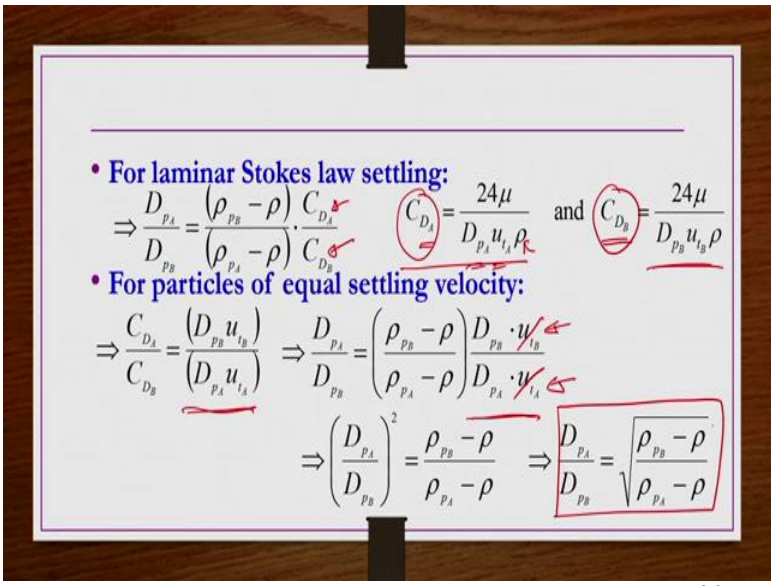

Similarly for laminar Stokes settling velocity, then we know that C_D is $\frac{24}{Re} \cdot \frac{24\mu}{D_p u_{tA} \rho}$. So now for A, D_{pA}, u_{tA} and then this should be rho is a kind of density of the liquid. It is not for the solid, so you cannot have a ρ_A kind of thing, it is for the liquid. ρ_A and ρ_B are for the A and B type particles.

Similarly $C_{DB} = \frac{24\mu}{D_{pB} u_{tB} \rho}$. So when you substitute this generalized expression, $\frac{D_{pA}}{D_{pB}} = \frac{(\rho_{pB} - \rho) C_{DA}}{(\rho_{pA} - \rho) C_{DB}}$. Here you substitute the C_{DA}, here you substitute C_{DB} and simplify. Then particles of equal settling velocity we can have this equation. From here we need $\frac{C_{DA}}{C_{DB}}$. So $\frac{C_{DA}}{C_{DB}}$ if you do, you will get $\frac{D_{pB} u_{tB}}{D_{pA} u_{tA}}$. So in place of $\frac{C_{DA}}{C_{DB}}$, now we are writing $\frac{D_{pB} u_{tB}}{D_{pA} u_{tA}}$. So now DpA we take the other side so that $\left(\frac{D_{pA}}{D_{pB}}\right)^2 = \frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}$. And then this $\frac{u_{tA}}{u_{tA}}$ are cancelled out because they are for these equations are for the equal settling velocity.

There are some particles of A of size let us say D_{pA} whose terminal velocity is same as particle type B whose size is let us say D_{pB}. So that is what let us not confuse, D_{pA} are having u_{tA} particle, some size we do not know no what it is. D_{pB} is also having some terminal velocity, so let us say u_{tB}. So there will be some value of D_{pA}. So let us say here depending on the difference this is heavier, this is lighter. So then some not the largest size, in between size particles you know, medium size of D_{pA} and medium size of D_{pA}. You know they may be having some kind of range or some kind of value at which this velocities maybe become equals to each other. What is that D_{pA} D_{pA} value, that is what we are going to find out for different regimes.

So Newton's regime we have already found this, this ratio of $\frac{D_{pA}}{D_{pB}} = \frac{(\rho_{pB}-\rho)}{(\rho_{pA}-\rho)}$. Now under the case of Stokes regime, we can see this ratio is nothing but $\sqrt{\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}}$

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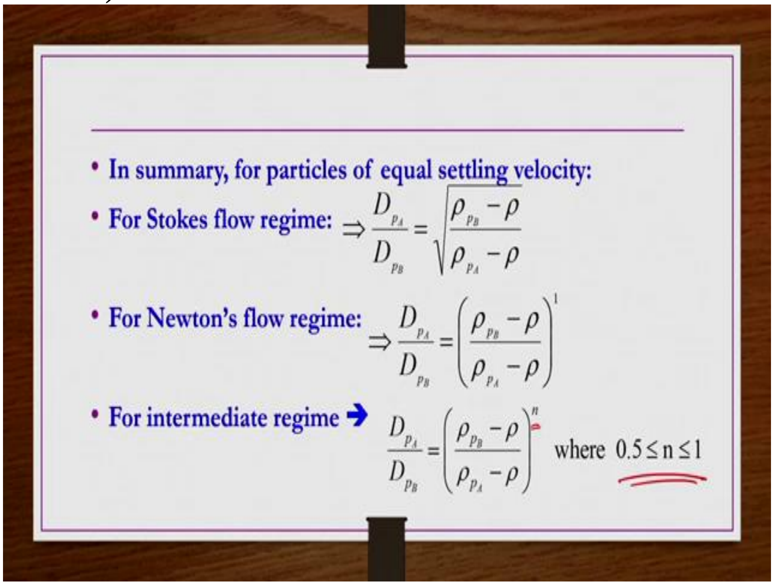

So if you summarize, in Stokes regime $\frac{D_{pA}}{D_{pB}}$ is going to be $\sqrt{\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}}$. Under Newton's flow regime $\frac{D_{pA}}{D_{pB}} = \frac{(\rho_{pB}-\rho)}{(\rho_{pA}-\rho)}$. That means and that the intermediate range, if you write it as like $\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}\right)^n$, so then n is going to vary between 0.5 to 1. 0.5 for Stokes regime, 1 for the Newton's regime, in between it is going to be 0.5 to 1 for the intermediate regime.

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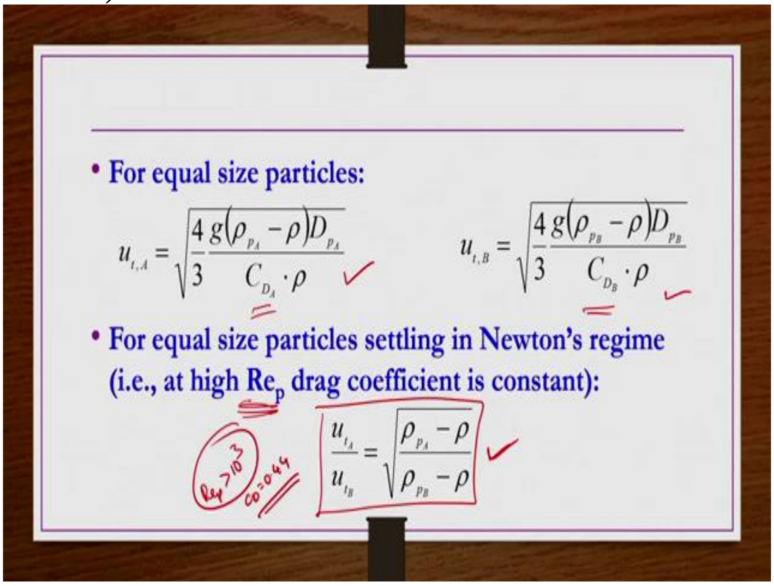

Let us say equal size particles, equal size particles then u_{tA} is same equation, u_{tB} is also same equation. But now we are taking equal size particles settling in Newton's regime at higher Re_p higher Re_p, drag coefficient is constant, so C_{DB} is equals to $C_{DA} = 0.44$. That is cancelled out from this equation, what we get? At higher Re_p, at higher Re_p, Reynolds number that is Re_p>10^3. We have a Newton's flow regime, so under that condition C_D is going to be 0.44. So it is going to be same 0.44 for both A and B type particles.

So then we can write $\frac{u_{tA}}{u_{tB}}$, previously when the equal settling velocities are there then we have seen the ratio of you know particle sizes which are having the equal settling velocity. Now here what we are trying to do? Here we are trying to find out the equal size particles. Size of the particles same when you keep same. So then their corresponding terminal velocities are going to be different anyway because of the density difference. So what is the ratio of those two terminal velocity, that is what this one. This is for the Newton's flow regime.

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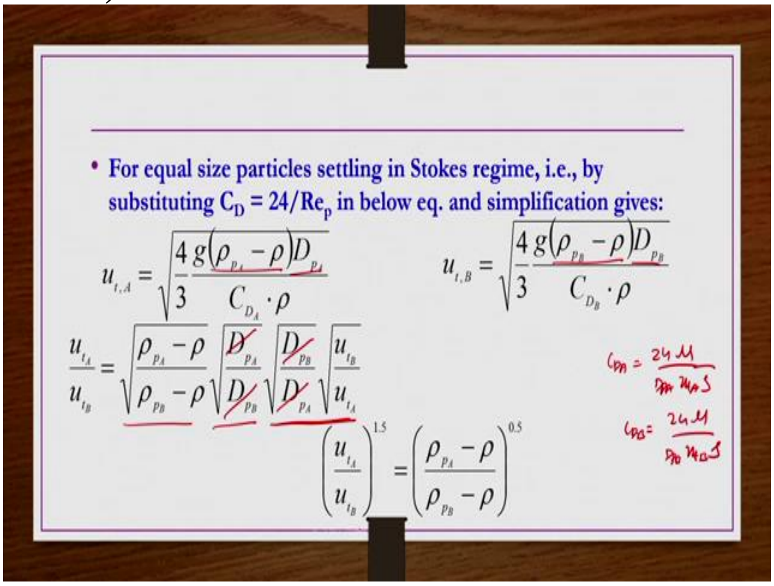

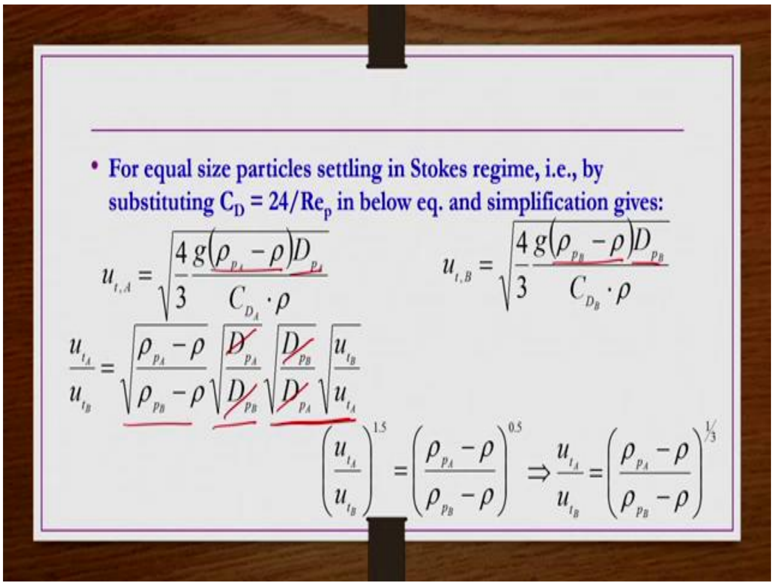

So now Stokes flow regime, for equal size particles settling in Stokes regime, that is by substituting C_D is equals to $\frac{24}{Re_p}$ in below equation and simplifying on simplification will give you this thing. So here what we do? We take $\frac{u_{tA}}{u_{tB}}$. So $\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}$ would be as it is and $\frac{D_{pA}}{D_{pB}}$ would be as it is like this and then this square root of this $\frac{C_{DB}}{C_{DA}}$, whatever will come here. So that if you substitute C_{DA} is equals to $\frac{24\mu}{D_{pA}u_{tA}\rho}$ you substitute. And then C_{DB} is equals to $\frac{24\mu}{D_{pB}u_{tB}\rho}$ you substitute and simplify. So then that ratio will come out to be $\sqrt{\frac{D_{pA}}{D_{pB}}}$, $\sqrt{\frac{u_{tB}}{u_{tA}}}$ so that $\left(\frac{u_{tA}}{u_{tB}}\right)^{1.5}$ should come out as $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{0.5}$.

When you substitute cancel out like these terms, this is same, this is same, cancelled out. This one, this one is cancelled out. And then this $\sqrt{\frac{u_{tB}}{u_{tA}}}$ you should take to the left hand side, you have $\left(\frac{u_{tA}}{u_{tB}}\right)^{1.5}$. And then right hand side $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{0.5}$ is as it is. So that you know $\frac{u_{tA}}{u_{tB}}$ what you will get, you will get $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{1/3}$. This is for the Stokes regime.

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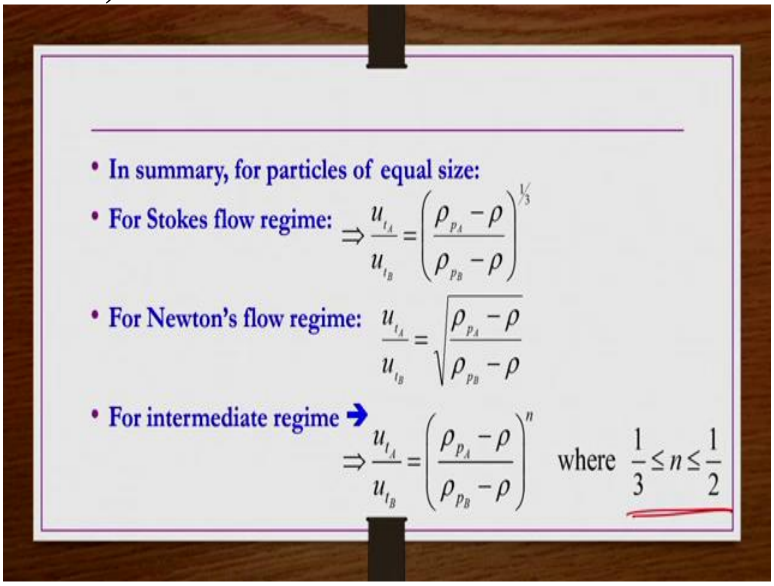

So in summary, Stokes flow regime we have seen that $\frac{u_{tA}}{u_{tB}}$ is equals to $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{1/3}$. If it is Newton's flow regime we have seen that $\frac{u_{tA}}{u_{tB}}$ is equals to $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{0.5}$. So in the intermediate range if you write let us say $\frac{u_{tA}}{u_{tB}}$ is equals to $\left(\frac{\rho_{pA}-\rho}{\rho_{pB}-\rho}\right)^{n}$, so that n varies between 1/3 to 1/2 for intermediate range.

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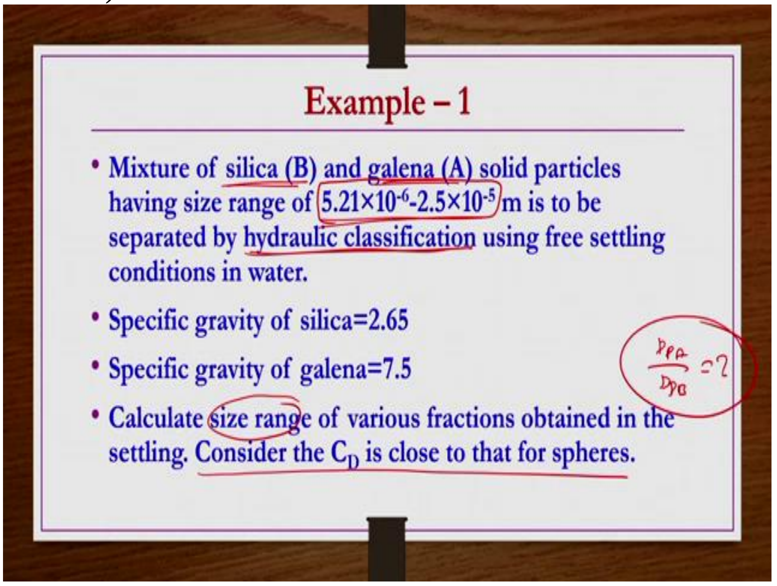

So now we take an example problem to understand how to use this equation. Mixture of silica and galena solid particles having size range 5.21 x 10^{-6} to 2.5 x 10^{-5} meters is to be separated by hydraulic classification using free settling conditions in water. Hydraulic classification, that means we have to use water as a kind of liquid and this both silica and then galena particles are in this range. This range of particle is there that is for both of the particles. Both type of particles are there in this range.

It is not for only anyone kind of thing. Specific gravity of silica is given, specific gravity of galena is given. Calculate the size range of various fractions obtained in the settling. Consider the C_D is close to that of spheres. So we need to find out the size range. That means $\frac{D_{pA}}{D_{pB}}$ we have to find out depending on which range, which regime the particles are settling. So first you have to find out which regime of particles are they settling because it is Newton's, then this ratio is different. If it is Stokes regime then it is different. So we have to find out that regime first.

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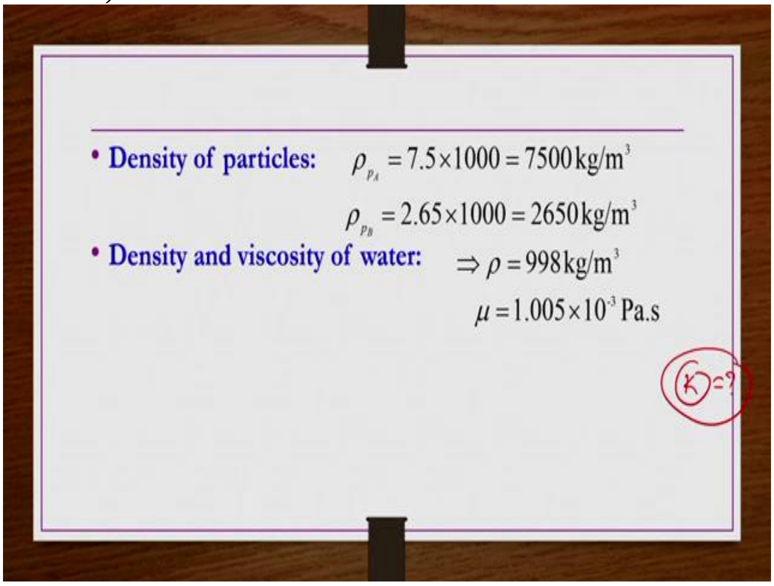

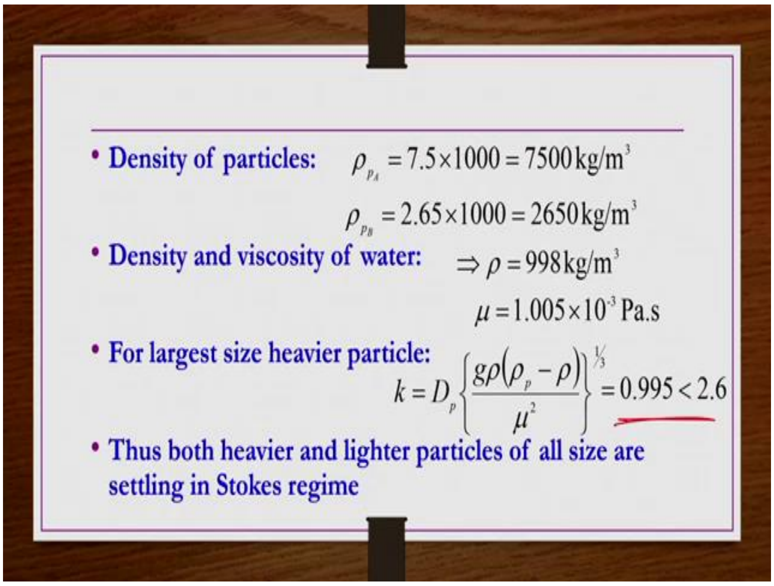

So density of particles if you multiply by density of, because you know the specific gravity are there given. So if you multiply by the water density then you will get the density of these particles. So then ρ_{pA} is equal to 7500 kg/m^3. ρ_{pB} is equals to 2650 kg/m^3. Density and viscosity of water or ρ is 998 kg/m^3, µis equals to 1.005 x 10^{-3} Pa-s for larger size heavier particles.

So now first we have to find out K, what is this K range? So in order to, for which particles you have take? You have to consider largest settling particles. If the largest settling particles under the Stokes regime, so whatever the remaining particles will also be settling, smaller particles will also settle in the Stokes regime. That is the regime for the largest and then heavier particles you have to see the K value. So for largest size heavier particle, K if you calculate it comes out to be 0.995 which is less than 2.6, that means both heavier and then lighter particles of all size will be settling in Stokes regime. Because the heavier particles are going to have the largest settling velocity compared to the lighter particles.

And then out of these heavier particles largest particle size whatever is there, that is going have a kind of highest settling velocity compared to all other particles. So for that case, that is largest size of heavier particles, that itself is in the Stokes regime. So then all the type of material both heavier as well as the lighter particles you know they will (set) settle under the Stokes regime only. Because their size range and the density such a way that their combination will always be providing that K value less than 2.6 only. Because the K value for the largest heavier particle is less than 2.6.

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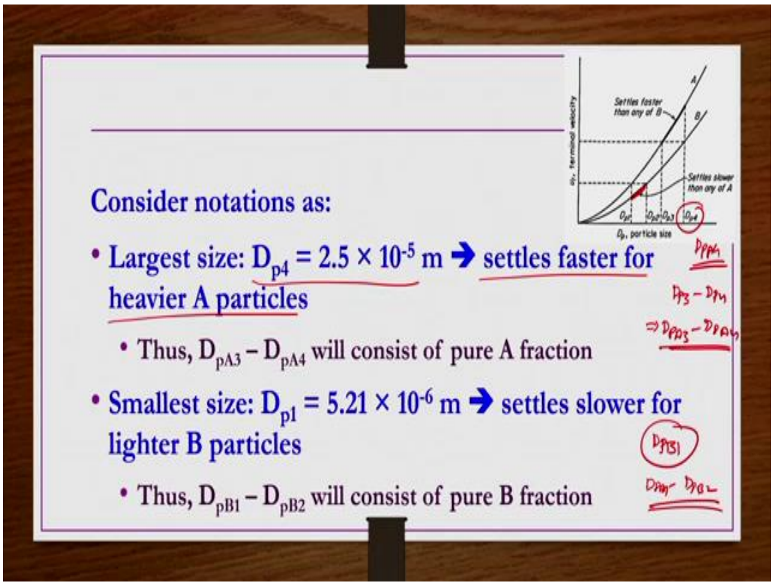

So from these particles from here this we consider largest size range is 2.5 x 10^{-5} m, settles faster for heavier particles. So now here this as I mentioned this D_{p1}, D_{p2}, D_{p3}, D_{p4} all are valid for a kind of you know A and B type both the type. So now the largest particle D_{p4} that should be can be given as a kind of D_{pA4}, which will be settling faster than any other particle in the mixture. Thus so whatever the D_{p3} whatever the D_{p3} to D_{p4} is there, that is going to be pure A fraction D_{pA3} to D_{pA4}. Because these are the largest particle and then heavier larger size particle.

Heavier particles are the A type of particles. And then out of those heavier particles D_{p4} size is the largest one. So D_{p4} should be designated as a kind of D_{pA4} because that is going to have a kind of largest velocity. The next one is going to be D_{pA3}. So this ratio we have to find out. D_{pA4} is already known. This is 2.5 x 10^{-4}m. Similarly smallest size lighter ones slower for the smallest size whatever theD_{p1} is there, so that settles slower for B type. So this is these two one. So this whatever the D_{p1} is given that should be D_{pB1}. And then D_{pB1} to D_{pB2} is going to be the fraction which is having the slower lower settling velocity which may be forming a kind of almost pure B fraction at the top which is having the lower density. So D_{pB1} to D_{pA3} will consist pure B fraction.

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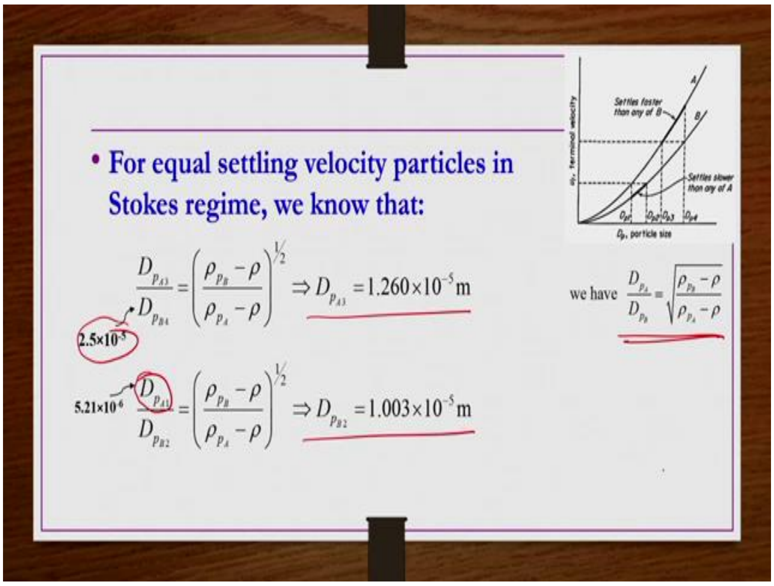

So this ratio we are going to find out here. So we have this equation for Stokes regime $\frac{D_{pA}}{D_{pB}}$ is equals to $\sqrt{\frac{\rho_{pB}-\varrho}{\rho_{pA}-\varrho}}$. So for equal settling velocity particles in Stokes regime we have this one. So now $\frac{D_{pA3}}{D_{pB4}}$ so D_{pB4} is known, so that is nothing but 2.5 x 10^{-5}. So from here you can find out D_{pA3}. Similarly $\frac{D_{pA1}}{D_{pB2}}$ is equals to $\sqrt{\frac{\rho_{pB}-\varrho}{\rho_{pA}-\varrho}}$. Here again D_{p1} is known, this is the 5.21 x 10^{-5}. So D_{pB2} you can find out.

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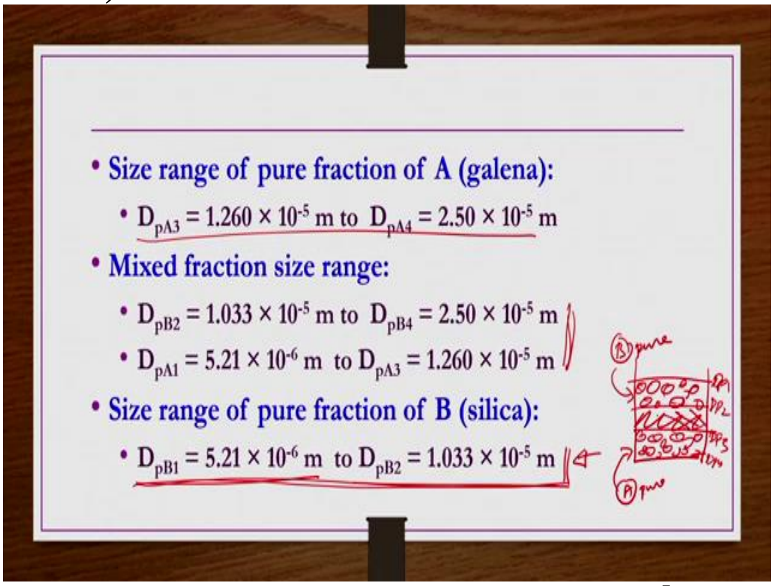

So now what we have? Pure fractions of A, D_{pA3} that is 1.26 x 10^{-5} m to D_{pA4}, that is 2.5 x 10^{-5} m sized particles of A type particles will be settling at the bottom, will be settling at the bottom because these particles are going to have a kind of a larger settling velocity compared to any of the B type particle, any of the B type particle. So pure A fraction is forming at the bottom of having size range this one and then then size range of pure fraction of B, that is D_{pB} that is 5.21 x 10^{-6} meter to D_{pB2} 1.033 x 10^{-5} m. That is the you know finest particles or the smallest smaller size particles. But you know lighter density materials.

So they will be forming a kind of almost pure B fraction at the top because no A type particles are going to have a kind of settling velocity as low as the particles of this range of B type particles are having. So all the A type particles are having the velocity higher than the these B type particles of this size range. So that is the reason these particles are going to form a kind of almost pure fraction layer at the top and in between there may be a kind of mixed fraction of these ranges.

So in this range you may be having a kind of D_{p1}, D_{p2}, D_{p3}, D_{p4}. So here so this is at the bottom of size of the particles whose range is between D_{p3} to D_{p4} of A type will be forming a kind of pure A fraction. And then particle size whose range in D_{p1} to D_{p2} of B type forming a kind of pure fraction at the top and in between we can have a kind of mixed fractions.

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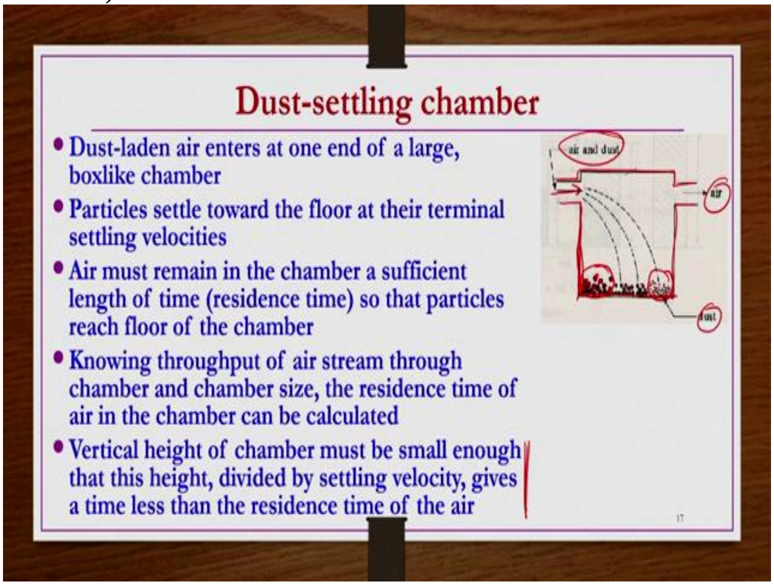

Now we will be discussing a few classifier equipment. Let us start with dust settling chamber. So dust settling chamber is nothing but a kind of box like chamber like this. It is a kind of bigger size something like this. We have a box like chamber. In this one dust laden air or gas whatever is there that is allowed to pass through. Then this air is having the dust particles, these particles are having different size distributions. So there may be some fine particles and there may be some kind of coarse particles are maybe there in this air.

So the particles which are having a kind of bigger size and then heavier density they may be having a kind of higher settling velocity. So obviously when they enter into the chamber, they maybe settling close to the wall of the inlet side of the chamber. This is the inlet side of the chamber, towards this one the coarse particles heavier particle will be settling because their settling velocity is very large. So there maybe some particles which are very fine and then their density maybe low. So those particles what happens you know, their settling velocities is very small. It is very small compared to the other particles.

Sothey may be travelling, taking long time to settle and then in that long period of time they will be settling at the other end. Other end like in the sense towards the exit side whatever the chamber wall is there, that side the particles the fine particles will be collected. And in between the intermediate particles would be collected. So that all the particles would be settled here and almost clear air will be going out.

So this is the basic concept and then solids are in a kind of settled at the bottom and then clear air is passing through from the top. This is the basic principle. So here again one has to do the proper calculation what should be the height of the chamber, what should be the length of the chamber and then what velocity the air should be coming, all those calculations one has to see so that these particles settle according to this manner.

Particles settle towards the floor at their terminal settling velocities. Air must remain in the chamber a sufficient length of time, that is known as the residence time so that particles reach floor of the chamber. Knowing throughput of airstream through chamber and chamber size, the residence time of air in the chamber can be calculated. Vertical height of chamber must be small enough that this height divided by the settling velocity gives the time less than the residence time of the air. So this is condition, how to design, on what basis we should design. Vertical height of the chamber has to be small so that you know whatever the height divided by the settling velocity is there, that will be giving the time unit. That time unit must be less than the residence time of the air so that particles will settle at the bottom.

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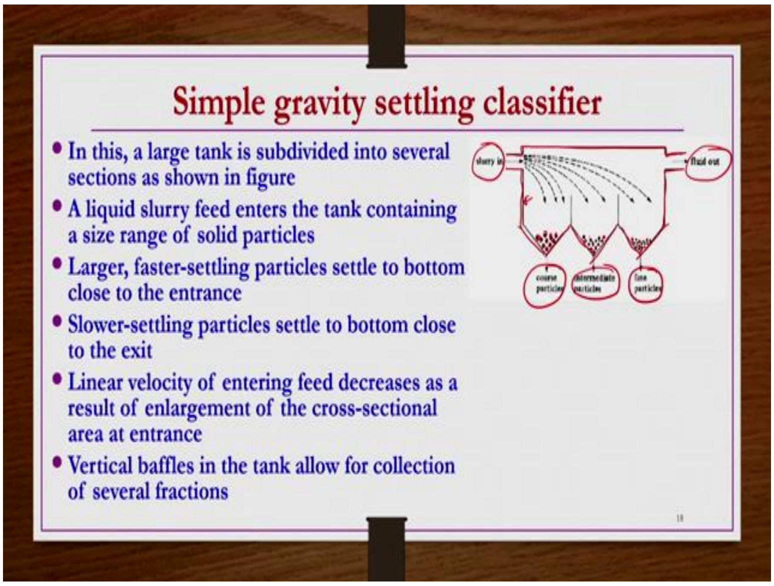

Another equipment is simple gravity settling classifier. Here what we have, we have a large tank like this. But that is divided into several sections. I have shown only three sections, it can be more than three sections also. So whatever the slurry which is containing the particles that comes through here in the left hand side with a certain velocity. Then that slurry again having the particles of different size distribution, so then whatever the particles which are largest size you know, obviously they will be having a kind of higher settling velocity, so they will be settling towards the entrance wall as a kind of coarse particles here.

Whatever the fine particles of the slurry, they will be having the lowest settling velocity compared to all other particles of this slurry. So they will be taking long time to settle and then they will settle at the end of this container towards the exit wall of the tank. So here we get the fine particles, in between we may get kind of intermediate size particles, you may get like this. Almost clear fluid can be taken out as a kind of outlet from the top there. This is again the working principle is same, that is the gravity setting or the setting of the particles under the gravity is the working principle for this classifier. So the same process, only designs are different here.

In this, a large tank is subdivided into several sections as shown in the figure. A liquid slurry feed enters the tank containing a size range of solid particles. Larger, faster setting particles settle to bottom close to the entrance. Slower settling particles settle to bottom close to the exit. Linear velocity of entering field decreases as a result of enlargement of the cross-sectional area at the entrance. Vertical baffles in the tank allow for collection of several fractions.

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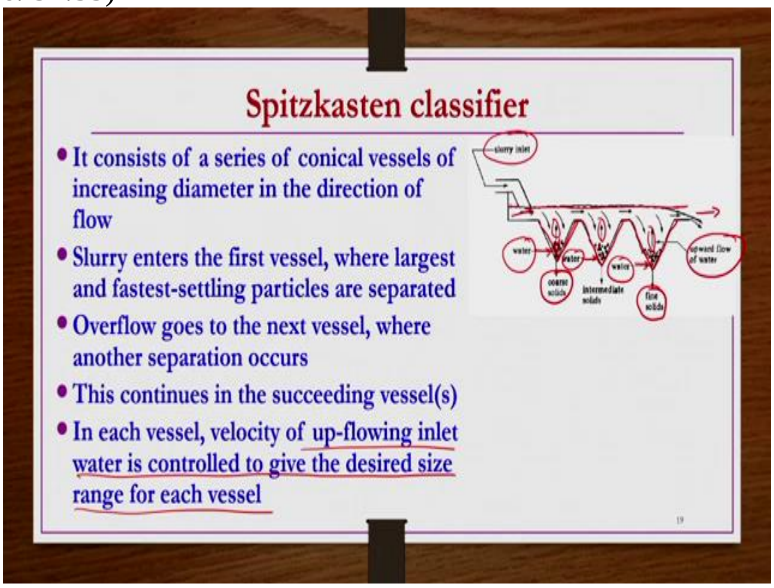

Spitzkasten classifier is the other kind of equipment, here what we have? We have different cones, we have kind of equipment in which you know we have different cones. The size of the cones is increasing in the direction of the flow. In the direction of the flow the size of these cones are increasing. So here the smaller cone, next is intermediate cone. And then at the last is the largest cone is there. Whatever the slurry that is having the solid particles that passes through as an inlet from here. The same settling of the particles under gravity, that same principle is valid here also.

So whatever the particles are there, you know larger particles having the faster settling velocity they will be settling immediately at the first cones in the kind of coarse solids. Now if the particles which are having you know lowest terminal velocity settling velocity under the gravity, you know they will be settling slowly. So they will be settling in the last cone as a kind of find solids. And then in between intermediate solids are collected. So here what is the additional thing, there is a kind of provision for water inlet at the bottom of this cone. And then this water is allowed to flow up so that the particles are settling as per the requirements as there here.

So this upward flow of water is additionally provided here. So that the separation is efficiency, efficiently is done and then almost clear liquid or fluid is taken from the top like this.

It consists of a series of conical vessels of increasing diameter in the direction of flow. Slurry enters the first vessel where the largest and fastest settling particles are separated. Overflow goes to the next vessel where another separation occurs. This continues in the succeeding vessels as well. In each vessel, velocity of up-flowing inlet water is controlled to give the desired size range for each vessel. This flow rate of up-flowing water whatever is there, it has to be tuned in such a way that you know, in each vessel the particle which are being collected they are of desired size. So accordingly that flow rate has to be calculated and then provided.

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So the references for this lecture, we have these many references. You can find these topics in any of these reference books. Thank you.