**Mechanical Unit Operations**
**Professor Nanda Kishore**
**Department of Chemical Engineering**
**Indian Institute of Technology Guwahati**
**Lecture 33 – Gravity Sedimentation – Design of Thickeners - 2**

Welcome to the MOOCs course, Mechanical Unit Operations. This is the continuation lecture of previous lecture that is design of thickeners we are discussing. Title of this lecture is Gravity Sedimentation - Design of Thickeners part 2. Design of thickeners, since it is a continuation of previous lecture let us see what we have seen in the previous lecture related to the design part of the thickeners.

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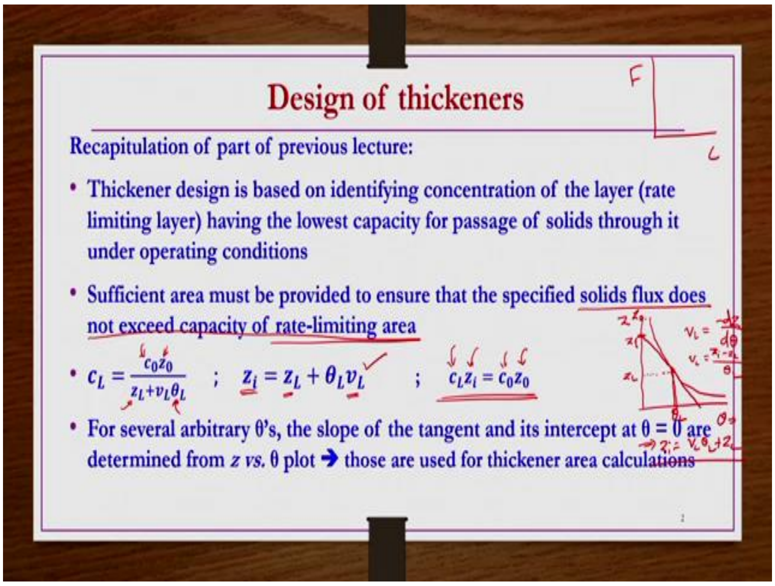


So thickener design based on identifying concentration of the layer having the lowest capacity for passage of solids through it under operating conditions. The thickener design calculation whatever we started with in previous lecture, we started with you know finding a, identifying a kind of layer which is having the lowest capacity for passage of solids through it and then that layer is initially assumed to be at the bottom of the thickener and then it gradually moves up with a kind of velocity $\bar{v}_L$ and then after that you know what we have found? We have found the concentration of solids in the particular layer which is moving up and then what is the settling velocity of the particles that are passing this limiting layer.

This layer we identified it as a kind of a rate limiting layer and then according to this method sufficient area must be provided to ensure that the specified solid flux does not exceed capacity of this rate limiting layer. So from this what we understand actually, whatever the area or expression for area calculation for a given thickener that we are going to develop, that is based on a kind of solids flux. There will be a kind of layer which having a kind of a lowest solid flux and then corresponding to that we have to find out the area requirement for the you know thickener design and then that area is going to be minimum area requirement.

And then what we have done? We have done a kind of a material balance, then we have found that you know this $C_L$ the concentrations solids in that rate limiting layer is nothing but $\frac{c_0 z_0}{z_L+(v_L\theta_L)}$, this c naught is nothing but the concentration of solids in the initial slurry, $z_0$ is the initial interface height, $z_L$ is nothing but the you know height of that layer. Interface height when this $C_L$, when this rate limiting layer has been identified. At that particular time $\theta_L$, let us say at settling time $\theta_L$ you know the sedimentation time $\theta_L$ the concentration of solids in that particular layer, $\theta_L$ time is nothing but the $C_L$ and then corresponding interface height is $z_L$.

And then this $z_i$ is nothing but the intercept, it is not the interface height. So what we have, we have these curves like this. Sedimentation curve that interface height z versus θ if you plot, you will get curve like this, the interface height gradually decreases as particles settling down with respect to the time of sedimentation process. So now let us say if this is time $\theta_L$, at this $\theta_L$ whatever the interface height is there that is nothing but $z_L$. But what is $z_i$? So at this interface if you draw a kind of tangent like this, so whatever this tangent at $\theta = \theta_L$ that has been drawn on this curve, so that tangent may be intercepting this y axis at some location, that intercept is $z_i$.

That $z_i$ is nothing but $z_L + (v_L\theta_L)$, $v_L$ is nothing but the settling velocity of the particles that is passing through this layer. Or that settling velocity you can simply find it out, $v_L$ is equals to $\frac{-dz}{d\theta}$, simply $\frac{-dz}{d\theta}$ if you do you will get the $v_L$ the settling velocity of the particle at time $\theta_L$ here. So that we can find it out from here you know $z_i - z_L$ and then this time $\theta_L$. So actually that $z_i$ information whatever $z_i = z_L + \theta_L v_L$ information is there that is coming from this slope at this point. So that is what we get this. And $z_i$ is intercept at $\theta_L$ and then $v_L$ is nothing but the slope of the tangent at $\theta_L$.

And then finally we after doing few more simplification what we understand that $C_L$, $z_i$ is equals to c naught, $z_0$. Now $C_L$ is the concentration of solids of that layer, rate limiting layer and $z_i$ is intercept as shown here. And c naught is initial solids concentration, $z_0$ is initial interface height. Let us say this is the initial interface height that must be $z_0$ kind of thing.

And then for several arbitrary thetas the slope of the tangent and its intercept at $\theta$ is equals to 0 are determined from z versus $\theta$ plot. Those are used for thickener area calculations. Till this point we have already seen in the previous lecture.

Now what we understand from here? So the solids flux, solids flux does not exceed the capacity of rate limiting layer. So what does it mean? So what we do, if we find out somehow the flux, solid flux F and then plot it against the concentration, solids concentration of the slurry at different times that C if you plot it then you may be having a kind of layer or location where the flux is a kind of minimum one. The flux is a kind of minimum one and then corresponding to that minimum flux whatever the $F_L$ is there, you can note down and then from there we can get the required area for a thickener. So that is what we are going to do now.

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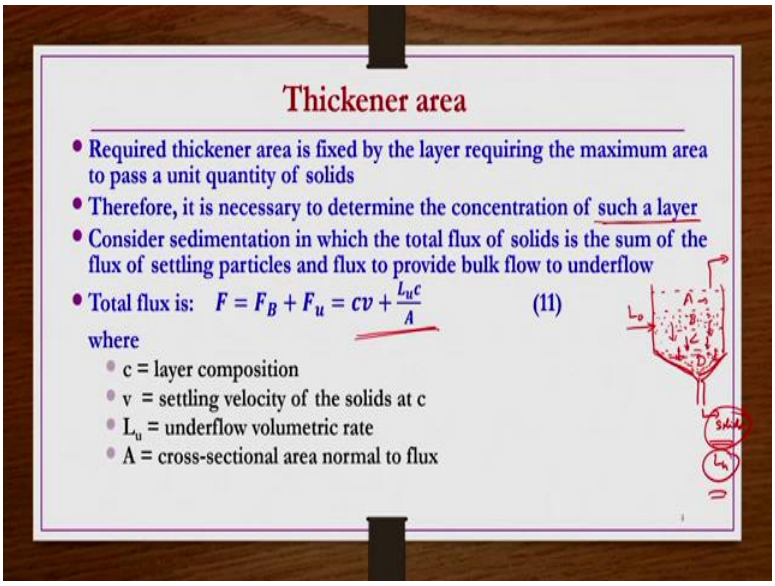


So thickener area. So required thickener area is fixed by the layer requiring the maximum area to pass a unit quantity of solids. Therefore, it is necessary to determine the concentration of such layer, so we need to know what is $C$, what is $C_L$. We graphically we can know and then also we already know it as $C_L$ is equals to $\frac{c_0 z_0}{z_i}$, that we have already obtained. So but is that information is sufficient? Not at all, so we need more information. We need to know the you know solids flux, we need to know the solids flux with respect to the concentration for different time intervals.

So once we have that one, from there we can get the you know required area for the design of the thickener. Till now what we have found, we have found only the concentration of solids of that rate limiting layer and then settling velocity of the solids that are passing through this rate limiting layer $v_L$, that is $C_L$ and $v_L$ that we have found as function of $z_i$. From here also we found that you know the settling velocity of the particle is function of concentration, is function of concentration that is what we understand from previous derivations anyway, the previous slide that we have seen.

So now consider sedimentation in which the total flux of solids is the sum of the flux of settling particles and flux to provide bulk flow to underflow. Let us say what we have, we have a kind of continuous thickener something like this. Once this steady state is achieved you know with these layers different zones A zone, C zone, B zone, D zone etcetera are there, so these layers will be developed. A zone is a clearly liquid zone and then that is taken as a kind of overflow. D zone is a kind of concentrated slur zone from where you collect a kind of solids out, solids discharge you take. And then feed is coming somewhere in, somewhere in let us say at the middle of this sedimentation tank.

Once this steady state is you know established, whatever the feed rate is coming and that feed rate is equal to the whatever the overflow rate is going out and whatever the solids are going out. So that these layers whatever the layers are developed, you know different A, B, C, D zones are formed, they are not disturbed, they are not disturbed. So that condition we know as a kind of a steady state condition that we know. So now what happens, from each layer let us say A is almost a kind of clarified liquid from each layer.

B is a kind of transition zone kind of thing, it is having different concentration. So from each layer you know solids are continuously settling and then going towards the bottom zone that is the D zone where the sludge is being collected from there. So now when the solids are settling, so there is a kind of batch sedimentation of the solids because of the settling velocity of these particles. There is a flux, one flux is there, flux for the solids is there because of the settling of this particle.

Another flux is there. The solids being discharged, taken out continuously from the bottom through the underflow discharge. So there is a kind of solid flux here also. So the overall solid flux of the system, this sedimentation experiment if you wanted to get, you have to obtain this 2 information and then add them together so that you can have a kind of total flux $F = F_B + F_u$ is nothing but $cv + \frac{L_u c}{A}$.

Let us say if you are obtaining flux at time you know $\theta_1$, so then it should be $c_1 v_1 + \frac{L_u c_1}{A}$ like that. If you are obtaining the flux at $\theta$ is equals to 2 because now the solids are continuously settling until a kind of final you know total or almost complete sedimentation is taken over, until that point you know solids are continuously settling and then at each layer, different layer, different times the solid flux are different. So this solid flux if you calculate at different times so then you can have a kind of F versus concentration information because the concentration of this layer is also changing with the time.

So initially let us say all the solids concentration everywhere uniform that is $c_0$ concentration was there before the sedimentation starts. So later on what happens, gradually the concentration of solids in A is almost negligible or almost close to zero and then concentration of the solids in the D zone is very high. So from layer to layer as well as with respect to time this concentration is changing, that is what we understand. So then this concentration with respect to the settling time we have to find out and from there the flux with respect to the settling time we have to find out.

So then with respect to the time we will be able to calculate flux as well as the concentration of solids at different time intervals. So that information F versus C we can plot and get the solids flux of that rate limiting layer. So it is a quiet simple and straightforward, need not to worry, we will be doing some problems also. So here c is nothing but layer composition whichever layer we have taken. So that is the reason we have not designated anywhere $c_1$, $c_2$, $c_L$, $c_i$ something like that, nothing is being designated because it is a generalized one.

Whichever layer for whatever the layer you are trying to calculate the flux, for that layer you know the concentration of the solids in that layer is c and then v is the settling velocity of the solids at that particular c. We already understood the settling velocity of the particles is a kind of concentration, function of the concentration of the solids. So $L_u$ is the underflow volumetric rate, whatever the volumetric rate that you are taking through the D zone the solid sludge that you are taking out through this bottom zone, D zone you know that is Lu and then A is the cross-sectional area normal to the flux.

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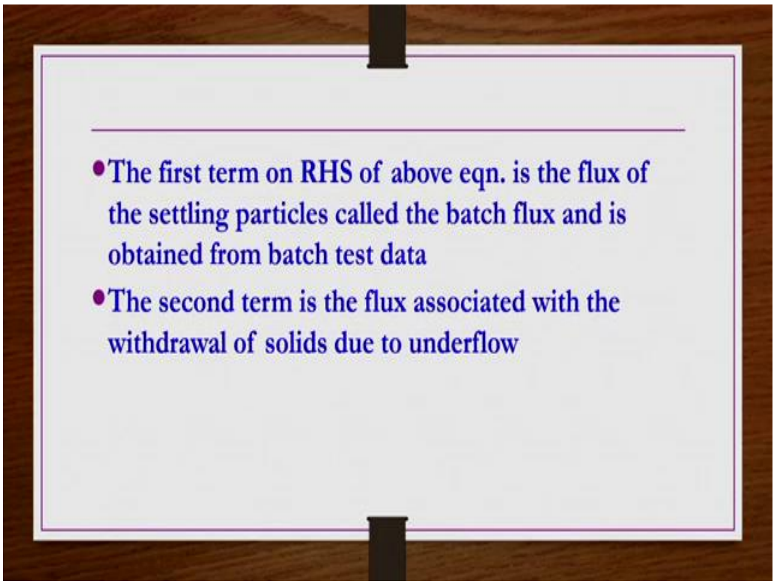


So the first term of RHS of the above equation is the flux of the settling particles called the batch flux, that is c $cv$ is nothing but the batch flux and is obtained from the batch test data. The second term is flux associated with the withdrawal of the solids through the bottom due to underflow that is known as a kind of underflow flux.

So now let us say for a given system you already have z versus theta information, that is in interface the clear liquid A, B interface versus theta information that you are having. From there you are able to find out the $c_L$ concentration from you know $\frac{c_0 z}{z_i}$, information for different theta information, for different times. So that means you are able to calculate the concentrations also for different layers. Once you have the concentration for different layers then you are able to generate the flux, batch flux as well as the underflow flux. Let us say that information is completely available then you plot.

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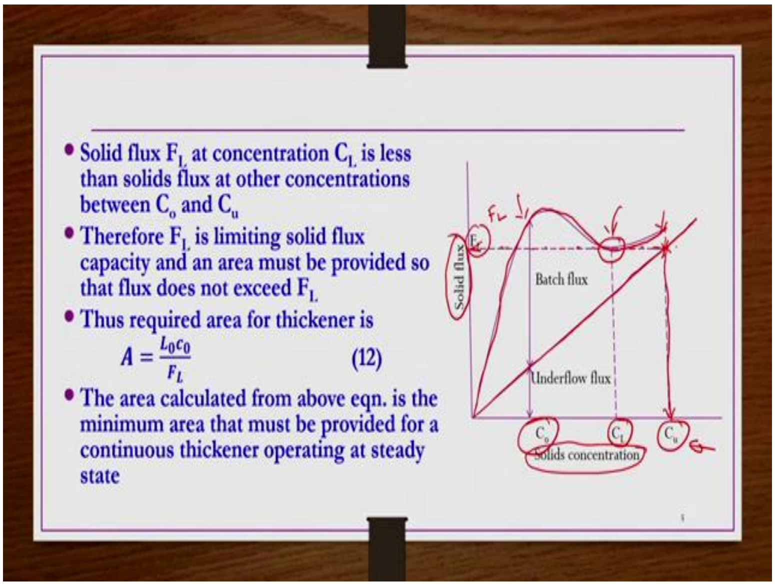


So then when you plot it, the solid flux whatever the total flux that is there, so that if you plot you know against the solids concentration and the slurry, so you may have a kind of curve like this. So this is the total flux and then if you draw this underflow flux so then you may have a straight line like this. So now you can see there is a region where the flux is minimum, where the flux is minimum. So the solids flux corresponding to this minimum location is known as the $F_L$ is known as the $F_L$ or the solids flux of the rate limiting layer whichever we have considered and then corresponding concentration is $C_L$.

Sometimes you need to know the underflow sludge concentration also, sometimes in problem it is available or it is requirement, so you wanted to collect the sludge at such so and so concentration like that. So if it is not there, so under this flux you know whatever the when this minimum flux is established, under such conditions you know you start taking the underflow like that. So what is the corresponding underflow concentration if you wanted to know, this line this $F_L$ line if you extend to further in the increasing x direction, so that may be intersecting at this point with the underflow flux and then corresponding concentration whatever is there that is nothing but $C_u$, that is nothing but $C_u$.

So this is what the flux, solid flux versus concentration information, this is what you get it here. So solids flux $F_L$ at concentration $C_L$ is less than solids flux at other two concentrations $C$ and $C_u$. Whatever at $c_0$ if you calculate the solids flux and then whatever the concentration at $C_u$ if you find out the solid flux, whatever the solids flux are there they are going to be the, they are going to have a kind of higher value of the solid flux compared to the kind of solid flux at the $C_L$ concentration.

Therefore $F_L$ is limiting solid flux capacity and an area must be provided so that flux does not exceed $F_L$. Therefore, required area for the thickener we can get it as $A = \frac{L_0 c_0}{F_L}$. So now we can see it is so simple, for a continuous thickener if you able to find out this solid flux of the rate limiting layer, so whatever the limiting solid flux capacity or the solid flux of the rate limiting layer $F_L$ if you can find out, you do not need any other information to calculate the required area for that particular thickener.

So because in general $L_0$ $c_0$ are given, $L_0$ is nothing but initial feed slurry inlet whatever the inlet of the feed slurry we are giving to the continuous thickener, so in volumetric flow rate that is $L_0$. That is $L_0$, $c_0$ is the initial feed slurry concentration, initially before sedimentation what is the concentration of the solids uniform solids concentration before the start of a sedimentation. That $c_0$ is also known, so $F_L$ if you find out then directly you can calculate the area using L naught, c naught divided by $F_L$.

So the area calculated from above equation is the minimum area that must be provided for a continuous thickener operating at steady state. So now this is what one of the ways to get the area requirement for the thickener, that is the design of thickener. This is how we do in general, design of thickener means what? You should able to obtain a kind of required cross-sectional area and depth of the container that you are using for a given sedimentation duties.

So you can fix the depth and then accordingly you can calculate the area. Now we go to a different method to get this area of a thickener, how to calculate area of thickener from a different approach we can do which is quite similar and straight forward as the previous one.

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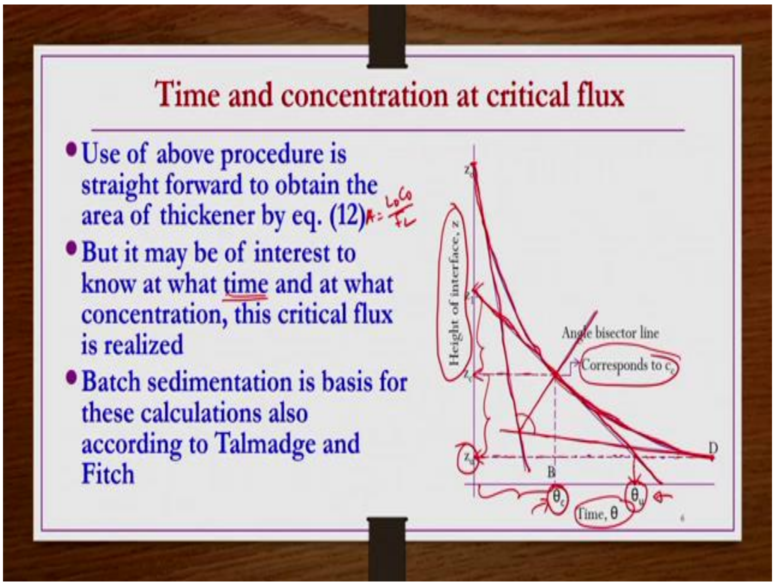


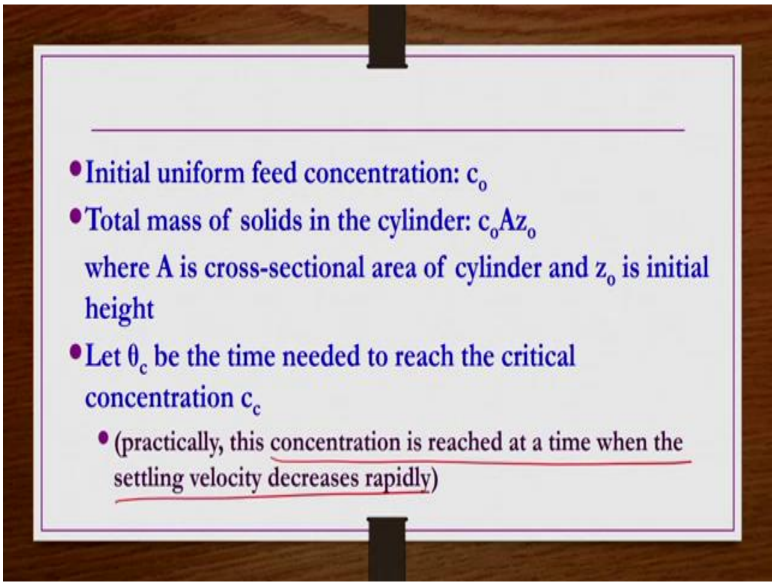


So time and concentration at critical flux. So use of the above equation that whatever $A = \frac{L_0 c_0}{F_L}$, this equation whatever is there is straightforward to obtain the area of thickener. So but however it may be of interest to know at what time, at what time and at what concentration this critical flux is realized. So that you can if you can find out, that is going to be a kind of additional information. Indeed from that additional information also we can calculate the area, required area for a given thickener duty. So that we are going to see now.

Remember whatever the $C_L$ is there, that is not a kind of critical concentration, so what concentration this critical flux is realized that is what we are going to see. So the $C_L$ is the kind of a layer that is moving up, so now that is different and this critical flux also different. So whatever this critical time or the concentration at critical flux that we are going to calculate, that is the point from that time onwards the sedimentation of particle or the settling velocity of particle drastically decreases. Drastically decreases, pictorially we will be seeing now here anyway.

So batch sedimentation is basis for these calculations according to Talmadge and Fitch. So what we do, we have this height of the interface versus time data. You do a batch study, batch sedimentation study and then get this height versus time information. So simply you take a cylindrical column and then you take initial feed slurry which is having certain $c_0$ concentration and then with respect to time you find out how this interface is dropping down with respect to time. Because obviously as the time progresses solids are settling down so then the clear liquid interface is also falling down. So that information z versus $\theta$ that interface height we call Z and then time of sedimentation at which we noted that interface height is the theta.

So when you plot it, so you get a curve like this, you get a curve like this. So this is the initial $z_0$, initial height $z_0$ and then let us say the last point z whatever is there at this point. At these two points what you do, you draw tangents, you draw tangents like this at this two points, that initial $z_0$ and final data point whatever that you have, you draw a tangent like this.

So when you draw this tangent and extends them so they may be intersecting each other at certain point. And then when they are interesting they are making some angle, this angle if you bisect into equal parts and then draw a kind of angle bisector line like this and then extend it towards the z versus $\theta$ curve, towards the z versus $\theta$ curve you just extend it. So this angle bisector line is intercepting this curve at certain point, that point corresponds to you know critical flux point and then corresponding time is known as critical time and then corresponding height is $z_c$.

So critical time that you got right now, so we can get the critical concentration also, how to get the critical concentration? At this point if you at this theta is equals to theta c if you draw a tangent, that will be intersecting this y axis at z is equals to $z_1$. So if you wanted to know the settling velocity of the particles at $\theta = \theta_c$ that should be $\frac{z_1 - z_c}{\theta_c}$. So let us say the settling is occurring from $\theta_c$ to $\theta_u$ time, then what should be the settling velocity of those particles?

So the particles in that settling velocity of the particle between these two time zone would be nothing but $\frac{z_c - z_u}{\theta_u - \theta_c}$, because v is nothing but $\frac{-dz}{d\theta}$.

So how to get this $\theta_u$, how to get this theta you, so that is also important. So now when you, when you drawn this tangent at $\theta = \theta_c$, you extend it and then at final point you know whatever the final data point that you have at let us say that point d, so at point what you do, you draw a kind of horizontal line. So wherever this horizontal line is intersecting on y axis is nothing but $z_u$ and then this horizontal line is intersecting with the tangent at $\theta = \theta_u$ so the corresponding time is nothing but $\theta_u$, the corresponding time is nothing but $\theta_u$.

Actually this $\theta_u$ that is what we are going to use for area calculation now anyway. So this is additional information before going for area calculation, so you know this theta u anyway useful for area calculation but in addition to that one when we do this process we are able to find out a critical time and then concentration at critical flux. So that we are able to find out through these two points. So what does it mean? The critical flux so that is the critical point, what does mean by critical point? Critical point is the point from that point onwards the settling velocity of the particles is drastically decreasing, is drastically decreasing.

So let us say from z naught to $z_c$, or from 0 to $\theta_c$ time the settling velocity is you know sufficiently is there. So after that the settling velocity change in settling velocity is not that much in general. So that is what meant by that point at which the settling velocity is going to decrease drastically is known as the critical point. So now we try to get the additional information, now initial uniform feed concentration is $c_0$, total mass of solids in the cylinder is $c_0 A z_0$ forward because a is cross-sectional area of the cylinder and $z_0$ is initial height.

Let $\theta_c$ be the time needed to reach the critical concentration $c_c$, then practically this concentration is reached at a time when the settling velocity decreases rapidly.

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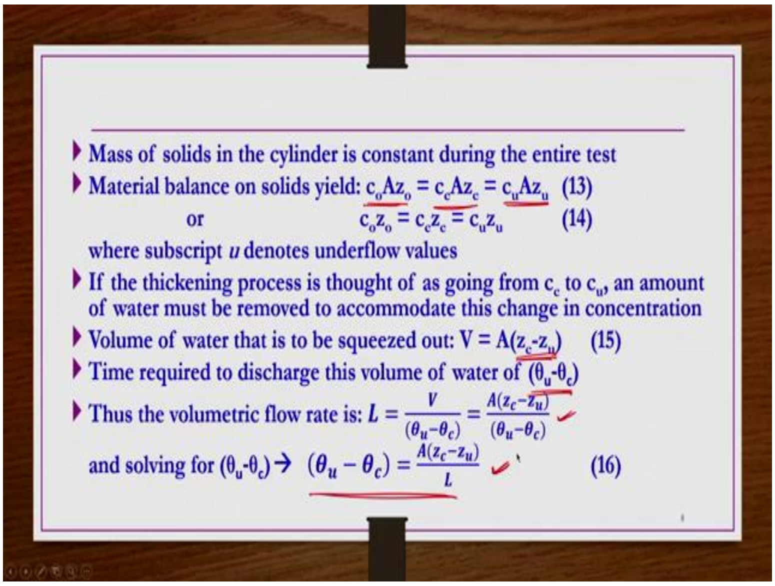


Then mass of the solids in the cylinder is constant during the entire test. Obviously because there is no mass loss. Material balance on solids if you do different times you know, $c_0Az_0$ is equals to $c_cAz_c$ is equals to $c_uAz_u$ like that you get it because the total mass of the solids is same in the cylinder. So material balance on solids if you do at different times $c_0$, $z_0$ at the initial point like you know before starting the sedimentation $c_c$, $z_c$ are corresponding to the critical point. At that point also the total number of solids are same and then is like you know at $c_u$, $z_u$ are corresponding to the you know the underflow sludge concentration.

So at that point also total solids are same in the system. So all these things are same, only thing like you know a is fixed here; $A_0$, $Az_0$ is fixed here all the cases $c_0$, $z_0$, $c_c$, $z_c$, $c_u$, $z_u$ are changing in a combination such a way that this total value of product value whatever $c_0$, $A_0$, $z_0$, $c_c$, $Azc$, $c_uAz_u$ are there, that is going to be constant. So since area is constant the same equation you can write it as $c_0z_0 = c_cAz_c = c_uAz_u$ so where you denote underflow values.

If the thickening process is thought of as going from $c_c$ to $c_u$, an amount of water must be removed to accommodate this change in concentration. So what is that amount of water? So volume of water that is to be squeezed out is nothing but the cross-sectional area A multiplied by the time difference, the height difference. Cross-sectional area A multiplied by the height difference from $z_c$, to $z_u$ that is $z_c - z_u$ because we are assuming this rapid change is taking place the rapid decrease in settling velocity is taking place from theta c onwards, so $\theta_c$ to $\theta_u$ time whatever the corresponding heights are there interface heights are there, that we should take as difference $z_c - z_u$ and multiplied by the area.

Then you will get the volume of water that is to be squeezed out. Then time required to discharge this volume of water obviously $\theta_u - \theta_c$ from graphically we have seen. So then volumetric flow rate L would be $\frac{V}{(\theta_u - \theta_c)}$, so then we get $\frac{A(z_c - z_u)}{(\theta_u - \theta_c)}$. Simply by substituting here for V, A is equals to $z_c$, $z_u$ if you substitute you will get this L volumetric flow rate. And then solving for this $\theta_u - \theta_c$ that means $\frac{A(z_c - z_u)}{L}$ that if you do, you will get $(\theta_u - \theta_c)$.

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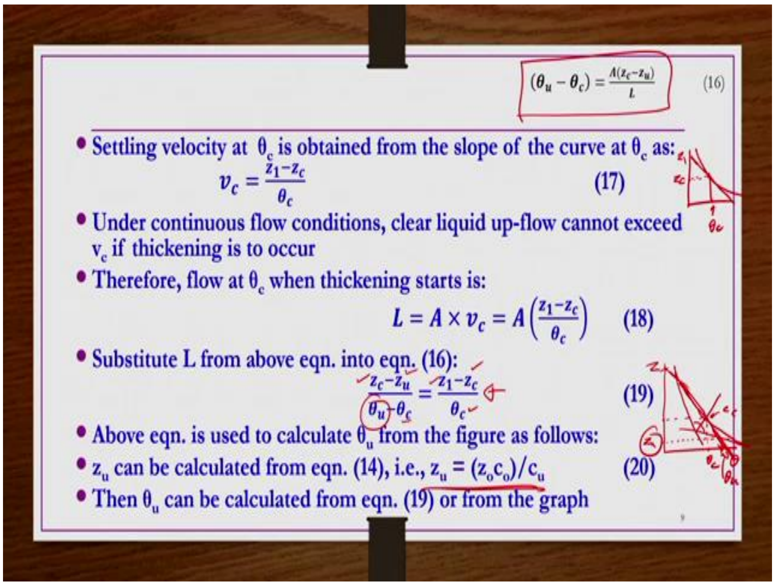


Settling velocity at $\theta_c$ is obtained from the slope of the curve at $\theta_c$ and then as already did in the graphical approach there, so if it is like $\theta_c$ point, at this point if you draw this intercept, so what happens? It will be intercepting somewhere this y axis and then that is known as the $z_1$ and then the interface height corresponding to $\theta_c$ is nothing but $z_c$. So $\frac{z_1 - z_c}{\theta_c}$ if you do you will get settling velocity at $\theta_c$. So under continuous flow conditions clear liquid up flow cannot exceed $v_c$ if thickening is to occur. So we do not want these solids to go into the clear liquid, we want the solids to be settled. So then under the continuous flow conditions clear liquid up flow should be maintained such that it cannot exceed the $v_c$ if thickening is to occur.

Up flow whatever the clear liquid overflow that you are taking, its flow rate should be less than the $v_c$ if thickening is to occur, otherwise in that clarified liquid particles will come in. That we do not want. Therefore flow at theta c when thickening starts is nothing but L is equals to $Av_c$, $Av_c$ just now we have seen $\frac{z_1 - z_c}{\theta_c}$. So now substitute L from above equation in equation number 16 this is the equation number 16 that previous slide we have seen. So when you substitute here you get $\frac{z_c - z_u}{\theta_c}$ is equals to $\frac{z_1 - z_c}{\theta_c}$.

So above equation used to calculate theta u from the figure as follows, as I already explained. So let us say once again we draw this z versus θ information, you draw a tangent at z naught, you draw a tangent to this curve at final data point so there may be intercepting, intersecting each other at some point. So that angle whatever is there you divide into half and then draw a kind of angle bisector line something like this, so that may be intercepting this z versus θ curve at some point. So that point is corresponding to critical point, corresponding concentration is $c_c$, corresponding time is $\theta_c$.

So now at this point you draw a kind of tangent at $\theta_c$ and then extend it like this. So at this $z_u$ information that we already calculated and $z_u$ information what you do, you draw a kind of horizontal line this thing and then $z_u$ line constant line if you horizontal line when you draw, so that is going to intercept this tangent at $\theta_c$ at some point and then corresponding time is nothing but $\theta_u$, that we have already seen in previous slide also.

So then for this we need to know $z_u$, so $z_u$ we can calculate $z_0$, $\frac{c_0}{c_u}$ because $c_0 z_0$, is equals to $c_u z_u$ is equal to $c_c z_c$ that is all we have already seen because the total solids in the system are going to remain constant. Then equation theta can be calculated from equation 19 as well, from this equation also you can calculate or from graphically also you can calculate if you know the $z_u$.

So because in this equation $z_c$ you already calculated, $\theta_c$ you already calculated, $z_1$ is nothing but the intercept of tangent at theta is equals to theta c, whatever that $z_1$ is also known. $z_c$ is there so $z_u$ if you calculate from here and then substitute here, so $\theta_c$ is also known, only unknown is $\theta_u$, only unknown is $\theta_u$. So once again for a kind of clarification because of these symbols what we have?

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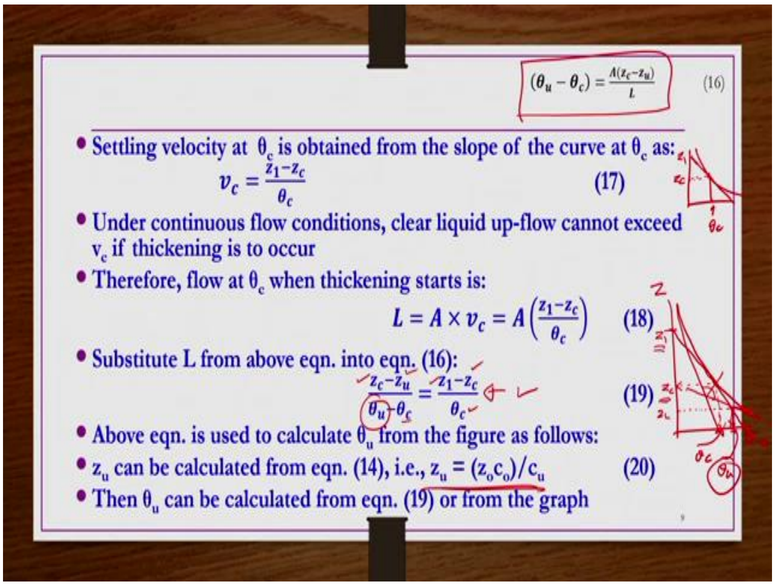


We have a kind of picture once again here, draw a kind of clear picture bigger ones probably. So these intercepts we have, z versus $\theta$ curve we have like this and this $z_0$ point and final data point we draw on the intercept the tangent lines and they are intersecting each other at some point so that intersect angle whatever is there that you bisect it, bisect it and then that point when you bisect this line like extend line bisector line like this, so that will be intersecting this z versus $\theta$ curve at certain point.

So that point is corresponding to, that point is corresponding to $\theta_c$ and then corresponding height interface height is $z_c$. So if you draw a kind of interface like, if you draw a kind of tangent at $\theta = \theta_c$, so that is intercepting this line y axis line at some point that is $z_1$, so that is $z_1$ is known, $z_c$ is also known. That is what I mean by, if you know the $z_u$, $z_u$ at that $z_u$ you draw a kind of constant horizontal line like this. So that line may be intercepting this tangent at $\theta = \theta_c$ and then corresponding time whatever you get, that is nothing but $\theta_u$, that is nothing but $\theta_u$ and then this point is nothing but $\theta_c$.

So everything is known except $\theta_u$, so theta u you can get graphically like this or in this equation also $z_1$, $z_c$, $z_u$ $\theta_c$ are known so theta you can calculate from equation number 19 also.

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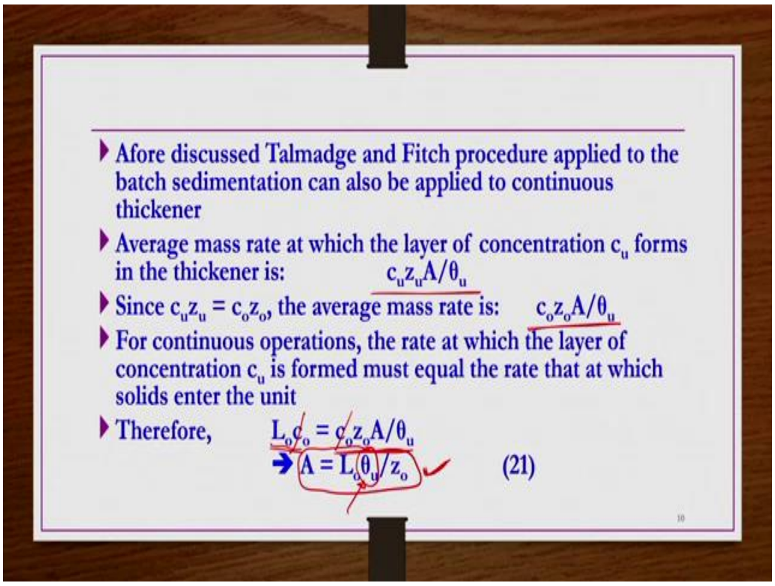


So whatever this information, that process that we are doing, that we are doing for a kind of batch sedimentation test. So information we are getting from the batch sedimentation test only but however this pressure can also be applied to continuous thickener according to the researchers who developed this method. So average mass rate at which the layer of concentration cu forms in the thickener is nothing but $\frac{c_u z_u A}{\theta_u}$, obviously. So $c_u$ is the concentration of solids that layer whatever the underflow that is there, $z_u$ is the corresponding interface height, A is the area and then theta u is the time required to reach that $z_u$ interface height.

Then since $c_u z_u = c_0 z_0$ the same average mass rate we can write it as $\frac{c_0 z_0 A}{\theta_u}$. Why we are doing? Because you know initially $c_u z_u$ are not known, we are going to obtain by this graphical method. So however we tried to reduce the work so that we can get the required information with minimum effort. So now here $c_u z_u$ is nothing but $c_0 z_0$, so $c_0 z_0$ are known initially up from the problem statement, so that we can write rather writing $c_u z_u$.

So then average mass rate is nothing but $\frac{c_0 z_0 A}{\theta_u}$. Then for continuous operation the rate at which the layer of concentration cu is formed must equal the rate that at which solids enter the unit. Because than only steady state conditions will prevail for a given continuous operation. So that is the rate at which the layer of concentration $c_u$ is formed, that must be equal to the rate that at which solids enter the units.

So that means rate at which solids enter the unit is nothing but $L_0 c_0$, so that $L_0 c_0$ should be equal to this $\frac{c_0 z_0 A}{\theta_u}$. So this $c_0$, $c_0$ you cancel out and then from here you can write an expression for A as you know $\frac{L_0 \theta_u}{z_0}$, $\frac{L_0 \theta_u}{z_0}$ is nothing but the area required for that continuous thickener.

So now $L_0$ is what? Initial feed rate whatever the feed rate at particular volumetric rate that is entering to the continuous thickener that is $L_0$ and $z_0$ is nothing but initial interface height, $\theta_u$ is nothing but the time required that interface to decrease from $z_0$ to $z_u$ height. So here also straightforward, only one thing is required is $\theta_u$, only one thing is required is theta you, so this $\theta_u$ you can get from graphically as by the equation number 19 previous slide that we have seen.

So here also the area of continuous thickener you can apply, you can obtain by this method. Previous method we required solid flux of rate limiting layer, here we need that time required to reach initial interface to $z_u$ interface so that whatever the time theta u requirement is there, so that if you know you can calculate the area of the continuous thickener. So now we see couple of example problems so that to understand how to use this equation and get the area requirement.

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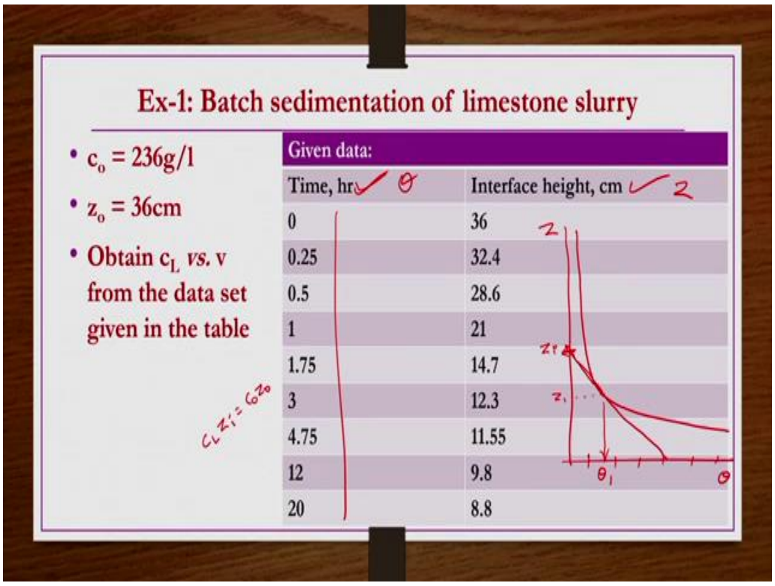


Example number 1, batch sedimentation of limestone slurry, so here the initial concentration $c_0$ is given as 236 g/L, $z_0$ is 36 centimeters. So obtain $C_L$ versus V from the data set given in the table. So in the table theta and then z, this is $\theta$ this is z, $\theta$ versus z information is given.

How the height of the interface is decreasing with different time interval that is given here. From this information what we have to do, we have to obtain $C_L$ versus V information. So now we use the first equation that we developed yesterday, that what we have developed? $C_L$ $z_i = c_0 z_0$. So what we are going to do, we are going to draw this z versus θ information graphically like this something like this.

So at each point at different intervals you take like this. Different intervals in the time, at each point also you can do. So you can take around 8 to 10 intervals of time, not necessarily same as like this. So from the graph let us say at two hours, this is the point at two hours or three hours something like that. At this point if you know if you wanted to know the, at this point of sedimentation time the x axis is nothing but sedimentation time. So at this point of sedimentation time if you wanted to know the concentration of that layer, you know what you do, you draw a kind of tangent like this at this point.

So it may be intersecting somewhere, let us say if it is $\theta_1$ so then corresponding this one is nothing but $z_1$ and then this is $z_i$. If you wanted to know the velocity it should be $\frac{z_i - z_1}{\theta_1}$. Then that will give the settling velocity. So v is known, so then after that c you can calculate because the $z_i$ is also known. So $\frac{c_0 z_0}{z_i}$ if you do, you get this $C_L$ information, so both of the information are known, so that is just a graphical work that we have to do here.

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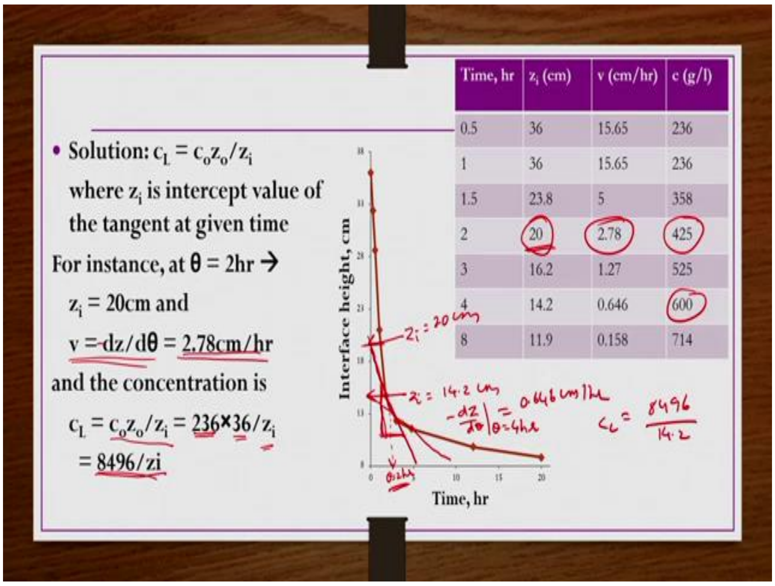


So solution that $C_L$ is equals to $\frac{c_0 z_0}{z_i}$ where $z_i$ is the intercept value of the tangent at a given time. So let us say that data if you plot you will get a graph like this. So now what you do, let us say at 2 minutes, at 2 minutes of time you draw a kind of tangent like this. So let us say this is the point corresponding to theta is equals to 2 hours, not 2 minutes it is given in 2 hours. So at 2 hours if draw a kind of tangent, so that may be intercepting this y axis at 20 centimeters, so this $z_i$ corresponding to this one is nothing but $z_i$ is equals to 20 centimeters.

So now from here if you do at this point, if you take the slope of this curve, slope of this tangent whatever that you have taken at this point on the curve at $\theta$ is equals to 2 hours if you take $\frac{-dz}{d\theta}$, $\frac{-dz}{d\theta}$ then you will get 2.78 cm/h. So that is nothing but the, that is nothing but the settling velocity of the particles at that particular time. So then after that concentration $C_L$ you can find out $\frac{c_0 z_0}{z_i}$. $c_0$ is given as 236, $z_0$ is given as 36, $z_i$ at this point of time we have got it only as a kind of 20.

So overall $C_L$ final information is that 8496 divided by $z_i$, $z_i$ this is what we have. So now different time intervals you find out different $z_i$'s and then you substitute here so you will get with respect to time the $C_L$ information you get and then the slope of those tangents at different time intervals whatever you get that is nothing but the v information settling velocity of the particle. So when you do that one, you will get this one, you will get this one.

Let us say $C_L$ at 2 hours time you know, $z_i$ we already got this as 20. So 8496 divided by 20 if you do it, it is coming to be something like 425 roughly. And then v at this point, $\frac{dz}{d\theta}$ at this point, theta is equals to 2 hour we have already seen it is 2.78. Let us say if you do it at 4 hours time, at this point if you draw a kind of tangent like this at this 4 hours point time like this, so this here now $z_i$ is going to be something like 14.2 centimeters.

And then $\frac{-dz}{d\theta}$ of this line at theta is equals to 4 hours is nothing but 0.646 centimeter per hour and then $C_L$ for this time is nothing but it is going to be 8496 divided by, $z_i$ is 14.2, so that is roughly coming out to be 600 g/L. So this is how we can get, we can construct c versus v information.

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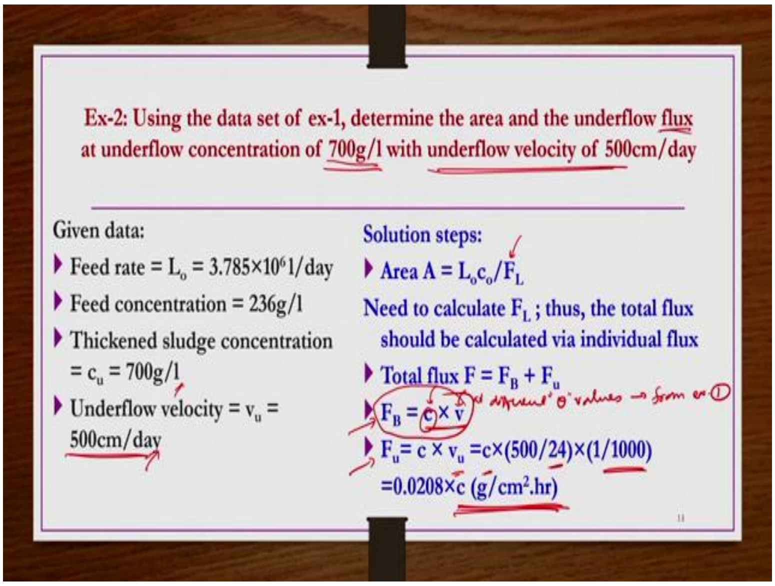


Now we take another example problem. Using the data set of example 1 determine the area and underflow flux at underflow concentration of 700 g/L with underflow velocity of 500 centimeter per day something like that. So then given data is feed rate $L_0$ is given, initial feed rate entering the slurry at what rate it is entering this continuous thickener that is given, $L_0$ is given in liters per day, feed concentration that is $c_0$ is given same as previous problem, that is 236 g/L and sludge concentration here it is given cu is given that is 700 g/L.

That is taken as a kind of underflow and then $v_u$ is also given and then $v_u$ is also given that is underflow velocity is also given 500 cm/day. So that means underflow flux you can directly calculate easily here. So now what we do, you need to have a kind of consistency of units; day, liters, centimeters, different units are there. You should be very careful about these units. So now solution steps:

Area because now here flux are required, the part of the question is that you have to determine the area and then underflow flux. So the flux calculations if you do, whatever the limiting layer flux $F_L$ is there once you get that you substitute here $\frac{L_0 c_0}{F_L}$, then you will get the area requirement. So $F_L$ we need to calculate, so total flux $F = F_B + F_u$, $F_B$ is nothing but $cv$. C for different times, v for different times you know at different theta values we have already done in the previous example problem 1, so from there we can get.

That means $F_u$ is nothing but multiplication of whatever the c and v columns of previous example whatever the c and v information that we got as a kind of solution of the previous problem. And then $F_u$ is nothing but $cv_u$;, it is $cv_u$, so then at different times c that also we have already got it here in the previous problem. So that $cv_u$ will get $F_u$. So now flux we wanted to have in g/cm$^2$-hr, so the liters whatever is there 1/1000 we have done, so then liters are also converted into the cm$^3$.

And then here centimeter per day $v_u$, so what we have done, divided by 24 so that it is comes into the centimeters per hour. So that you know overall these units are going to be gram per centimeter square hour. So what we have to do, first step whatever the $F_u$, $F_B$ values we have to calculate it. We have to calculate, $F_B$ value is straightforward is the nothing but the solution of the previous example c and v are the solution of previous example problem. Or c versus v information for different time values is a kind of a solution for the previous problem that we already got. You just simply multiply those values, 2 values for different times. So then for different times $F_B$ you will get it.

Similarly different times concentration or solids concentration we know, that if you multiply it by $v_u$ then you will get the underflow flux. Then when you add them together you may get a kind of something like total flux whatever required flux.

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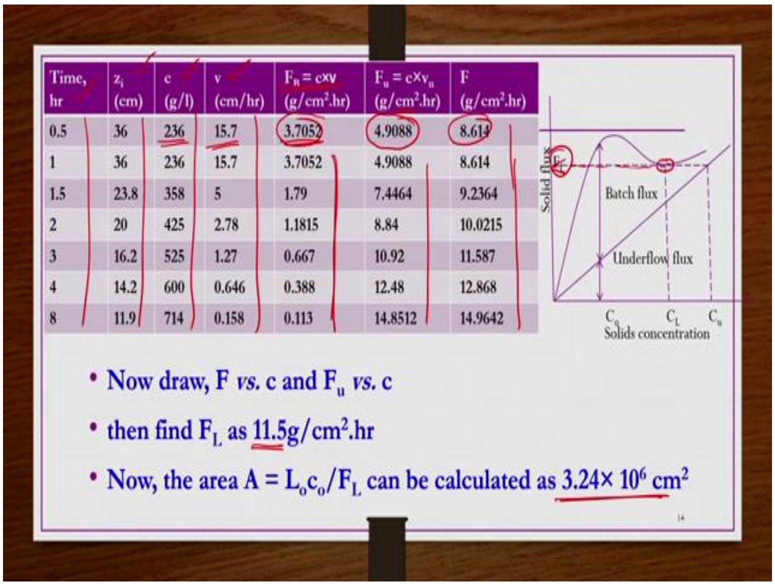
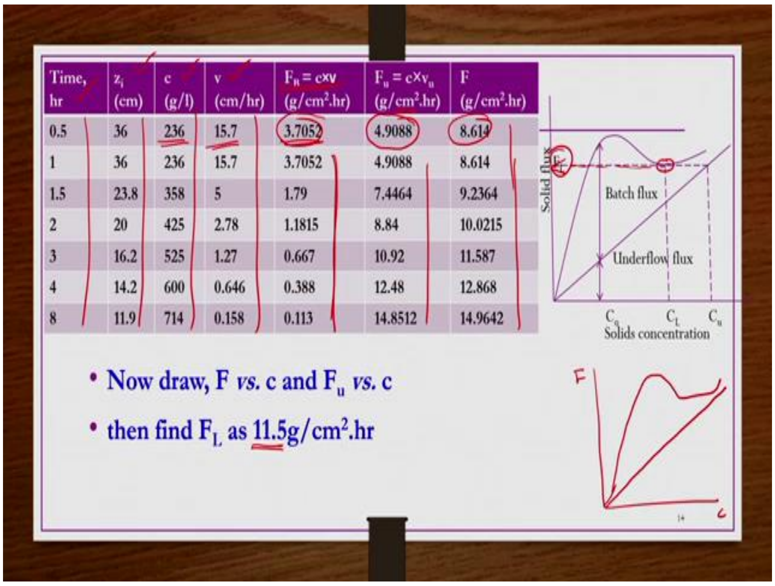
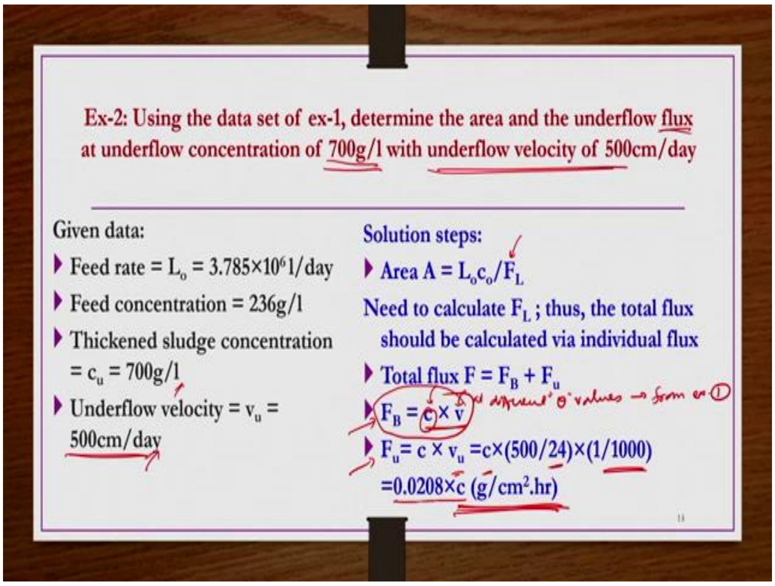


| Time, hr | $z_i$ (cm) | c (g/l) | v (cm/hr) | $F_B = c \times v$ (g/cm$^2$.hr) | $F_u = c \times v_u$ (g/cm$^2$.hr) | F (g/cm$^2$.hr) |
|---|---|---|---|---|---|---|
| 0.5 | 36 | 236 | 15.7 | 3.7052 | 4.9088 | 8.614 |
| 1 | 36 | 236 | 15.7 | 3.7052 | 4.9088 | 8.614 |
| 1.5 | 23.8 | 358 | 5 | 1.79 | 7.4464 | 9.2364 |
| 2 | 20 | 425 | 2.78 | 1.1815 | 8.84 | 10.0215 |
| 3 | 16.2 | 525 | 1.27 | 0.667 | 10.92 | 11.587 |
| 4 | 14.2 | 600 | 0.646 | 0.388 | 12.48 | 12.868 |
| 8 | 11.9 | 714 | 0.158 | 0.113 | 14.8512 | 14.9642 |




So this time $z_i$, c, v L this 4 are you know adopted from the previous problem because it is a continuous of the previous problem, this we have already done. Then now here this $F_B$ is nothing but $cv$, so this 236 multiplied by 15.7 if you do that comes out to be 3705.2 something like that. That would be in liters so but you know we have to convert this one liters as kind of in terms of centimeter cube. So then if you divide by 1000 you will get 3.7052 gram per centimeter square hour as a kind of batch flux and then corresponding flux here underflow flux is nothing but c multiplied by $v_u$, so 236 multiplied by $v_u$ information for the first case.

This is 236 multiplied by 0.0208 if you do you will get, you will get 4.9088 g/cm$^2$-h. You should be careful about these units because these units if for concentration is given in g/L and then flux is in g/cm$^2$-h. So the total flux should be the addition of these two, like that for other time values also you calculate the batch flux, underflow flux and then total flux. So total solid flux you can get it. This total solid flux if you plot F versus concentration then you will have this kind of curve.

We may not have exactly the similar nature of the curve because it is a kind of ideal condition I have shown like this. If you plot like this you may have something like this. And then underflow flux if you draw like this, you will have like this. So the corresponding to the point where the minimum flux is there, there would be some point where the flux should be the minimum. Like the lowest flux point that is nothing but the rate limiting layer solid flux or the solid flux of the rate limiting layer $F_L$.

So that $F_L$ you simply substitute and then here in this case you will get as 15.5 g/cm$^2$-h. So simply area is going to be $\frac{L_0 c_0}{F_L}$, so that is going to be 3.24 x 10$^6$ cm$^2$, so this is the minimum area required for the continuous thickener which has to perform this duty according to whatever the $c_u v_u$ et cetera are given.

Now we take another example problem, that problem will be calculating the critical time etcetera, that approach we follow and then we obtain the required area for a continuous thickener.

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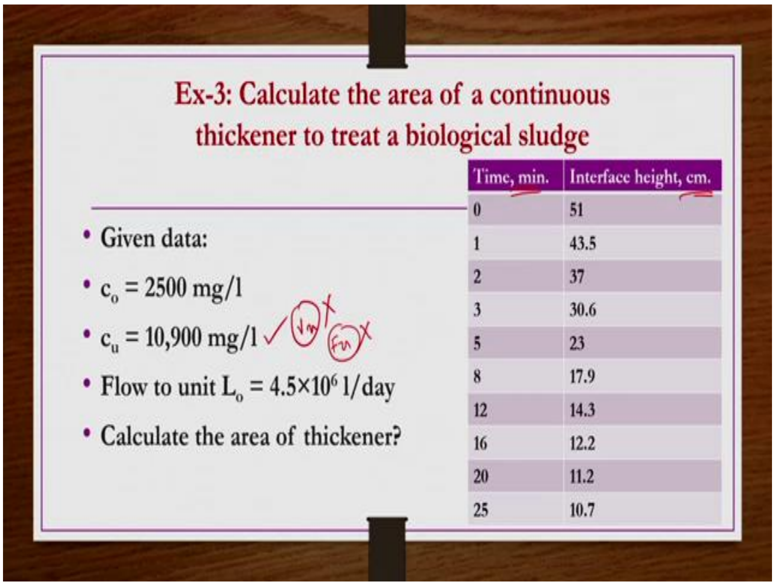


Calculate the area of the continuous thickener to treat a biological sludge. Given data: c naught is given, $c_u$ is given, $L_0$ is given; calculate the area of the thickener. Here $v_u$ is not given. If $v_u$ is not given so then you cannot obtain the underflow flux. So the flux method you cannot use for this problem, you will not be able to solve because v u is not given, cu is given but $v_u$ is not given. If it is, had it been given so then this problem could also being solved as a kind of example number 2.

Since it is not given here so underflow flux $F_u$ you cannot calculate. So solids flux versus concentration plot from there getting $F_L$ and then getting A as $\frac{L_0 c_0}{F_L}$ is not going to be possible. So what we have to do? We have to find out theta u and then use the critical time approach to get this area requirement. So the data time versus interface height is given in, time is given in minutes and then interface height is given in centimeters. So first what we have to do?

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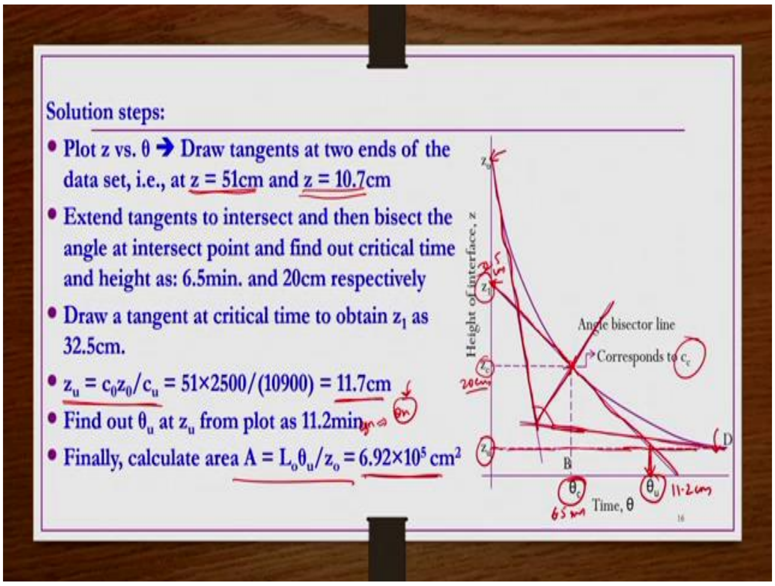


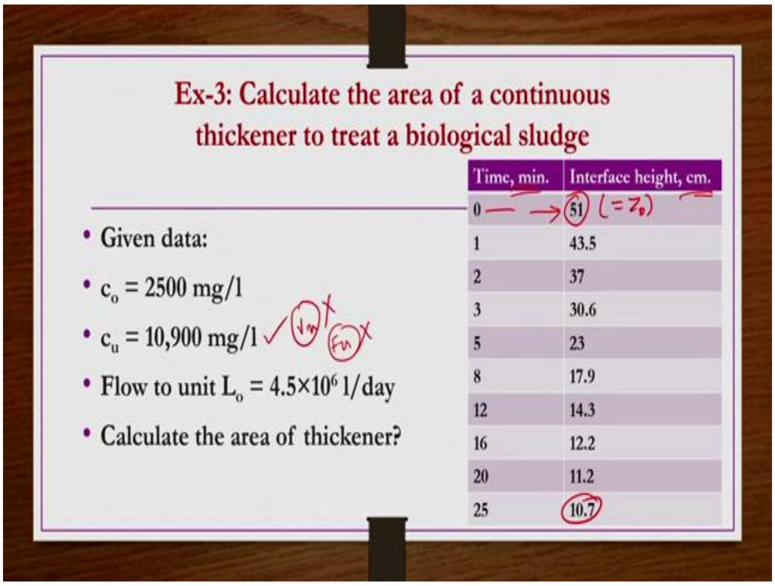

We have to plot z versus $\theta$. z versus $\theta$ you have to plot and then two ends of data points are z is equals to 51 centimeters and then z is equals to 10.7 centimeters. So $z_0$ is 51 centimeters as per this data points given, $z_0$ is nothing but 51 centimeters because it is at 0 time. So at the last data point that is at 10.7 and at this first data point at z is equals to 51, at these two points we have to draw two tangents to this z versus θ curve like this. So this is$z_0$ point, this is let us say whatever the 10.7 or something like that.

At these 2 points you draw a kind of tangent like this which we have already done something like this, so and then extend this tangent so that they intersect. So when they may be intercepting at certain angle, so that angle you divide into two equal and then draw a kind of angle bisector line like this. And extend it towards the z versus $\theta$ curve, so this angle bisector line is you know intercepting this curve at certain point, that point is corresponding to critical concentration or critical point, then time is known as a kind of a critical time.

So the corresponding interface height is $z_c$, at this point at theta is equals to $\theta_c$ now again you draw a kind of a tangent like this and then this tangent at theta is equals to $\theta_c$ is intercepting y axis at $z_1$. So but we need $\theta_u$, we need theta u information.

$z_u$ is given here in the problem, $z_u$ is not given here, so we will see anyway. So by doing this one we get theta c as 6.5 min and then $z_c$ is something approximately 20 cm that we get here. Now when you draw a tangent at theta c then z 1 we get it as a kind of 32.5 cm as a $z_1$ kind of thing and then $z_u$ we need to find out, $z_u$ is nothing but it is not given, so we have to find out, that is nothing but $\frac{c_0 z_0}{c_u}$. When you do it you will get 11.7 cm.

So equation number 19 we can use to get theta u information because $\theta_u$. is the required information in order to get the area. That way you can do or graphically also you can do. At $z_u$ is equals to 11.7 cm you draw a kind of horizontal line like this and then this line may be intersecting with the tangent at $\theta$ is equals to $\theta_c$ at certain point and then corresponding time is nothing but $\theta_u$. And then here that comes out to be 11.2 cm. So that is it, area you can calculate it, area you can calculate it as $\frac{L_0 \theta_u}{z_0}$ that comes out to be 6.92 x $10^5$ $cm^2$.

This is how you can calculate the required area of a continuous thickener using either of the approaches.

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So the references; we have the several references for this course. So all of these references are going to be very important and all of them must be having some amount of information about these topics that we have discussed in this lecture. But especially this is the new reference I have added, so this is a very good book for you know this thickener design, whatever the thickener design and then problems that we have solved so those things you know some material we can find out from this reference book. Thank you!