Mechanical Unit Operations
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Lecture 32 - Gravity Sedimentation – Design of thickeners

Welcome to the MOOCs course Mechanical Unit Operations. The title of this lecture is Gravity Sedimentation – Design of thickeners. Under the gravity separation processes, we have discussed about classifiers in the previous lecture. This lecture we will be discussing about the clarifiers. What does it mean? Let us say if you have a concentrated suspension, then how to separate it into clarified liquid and a thick and sludge kind of thing, so that is what to be done is the kind of clarifier.

So, how to do that one and what are the design calculation, et cetera, those things we are going to discuss in this particular lecture. Again the principle is same, it is based on the gravity. Separation is based on the gravity as in the case of in the classification also. In the classifiers, we separate the particles into two fractions, two solid fractions, pure A, pure B fractions like that. But here in what we do in clarifiers, virtually almost all solid particles are separated as a kind of sludge and giving a kind of a, almost clear or almost free from solid kind of clear liquid you can get as a kind of a clarified liquid.

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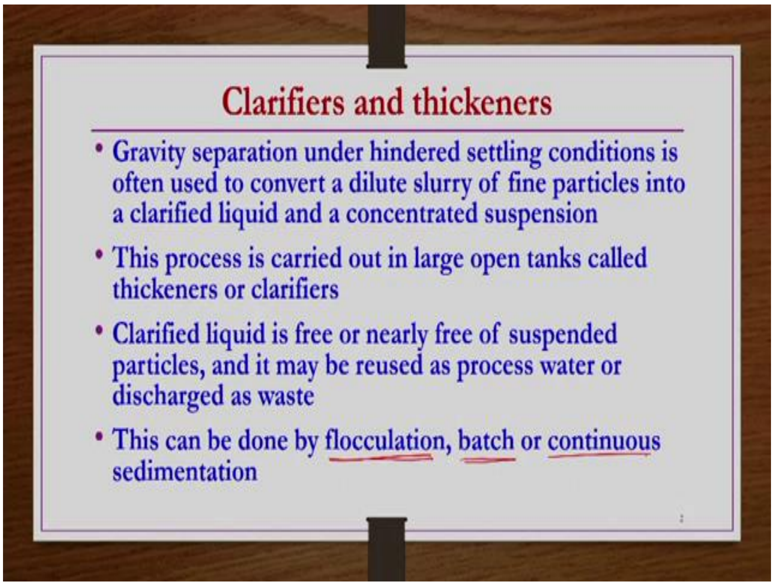

Clarifiers and thickeners

- Gravity separation under hindered settling conditions is often used to convert a dilute slurry of fine particles into a clarified liquid and a concentrated suspension
- This process is carried out in large open tanks called thickeners or clarifiers
- Clarified liquid is free or nearly free of suspended particles, and it may be reused as process water or discharged as waste
- This can be done by flocculation, batch or continuous sedimentation

Clarifiers and thickeners – Gravity separation under hindered settling conditions is often used to convert a dilute slurry of fine particles into a clarified liquid and a concentrated suspension. So, here the separation is taking based on the settling velocity of the particle but settling velocity is not kind of one single particle, so there are large number of particles are there, so a kind of hindered settling conditions may be prevailing.

However, there will be a kind of different regions of settling where this velocity may be constant or rapidly decreasing to a certain extent and there may be some other extent where this may be decreasing very slowly. This process is in general carried out in a large open tanks called clarifiers or thickeners. Clarified liquid is free or nearly free of suspended particles in general.

And it may be reused as process water or may be discarded as a kind of waste. Either of way it is useful. So, this can be done by flocculation, batch or continuous sedimentation. So, this can be done by flocculation, batch or continuous sedimentation. So, we are going to discuss each of these three things; flocculation, batch sedimentation and then continuous sedimentation.

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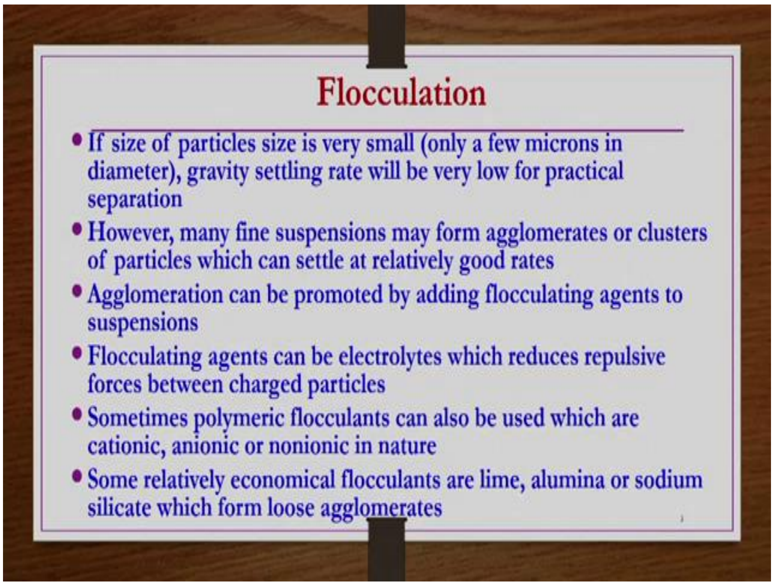

Flocculation

- If size of particles size is very small (only a few microns in diameter), gravity settling rate will be very low for practical separation
- However, many fine suspensions may form agglomerates or clusters of particles which can settle at relatively good rates
- Agglomeration can be promoted by adding flocculating agents to suspensions
- Flocculating agents can be electrolytes which reduces repulsive forces between charged particles
- Sometimes polymeric flocculants can also be used which are cationic, anionic or nonionic in nature
- Some relatively economical flocculants are lime, alumina or sodium silicate which form loose agglomerates

Flocculation – In general, what happens in suspension if you have fine particles, it becomes very difficult to settle for those particles by the gravity or they take very long time to settle due to the gravity field. If at all the particle size is very fine in some micron size in some micron something like that, so such fine particles obviously will take long-long time because their settling velocity is, in general, under the stokes region where the settling velocity may be out of 10^{-5} or 10^{-6} m/s.

When the settling velocity of the particle is so small then obviously it is going to take a long time to settle all the particles and then form a kind of sludge at the bottom of the container, so that is the problem in general. If you have very fine particles of size microns in the slurry which you wanted to separate in a thickener as a kind of thick sludge as well as a kind of a clear liquid.

So, what we do in general, agglomeration of these particles may be done. So if the particles may be agglomerated then they may be having a kind of slightly bigger size, then the same process can be carried out. That is what basically happens in flocculation. If size of particles is very small, that is only a few microns in diameter, gravity settling rate will be very-very slow for practical separation because their settling velocity is out of 10^{-5} or 10^{-6} m/s, such small settling velocity these particles in general, have.

However, many fine suspensions may form agglomerates or clusters of particles which can settle at relatively good rates. How can it be possible? Agglomeration can be promoted by adding flocculating agents to the suspension, right. When you add some kind of flocculating agents, so then some particles will form a kind of agglomerates and then form a kind of bigger particles. So, like that the size of the particles may be increased and then automatically if the size of the particle increased, then obviously their settling velocity will also increase.

So the obviously the separation by this flocculation, or separation by this thickness will be fast or the rate of separation will increase obviously. Flocculating agents can be electrolytes, which reduces the repulsive forces between the charged particles. Sometimes polymeric flocculants can also be used which are cationic, anionic or nonionic in nature, in general. Some relative economical flocculants are lime, alumina or sodium silicate which form loose agglomerates.

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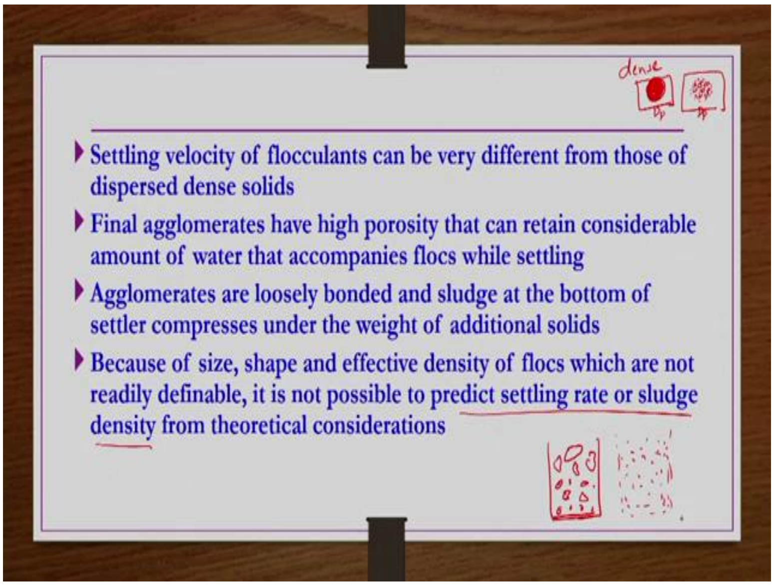

- Settling velocity of flocculants can be very different from those of dispersed dense solids
- Final agglomerates have high porosity that can retain considerable amount of water that accompanies flocs while settling
- Agglomerates are loosely bonded and sludge at the bottom of settler compresses under the weight of additional solids
- Because of size, shape and effective density of flocs which are not readily definable, it is not possible to predict settling rate or sludge density from theoretical considerations

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- Design of thickeners is generally based on measurements of settling rates obtained from batch tests in laboratories

Settling velocity of flocculants can be very different from those of dispersed dense solids. Because final agglomerates have high porosity that can retain considerable amount of water that accompanies flocs while settling, that is the reason settling velocity of flocculants, in general, very different from those dispersed dense solids. Because if you have dispersed dense solids, individual particles, bigger size kind of particles, in general, so those particles may not be having any kind of porous structure kind of thing.

Or you know, obviously, when we say that dense particles, so they will not be having a kind of porous structure, so then they can form a kind of a…They can settle at kind of a quite faster size but the same size flocculants if you are calculating the settling velocity of a two particles, let us say one particle dense solid particle which is not having any kind of porous structure.

And then there is another particle of the same size, let us say like this, so here D_p, here also it is D_p, but these particles flocculants, you know, because of adding flocculants agents this is a kind of a particle having this kind of nature, right. So, whereas here this is a kind of a dense solid, it is not having any kind of porous structure, it is a pure dense particle, so then obviously its settling velocity is going to be much higher compared to the settling velocity of the same size flocculants particle like this.

Because here it is forming because of some fine particles and then between the fine particles which are forming a kind of flocculants, they are having agglomerates which are having kind of, some kind of porous structure, interstitial space is there, so those interstitial spaces are occupied by the water or the liquid in which these are settling. Okay, that is the reason because of this one, the effective volume of the particle in case of flocculent is a kind of a lesser compared to the dense particle of the same size here.

So, that is the reason, you know, their settling velocity can be very different from those of dense particles. Agglomerates are loosely bonded and sludge at the bottom of settler compresses under the weight of additional solids. So, because of the size, shape and then effective density of flocs which are not readily definable, it is not possible to predict the settling rate or sludge density from theoretical consideration, in general, for flocculation.

So, it has this kind of flocculation or sedimentation or thickening of the fine particles forming a suspension if you want, that is if you have a suspension which is having particles of very small size like in some micron size, so then having theoretical calculation in order to get the settling rate etc. or sludge density etc. from theoretical consideration is a kind of a difficult because of the size, shape, and effective density of flocs which are not readily definable, because they are forming.

What we are doing here? We are adding some kind of flocculation agents so that agglomerates are forming, so these, when you are adding these flocculating agents, the agglomerates, whatever are forming they may be of different size and shape and then effective density may also be different. It is not necessary that you know, you have these n number of finer particles, let us say very-very small particles like this, right.

So, here the effective volume is different of each particle in general. So, now if you have done a kind of a flocculation, agglomeration by adding flocculent, so then they may be forming a kind of agglomerates like this of different size and different shapes, something like this. Because we are in situ in the process, so the size and shape are different, and their effective volume would also be different because of, you know, the nature how they form, they form by loosely packing this kind of individual particles.

So, since, because of these many variations are there in their properties, getting their properties readily defining or readily defining the size, shape and then effective density of such flocculants is very difficult. Because of that theoretical calculations in order to get the settling rate or sludges or sludge density etc. are not possible in kind of a flocculation. Okay? Then design of thickeners is generally based on measurements of settling rates obtained from batch tests in laboratories in general.

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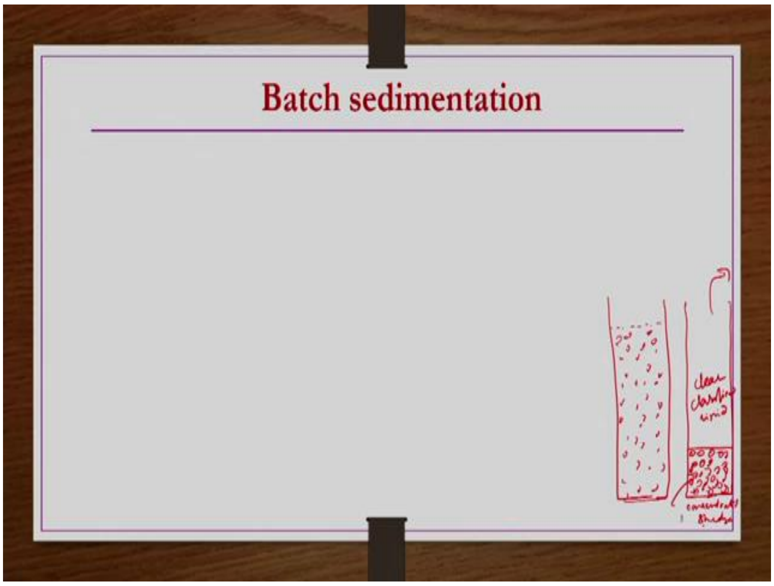

Batch sedimentation

So, what is batch sedimentation? Simply, if you see basically batch sedimentation like you know, it is nothing but you have a kind of cylindrical container in which you take a kind of slurry, whatever the slurry that you have or concentrated suspension that you are having, so that you feel here in this container, right. So, then what happens?

If you allow sufficiently large time for this, you know, suspension to settle, after some time what you can see, after some time what you can see, you can see kind of all the particles are being settled and then forming a kind of a sludge at the bottom like this. Leaving behind almost clear or clarified water or liquid kind of thing, you can see on the top of it, and then concentrated sludge you can see as a kind of separate space at the bottom.

This is what happens in general in batch sedimentation, but from this initial stage to this final stage you go, there are intermediate different stages that we are going to discuss. This is what basic principle, basically what is happening in batch sedimentation is that, you take a cylindrical container, in that you take a the slurry or the concentrated suspension that you wanted to separate into concentrated sludge and then clarified liquid has two different phases and then allow it to settle.

Because now it is only the settling is the process, gravity settling is the even which is causing this particle to settle but those particles may be taking sufficient time to settle depending on their size, density and then properties of the fluid etc. all those kind of things may be acting a kind of role in this kind of settling. Okay. So, as we have seen settling of particles they are affected by the particle size, they are affected by the particle density, they are also affected by the fluid properties like fluid viscosity and density.

So, we have to allow sufficient of enough time for this suspension to settle into sludge and then give a clear clarified liquid as a separate phase like this. Since, it is a batch process, once this stage has come, one can decant the clear liquid from the top and then sludge can be taken out from the bottom. Okay.

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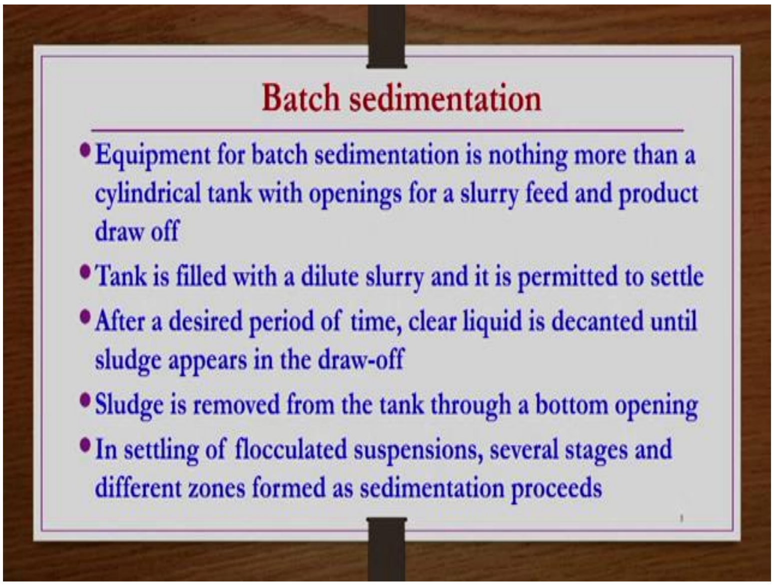

Batch sedimentation

- Equipment for batch sedimentation is nothing more than a cylindrical tank with openings for a slurry feed and product draw off
- Tank is filled with a dilute slurry and it is permitted to settle
- After a desired period of time, clear liquid is decanted until sludge appears in the draw-off
- Sludge is removed from the tank through a bottom opening
- In settling of flocculated suspensions, several stages and different zones formed as sedimentation proceeds

So, what are these steps in between they are existing from initial uniform concentration of the slurry to the final stage of a concentrated sludge and clarified liquid? There are several stages, those several stages we see pictorial anyway. Equipment for batch sedimentation is nothing more than a cylindrical tank with opening for a slurry feed and product draw off. Tank is filled with a dilute slurry and it is permitted to settle.

After a desired period of time, clear liquid is decanted until sludge appears in the draw off. Sludge is removed from the tank through a bottom opening. In settling of flocculent suspensions, several stages and different zones formed as sedimentation proceeds. Okay, there are several stages and different zones are there, so those things we are seeing.

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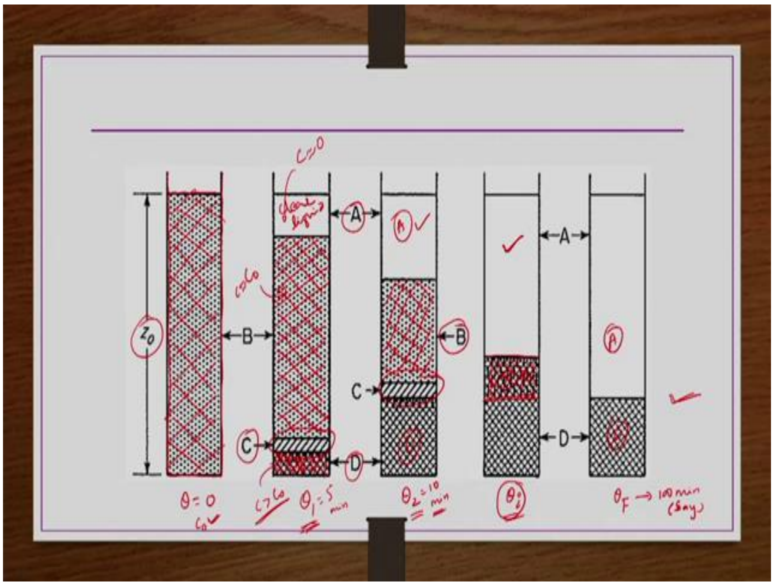

So, let us say, at T is equals to 0 or θ is equals to 0, let us designate time as θ, so now you have a kind of cylindrical tank like this or cylindrical container you have taken if you are doing a kind of laboratory test. If you are doing industrially then it is a kind of big tank, cylindrical tank like this, so in this one you have taken the initial slurry, initial slurry whatever the slurry that you wanted to separate into two phases, so that you have taken.

And then it is height of the, this interface, from here there is nothing above, so then we call it interface, this height is Z_0. Initial height or initial interface height of the liquid that we have taken is Z_0. Now, at some θ_1 time, you allowed some θ_1 time, you allowed these solids to settle, then what happens, at the bottom, some solids are settling as a kind of concentrated sludge and there is a kind of clear liquid is forming a zone at the top.

Because now the particle started settling so gradually they have started settling, so at the bottom more and more particles are being accumulated as a kind of sludge. Since, some particles are settled so obviously at the top we will have a kind of clear liquid phase, and that clear liquid phase is almost free from the particles, nearly free from the particles or virtually clear liquid phase we can see and that zone we are calling it as a kind of A zone.

And whatever the sludge that are forming as a concentrated sludge which is forming at the bottom, that we call it as a kind of D zone, it is just a kind of nomenclature, they do not have any kind of specific reference by abbreviations. Clear liquid you can say, zone A and then sludge that is forming zone D. Now, this B zone is the one which is having a kind of a concentration same as the initial concentration of the feed slurry that you have taken.

Whatever the feed slurry you have taken, let us say feed slurry concentration is C_0, initially. Whatever theta is equals to 0, the feed concentration is C_0 or slurry concentration in the feed is the kind of C_0. So, the B zone is the zone in which the concentration is same as C_0, same as C_0 here also, and then here in clear liquid, the concentration C is approximately 0.

In the B zone the concentration is approximately equals to C_0. In the D zone, this C is much greater than the C_0, some high concentration of the sludge, only solids are there. Only whatever the intestinal spaces between the particles are there, you know that interstitial spaces is only occupied by the liquid kind of thing, it is just a kind of packed bed or K kind of thing that we have seen previously.

And then this zone C whatever is there is a kind of transition zone, is a kind of transition zone which is you know having the different concentration from D from B. Let us say initially first settling time zone, you set as a kind of θ_1 is equals to 5 minutes. After 5 minutes if you take instantaneous picture, probably you can say this kind of thing.

So, where you can see a kind of concentrated sludge at the bottom and then onto some portion and then where you can see a kind of a clear liquid at the top or to some extent, to some height of the container. And you can see a clear interface between A zone and B zone, B zone is maybe having the concentration similarly as it was initially, like by appearance also you can see, so that you can see.

So, interface, whatever interface between A and B zone is there that can be very clear at this point of time. But interface between B and C zones is not very clear, it is not very distinguishable. If you further increase the time, settling time to θ_2, let us say 10 minutes. Now, initially it was θ_1 is equals to 5 minutes. Now, θ_2 let us say 10 minutes.

For example, I am saying, you allow another 5 minutes to settle continuously like that, so then you can see this clear liquid zone A, its size is increasing. Similarly, D size, whatever the sludge that is forming at the bottom, its size is also increasing or its thickness is also increasing. The height of the zone D is increasing, height of the zone A is also increasing, whereas the height of the B zone it is decreasing, because now most of the, this B zone is actually having the concentration same as a kind of initial concentration.

So, what happens, continuously as the particles are going down to D zone and then liquid moving up to the A zone, so obviously its height should decrease, so B zone height is decreasing. Whereas the C zone, whatever the transition zone is there, so that is remaining same here also, that does not change. Then if you let us say, allow settling for some other time θ_i, maybe 30 minutes or 40 minutes something like that, then you can see clearly A zone you can see, clear A zone you can see, clear D zone you can see, their heights are sufficiently larger compared to the previous θ_1, θ_2 size, and then it becomes very different to distinguish this C zone.

Actually C zone now, from this point onwards, it starts disappearing and then finally if you further allow θ_F, let us say final settling time, let us assume if this is θ_F let us say 100 minutes. By 100 minutes let us say it is completely separating into 2 phases. By that time, you know, what you can see, you can see only A and then D zones and there will be whatever the turbid C zone kind of transit, C zone kind of thing is there, that also disappears completely.

Here in this picture, θ_i at time θ_i, what we can see here, there is a kind of concentrated sludge you can see, and then liquid you can see at the top. In between there will be a kind of some region which is a kind of very turbid kind of thing that does not have a kind of connection or appearance, it does not look like A zone in appearance, it does not look like D zone in appearance kind of thing.

So, that is a point where B and C zones are you know kind of, actually B zone is completely disappeared, the C zone is a kind of a mixing up with D zone, or slowly it started decreasing and then there will not be C zone at all after that also. Then we get this clarified liquid and then concentrated sludge as a kind of two separate phases. This is what happens in batch sedimentation, these are the steps in general they are having.

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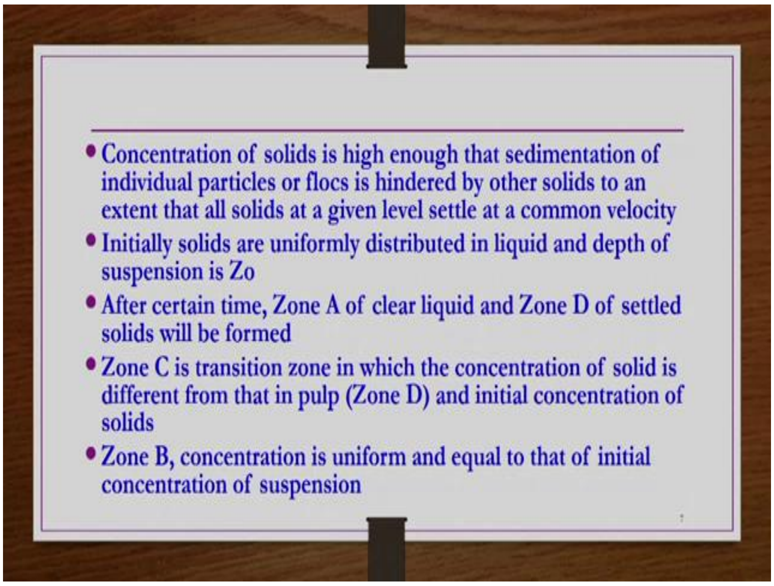

- Concentration of solids is high enough that sedimentation of individual particles or flocs is hindered by other solids to an extent that all solids at a given level settle at a common velocity
- Initially solids are uniformly distributed in liquid and depth of suspension is Zo
- After certain time, Zone A of clear liquid and Zone D of settled solids will be formed
- Zone C is transition zone in which the concentration of solid is different from that in pulp (Zone D) and initial concentration of solids
- Zone B, concentration is uniform and equal to that of initial concentration of suspension

So, the same thing is provided as a kind of notes here. Concentration of solids is high enough, that sedimentation of individual particles or flocs is hindered by other solids to an extent that all solids at a given level settle at a common velocity individually, initial stages. Initial solids are uniformly distributed in liquid and depth of suspension Z_0.

After certain time, Zone A of clear liquid and Zone D of settled solids will be formed. Zone C is transition zone in which the concentration of solid is different from that in pulp Zone D and then initial concentration of solids that is zone B. Okay. Zone B, concentration is uniform and equal to that of initial concentration of the suspension.

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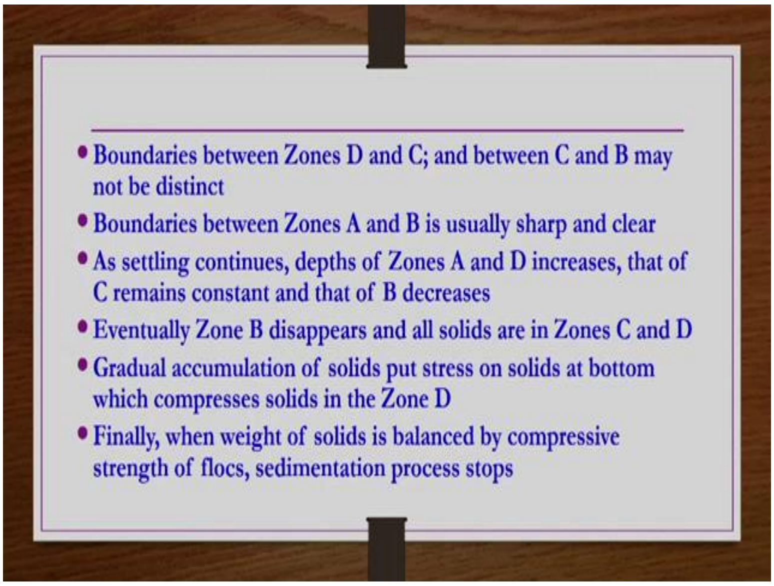

- Boundaries between Zones D and C; and between C and B may not be distinct
- Boundaries between Zones A and B is usually sharp and clear
- As settling continues, depths of Zones A and D increases, that of C remains constant and that of B decreases
- Eventually Zone B disappears and all solids are in Zones C and D
- Gradual accumulation of solids put stress on solids at bottom which compresses solids in the Zone D
- Finally, when weight of solids is balanced by compressive strength of flocs, sedimentation process stops

Boundaries between Zones D and C and C and B may not be distinguishable in general easily. However, boundaries between Zones A and B is usually sharp and clear. As settling continues, depths of zones A and D increases, that of C remains constant, whereas the depth of B decreases, eventually zone B disappears and all solids are in zones C and D only. Gradual accumulation of solids puts stress on the solids at bottom which compresses solids in the zone D.

Finally, when weight of solids is balanced by the compressive strengths of flocs, sedimentation process stops and there will be only 2 zones, zone A and zone D. Zone A is a clear, clarified liquid which is almost free from the solid particles and zone D is a kind of concentrated sludge which is having only solid particle. But however, interstitial spaces may be occupied with some amount of the liquid.

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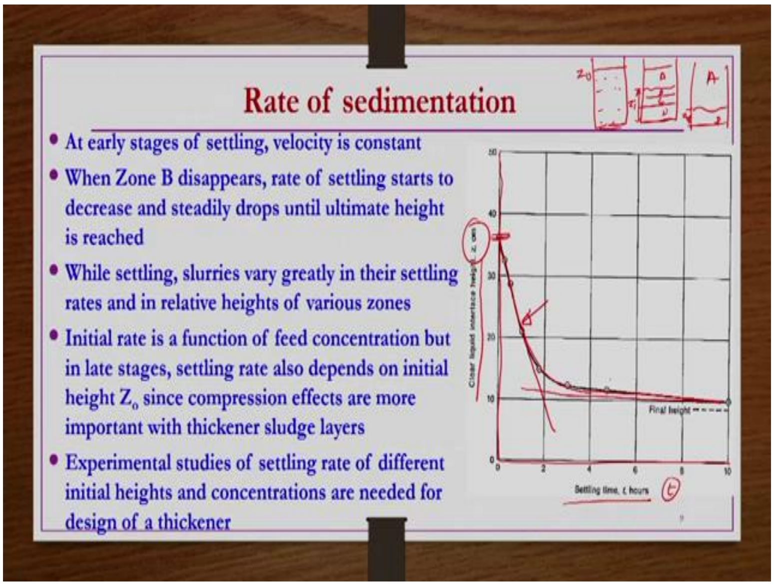

Rate of sedimentation

- At early stages of settling, velocity is constant
- When Zone B disappears, rate of settling starts to decrease and steadily drops until ultimate height is reached
- While settling, slurries vary greatly in their settling rates and in relative heights of various zones
- Initial rate is a function of feed concentration but in late stages, settling rate also depends on initial height Z_0 since compression effects are more important with thickener sludge layers
- Experimental studies of settling rate of different initial heights and concentrations are needed for design of a thickener

So, now rate of sedimentation if you see in this process, let us say clear liquid interface height whatever we have taken, that interface how it is decreasing with respect to settling time. Settling time if you are let us say T hours and then clear liquid interface height Z centimetres, let us say initially you have something like you know 35 centimetres, so then as time progresses, that initial interface height, whatever Z_0 is there decreases.

Z_0 is decreases gradually like this and then it decreases like this. What we can see here, initially it decreases very rapidly, very rapidly decreases which is kind of a, then after on, after certain kind of sedimentation rate, again it becomes a kind of a very constant kind of rate would become. What does mean by this clear liquid interface? Like you know, you have initially the let us say the Z_0.

Initially filled up to this one, let us say, after some time this is forming a kind of two layers that is A and there is a D, and then C, B and C layers are there. Whatever the interface between this B and A layer is there, that height is nothing but Z_i, so that is decreasing. So that Z clear liquid interface height, that is interface between this A and B zone, whatever is there, that is between the clear liquid.

And then zone B, where the initial, where the concentration of solid is M is as a kind of initial concentration of the solids, so whatever the interface is there, that interface is Z. So, that interface gradually what obviously we have seen, that is decreasing, that is decreasing you know when we completely having a kind of final A and D zones. This is the minimum depth that is going to have, that is Z_F whatever we indicate.

So, that Z how it is decreasing with respect to time is given here. At early stages of settling, velocity is constant. When zone B disappears, rate of settling starts to decrease and rapidly drops until ultimate height is reached. While settling, slurries vary greatly in their settling rates and relative heights of various zones also varies. Initial rate is a function of feed concertation but in late stages, that is later stages final settling stages kind of thing, settling rate also depends on initial height Z_0 since compression effects are more important with thickener sludge layers.

Experimental studies of settling rate of different initial heights and concentration are needed for design of a thickener. So, this information if you have, we can do kind of designing of a thickener.

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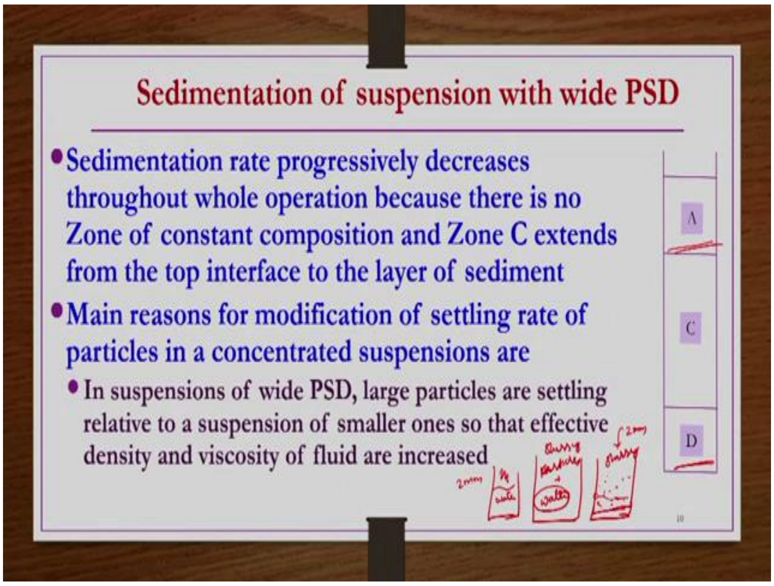

Sedimentation of suspension with wide PSD

- Sedimentation rate progressively decreases throughout whole operation because there is no Zone of constant composition and Zone C extends from the top interface to the layer of sediment
- Main reasons for modification of settling rate of particles in a concentrated suspensions are
  - In suspensions of wide PSD, large particles are settling relative to a suspension of smaller ones so that effective density and viscosity of fluid are increased

Before going into the design calculations, sedimentation of suspension with wide PSD. So, whatever things that we have seen, so we assume that in a kind of particles are of a uniform size kind of thing and then made of same particles. So, those sedimentation characteristics whatever we have seen like A, B, C, D zones are forming etc., that kind of separation we can see and then different zones we can clearly see.

But if you have the feed, if the feed slurry is having particles of very wide particle size distribution. If the feed slurry particles are whatever the particles present in the feed slurry, their size distribution is very wide, so then do we get the same zones like settling zones like A, B, C, D, like we have got under the case of uniform particle with the slurry which are having uniform particle size, so that in general not possible.

In general, what we get? We get under slurry having the wide particle size distribution, then we have our A, D and then C kind of zones. B kind of zones in general, we do not find because of the several inherent reasons as we will be discussing. Sedimentation rate progressively decreases throughout whole operation because there is no zone of the constant composition and zone C extends from the top interface to the layer of the sediments.

So, there will not be any constant composition zone because the particle size itself is varying, particle size distribution of the feed material itself is very wide. Whatever the slurry that we have taken in that, you know, whatever the particles are there their particle size distribution itself is very wide, maybe varying from few microns to few mm, so you may not expect to have a kind of a uniform concentration at any level, at any level you may not expect to have a uniform concentration within the sedimentation tank.

So, because of that reason it is not possible to have a kind of a B zone, especially if you are separating slurry having particles of very wide size distribution, under such condition we may be having A and D zones whereas the C zone maybe extending from the top layer of the slurry to the bottom layer of the clarified liquid as shown in the picture. Main reasons for modification of settling rate of particles in a concentrated suspensions are as below:

In suspension of wide PSD, large particles are settling relative to a suspension of smaller ones so that effective density and viscosity of fluid are increased. Let us say, we have a kind of a you know slurry in which the particles size distribution is very wide, few microns to few mm let us say 1 micron to 2 mm size, something like that. So, whatever the micron size particles are there, you know if they are mixed with a kind of, in the feed it is a slurry so they are mixed kind of things.

So, these particles, very fine particles, you know their settling velocity is very small. So when they are mixed with a kind of liquid, they will be forming a kind of new suspensions kind of thing whose effective density may be very much higher than the initial liquid whatever you have taken. Let us say you have slurry making in the process, slurry, let us say laboratory you are doing, so you have to make a slurry.

So, you have a water, as a liquid you have taken water and then let us say you have taken some particles. So when you mix it, you get a kind of slurry. So, now here in this one, the particles you know, some of the particles are very fine. So, this slurry is going to have a kind of suspension nature, a kind of suspension nature to some extent, so the suspension viscosity and density would be obviously higher than the pure water viscosity density or the liquid whatever you have taken here, as we have already seen in some kind of previous lectures.

So, they will make a kind of stable suspensions of having higher density and viscosity, so obviously in such kind of suspensions, settling of the other particles, other particles, other bigger particles would be very much smaller, very much smaller. Let us say, 2mm particle if it is settling in pure water, whatever the settling velocity would be there here in water, pure water, that must be very high compared to the settling velocity of same 2mm particle in a kind of concentrated suspension.

So, because this concentrated suspension, that is fine particles mixed with water, they are forming a kind of concentrated suspension, so far those concentrated suspensions density and viscosity are increasing. So, because of the increased density and viscosity of the fluid, the settling velocity of the particle will be reduced, will be obviously smaller compared to settling velocity of the same particle in a clear liquid.

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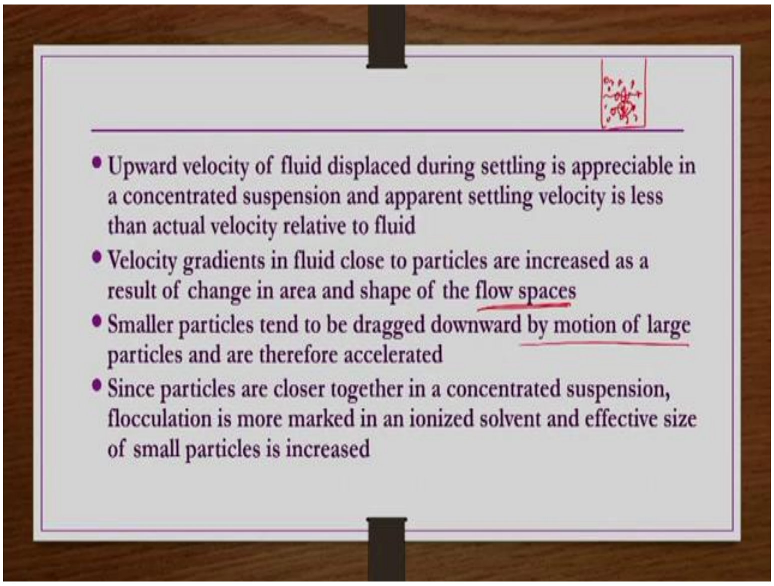

- Upward velocity of fluid displaced during settling is appreciable in a concentrated suspension and apparent settling velocity is less than actual velocity relative to fluid
- Velocity gradients in fluid close to particles are increased as a result of change in area and shape of the flow spaces
- Smaller particles tend to be dragged downward by motion of large particles and are therefore accelerated
- Since particles are closer together in a concentrated suspension, flocculation is more marked in an ionized solvent and effective size of small particles is increased

Upward velocity of fluid displaced during settling is appreciable in a concentrated suspension and apparent settling velocity is less than the actual relative velocity or actual velocity relative to the fluid. What happens, let us say you doing this separation here, okay, so now these particles are being settled, settling, so now here the particles are having different size distribution.

So obviously the particles, let us say bigger particle is settling here, so whatever the fluid was present underneath that particle, that must be moving up because of the buoyancy, displacement, fluid displacement is taking place. Particle, the bigger size particle displacing the fluid underneath and settling one level down let us say, so that fluid should be going up, moving up.

So, that the motion of the fluid upward, that is upward motion of the fluid is much appreciable, it cannot be very small in as a kind of previously other cases we have seen. So, that appreciable velocity because of appreciable upward moving velocity of the liquid, the actual relative velocity between the fluid and particle is going to be much smaller compared to the apparent settling velocity.

Whatever, the apparent settling velocity is there for this particle that is going to much smaller compared to the actually relative velocity, so that is because of the upward motion of the fluid. Obviously, when a particle is settling, let us say there is a particle here, now there is underneath fluid, this is settling and then coming this region here. So, whatever the fluid was there here in this region, that would be moving up.

So, when the particles are settling are in a kind of fluid so then there would be a kind of upward motion of a fluid that we have already seen in hindered settling case. So that upward motion is a kind of a very appreciable in such kind of, in such kind of suspension. So because of that one apparent settling velocity of the suspension here in this case having wide particle size distribution is going to be very much different when compared to the actual relative velocity between the fluid and particles.

Velocity gradients in the fluid close to particles are increased because of these things as a result of change in area and shape of the flow spaces. Further what happens, velocity gradients in fluid close to particles are increased as a result of change in area and shape of the flow spaces. Right now, different types of particles are there so then different types of you know velocity gradients would be there.

But in general if you have a kind of the particles of wide size and shape then their area particles, surface area etc. those kind of thing would be there, their shape would be different. So because of that one velocity gradients in fluid close to particles will also increase as a result of change in area and shape of the flow spaces, flow spaces, that is the flow available for the particle to settle.

Smaller particles tend to be dragged down by motion of larger particles and therefore accelerated. In general smaller particles they may be settling very slowly but you know bigger particle is coming, let us say one big particle is coming settling like this, so whatever the finer smaller particle is that will be also be pushed downwards and then that will also be settled. Since particles are closer together in concentrated suspension, flocculation is more marked in an ionized solvent and effective size of small particles is increased.

So, these are the reason that you know in case of sedimentation of slurry having wide particle size distribution, it is not possible to kind of, having a kind of B zone rather we have only A and D zone with C zone, which is very wide spread. Actually C zone, thickness of C zone is very small in a kind of sedimentation process. But here in this kind of thing it is very thick. It is very thick and then it is extending from the top layer of the sludge to the bottom layer of the clarified liquid without giving any space for B zone to form. Indeed B zone will not be there in kind of suspensions having wide particle size distributions. So, that is what about flocculation and then batch sedimentation we have seen.

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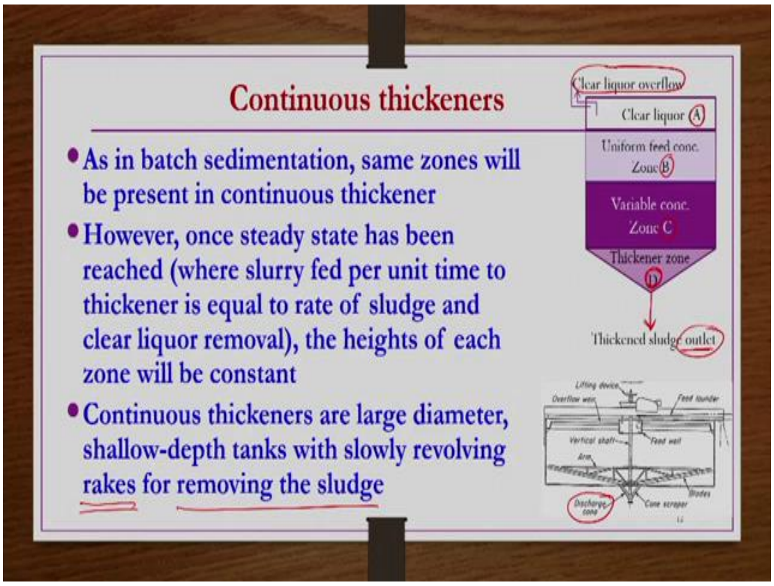

Continuous thickeners

- As in batch sedimentation, same zones will be present in continuous thickener
- However, once steady state has been reached (where slurry fed per unit time to thickener is equal to rate of sludge and clear liquor removal), the heights of each zone will be constant
- Continuous thickeners are large diameter, shallow-depth tanks with slowly revolving rakes for removing the sludge

Now, we see continuous sedimentation or continuous thickeners. What happens here? So, as in batch sedimentation, same zones will be present in continuous thickener also. If we see the continuous thickener pictorially, we can see, we can have a kind of image like this, so here a kind of cylindrical rake is there, here we have a cylindrical container.

So, now here we can see as in the kind of batch sedimentation there is a thickener zone at the bottom, thickener zone at the bottom and then clear liquid zone at the top A and then variable concentration zone C and then uniform feed concentration zone B. This picture is drawn at the kind of certain time during the sedimentation process, right? So, now here in this continuous process, what we do?

We continuously take out the thick and sludge from the outlet provided at the bottom. Similarly, we take continuously clarified liquid from the top as a kind of overflow. So, since in this case both the clarified liquid as well as the concentrated sludge are continuously taken out from this thickener, so this process is a kind of a continuous thickener. Here the feed is also continuously being allowed to enter into the thickener.

So, how does it happen, those things we are going to see. Once the steady state has been reached, that is where the slurry fed per unit time to thickener is equal to the rate of sludge and clear liquor removal then we can say a kind of a steady state has achieved. The heights of each zone will be constant. Whatever the heights of zone, let us say pictorially shown here, A, B, C, D, they will be remaining constant at any time in a kind of continuous thickener once this steady state has achieved.

Steady state here in the case, in this case, what does mean by steady state? Whatever the rate of feed is allowed, whatever the rate of feed allowed into the thickener, that must be equal to rate of a sludge and clear liquor removal. Whatever the sludge we are removing, rate of sludge removal and then rate of liquor removal is there, if that is balanced by the rate of feed into the thickener then we can say a kind of a steady state has achieved. Under such steady state conditions, the height of individual zones will remain constant in continuous thickener.

So, industrially thickeners are provided like this, one industrial thickener picture have been shown here, it is same like whatever we have drawn above here, but only thing that there are a kind of rakes are provided at the bottom so that that can be taking out the sludge, continuously sludge can be taken out. Continuous thickeners are large diameter, shallow-depth tanks with slowly revolving rakes for removing the sludge.

These rakes are provided at the bottom, they are slowly revolving and then slowly collecting the concentrate from the bottom, while it is revolving, what happens? Whatever the liquid that is trapped between the particles within the sludge zone, D zone that will be released and that liquid may be moving towards in the C zone side.

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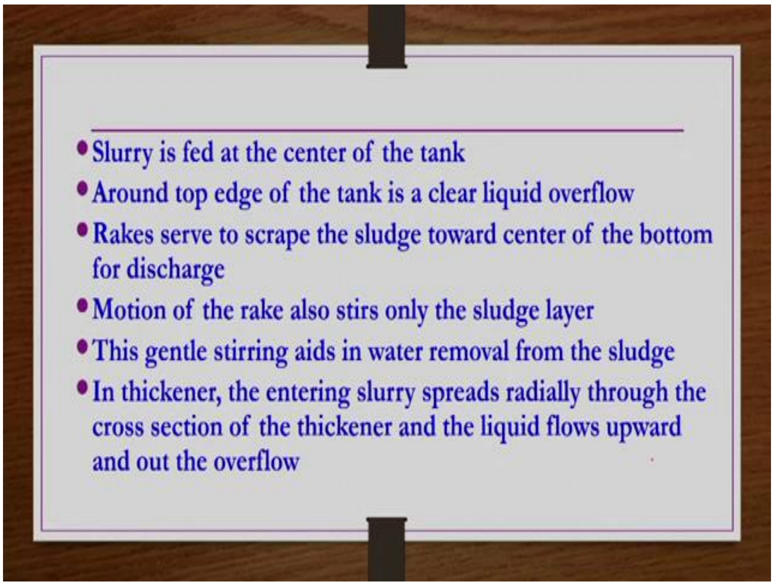

- Slurry is fed at the center of the tank
- Around top edge of the tank is a clear liquid overflow
- Rakes serve to scrape the sludge toward center of the bottom for discharge
- Motion of the rake also stirs only the sludge layer
- This gentle stirring aids in water removal from the sludge
- In thickener, the entering slurry spreads radially through the cross section of the thickener and the liquid flows upward and out the overflow

Slurry is fed at the centre of the tank. Around top edge of the tank is clear liquid overflow. Rakes serve to scrape the sludge toward center of the bottom for discharge. Motion of the rake also stirs only the sludge layer. Whatever the rake that is rotating at the bottom, that does not disturb C, B, A zones, that will disturb only material that is present in the sludge zone only, that is in the D zone. This gentle stir aids in water removal from the sludge as I mentioned. In thickener, the entering slurry spreads radially through the cross section of the thickener and the liquid flows upward and out the overflow.

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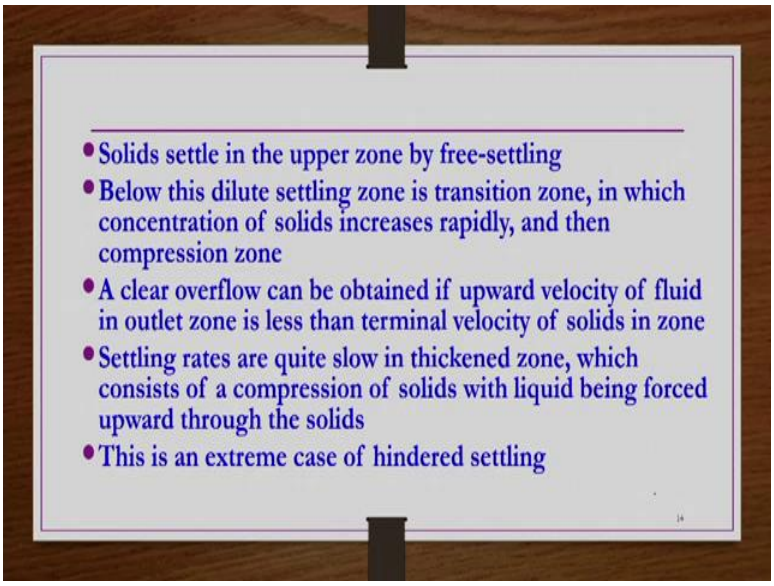

- Solids settle in the upper zone by free-settling
- Below this dilute settling zone is transition zone, in which concentration of solids increases rapidly, and then compression zone
- A clear overflow can be obtained if upward velocity of fluid in outlet zone is less than terminal velocity of solids in zone
- Settling rates are quite slow in thickened zone, which consists of a compression of solids with liquid being forced upward through the solids
- This is an extreme case of hindered settling

Solids settle in the upper zone by free-settling. Below this dilute settling zone is transition zone, in which concentration of solids increases rapidly and then compression zone would be there. A clear overflow can be obtained if upward velocity of the fluid in outlet zone is less than the terminal velocity of solids in the zone.

Obviously when you are taking the clear liquid from the top, the liquid rate, at what rate you are taking liquor out that should be kind of, having a kind of less velocity compared to the terminal velocity of the solids, otherwise solids will also be taken away along with the kind of this clear liquid, so that we do not want. So, that whatever the overflow liquid that we are taking, that velocity must be smaller than the terminal velocity of the solids in the A zone.

Because A zone is almost clarified liquid but it is not possible to have a kind of free from particles, obviously, there will be a few particles but nearly free from particles. So, if you do not want those particles into the overflow to come into the clarified liquid, then this condition has to be maintained. That is the upward velocity of fluid and outer zone should be less than terminal velocity of solids that in the zone, that is what it mean by.

So, that is a clear overflow can be obtained if upward velocity of fluid in outlet zone is less than terminal velocity of solids in the zone. Settling rates are quite slow in thickened zone, that is in the D zone at the bottom which consists of a compression of solids with liquid being forced upward through the solids. This is a kind of extreme case of hindered settling.

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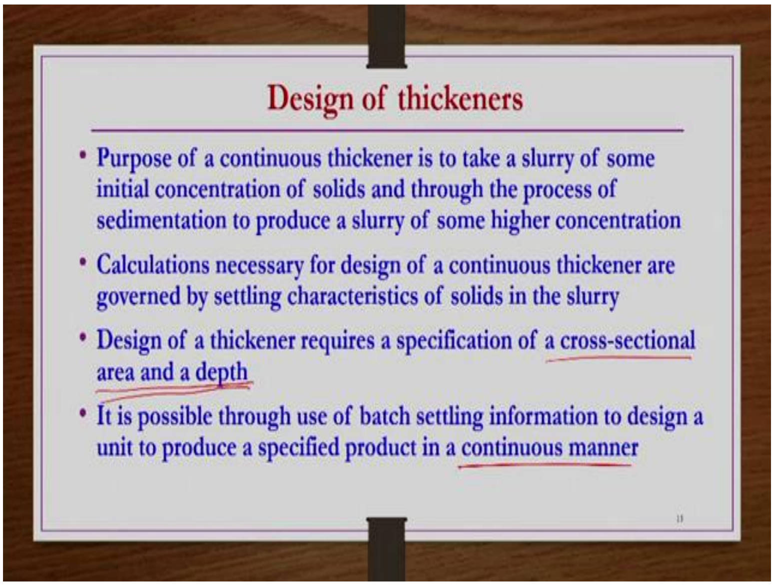

Design of thickeners

- Purpose of a continuous thickener is to take a slurry of some initial concentration of solids and through the process of sedimentation to produce a slurry of some higher concentration
- Calculations necessary for design of a continuous thickener are governed by settling characteristics of solids in the slurry
- Design of a thickener requires a specification of a cross-sectional area and a depth
- It is possible through use of batch settling information to design a unit to produce a specified product in a continuous manner

Now, we see design of thickeners. So, what should we have a kind of to get the design of thickeners? We should have a kind of information from laboratory test, from laboratory test we should have a kind of information between the clear interface height and time of sedimentation, that is Z versus T information we should have, or Z versus theta information we should have. So, from that information we should able to do the design calculation because that is what we get in general.

So, from this design calculations we will get the required area, what is the minimum required that is, that must be provided for container sedimentation tank so that the separation can be efficient, that is one. If we have a kind of a continuous thickener then the underflow, whatever the sludge underflow rate is there, so that should also come into the picture because that is also essential, very essential.

Purpose of a continuous thickener is to take a slurry of some initial concentration of solids and through the process of sedimentation to produce a slurry of some higher concentration obviously. So, calculation necessary for design of a continuous thickener are governed by settling characteristics of solids in the slurry. Design of a thickener required a specification of cross-sectional area and a depth.

Let us say if you are fixing the depth, so then what should be the cross-sectional area, that you should able to calculate from this design calculation whatever we are going to do. It is possible through use of batch settling information or batch sedimentation information to design a unit to produce a specified product in a continuous manner as well. So, whatever the experimental data or batch sedimentation data in experiment what we do, in general we have a kind of measuring jar, 1 litre measuring jar probably. So in that one you take calcium carbonate slurry of certain concentration.

Now, you, since it is a kind of a glass jar, so what you can see, you can clearly see the interface moving down. So with respect to time you know how much the interface has decreased that you note down. So, from there you know as per your requirement you can calculate the area requirement for a continuous sedimentation continuous thickener as well as that is what we are going to do, that is also possible. So, from the batch settling information, batch sedimentation information that you do in the laboratory course, that data you can use and then develop a kind of a specifications for a continuous thickener.

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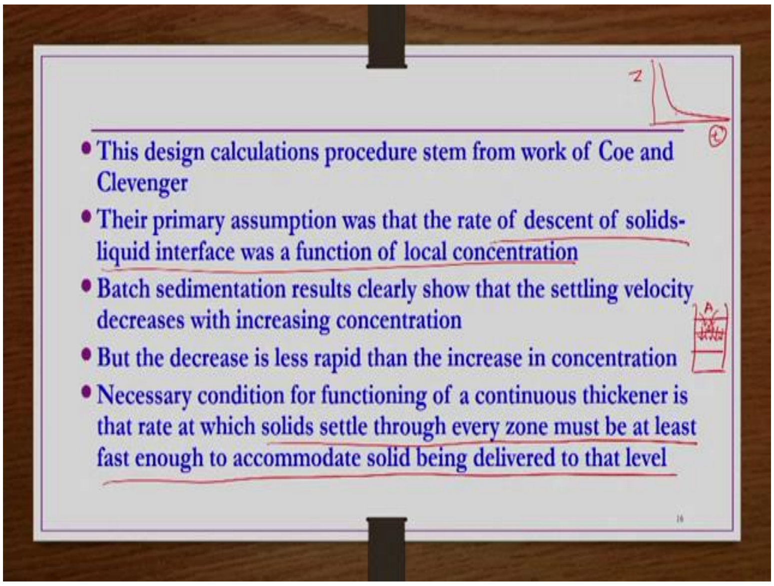

- This design calculations procedure stem from work of Coe and Clevenger
- Their primary assumption was that the rate of descent of solids-liquid interface was a function of local concentration
- Batch sedimentation results clearly show that the settling velocity decreases with increasing concentration
- But the decrease is less rapid than the increase in concentration
- Necessary condition for functioning of a continuous thickener is that rate at which solids settle through every zone must be at least fast enough to accommodate solid being delivered to that level

This design calculation whatever we are going to discuss is from the work of Coe and Clevenger, which is published by them long time back around 1960s or 70s. Their primary assumption was that the rate of descent of solids – liquids interface was a function of local concentration; obviously we have seen that it is a function of local concentration obviously because initially the particles are settling very fast.

Because but rather we have seen, the settling velocity curve that we have seen, initially the zone decreases the interface side gradually decreasing, this is what we have seen, Z versus T, one of the pictures we have seen. So initial times it gradually decreases very rapidly like this. So, that means you know, initially the concentration is slurry concentration whatever was there, and then gradually the concentration towards the other side, it is increasing.

Clarified liquid, as the clarified liquid is forming, so in the remaining zone the concentration of particle is increasing. So because of that one the sedimentation rate decreases subsequently as the sedimentation time progresses. That means, sedimentation, is whatever the sedimentation are the interface height that is decreasing, descent of solids liquid interface whatever is there that is function of a local concentration.

Batch sedimentation results clearly show that the settling velocity decreases with increasing the concentration. But the decrease is less rapid than the increase in the concentration. So, necessary condition for functioning of a continuous thickener is that rate at which solids settle through every zone must be at least fast enough to accommodate solid being delivered to that zone.

Let us say you have different zones; A, B, C, D zones, something like that. Now, let us say this is A zone, B zone. Now from this B zone particles are settling through, they are passing thorough. That whatever these particles are settling through this B zone, so that must be fast enough, how much fast enough, the rate at which the particles are entering that zone.

From A zone to the B zone, whatever the rate it is coming, so that much, that much rapidly the particle should also, should also go through, go out of that particular zone, otherwise what will happen? The size of that zone will increase, that we do not want. We want the particles to move down, not going up, we do not want any layer to go up, either of this B or C layers, we want only that A layer to increase and then D, depth of A layer to increase and depth of D layer to increase until the final separation is taken place.

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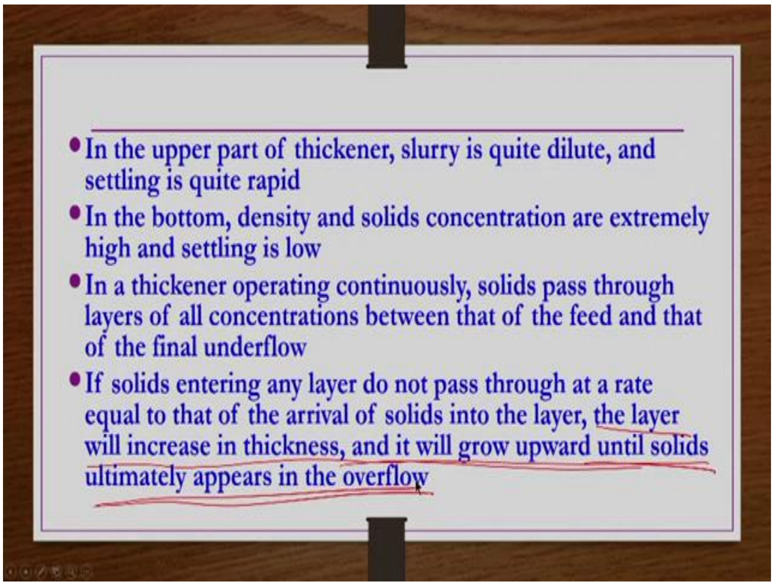

- In the upper part of thickener, slurry is quite dilute, and settling is quite rapid
- In the bottom, density and solids concentration are extremely high and settling is low
- In a thickener operating continuously, solids pass through layers of all concentrations between that of the feed and that of the final underflow
- If solids entering any layer do not pass through at a rate equal to that of the arrival of solids into the layer, the layer will increase in thickness, and it will grow upward until solids ultimately appears in the overflow

So, in the upper part of the thickeners, slurry is quite dilute and settling is quite rapid. In the bottom that is in the D zone, density and solids concentrations are extremely high and settling is low. In a thickener operating continuously, solids pass through the layers of all concentration between that of the feed and that of the final underflow that we have already seen.

If solids entering any layer do not pass through at a rate equal to that of the arrival of solids into the layer as I mentioned, then the layer will increase in thickness, the layer will increase in thickness and it will grow upward, and it will grow upward until solids ultimately appears in the overflow. So, that if whatever the particles are coming into one layer, at the same layer, at the same rate or even at higher rate those particles should be pass through that layer, otherwise the particles will or the layer, thickness of that layer will increase.

The thickness of layer will increase and then particles move upward and then eventually what we will see? We will see particle floating in the overflow region. That is the reason design calculations are essential, it is not that you have simply have a container and then take the slurry automatically suppression will take place, no it is not true. That is the reason one should be careful, and this is the primary criteria that they have found, this work, this work is based on this primarily this criteria.

So, this criteria and then from this point, they have taken a kind of a rate limiting layer. So, that we see. So, if solids entering any layer do not pass through at a rate equal to that of arrival of solids into the layer, the layer obviously will increase in thickness and it will grow upward which we do not want. And how long it will grow? It will grow until the solids ultimately appears in the overflow which also we do not want. We want solids to be there in the underflow in the sludge only, in the overflow we want clarified liquid.

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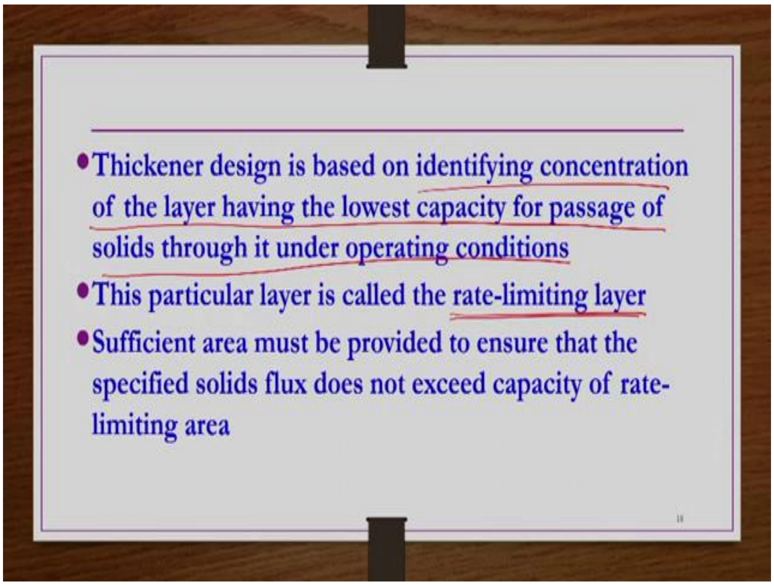

- Thickener design is based on identifying concentration of the layer having the lowest capacity for passage of solids through it under operating conditions
- This particular layer is called the rate-limiting layer
- Sufficient area must be provided to ensure that the specified solids flux does not exceed capacity of rate-limiting area

So, thickener design is based on identifying concentration of the layer having the lowest capacity for passage of solids through it under operating conditions. We have to identify a layer which is having the lowest capacity for passage of solids through it under whatever the operating conditions we have taken. So, that is the primary one. So, based on that one this work progresses. This particular layer is called a kind of rate-limiting layer.

Sufficient area must be provided to ensure that this specified solid flux does not exceed capacity of the rate limiting layer. In solids flux exceeds the rate of the capacity of this rate limiting layer then obviously what will happen? This particle will be entering into the upward, upper layer and then gradually they will appear in a kind of overflow. So, we have to find out area corresponding to this flux which is below the capacity of the rate limiting area, rate limiting layer.

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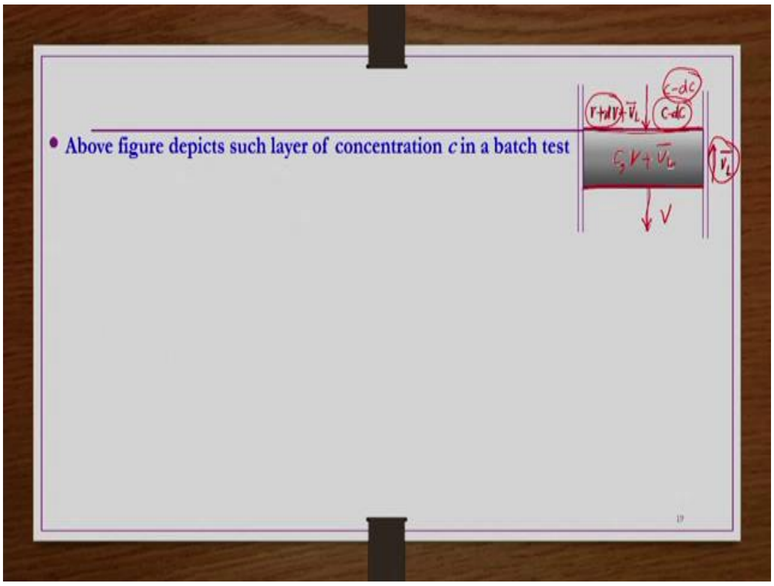

- Above figure depicts such layer of concentration *c* in a batch test

So, now this layer, we take this rate limiting layer, whatever is there. Let us say in the continuous, in the thickener we have different layers, we have taken one layer. Let us assume this is that layer and then whatever the rate limiting layer that we want, that is this one, and then it is moving upward $\bar{v}_L$, and it is moving upward $\bar{v}_L$. The concentration of solids in this layer is C and then the solids settling through this layer at velocity $v + \bar{v}_L$V with respect to the layer or simply v with respect to the whatever the column wall that we have.

The particles coming into this layer are having the concentration $c - dc$, because why minus dc? Because the particle concentration decreases underneath layers gradually. Because the particles settling downwards, so bottom layers will have the higher concentrations, and then upper layers will have the kind of lower concentration at any point of time of separation sedimentation process, you take, that is what you see. So, that is the reason the concertation of solids that are entering in this layer is $c - dc$.

And then the velocity of the particles that they are entering in this layer is v + dv with respect to the column or wall but if you obtain, if you write the velocity of settling velocity of the particles with respect to the layers, since layer is moving up, so then it should be added with $\bar{v}_L$. So, v + dv is nothing but, is nothing but the velocity, settling velocity of the particles, they are entering this layer and then v is nothing but the settling velocity of the particles, they are leaving this layer.

And then concentration of those particles entering this layer is $c - dc$, concentration of the particle in this layer is c. And then this layer is moving upward with $\bar{v}_L$. This $\bar{v}_L$ is the kind of a, this layer is a kind of a layer which is having the minimum passage of the solids as we mentioned like here in the previous case.

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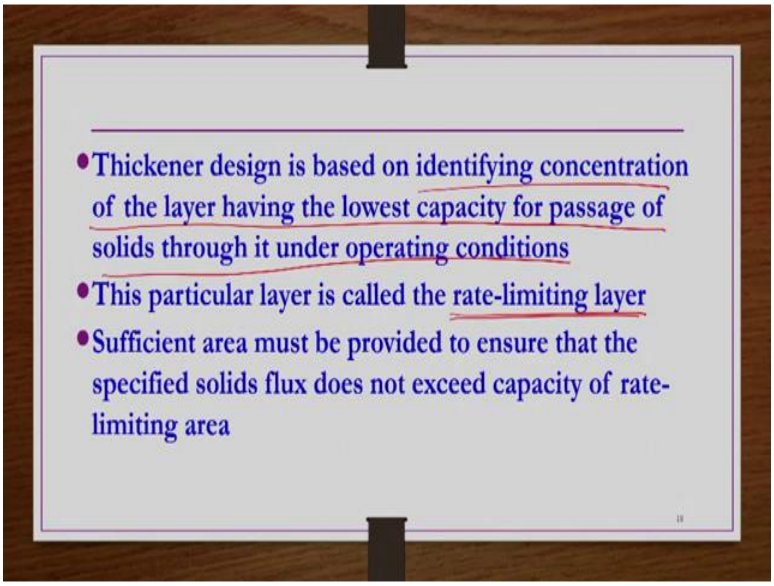

- Thickener design is based on identifying concentration of the layer having the lowest capacity for passage of solids through it under operating conditions
- This particular layer is called the rate-limiting layer
- Sufficient area must be provided to ensure that the specified solids flux does not exceed capacity of rate-limiting area

The identifying layer having lowest capacity for the passage of the solids through it under operating conditions, that is possible.

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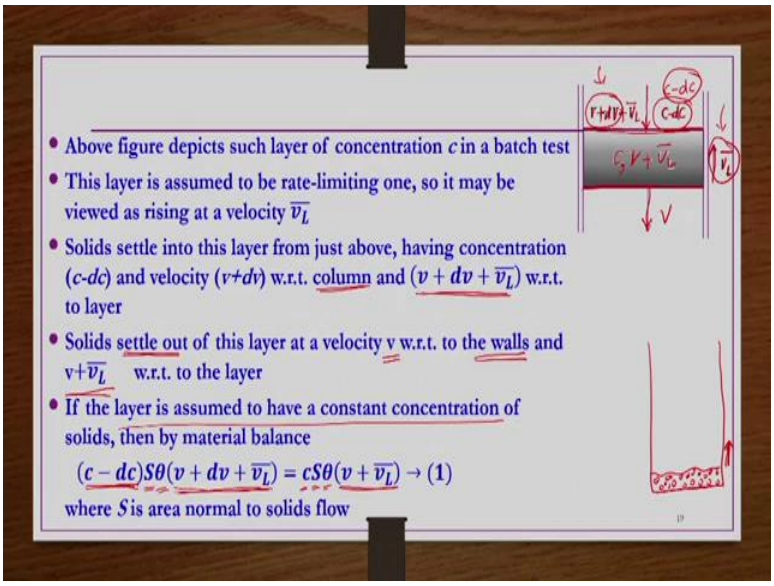

- Above figure depicts such layer of concentration c in a batch test
- This layer is assumed to be rate-limiting one, so it may be viewed as rising at a velocity $\overline{v_L}$
- Solids settle into this layer from just above, having concentration (c-dc) and velocity (v+dv) w.r.t. column and $(v + dv + \overline{v_L})$ w.r.t. to layer
- Solids settle out of this layer at a velocity v w.r.t. to the walls and v+$\overline{v_L}$ w.r.t. to the layer
- If the layer is assumed to have a constant concentration of solids, then by material balance

$$(c - dc)S\theta(v + dv + \overline{v_L}) = cS\theta(v + \overline{v_L}) \rightarrow (1)$$

where S is area normal to solids flow

Initially, at what conditions it is possible? So, if you have a kind of like this, so such kind of layer, obviously they will be at the bottom initially. They will be bottom initially and then that layer will be moving up. This layer, it is about the layer of suspension. How it is moving up?

So, but it is not the settling velocity of the particle, settling velocity of the particle is v and then rising velocity of the layer is $\bar{v}_L$.

So, that is how these layers are formed. So, above figure depicts such a layer of concentration c in a batch test. Whatever the limiting layer we have considered and then pictorially drawn that one particular like this. So, this layer is assumed to be rate limiting one, so it may be viewed as rising at velocity $\bar{v}_L$. Obviously, initially it is assumed that it is at the bottom and then it gradually raises up, gradually this layer raises up.

That is what we want so that the layer which is having the lowest capacity for the passage of solids, that moves up, that moves up as per this one, as per this one. Lowest capacity for passage of solids through it, you know that should be minimum kind of thing, that we have. So, initially the particle whatever settles, they will be having lowest capacity to move up. So, initially it is there, so whatever the particles are there, gradually they will be coming down, so then this layer moves up.

Solids settle into this layer from just above, having concentration C minus dc and then velocity v + dv with respect to the column, with respect to the column that is container wall and then v + dv plus layer velocity if you write with respect to the layer, so $v + dv + \bar{v}_L$ with respect to layer. Similarly, solids settle out that is pass through, go out through this layer that is passing through, passing down, sedimenting down.

Solids sedimenting down, sedimenting out through this layer at a velocity v with respect to the column walls and then $v + \bar{v}_L$ with respect to the layer. So, this is very important one. Once we understand this one, so then rest everything is mathematically simple one. If the layer is assumed to have a constant concentration of solids, if the layer is having a concentration which is constant, then by material balance what is the amount of mass that is entering in this layer?

That should be the velocity, entering velocity $v + dv + \bar{v}_L$ multiplied by the time, multiplied by the area normal to the flux, and then multiplied by the concentration of the solids that is the coming into the layer, that will give us the cages of solids entering into the layer. Similarly, so what is the, whatever this, these solids are entering, the entire solids has to go out because what we are assuming here the layer to have a constant concentration of solids.

So, if you wanted to have the constant concentration of solids, whatever the solids are entering, the same amount of solids should also leave. So, leaving is a kind of leaving velocity multiplied by time, multiplied by the area normal to the flux, multiplied by the concentration that is leaving, so then we have a mass of solids leaving this layer that is $cS\theta(v + \bar{v}_L)$, S is the area normal to the solids flow.

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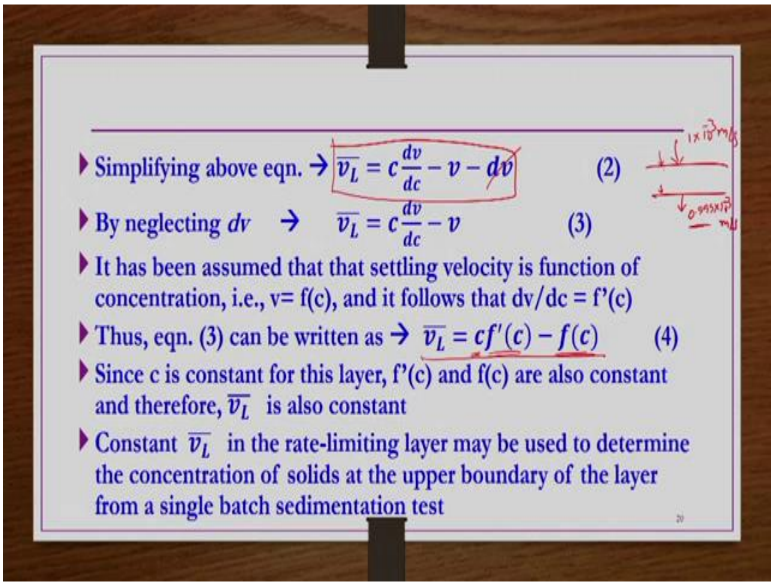

- Simplifying above eqn. → $\boxed{\overline{v_L} = c\frac{dv}{dc} - v - \cancel{dv}}$ (2)
- By neglecting dv → $\overline{v_L} = c\frac{dv}{dc} - v$ (3)
- It has been assumed that that settling velocity is function of concentration, i.e., v= f(c), and it follows that dv/dc = f'(c)
- Thus, eqn. (3) can be written as → $\underline{\overline{v_L} = cf'(c) - f(c)}$ (4)
- Since c is constant for this layer, f'(c) and f(c) are also constant and therefore, $\overline{v_L}$ is also constant
- Constant $\overline{v_L}$ in the rate-limiting layer may be used to determine the concentration of solids at the upper boundary of the layer from a single batch sedimentation test

1×10^{-3} m/s

0.995×10^{-3} m/s

Now, above equation you simply and then write for $\bar{v}_L$ because layer velocity we do not know. What is this layer velocity that we do not know. So, $\bar{v}_L = c\frac{dv}{dc} - v - dv$. In general, this layer is very thin. In general, this layer is very thin, so obviously the change in settling velocity of the particles within this layer is very small, so whatever the dv is there, that is in general is very small.

Because v is the settling velocity of the particles. So, $v + dv$ is the settling velocity of the particles just above that layer, so that since the layer is very thin the change in the settling velocity of from the upper edge of this layer to the lower edge of layer is going to be very small. So the change if you take it out as a kind of negligible compared to the v, so then we have $\bar{v}_L = c\frac{dv}{dc} - v$.

Let us say, whatever the layer here, the particles are coming here, so that may be having the velocity 1 x 10^{-3} meter per seconds. So when it is going out here, so here it may be having the velocity something like 0.999 x 10^{-3} m/s. So, whatever the dv, the change in settling velocity for the particles to move from this layer to this layer is going to be very very small in general especially when this layer is very thin, so dv we can cancel out.

Then we have this equation, it has been assumed that the settling velocity is function of concentration that is v is function of c, obviously that we have seen at the beginning itself and from this what we can find out that $\frac{dv}{dc}$ is going to be another function of concentration only. So, from this equation in terms of these functions, function of c and then f, another function of $c\frac{dv}{dc}$ if you write, $\bar{v}_L = cf'(c) - f(c)$.

$f'(c)$ is nothing $\frac{dv}{dc}$ which is nothing but function of c only. It is just a kind of designation here. Since, c is constant for this layer obviously $f'(c)$ and then $f(c)$ both will be constant. Since c is constant here, $f(c)$ is also constant and $f'(c)$ is also constant, so $\bar{v}_L$ is also going to be constant. Then for constant $\bar{v}_L$ in the rate limiting layer may be used to determine the concentration of solids at the upper boundary of the layer from a single batch test, how to do that one, that we will do now.

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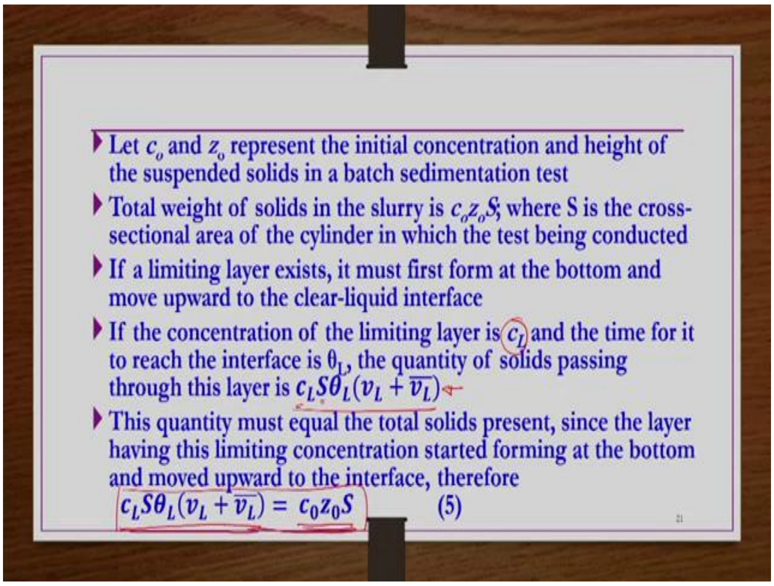

- Let c_o and z_o represent the initial concentration and height of the suspended solids in a batch sedimentation test
- Total weight of solids in the slurry is $c_o z_o S$; where S is the cross-sectional area of the cylinder in which the test being conducted
- If a limiting layer exists, it must first form at the bottom and move upward to the clear-liquid interface
- If the concentration of the limiting layer is c_L and the time for it to reach the interface is θ_L, the quantity of solids passing through this layer is $c_L S\theta_L(v_L + \overline{v_L})$
- This quantity must equal the total solids present, since the layer having this limiting concentration started forming at the bottom and moved upward to the interface, therefore

$$\boxed{c_L S\theta_L(v_L + \overline{v_L}) = c_0 z_0 S} \qquad (5)$$

Let c_0 and z_0 represent the initial concentration and height of the suspended solids in a batch sedimentation test. The total weight of solids in the slurry obviously would be $c_0 z_0 S$, where S is the cross sectional area of the cylinder in which the test is being done. If a limiting layer exist, it must form at the bottom obviously, as I mentioned and move upward to the clear liquid interface.

If the concentration of the limiting layer is c_L and the time for it to reach the interface is θ_L, the quantity of solids passing through this layer would be nothing but $(v_L + \bar{v}_L)c_L S\theta_L$. v_L is nothing but settling velocity of the particle through this layer and then $\bar{v}_L$ is nothing raising velocity of this layer, $\bar{v}_L$ is nothing raising velocity of this limiting layer and then multiplied by the θ that is the time for it to reach the interface.

It multiplied by the cross-sectional area of the cylinder whatever you have taken, so that will give the volume, and then if you multiplied by the solids concentration in this layer that is mass per volume in it so then that will be giving total mass of solids passing through this layer. This quantity must equal the total solids present, since the layer having this limiting concentration started forming at the bottom and moved upward to the interface.

So, whatever the solids there in the total, in the total slurry, so that should be balanced by the, that should be balanced by the solids present in this layer in this limiting layer because this layer initially is at the bottom and then it is moving all the way to the top so that all the solids are remaining same. They are not changing, only thing that they are setting at the bottom, the quantity is going to be same.

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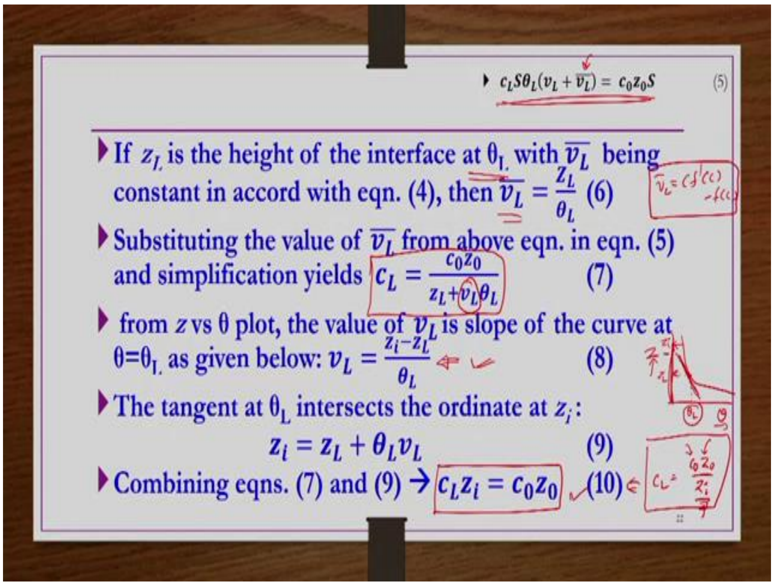

- $c_L S \theta_L (v_L + \bar{v}_L) = c_0 z_0 S$ (5)

- If z_L is the height of the interface at θ_L with $\bar{v}_L$ being constant in accord with eqn. (4), then $\bar{v}_L = \frac{z_L}{\theta_L}$ (6)
- Substituting the value of $\bar{v}_L$ from above eqn. in eqn. (5) and simplification yields $\boxed{c_L = \frac{c_0 z_0}{z_L + v_L \theta_L}}$ (7)
- from z vs θ plot, the value of v_L is slope of the curve at $\theta = \theta_L$ as given below: $v_L = \frac{z_i - z_L}{\theta_L}$ (8)
- The tangent at θ_L intersects the ordinate at z_i:

$$z_i = z_L + \theta_L v_L \quad (9)$$

- Combining eqns. (7) and (9) → $\boxed{c_L z_i = c_0 z_0}$ (10)

$\bar{v}_L = cf'(c) - f(c)$

$c_L = \frac{c_0 z_0}{z_i}$

So, now in this equation 5, if you substitute $\bar{v}_L$, what is $\bar{v}_L$? If Z_L is the height of the interface at θ_L, then $\bar{v}_L$ is being constant in accordance to equation number 4, then $\bar{v}_L = \frac{Z_L}{\theta_L}$. Equation number 4 what we understand, we understand that $\bar{v}_L = cf'(c) - f(c)$, according to this one it is a constant and that constant we can find out.

Height of the interface at $\frac{Z_L}{\theta_L}$, because now we are doing for the limiting layer. So, for this limiting layer we are doing this one, so $\bar{v}_L = \frac{Z_L}{\theta_L}$. So, now here, this $\bar{v}_L$, here in place of this one, you substitute $\frac{Z_L}{\theta_L}$ and then simplify this equation, so simply you will get the concentration of solids in that particular layer. $c_L = \frac{c_0 Z_0}{Z_L + v_L \theta_L}$.

Initially we started with assuming finding out this importance of this layer, and then we started out finding out what is the rising velocity of this layer and what is the concentration of the solids, so those things we are now known, now they are known. They are known $\bar{v}_L = \frac{Z_L}{\theta_L}$, assuming θ_L is the time that this layer required to reach the interface between the, interface at the clear liquid that is bottom layer of the clear liquid, so that time is θ_L, so then c_L is going to be $\frac{c_0 Z_0}{Z_L + v_L \theta_L}$.

C naught is the initial concentration of the solids, Z_0 is the initial height of the interface. So, we got this expression but still these expressions are incomplete. So if you wanted to know these equations, you should know what is θ_L, what is Z_L, otherwise it is not possible. Otherwise there is no use of this equation. So, further we have to simplify these equations.

From Z versus θ point the value of v_L is nothing but slope of the curve at $\theta = \theta_L$, so whatever we have this Z versus theta curve, the sedimentation curve like this, let us say at particular time theta here, at particular time theta here. If you wanted to know the settling velocity of this particular suspension, so then that is, settling velocity is nothing but v_L, so that you have to have kind of tangent curve like this here.

So, the value of v_L is slope of the curve at $\theta = \theta_L$ let us say. This is your θ_L. Then we can have this slope as like this, so let us say this is a curve and then you extend it like this, this curve. So, this point at this point which is corresponding to θ_L, at this point, at this point what you do, you draw a tangent like this and then you extend it, you extend it to the y axis so that you can find out this interface as a Z_i , as a Z_i.

Initially Z versus θ experimental information you have, at $\theta = \theta_L$ on this sedimentation curve, you draw a tangent. And then you extend to y axis or whatever the intercept that you will get, that is Z_i and then corresponding to θ_L whatever the Z is there, v_L, so $\frac{Z_i - Z_L}{\theta_L - 0 - 0}$ here. Because Z_i is corresponding to 0 so that we will get a kind of a settling velocity of the particle in negatives, but we should have a kind of a positive value, so $\frac{Z_i - Z_L}{\theta_L}$.

So, from here the slope of this curve is nothing but $\frac{Z_i - Z_L}{\theta_L}$, that would be the settling velocity of the particles, so this v_L. Now, if you can substitute here, in this equation number 7 and then simplify then you will get the required equation as a $c_L Z_i = c_0 Z_0$. So, in this equation number 7 here $v_L = \frac{Z_i - Z_L}{\theta_L}$ if you substitute you will get the final equation $c_L Z_i = c_0 Z_0$..

So, now c_L is also known from here. Now, it is a usable, this equation is a kind of from which we can use it now, this is, let us say at particular θ_L whatever the intercept that you can get from the experimental data, that you know, initial concentration you know, initial interface height you know, so c_L you can know from here. So, this equation is more useful compared to the equation number 7.

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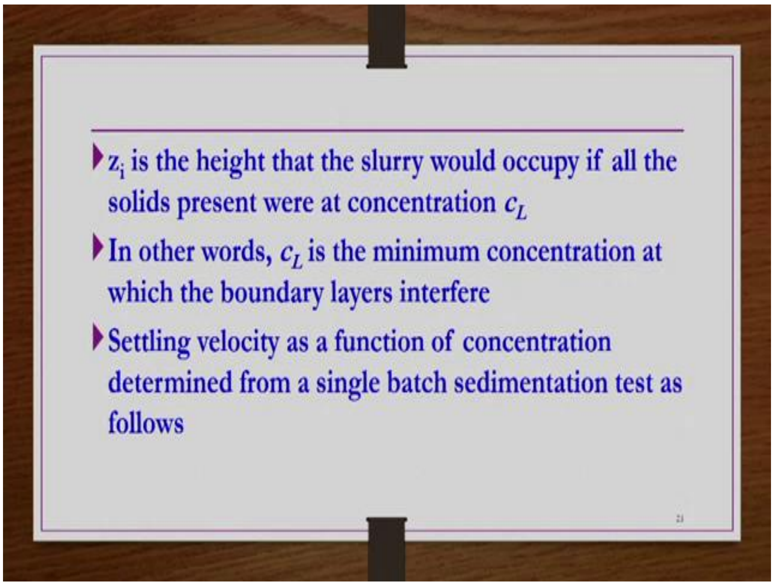

- z_i is the height that the slurry would occupy if all the solids present were at concentration c_L
- In other words, c_L is the minimum concentration at which the boundary layers interfere
- Settling velocity as a function of concentration determined from a single batch sedimentation test as follows

21

So, what is Z_i? Z_i is the height that the slurry would occupy if all the solids present were at concentration c_L, at concentration c_L. Other words, c_L is the minimum concentration at which the boundary layers interfere. So, boundary layers are interfering at Z_i distance. Settling velocity as a function of concentration determined from a single batch sedimentation test as follows:

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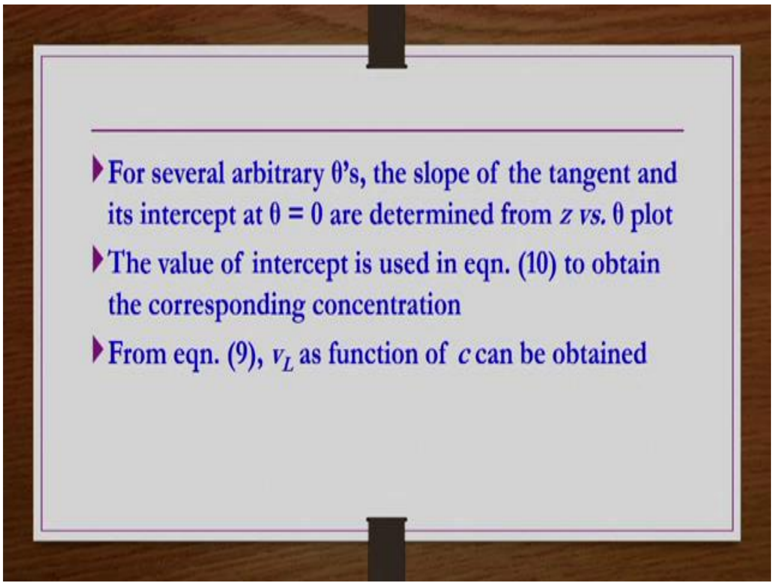

- For several arbitrary θ's, the slope of the tangent and its intercept at θ = 0 are determined from *z vs.* θ plot
- The value of intercept is used in eqn. (10) to obtain the corresponding concentration
- From eqn. (9), v_L as function of *c* can be obtained

So, what we do? For arbitrary θ's, the slope of the tangent and its intercept at theta is equals to 0 are determined from Z versus θ plot. The value of the intercept is used in equation number 10 to obtain the corresponding concentration c_L. From equation number 9, v_L as function of c can be obtained.

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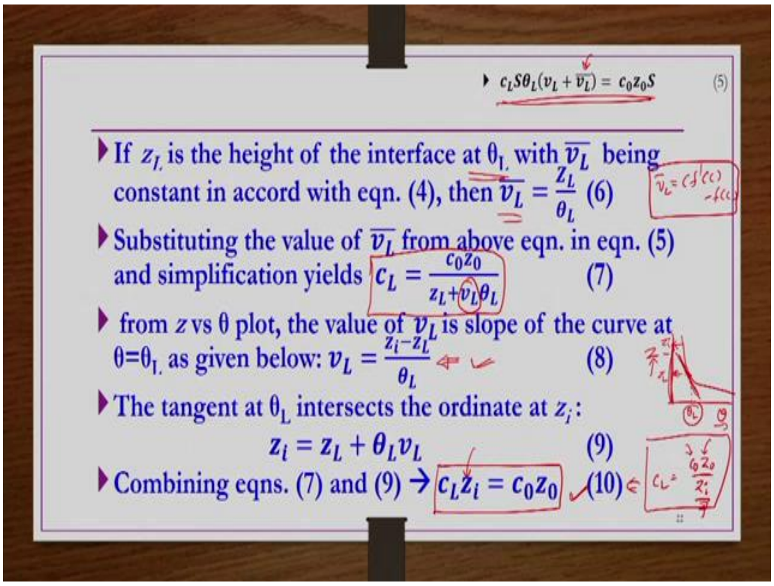

- $c_L S \theta_L (v_L + \overline{v_L}) = c_0 z_0 S$ (5)

- If z_L is the height of the interface at θ_L with $\overline{v_L}$ being constant in accord with eqn. (4), then $\overline{v_L} = \frac{z_L}{\theta_L}$ (6)
- Substituting the value of $\overline{v_L}$ from above eqn. in eqn. (5) and simplification yields $\boxed{c_L = \frac{c_0 z_0}{z_L + v_L \theta_L}}$ (7)
- from *z* vs θ plot, the value of v_L is slope of the curve at $\theta = \theta_L$ as given below: $v_L = \frac{z_i - z_L}{\theta_L}$ (8)
- The tangent at θ_L intersects the ordinate at z_i:

$$z_i = z_L + \theta_L v_L \quad (9)$$

- Combining eqns. (7) and (9) → $\boxed{c_L z_i = c_0 z_0}$ (10)

$\overline{v_L} = cf'(c) - f(c$

$c_L = \frac{c_0 z_0}{z_i}$

So, equation number 9 and 10, how we can use it here, like so from here as mentioned the intercept, whatever the intercept if you know, this c_L you can know. Once c_L is known then Z_L also can be known and then one can know whatever this v_L as well from equation number 8. So, what we understand from this equation number 10 that from this z versus θ plot, at any θ_L, whatever the θ_L that is not any θ , that is the time required for this layer to reach the interface.

At that time if you draw a tangent to this z versus θ curve, whatever the intercept Z_i is there, that you can substitute here and then you can find out c_L. And then if you substitute this c_L back here in either of these equations, equation number 7 or equation number 9 here, so you can get v_L as function of c, v_L as function of c you can get here.

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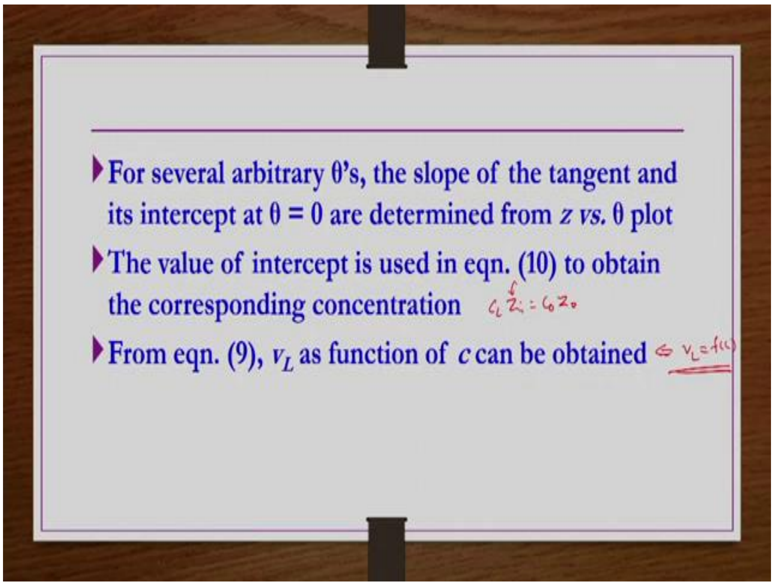

- For several arbitrary θ's, the slope of the tangent and its intercept at $\theta = 0$ are determined from *z vs.* θ plot
- The value of intercept is used in eqn. (10) to obtain the corresponding concentration $c_L \overset{\downarrow}{Z_i} = c_0 Z_0$
- From eqn. (9), v_L as function of *c* can be obtained $\Leftarrow \underline{v_L = f(c)}$

For arbitrary that is what, that is what it mean by, for arbitrary thetas the slope of the tangent and its intercept at θ is equals to 0 are determined from z versus θ plot. The value of intercept that is Z_i is used in equation number 10 to obtain the corresponding concentration c_L because $c_L Z_i = c_0 Z_0$ are the initial values known, Z_i is the intercept value that you know, so now you know the c_L concentration.

Now, this c_L if you substitute in equation number 9, so then what you get, you will get a expression v_L function of c, v_L function of c. And then we understand that the settling velocity of the particle is function of concentration. What function of the concentration, that we can know from this equation number 9. By substituting in equation number 9 whatever the c_L information, we can substitute and then get this information.

(Refer Slide Time: 71:11)

References

- W.L. McCabe, J. Smith and P. Harriot, *Unit Operations of Chemical Engineering*, 7th Ed., McGraw - Hill, International Edition, 2005.
- E. Ortega-Rivas, *Unit Operations of Particulate Solids: Theory and Practice*, CRC Press, FL, 2012.
- J.F. Richardson, J.H. Harker, *Coulson and Richardson's Chemical Engineering*, 2nd Volume, 5th Ed., Butterworth-Heinemann, 2003.
- C.J. Geankoplis, *Transport Processes and Unit Operations*, 4th Ed., Prentice Hall, India, 1993.
- G.G. Brown et al., *Unit Operations*, 1st Ed., CBS Publishers & Distributors, 2005.
- W. L. Badger and J. T. Banchero, *Introduction to Chemical Engineering*, Tata McGraw-Hill, International Edition, 1997.

Foust et al.

" principles of unit operatio

So, the references are given here. These are references we are following. But there is other Foust et al, title is Principles of Unit Operations. In this book all the details are given about the design calculations. Thank you.